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Blunt body paradox and improved application of transient growth framework

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Abstract

The “Reshotko-Tumin transition criterion” based on optimal transient growth successfully correlates laboratory measurements of roughness induced transition over blunt body configurations. Even though transient growth has not been conclusively linked to the measured onset of transition, the above correlation denotes the only available physics-based model for subcritical transition in blunt body flows, since the latter do not support any modal instabilities at typical experimental conditions. Unlike other established models based on empirical curve fits that are valid for a specific subclass of datasets, the optimal-growth-based transition criterion appears to provide a reasonable correlation with measurements in various wind tunnel and ballistic range facilities and for a broad range of surface temperature ratios. This paper is focused on optimal growth calculations that improve upon significant shortcomings of the computations underlying the Reshotko-Tumin correlation. The improved framework is applied to leeward transition over a spherical section forebody that was tested in the Mach 6 Adjustable Contour Expansion wind tunnel at Texas A&M University. The computed results highlight the significance of nonparallel basic state evolution, curvature terms, and an optimization procedure that varies both inflow and outflow locations of the transient growth interval. More important, the results indicate that the modified correlation is very close to its original form, and hence, that the accuracy of the transient-growth-based transition criterion is not compromised by using a more thorough theoretical framework. Yet the results also show that the optimal energy gain up to the predicted transition onset location can be rather small, highlighting the need to further investigate the optimal growth criterion for additional experimental configurations and to also uncover the in-depth physics underlying blunt body transition.

I. Introduction

Laminar-turbulent transition of boundary-layer flows greatly increases convective heat transfer to the body surface. Therefore, predicting the onset of transition is an important aspect of the design of hypersonic vehicles. In particular, the thermal protection system (TPS) of the vehicle must be designed to ensure adequate protection for the interior payload. The TPS can be of three types: (i) passive reusable, (ii) active reusable, and (iii) ablative, nonreusable. The ablative TPS is well-suited for reentry vehicles that encounter severe

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aerothermodynamic environments, particularly when it is not essential to consider reusable TPS. Most planetary entry, descent, and landing systems involve ablative TPS. The boundary-layer transition characteristics can influence both the surface heating rate (which determines the type of TPS to be employed) and the overall heat load (which controls the thickness of the TPS and, in turn, the associated weight penalty). Ablation leads to out-gassing and surface roughness, both of which are known to influence the transition process. The effects of out-gassing on disturbance amplification in boundary layer flows over hypersonic configurations were studied by Li et al.¹ The present paper seeks to isolate the effects of distributed surface roughness on blunt forebodies in the absence of any outgassing.

On slender hypersonic vehicles with smooth surfaces, the cause of boundary layer transition can usually be traced to the amplification of hydrodynamic instabilities, such as streamwise instabilities in the form of first and second mode waves, stationary and traveling crossflow instabilities, and centrifugal instabilities (i.e., Görtler vortices) in the case of vehicles with a significant region of concave surface curvature. In contrast, boundary-layer flows over blunt vehicles such as a reentry capsule do not support the growth of modal instability waves. First mode waves are stable because of the strong favorable pressure gradient associated with stagnation point flow. Mach number values at the edge of the boundary layer remain subsonic or low supersonic, so that the second mode instabilities cannot come into play. The convex curvature of the geometry prevents the growth of Görtler disturbances. Furthermore, the crossflow velocities remain small and crossflow modes do not become important, especially for forebody shapes that correspond to spherical segments. Yet, several experimental measurements across a broad set of flow conditions have demonstrated that transition does occur on blunt-body configurations.^{2–6} This conundrum is known as the “blunt-body paradox”.⁷ Surface roughness is known to play an important role in blunt-body transition, but the underlying physical mechanisms have not been established as yet.

As a result of the blunt-body paradox, transition onset over the nose tips of reentry vehicles is usually predicted via empirical correlations. One such well-known correlation is based on the extensive set of measurements carried out under the PAssive Nosetip Technology (PANT) Program.^{8,9} In its original form, the PANT correlation was expressed as

$$Re_{\theta} \left(\frac{k \bar{T}_e}{\theta \bar{T}_w} \right)^{0.7} = \begin{cases} 255 & \text{at } M_e = 1: \text{ transition onset,} \\ 215: & \text{onset location,} \end{cases} \quad (1)$$

where Re_{θ} is the boundary-layer momentum-thickness Reynolds number, k is the peak-to-valley roughness height, θ is the momentum thickness, $\bar{T}_e$ is the edge temperature, $\bar{T}_w$ is the wall temperature, and M_e is the edge Mach number. The PANT correlation for transition onset is based on the observation that transition onset only occurred in cases where the transition parameter on the left hand side of Eq. (1) exceeded a value of 255 at the sonic point (i.e., the location where the edge Mach number becomes unity) and the onset location itself correlated with an upstream position corresponding to a subsonic value of edge Mach number and where the Re_{θ} parameter became equal to 215.

The PANT correlation was examined by Reda & Leverance¹⁰ and Reda¹¹ for actual conditions of the reentry environment as simulated in a ballistic range. Significant

discrepancies were noted between the PANT prediction and the experimentally observed transition zone behavior in ballistic range facilities. By using the same form of Eq. (1), a modified correlation was obtained through a curve fitting of the ballistic range data,

$$Re_{\theta} \left(\frac{k}{\theta} \frac{\bar{T}_e}{\bar{T}_w} \right)^{1.30} = 574. \quad (2)$$

Reda¹¹ also noticed that the momentum thickness Reynolds number at transition onset location, Re_{θ} , correlated better with the inverse of the roughness-height parameter, k/θ , i.e., $Re_{\theta} \propto (k/\theta)^{-1}$.

Even when a boundary layer flow does not support any unstable eigenmodes, disturbances can still be amplified via nonmodal processes that are associated with the non-normality of the disturbance equations.¹² In particular, a transient, algebraic growth can occur via a so-called lift-up mechanism when suitable disturbances can be excited within the boundary layer flow. In general, the maximum growth in disturbance energy is associated with inflow conditions that correspond to streamwise vortices. Small-amplitude, initial disturbances in this optimal form can induce significantly stronger streamwise velocity perturbations via the lift-up process and an upper bound on the associated amplification in disturbance energy can be obtained by using optimization theory. Several works describing such optimal-growth calculations have established that the maximum energy amplification occurs for purely stationary disturbances.^{12,13}

In recent years, the transient growth mechanism has emerged as a potential explanation for many cases of subcritical transition,¹⁴ where the hydrodynamic instability paradigm does not apply. In this regard, two key developments related to transition in external flow systems correspond to the works of Andersson et al.¹⁵ and Reshotko & Tumin.¹⁶ Andersson et al.¹⁵ investigated subcritical (bypass) transition in a flat plate boundary layer due to moderate-to-high levels of freestream turbulence (FST). They were able to correlate an existing set of measurements with a semi-empirical criterion that linked a critical, i.e., threshold disturbance amplitude required for the onset of transition with the intensity of FST by using the optimal growth framework. Reshotko & Tumin¹⁶ extended the ideas from Andersson et al. to roughness-induced transition over blunt-body configurations. Specifically, they developed a similar transition criterion by relating the threshold disturbance amplitude to roughness height via a transfer function associated with the optimal growth of disturbances due to distributed surface roughness.

The physical relevance of optimal-growth theory to blunt-body transition still remains to be established. Indisputably, however, the Reshotko & Tumin¹⁶ work provides the first and only physics-based model toward a potential resolution of the blunt-body paradox. Furthermore, their correlation has been reported to do a better job than the existing, purely empirical curve fits at reconciling the various established datasets. In particular, the RT correlation was originally calibrated with the PANT wind-tunnel database ($\bar{T}_w/\bar{T}_e \approx 0.5$); but it was also shown to correlate well with Reda's ballistic-range data ($\bar{T}_w/\bar{T}_e \approx 0.33$) and, recently, with the transition measurements over a scale model of the Orion Crew Exploration Vehicle

(CEV), which was tested in the Adjustable Contour Expansion (ACE) wind tunnel at Texas A&M University^{5,6} ($\bar{T}_w/\bar{T}_e \approx 0.95$). The RT (Reshotko & Tumin) correlation is written as

$$Re_\theta \left(\frac{k}{\theta} \right) \left(\frac{\bar{T}_e}{\bar{T}_w} \right)^{1.27} = 434. \quad (3)$$

The transition parameter on the left hand side of the RT correlation depends linearly on k/θ , because they assumed the initial disturbance amplitude to scale linearly with the roughness-height parameter. The form of this correlation is very similar to that of Reda from Eq. (2), with the main difference being in the respective exponents of the roughness-height parameter on the left hand side of each correlation.

Despite the favorable agreement between the correlation from Eq. (3) and the available experiments, the accuracy of the transient-growth predictions underlying Eq. (3) remains questionable because of several shortcomings. Most important, Reshotko and Tumin¹⁶ used the laminar basic state profiles at a single streamwise station to estimate the optimal energy gain over the body surface. In reality, the disturbances would evolve along a nonsimilar flow that accelerates continuously from the stagnation point up to the shoulder of the capsule. Furthermore, their calculations implicitly assumed that the transient growth begins near the stagnation point. However, recent analysis of transient growth over a hemispherical forebody¹⁷ has shown that, in general, varying both the inflow and outflow locations to estimate the maximum transient amplification is necessary. Indeed, calculations for the hemispherical configuration have revealed that the maximum transient growth in disturbance kinetic energy growth is associated with a short spatial interval that is located just upstream of the sonic location. Other, less significant limitations of the calculations behind the RT correlation include the use of a self-similar approximation to the actual, nonsimilar boundary layer flow and the neglect of surface curvature effects on the transient growth. Given these shortcomings, one must determine if the optimal-growth correlation would still correlate the available datasets equally well after the abovementioned calculations have been replaced by an improved set of predictions that provide more accurate characterization of the magnitude of optimal growth over blunt body configurations. The objective of this paper is to perform such improved optimal-growth computations for one specific configuration of practical interest, namely, the CEV configuration from the experiments of Leidy et al.^{5,6} that was mentioned as one of the datasets for which the RT correlation has worked well.

The remaining parts of this manuscript are organized as follows. Section II provides a summary of the optimal-growth theory based on the parabolized stability equations (PSE). The numerical results for the CEV configuration in the ACE tunnel are presented in Section III. First, the basic state characteristics for the laminar flow over the CEV geometry are described in Subsection III.A. Next, the optimal growth predictions based on the improved framework with a dual optimization involving both inflow and outflow locations are presented in Subsection III.B. These results address the effects of basic state evolution, potential variation in azimuthal (i.e., spanwise) disturbance wavelength along the symmetry plane, the unit Reynolds number, and wall temperature. In Subsection III.C, the above

transient-growth predictions are used to revisit the RT correlation for roughness-induced transition. Conclusions are presented in Section IV.

Even though this paper does not address the physical connection between the blunt body paradox and optimal growth, it still represents an important exercise by addressing the question of whether the shortcomings of the underlying calculations may have contributed to the success of the RT correlation. Because the RT correlation denotes the only available physics-based, semi-empirical methodology to correlate the onset of subcritical transition over blunt bodies over a broad range of Reynolds numbers and surface temperature ratios, a negative outcome to this exercise would indicate that the previously reported success may have been fortuitous in nature, whereas a positive outcome would underline the importance of further studies to examine the correlation in the context of additional experimental data and, more important, to reverse engineer the success of this correlation.

II. Theory

Optimal growth calculations are performed by using the stationary form of linear PSE as explained in Refs. [18,19]. The only significant difference from the theoretical framework employed by Paredes et al.¹⁹ is that the spanwise wavelength of the disturbance is allowed to be an arbitrary but prescribed function of the location along the leeward symmetry plane. For completeness, a brief summary of the optimal growth framework is presented in this section.

We introduce a suitably nondimensionalized, orthogonal, curvilinear coordinate system (ξ, η, ζ) , where ξ denotes the distance from the stagnation point along the symmetry plane, η represents the wall-normal coordinate, and ζ varies along the azimuthal direction. The velocity components along the above three axes are denoted by (u, v, w) , respectively; the density is denoted by ρ , and the temperature by T . The Cartesian coordinates centered on the nose of the CEV configuration are represented by (x, y, z) . The vector of basic state variables corresponding to unperturbed boundary layer flow in the (ξ, η, ζ) system is denoted by $\bar{\mathbf{q}}(\xi, \eta) = (\bar{\rho}, \bar{u}, \bar{v}, \bar{w}, \bar{T})^T$, whereas the corresponding perturbation quantities undergoing (stationary) transient growth are indicated by $\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = (\tilde{\rho}, \tilde{u}, \tilde{v}, \tilde{w}, \tilde{T})^T$. The perturbations are assumed to have the form

$$\tilde{\mathbf{q}}(\xi, \eta, \zeta) = \hat{\mathbf{q}}(\xi, \eta) \exp(i\beta\zeta) + \text{c.c.}, \quad (4)$$

where c.c. denotes complex conjugate and $\hat{\mathbf{q}}(\xi, \eta) = (\hat{\rho}, \hat{u}, \hat{v}, \hat{w}, \hat{T})^T$ represents the vector of amplitude functions. The parameter β corresponds to the wavenumber along the azimuthal (ζ) direction, so that $\lambda \equiv 2\pi/\beta$ denotes the azimuthal wavelength of the transient growth disturbance. The wavelength parameter λ is a slowly varying function of the surface coordinates. For the axisymmetric basic state analyzed in Ref. [17], the spanwise disturbance wavelength $\lambda(\xi)$ could be assumed to increase in proportion to the distance from the stagnation point, corresponding to a fixed number of vortices (or streaks) at each streamwise location. No such assumption can be made for the disturbance wavelength in the present case with a fully three-dimensional flow and the variation of λ along the streamwise direction must be prescribed in an ad hoc manner. In reality, the $\lambda(\xi)$ variation is determined

by the surface geometry as well as the specific details of the disturbance source, which are usually suppressed in transient growth analyses. The latter choice has both its pros and cons. On one hand, the optimal growth theory can provide information about the transient growth properties of the basic state independently of the disturbance environment (analogous to modal instability characteristics such as the N -factor). On the other hand, the above choice also limits the practical utility of optimal growth theory because of issues related to the realizability of the predicted optimal growth. As mentioned in the Introduction, addressing the realizability issue is beyond the scope of this paper, which seeks to address the viability of the RT correlation. Thus, to help assess the sensitivity of optimal growth predictions with respect to the prescribed variation in disturbance wavelength, we adopt the pragmatic approach of performing computations for multiple plausible definitions of $\lambda(\xi)$. Further discussion of this topic is deferred until the presentation of the numerical results in Section III.

After inserting the perturbation form from Eq. (4) into the linearized NS equations and invoking the slow streamwise dependence of both the basic state and the amplitude functions to neglect the viscous derivatives in ξ , one obtains the linear PSE:

$$\mathbf{L}\hat{\mathbf{q}}(\xi, \eta) = \left(\mathbf{A} + \mathbf{B} \frac{\partial}{\partial \eta} + \mathbf{C} \frac{\partial^2}{\partial \eta^2} + \mathbf{D} \frac{1}{h_\xi} \frac{\partial}{\partial \xi} \right) \hat{\mathbf{q}}(\xi, \eta) = 0. \quad (5)$$

The linear operators $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are given by Pralits et al.¹⁸ and h_ξ is the metric factor associated with the streamwise curvature. The metric factor h_ζ that is associated with azimuthal, or equivalently, spanwise curvature is contained within the linear operators. For the purely stationary disturbances of interest in the present work, Refs. [20, 21] have shown that the streamwise pressure gradient term $\partial \hat{p} / \partial \xi$ can be omitted from the streamwise momentum equation without any significant loss in accuracy.

The optimal initial disturbance, $\tilde{\mathbf{q}}_0$, is defined as the set of initial (i.e., inflow) disturbance profiles at ξ_0 that maximize a suitably defined objective function, $J(\tilde{\mathbf{q}})$. The objective function is defined as the energy gain of the perturbation up to a specified position, ξ_1 ($\xi_1 > \xi_0$). Paredes et al.¹⁷ performed a thorough analysis of how the selections of the energy norm and objective function influence the optimal growth characteristics of the boundary layer flow over a blunt body. Two definitions of the energy gain are commonly used; namely, the outlet energy gain,

$$G_{out} = \frac{E(\xi_1)}{E(\xi_0)}, \quad (6)$$

and the mean energy gain,

$$G_{mean} = \frac{1}{\xi_1 - \xi_0} \frac{\int_{\xi_0}^{\xi_1} E(\xi') d\xi'}{E(\xi_0)}, \quad (7)$$

where E denotes the energy norm of $\tilde{\mathbf{q}}$. The energy norm is defined as

$$E(\xi) = \frac{1}{L_\xi} \int_{\xi} \int_{\eta} \tilde{\mathbf{q}}(\xi, \eta, \zeta)^H \mathbf{M} \tilde{\mathbf{q}}(\xi, \eta, \zeta) h_\xi h_\eta d\eta d\zeta, \quad (8)$$

where $\mathbf{M}$ is the energy weight matrix and the superscript H denotes conjugate transpose. The positive-definite energy norm used here was proposed by Mack²² and Hanifi et al.²³ and is defined as

$$\mathbf{M} = \text{diag} \left[\frac{\bar{T}(\xi, \eta)}{\gamma \bar{\rho}(\xi, \eta) M^2}, \bar{\rho}(\xi, \eta), \bar{\rho}(\xi, \eta), \bar{\rho}(\xi, \eta), \frac{\bar{p}(\xi, \eta)}{\gamma(\gamma - 1) T(\xi, \eta) M^2} \right]. \quad (9)$$

This energy norm definition includes both the kinetic energy and the thermodynamic energy of the disturbance.

The variational formulation of the problem to determine the maximum of the objective functional J leads to an optimality system,^{18,24,25} that is solved in an iterative manner, starting from a random solution at ξ_0 that satisfies the appropriate homogeneous boundary conditions at the surface and in the free stream. The continuous adjoint approach followed by Pralits et al.¹⁸ and Tumin & Reshotko²⁴ for compressible boundary layers is used here. In brief, the linear PSE, $\mathbf{L}\tilde{\mathbf{q}} = 0$, are used to integrate $\tilde{\mathbf{q}}$ from $\xi = \xi_0$ to $\xi = \xi_1$, where the optimality condition at the outflow station is used to obtain the initial condition for the backward integration of the adjoint PSE. At ξ_0 , the adjoint solution is used to calculate the new initial condition for the forward PSE integration with the initial optimality condition. The iterative procedure is terminated when the value of J has converged up to a specified tolerance, which was set to a relative error of 10^{-4} in the current computations. Additional details of the current implementation are found in Paredes et al.^{17,19}

Nonuniform, stable, high-order, finite-difference schemes (FD-q)^{26,27} of sixth order are used for discretization of the PSE along the wall-normal coordinate. The discretized PSE are integrated along the streamwise coordinate by using second-order backward differentiation. The number of discretization points in both directions was varied in selected cases to ensure grid convergence of the optimal gain predictions. The wall-normal direction was discretized using $N_y = 161$, with the nodes being clustered toward the wall.²⁷ No-slip, isothermal boundary conditions are used at the wall, i.e., $\hat{u} = \hat{v} = \hat{w} = \hat{T} = 0$. Unless stated otherwise, the amplitude functions are forced to decay at the farfield boundary by imposing the Dirichlet conditions $\hat{p} = \hat{u} = \hat{w} = \hat{T} = 0$. The farfield boundary is chosen to be just below the shock layer. Verification of the present optimal-growth framework against available results from the literature is presented in Refs. [19,28].

III. Results

Transient-growth results are presented for a spherical-section capsule at an angle of attack with respect to a hypersonic free stream. The characteristics of the unperturbed, laminar boundary layer flow, i.e., the basic state, are analyzed first. Then, the optimal growth characteristics are described with an emphasis on the effects of (i) a nonsimilar, spatially evolving basic state, (ii) streamwise variation in spanwise wavelength of the stationary disturbance, (iii) unit Reynolds number, and (iv) wall temperature. Finally, the implications

of optimal growth predictions on the transition correlation of Reshotko & Tumin¹⁶ are investigated.

III.A. Basic state

Transient growth analysis is performed for a blunt, spherical-section forebody at 28° angle of incidence with respect to a Mach 6 free stream. The forebody configuration models the Orion CEV capsule geometry.²⁹ The face diameter is $D = 2R = 0.0762$ m and the remaining dimensions are scaled according to Hollis,²⁹ resulting in a sphere radius of $R_s = 0.09144$ m. The flow conditions match those of a wind-tunnel experiment in the Adjustable Contour Expansion (ACE) facility at the National Aerothermochemistry Laboratory (NAL) of Texas A&M University.^{5, 6} Computations are performed for four freestream unit Reynolds numbers, namely, $Re' = 3.4 \times 10^6/\text{m}$, $4.4 \times 10^6/\text{m}$, $5.4 \times 10^6/\text{m}$, and $6.4 \times 10^6/\text{m}$. The freestream temperature is set to $\bar{T}_\infty = 54.69$ K and the surface temperature corresponds to $\bar{T}_w = 391.0$ K. To investigate the effects of surface temperature on the transient-growth characteristics, computations are also performed for additional, cooler surface temperatures corresponding to $\bar{T}_w = 300.0$ K, 195.5 K, and 130.33 K, respectively, with the unit Reynolds number held fixed at $Re' = 4.4 \times 10^6/\text{m}$.

The basic state, laminar boundary-layer flow over the forebody was computed by using a second-order accurate algorithm as implemented in the finite-volume compressible Navier-Stokes flow solver VULCAN-CFD (see Ref. [30] and <http://vulcan-cfd.larc.nasa.gov> for further information about the solver). The VULCAN-CFD solution is based on the full Navier-Stokes equations and uses the solver's built-in capability to iteratively adapt the computational grid to the shock.¹ The axial and wall-normal grid distribution was based upon a prior study of a similar blunt-body configuration involving axisymmetric flow over a hemispherical body at zero angle of attack,^{1, 17} wherein the computed solution was shown to be insensitive to changes in grid resolution. In the present case with a fully three-dimensional flow field, similar grid resolution was achieved along both surface coordinates and the singularity along the geometric axis of the spherical-section forebody was avoided by using a surface mesh with nonsingular topology. Using the symmetry of the flow field, only one half of the overall flow domain was discretized to reduce the overall computational cost. In total, the surface mesh consisted of 45,184 quadrilaterals and the wall-normal domain was resolved with 353 points, out of which at least 50 points were located inside the boundary layer. The resolution of the grid was checked for convergence by increasing the number of points by 1.5 times in each direction. The transient growth results obtained for the basic state with the higher resolution grid differed by less than 1% from the optimal gain values based on the baseline grid.

Figure 1(a) shows a three-dimensional view of the entire geometry, with isocontours of static pressure along the wall and also along the symmetry plane. The near-wall streamlines show the spreading of the flow away from the stagnation point that is located within the lower half of the body surface. As mentioned previously, the stagnation point is taken as the reference streamwise position within the symmetry plane, with $\xi_{st} = 0$. The Mach number contours along the symmetry plane are shown in Fig. 1(b). The measurements of Refs. [5, 6] focused on the leeward portion of the flow along the symmetry plane. Accordingly, the analysis

throughout the present paper is similarly restricted to the flow region above the stagnation point in Fig. 1(b), which accounts for a majority of the surface length along the symmetry plane. The Mach number at the edge of the boundary layer flow remains subsonic over a major portion of the body surface, and the largest value of the edge Mach number within the spherical part of the model surface remains within the low supersonic range.

In what follows, all reported flow variables are nondimensionalized with freestream values. The streamwise coordinate is represented by an angular coordinate defined as $\phi = \xi/R_s$. The Reynolds number based on face radius is defined as $Re_R = \rho u_\infty R/\mu_\infty$. The spanwise disturbance wavelength is normalized by length parameter $L_R = R/Re_R$, which is of the same order of magnitude as the boundary layer thickness in the limit of $Re_R \rightarrow \infty$.

The streamwise evolution of basic state Mach number M_e , streamwise mass flux $\bar{\rho}_e \bar{u}_e$, and boundary-layer edge to wall-temperature ratio $\bar{T}_e \bar{T}_w$ is plotted in Fig. 2(a) for $Re' = 4.4 \times 10^6/\text{m}$, with $\bar{T}_w = 391.0$ K. The M_e , $\bar{\rho}_e \bar{u}_e$, and $\bar{T}_e$ distributions are identical for all surface temperatures and therefore the wall-temperature ratio varies inversely with $\bar{T}_w$. The boundary-layer edge, $\eta_e = \delta_h$, is defined as the wall-normal position where $h_t/h_{t,\infty} = 0.995$, with h_t denoting the total enthalpy, i.e., $h_t = h + 0.5(\bar{u}^2 + \bar{v}^2 + \bar{w}^2)$, and h denoting the static enthalpy. The edge Mach number increases monotonically with distance from the stagnation point, whereas the edge temperature displays the opposite trend. In agreement with the inviscid flow theory, the peak of the streamwise mass-flux is located at the sonic point, $M_e = 1$. Similarly, the streamwise distributions of the Mach number and the mass flux are determined by the inviscid theory to the leading order, and hence, they display little variation when the surface temperature is varied.

The evolution of boundary layer thickness δ_h and the momentum-thickness Reynolds number $Re_\Theta = \bar{\rho}_e \bar{u}_e \Theta / \bar{\mu}_e$ along the symmetry plane for three surface temperature are plotted in Figs. 2(b) and 2(c), respectively, for $Re' = 4.4 \times 10^6/\text{m}$ and $\bar{T}_w = 391.0$ K, 195.5 K, and 130.33 K. Although not shown here, the momentum thickness is approximately 10 times smaller than the boundary layer thickness δ_h . The boundary layer thickness δ_h is larger for $\bar{T}_w = 195.5$ K than for $\bar{T}_w = 391.0$ K, although it is slightly reduced for the $\bar{T}_w = 130.33$ K case. The basic state profiles at $\phi = 7.83^\circ$ and $\phi = 26.2^\circ$ are shown in Figs. 3(a) and 3(b), respectively. Each figure shows local profiles for the streamwise velocity $\bar{u}$, temperature $\bar{T}$, and total enthalpy h_t , wherein each quantity is normalized by its freestream (i.e., pre-shock) value. The nonmonotonic trend of the boundary layer thickness δ_h with the wall temperature is attributed to the variation in the temperature profiles. While the boundary layer thickness seems to decrease for cooler walls based on the velocity profile, the opposite is true for the temperature profiles. The variation in total enthalpy (which was used to determine δ_h) is jointly determined by the shapes of the $\bar{u}$ and $\bar{T}$ profiles.

III.B. Transient-growth characteristics

The optimal growth computations are performed within the (ξ, η, ζ) frame that defines an orthogonal, body-fitted coordinate system. The metric factors are defined as

$$h_{\xi} = 1 + \kappa_{\xi}\eta, \quad (10)$$

$$h_{\zeta} = (1 + \kappa_{\zeta}\eta)\frac{\lambda}{\lambda_0}, \quad (11)$$

where κ_{ξ} denotes the streamwise curvature, κ_{ζ} denotes the azimuthal (i.e., spanwise) curvature, and λ/λ_0 denotes the prescribed evolution of the spanwise disturbance wavelength, normalized by the wavelength at a reference position that is chosen to be the inflow station $\xi = \xi_0$ of the transient growth interval. Because of the spherical geometry of the body surface, the streamwise and azimuthal (i.e., spanwise) curvatures are equal to each other and remain constant along the spherical section, $\kappa_{\xi} = \kappa_{\zeta} = 1/R_s = 10.936/\text{m}$.

As mentioned in Section II, the variation of $\lambda(\xi)$ along the streamwise direction must be prescribed to compute the optimal growth factors. To help assess the sensitivity of those predictions with respect to the prescribed variation, calculations are performed for three different variations, namely, a spherical divergence in disturbance wavelength similar to the axisymmetric case in Ref. [17], a constant wavelength ($\lambda/\lambda_0 = 1$), and a third case corresponding to an intermediate value of disturbance wavelength that increases according to the divergence of the inviscid streamlines near the symmetry plane. Figure 4 shows the evolution of spanwise wavelength for these three cases. The variation in $\lambda(\xi)$ is appreciable when the transient growth interval is large. However, subsequent results will show that the intervals corresponding to the overall maximum in the transient growth factor are relatively short. Thus, the $\lambda(\xi)$ variation across those intervals is relatively modest in comparison with the overall range of Fig. 4. One may also observe that, for the most part, the $\lambda(\xi)$ variation based on streamline divergence is bracketed by that in the other two cases, and therefore, the streamline divergence case can be chosen to characterize the scaling of optimal gain with respect to significant flow parameters.

Effect of basic state evolution—The main purpose of this paper is to revisit the transition correlation based on optimal growth theory,¹⁶ which provides the only physics-based transition prediction criterion for blunt body transition. The final form of this correlation, which has been used in all comparisons with the measurement database, was based on transient growth predictions that assumed the same basic state profile at every streamwise station. This profile was obtained by using a self-similar approximation to the boundary layer equations and a local, parallel theory was used to predict the optimal energy gains based on this single profile. As described in Section II, the present framework replaces the above ad hoc procedure by the PSE method, which accounts for the spatial development of the rapidly accelerating boundary-layer flow along the symmetry plane and its effect on the linear transient growth.

Figures 5(a) through 5(f) indicate the effect of including the basic state evolution in the transient-growth analysis on the outlet energy gain, G_{out} and the corresponding optimal disturbance wavelength, λ_0 . Three representative inflow locations are chosen for the purpose of this illustration, namely, $\varphi_0 = 7.83^\circ$, 12.5° , and 26.2° . These inflow locations correspond to local edge Mach numbers of $M_e = 0.28$, 0.42 , and 0.80 , respectively, and hence, lie within

the range of transition locations observed in the experiments ($M_e \in [0.2, 0.8]$) as indicated in Ref. [16]. The other flow parameters correspond to $Re' = 4.4 \times 10^6/\text{m}$ and $\bar{T}_w = 195.5$ K.

The parallel results are calculated by replicating the initial boundary layer profile at every downstream station (i.e., by ignoring the rapid acceleration of the flow) and by using a constant spanwise wavelength. Similar to Ref. [16], the parallel predictions exclude the effects of the streamwise curvature terms. Results show that the discrepancy between the parallel and nonparallel predictions for the peak values of optimal outlet energy gain values is relatively small when φ_0 is large (5(c)). However, the differences can be rather substantial when the inflow station is located farther upstream (Figs. 5(a) and 5(b)). Also, the magnitude of this discrepancy increases with the length of the transient growth interval. The variation in optimal values of the initial disturbance wavelength λ_0 corresponding to the maximum energy gain in Figs. 5(a), 5(b), and 5(c) is shown in Figs. 5(d), 5(e), and 5(f), respectively. The differences between parallel and nonparallel predictions for λ_0 indicate similar trends as the differences in optimal gain predictions, namely, the discrepancy is larger for upstream initial stations and increases in magnitude with the length of the transient growth interval.

Optimum interval for maximum transient growth—Paredes et al.¹⁷ showed that the analysis of optimal growth characteristics of a nonsimilar boundary layer requires one to optimize both the inflow and outflow locations of the transient growth interval in addition to the azimuthal wavelength of the disturbance (λ_0 in the present case). Furthermore, because an objective function based on the mean energy gain accounts for a possible overshoot in the disturbance energy evolution that is not accounted for by the outlet energy gain, the former objective function was suggested as a more appropriate metric for characterizing the transient growth magnitude for blunt body configurations with strongly accelerating boundary layers. The highest optimal gain in the mean energy, G_{mean} for $Re' = 4.4 \times 10^6/\text{m}$ is plotted in Figs. 6(a) and 6(b) for $\bar{T}_w = 391.0$ K and $\bar{T}_w = 130.33$ K, respectively. The domain of each contour plot is limited by the line $\varphi_1 = \varphi_0$ (i.e., the limiting case of zero length optimization interval) on the bottom and by the line $\varphi = 43.9^\circ$ ($\xi = 0.07$ m) on the top. The excluded region corresponding to small φ_0 and large φ_1 is deemed unnecessary for the present analysis because the optimal perturbations within that range extend far outside of the boundary-layer edge and are unlikely to be excited by wall roughness; see Paredes et al.¹⁷ for more details on the perturbation shapes. The highest mean energy gain occurs for relatively short optimization intervals for both cases. The wall temperature is seen to have a strong influence on the magnitude of the optimum energy gain, with peak values for $\bar{T}_w = 130.33$ K being larger by nearly one order of magnitude. The other major difference between the results for the two wall temperatures is that, for $\bar{T}_w = 391.0$ K, there exists an optimum set of parameters that lead to a maximum G_{mean} in the interior of the domain, in the vicinity of the sonic point (i.e., $\varphi \approx 33^\circ$ as shown in Fig. 2). On the other hand, the maximum for $\bar{T}_w = 130.33$ K is found to lie closer to the stagnation point. The reason behind this difference is that for the $\bar{T}_w = 391.0$ K case, the wall temperature is nearly equal to the edge temperature ($\bar{T}_w/\bar{T}_e \approx 1$) as shown in Fig. 2(a). Because of this, the thermodynamic component of the overall energy norm is relatively insignificant and the kinetic energy gain represents most of the energy growth for this case. Paredes et al.¹⁷ showed that the

maximum kinetic energy gain is achieved for initial locations in the vicinity of the sonic point. Although not shown here, the contours of optimum gain in mean kinetic energy for $\bar{T}_w = 391.0$ K and $\bar{T}_w = 130.33$ K are qualitatively similar to the total energy gain for $\bar{T}_w = 391.0$ K in Fig. 6(a). Quantitatively, however, the maximum gain in mean kinetic energy increases as the wall temperature is lowered; specifically, $\max(G_{mean}^{opt}) = 12.05$ for $\bar{T}_w = 391.0$ K and $\max(G_{mean}^{opt}) = 28.18$ for $\bar{T}_w = 130.33$ K. Finally, at both surface temperatures, the optimal length of the transient growth interval is seen to be rather short, i.e., well under 10 degrees.

From here on, the overall optimal growth characteristics of the flow are characterized in terms of the combination of spanwise wavelength and initial location that lead to the maximum value of the energy gain for a specific outflow location. The effect of assumed variation in spanwise wavelength, unit Reynolds number of the base flow, and the wall temperature on the maximum value of the optimal mean energy gain G_{mean}^{opt} , the optimal wavelength λ_0^{opt} , and optimal growth interval $(\varphi_1 - \varphi_0)^{opt}$, is analyzed next.

Effect of ansatz for streamwise variation of spanwise disturbance wavelength

—Figure 7 shows the effect of assumed streamwise variation in the spanwise wavelength of the disturbance for $Re' = 4.4 \times 10^6/\text{m}$ and $\bar{T}_w = 391.0$ K. Results are shown for the three representative cases from Fig. 4, namely, wavelength variation based on streamlines divergence, axisymmetric flow, and a constant value, respectively. Figure 7(a) shows that the predicted value of optimal mean energy gain for these three cases fall within a 20% of the mean, with the largest values corresponding to the axisymmetric definition (which corresponds to most rapid increase in disturbance wavelength). The initial optimal wavelength λ_0^{opt} is found to be nearly the same in all three cases (Fig. 7(b)), but Fig. 7(c) shows that the corresponding lengths of optimal growth intervals $(\varphi_1 - \varphi_0)^{opt}$ diverge for initial locations closer to the stagnation point. For the most part, the optimum length of the transient growth interval is small, less than approximately 8 degrees. As a result, the corresponding variation in $\lambda(\xi)$ is rather small, which explains the robustness of predictions for the peak optimal gain with respect to the assumed variation in $\lambda(\xi)$.

Effect of unit Reynolds number—The scaling of optimal gain values with the unit Reynolds number is presented in Fig. 8 for isothermal wall condition with $\bar{T}_w = 391.0$ K and $\lambda(\xi)$ variation based on streamline divergence. Figure 8(a) shows a nearly linear scaling of the optimal gain with the unit Reynolds number ($Re_R = R Re'$). Because the dimension of the body is kept constant when the unit Reynolds number is varied, the observed, small deviations from the linear trend are attributed to the differences in the ratio of boundary-layer thickness to the radius of surface curvature. As shown in Fig. 8(b), the scaling of the optimal wavelength with the boundary layer scale L_R is nearly perfect, indicating that the wavelength scales with respect to the boundary layer thickness δ_h . However, Fig. 8(c) shows that changes in unit Reynolds number have a small but observable effect on the optimal interval of transient growth $(\varphi_1 - \varphi_0)^{opt}$.

Effect of wall temperature—Finally, the effect of wall temperature on the optimal-growth characteristics is shown in Fig. 9. For this analysis, the unit Reynolds number is set to $Re' = 4.4 \times 10^6/m$ and the wavelength definition is based on streamline divergence. As mentioned previously, the latter choice denotes an intermediate case in terms of wavelength variation along the symmetry plane. As reported by Reshotko & Tumin,^{7, 16} and also observed in Fig. 6, the wall temperature has a strong effect on the optimal energy gain. Fig. 9(a) shows that the overall maximum of the optimal gain G_{main}^{opt} is about one order of magnitude larger for the $\bar{T}_w = 130.33$ K case in comparison with the corresponding values for the $\bar{T}_w = 391.0$ K case. Although not shown here, the kinetic energy fraction of the total energy gain is also larger for the cooler wall ($\bar{T}_w = 130.33$ K), with an increase of roughly 2.3 times the corresponding fraction for the $\bar{T}_w = 391.0$ K case. The effect of wall temperature on the optimal spanwise wavelength λ_0^{opt} and the optimal interval $(\varphi_1 - \varphi_0)^{opt}$ is shown in Figs. 9(b) and 9(c), respectively. The variation in optimal wavelength is less than 10% of the mean value whereas the corresponding changes in the length of the optimal interval are less than 20% of the mean length. Figures 10(a), 10(b), and 10(c) are equivalent to Figs. 9(a), 9(b), and 9(c), respectively, but with the alternative selection of the objective function corresponding to the outlet energy gain, $J = G_{out}$.

The results corresponding to the outlet energy gain are used in Subsection III.C to revisit the RT correlation for transition onset over blunt bodies. As expected, the values of the optimal outlet energy gain (Fig. 10(a)) are larger than those corresponding to the optimal mean energy gain (Fig. 9(a)), because the former values are close to the peak of the energy evolution curves, which is larger than the mean of the energy gain curve. Still, the trends in both objective functions are very similar to each other. The maximum value of the outlet energy gain corresponds to an N -factor of $N = 0.5 \ln(G_{out}) \approx 2.6$, where N denotes the logarithmic increase in disturbance amplitude across the leeward plan interval of interest. The optimal values of the azimuthal wavelength are nearly identical for both measures of transient growth. However, the length of the optimal interval for transient growth based on the outlet energy gain is nearly one half of the corresponding length associated with mean energy gain.

III.C. Revision of transient-growth based transition correlation

Herein, the previously presented results for optimal-growth characteristics are used to reexamine the successful transition criteria of Reshotko & Tumin.¹⁶ As described in Ref. [16], the input disturbance energy induced by the surface roughness scales according to $E_0 = \bar{\rho}_k \bar{u}_k^2$, if one assumes that the roughness-induced velocities are proportional to the roughness peak-to-valley height $\bar{u}_k/\bar{u}_{e,0} = k/\theta_0$, and that $\bar{T}_{e,0}/\bar{T}_k = \bar{\rho}_k/\bar{\rho}_{e,0}$, where $\bar{T}_k = \bar{T}_w$.

The onset of laminar-turbulent transition may be assumed to be associated with a critical value of local, nondimensional disturbance amplitude $A_{tr} = \sqrt{E_{tr}/(\bar{\rho}_{e,tr} \bar{u}_{e,tr}^2)}$, where the outlet disturbance energy at the transition location, E_{tr} is related to the induced disturbance energy E_0 by the energy gain associated with optimal growth, i.e., $E_{tr} = GE_0$. Therefore, introducing the linear dependence of the energy gain G with the unit Reynolds number, as shown in Fig.

8(a), or equivalently, with the square of the momentum-thickness Reynolds number, we can write

$$A_{tr} = (G^{1/2}/Re_{\theta,0})Re_{\theta,0}\left(\frac{k}{\theta_0}\right)\left(\frac{\bar{T}_{e,0}}{\bar{T}_w}\right)^{1/2}\left(\frac{\bar{\rho}_{e,0}\bar{u}_{e,0}^2}{\bar{\rho}_{e,tr}\bar{u}_{e,tr}^2}\right)^{1/2}. \quad (12)$$

Figure 11(a) shows the transition onset parameter, A_{tr} , scaled by the nondimensional roughness height k/R for the present CEV configuration as a function of the transition onset location φ_{tr} . The data plotted in this figure corresponds to optimal gain in outlet energy for $Re' = 4.4 \times 10^6/\text{m}$, all selected wall temperatures, and a $\lambda(\xi)$ variation based on streamlines divergence.

Following Ref. [16], we assume a power-law variation of the optimum energy gain with respect to surface-to-edge temperature ratio and neglect the effects of the local edge Mach number and the curvature terms, which leads to

$$(G^{1/2}/Re_{\theta,0})\left(\frac{\bar{\rho}_{e,0}\bar{u}_{e,0}^2}{\bar{\rho}_{e,tr}\bar{u}_{e,tr}^2}\right)^{1/2} \propto \left(\frac{\bar{T}_w}{\bar{T}_{e,0}}\right)^{c_T}. \quad (13)$$

The neglect of the local edge Mach number was justified in Ref. [16] by the fact that measured transition locations usually fall within the subsonic portion of the flow, i.e., where the edge Mach number is relatively small ($M_{e,tr} \in [0.2, 0.8]$). The effect of the curvature factor corresponding to the radius of the spherical section is assumed to be secondary to the effect of the wall temperature. There is no strong justification for this assumption, except perhaps to keep the correlation as simple as possible. The neglect of the curvature parameter is partially supported by the recent work by Hollis,³¹ whose findings suggest that a more accurate curve fit to the measured data can be obtained by considering the edge Mach number but without considering the curvature parameter.

The computations based on local, parallel theory (i.e., $\xi_{tr} \equiv \xi_0$) in Refs. [7,16] led to a power law exponent in Eq. (13) of $c_T = -0.77$ when $\bar{T}_w/\bar{T}_{ad,w} \approx 0.5$. The constant in the transition correlation was evaluated by using the subset of the PANT database^{8,9} of series A and J corresponding to $\bar{T}_w/\bar{T}_e \approx 0.5$. The resulting RT correlation from Eq. (3) can be written as

$$Re_{\theta,tr} = 434\left(\frac{k}{\theta}\right)^{-1}\left(\frac{\bar{T}_e}{\bar{T}_w}\right)^{-1.27}. \quad (14)$$

In the present work, results similar to those in Fig. 10 are used to estimate the best fit exponent c_T at each one of a selected set of outflow locations across the range of expected transition locations. Recall that the results in this figure are based on the outlet energy norm for $\bar{T}_w = 300.0$ K, 195.5 K, and 130.33 K, i.e., $\bar{T}_w/\bar{T}_e \approx 0.75, 0.50$, and 0.33, respectively. For each value of wall temperature, the c_T values are calculated by using the parameter combinations that yield optimal outlet energy gain for selected outlet locations, which are

defined as the potential transition locations, i.e., $\xi_{tr} \equiv \xi_1$. Figure 11(b) shows the variation of c_T with the transition or outlet location ξ_{tr} for each of the three assumed variations in spanwise disturbance wavelength, i.e., based on streamlines divergence, axisymmetric flow, and uniform distribution, respectively. The values of c_T are contained within the range $c_T \in [-0.9, -0.7]$ and are nearly insensitive to the assumed variation in the spanwise disturbance wavelength definition.

The resulting set of exponents are averaged to obtain the mean exponent values for each choice of $\lambda(\xi)$ variation. The calculated mean values correspond to $c_T = -0.808$, -0.809 , and -0.804 , for spanwise wavelength variations based on streamlines divergence, axisymmetric flow, and uniform distribution, respectively. Although not shown here, these values are nearly identical for objective functions based on the outlet and mean energy gains, respectively; specifically, the values of c_T change by less than 1% when the objective function is changed. However, the selection of the energy norm has a great effect on the c_T values; specifically, when the kinetic energy alone is used as the energy norm instead of the complete energy norm defined by Eq. 9, the mean value of the exponent is $c_T \approx -0.20$, which does not correlate with the experimental data. The c_T values obtained above are remarkably close to the constant calculated by Reshotko & Tumin^{7,16} on the basis of parallel, transient growth calculations in the vicinity of $\bar{T}_w/\bar{T}_{ad,w} \approx 0.5$. The constant in the transition correlation based on the present calculations is determined by using Reda's data^{10,11} with $\bar{T}_w/\bar{T}_e \approx 0.33$, a subset of the PANT data^{8,9} corresponding to series A and J with $\bar{T}_w/\bar{T}_e \approx 0.5$, and the ACE data^{5,6} with $\bar{T}_w/\bar{T}_e \approx 0.94$, omitting an obvious outlier measurement. Correlation parameters ($Re_{\theta,tr}$ and k/θ_{tr}) for the former two datasets are based on the historical data,⁸⁻¹¹ whereas analogous values for the ACE data were computed from the transition locations reported in Refs. [7, 16] and the basic state computations described earlier in this section. The resulting transition correlation can be written as

$$Re_{\theta,tr} = 455 \left(\frac{k}{\theta} \right)^{-1} \left(\frac{\bar{T}_e}{\bar{T}_w} \right)^{-1.31}. \quad (15)$$

Figure 12 shows the relatively good agreement between transition onset parameters predicted with Eq. 15 and the measured data for blunt nosetips under actual conditions of reentry environment in ballistic range^{10,11} with $\bar{T}_w/\bar{T}_e \approx 0.33$ and the PANT Series A and Series J wind-tunnel data^{8,9} with $\bar{T}_w/\bar{T}_e \approx 0.50$. Furthermore, the wind-tunnel data^{5,6} corresponding to transition onset measurements on the present CEV configuration with uniformly distributed surface roughness with $\bar{T}_w/\bar{T}_e \approx 0.94$ are also included. Although not shown here, the differences between the RT correlation of Eq. (14) and the modified correlation based on the present computations Eq. (15) are nearly indistinguishable from each other. Whereas both physics-based correlations provide a reasonable fit to all three datasets, the purely empirical PANT correlation does not correlate very well with the trends in the ballistic range data. On the other hand, the PANT correlation does appear to work quite well for the CEV data from the ACE Wind Tunnel. Furthermore, the recent transition data on hemispherical nosetips by Hollis³¹ with $\bar{T}_w/\bar{T}_e \approx 0.79$ are included. The present transition correlation does not accurately predict these measurements; however, a good

agreement is found when the transition correlation constant of Eq. (15) is changed to 345. Therefore, the inverse linear trend, i.e., $Re_{\theta, tr} \propto (k/\theta_{tr})^{-1}$ is confirmed, although the discrepancy reflects that accounting for other parameters, such as the edge Mach number or the surface curvature, could improve the correlation.

IV. Summary and Conclusions

Optimal, transient-growth analysis has been conducted for the laminar flow based on the solution of the Navier-Stokes equations over a scale model of the Orion CEV geometry at 28° angle of attack. The freestream conditions correspond to an experiment involving this model in the Mach 6 ACE wind tunnel at Texas A&M University.^{5,6} Similar to that experiment, the present analysis is focused on the transition behavior along the symmetry plane of the spherical-section CEV capsule. A parameter study is performed to study the variation of optimal-growth characteristics with freestream unit Reynolds number ($Re' = 3.4 \times 10^6/\text{m}$ to $6.4 \times 10^6/\text{m}$) and wall temperature ($\bar{T}_w = 130.33 \text{ K} = 130.33 \text{ K}$ to 391.0 K).

The specific improvements and changes from the analysis underlying the RT correlation are:

1. Allowing for proper spatial evolution of disturbance growth via PSE rather than using parallel theory predictions based on the basic state profiles at a single location.
2. Optimizing for maximum transient growth by varying both inflow and outflow locations. This is important because the computational results confirm an entirely new aspect of transient growth process over blunt bodies, namely that the maximum transient growth can occur over a short region away from the stagnation point, rather than in the near vicinity of the stagnation point.
3. Assessing effects of variable azimuthal wavelength. Prescription of azimuthal-wavelength variation along the transient growth trajectory is an outstanding issue that is generic to all fully three-dimensional boundary layer flows. Indeed, it applies equally to both modal instabilities and the transient growth disturbances. Since there are no prospects of a rigorous theoretical model to define $\lambda(\xi)/\lambda_0$, we have explored several possibilities to ensure that the scaling of optimal gain factor with respect to surface temperature and unit Reynolds number is relatively insensitive to the ad hoc prescription of $\lambda(\xi)/\lambda_0$.
4. Inclusion of curvature effects on disturbance growth.
5. Full Navier-Stokes basic state solution.

The last two improvements exert a quantitative influence on the disturbance evolution and both of them have been implemented in previous literature (see, for instance, Ref. [17]), but neither had been used toward the definition of the RT transition correlation.

Even though each of the above improvements is important in its own right, their combined effect on the scaling of optimal gain with respect to unit Reynolds number and surface temperature ratio is found to be small. Thus, the improved correlation based on just the present transient-growth results for the CEV wind-tunnel configuration of Refs. [5,6] is

remarkably close to the original correlation of Reshotko & Tumin.¹⁶ This implies that a transition criterion based on optimal-growth correlation continues to provide successful correlation with the previous data sets (PANT and Reda). However, the transition correlation constant had to be adjusted from 455 to 345 to provide an accurate correlation with the recent transition data on hemispherical nosetips by Hollis.³¹ This suggests that further improvement in the correlation may entail the consideration of additional parameters. On the other hand, the magnitude of maximum optimal growth on all blunt body configurations is found to be rather small (equivalent N factors of 2.6 or less), which raises questions concerning the relevance of realizable transient growth (which would be even less than the optimal-growth factors used in the correlation) to the observed onset of transition. Additional work is therefore necessary to examine the trends in optimal growth for other experimental configurations and to delineate the transition mechanisms underlying the experimental observations.

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Nomenclature

D	capsule-face diameter
E	energy norm
G	energy gain
h_t	total enthalpy
h_ξ	streamwise metric factor
h_ζ	spanwise metric factor
J	objective function
K	peak-to-valley roughness height
L_R	length parameter based on face radius
M	Mach number
$\bar{q}$	vector of base flow variables
$\tilde{q}$	vector of perturbation variables
$\hat{q}$	vector of amplitude variables
R	face radius

$\mathbf{R}_s$	spherical-segment radius
$\mathbf{Re}'$	unit Reynolds number
$\mathbf{Re}_R$	Reynolds number based on capsule-face radius
$\mathbf{Re}_\theta$	Reynolds number based on momentum boundary layer thickness
$\bar{T}$	basic state temperature
$\bar{T}_w$	wall temperature
$(\mathbf{u}, \mathbf{v}, \mathbf{w})$	streamwise, wall-normal and spanwise velocity components
$\bar{u}$	basic state streamwise velocity
$(\mathbf{x}, \mathbf{y}, \mathbf{z})$	Cartesian coordinates
β	spanwise wavenumber
δ_h	boundary-layer thickness based on total enthalpy
κ_ξ	streamwise curvature
κ_ζ	spanwise curvature
λ	spanwise wavelength
ρ	density
μ	dynamic viscosity
θ	momentum boundary-layer thickness
(ξ, η, ζ)	streamwise, wall-normal and spanwise coordinates
Φ	angular coordinate
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{L}$	PSE matrix operators
$\mathbf{M}$	energy weight matrix
$\mathbf{e}$	edge value
mean	mean gain
out	outlet gain
∞	freestream value
0	inlet disturbance location
1	outlet disturbance optimization location

Tr	transition location
*	dimensional value
H	conjugate transpose

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Figure 1.

(a) Static pressure contours and near-wall streamlines of the basic state solution along the wall and the symmetry plane. (b) Mach number contours and streamlines along the symmetry plane. The thick, dashed lines denote the sonic lines, $M=1$.

Figure 2.

Streamwise evolution of basic state variables within the symmetry plane. (a) Boundary-layer edge quantities (M_e , $\bar{\rho}_e \bar{u}_e$, and $\bar{T}_e/\bar{T}_w$) for $\bar{T}_w = 391.0$ K. (b) Boundary-layer thickness, δ_h , and (c) Reynolds number based on momentum boundary-layer thickness, $Re_\theta = \bar{\rho}_e \bar{u}_e \theta / \mu_e$, for $\bar{T}_w = 391.0$ K, $\bar{T}_w = 195.5$ K, and $\bar{T}_w = 130.33$ K. The selected unit Reynolds number is $Re' = 4.4 \times 10^6/\text{m}$.

Figure 3.

Basic state profiles of streamwise velocity, temperature, and total enthalpy at (a) $\varphi = 7.83^\circ$ and (b) $\varphi = 26.2^\circ$, for selected wall temperatures. The selected unit Reynolds number is $Re' = 4.4 \times 10^6/\text{m}$.

Figure 4.

(a) Evolution of spanwise disturbance wavelengths, λ , based on streamlines divergence, axisymmetric flow, and constant value definitions. The wavelengths are normalized with $\lambda_0 = \lambda(\phi = 6.3^\circ)$.

Figure 5.

Effect of nonparallel terms on (a,b,c) outlet energy gain G_{out} and (d,e,f) corresponding optimal spanwise disturbance wavelength λ_0 , with (a,d) $\phi_0 = 7.83^\circ$, (b,e) $\phi_0 = 12.5^\circ$, and (c,f) $\phi_0 = 26.2^\circ$. The wall temperature is $\bar{T}_w = 195.5$ K and unit Reynolds number is $Re' = 4.4 \times 10^6/\text{m}$.

Figure 6.

Contours of optimal mean energy gain G_{mean} for $Re' = 4.4 \times 10^6/\text{m}$ with (a) $\bar{T}_w = 391.0$ K and (b) $\bar{T}_w = 130.33$ K, and $\lambda(\xi)$ varying according to local streamline divergence. The solid line in the contour plot indicates the value of ϕ_1 corresponding to maximum G_{mean} for a given ϕ_0 .

Figure 7.

Effect of ansatz for streamwise variation of spanwise disturbance wavelength on (a) optimal mean energy gain G_{mean}^{opt} and corresponding (b) optimal wavelength parameter λ_0^{opt} and (c) optimal growth interval $(\phi_1 - \phi_0)^{opt}$, for flow conditions $Re' = 4.4 \times 10^6/\text{m}$ and $\bar{T}_w = 391.0$ K.

Figure 8.

Effect of unit Reynolds number on (a) optimal mean energy gain G_{mean}^{opt} and corresponding (b) optimal wavelength λ_0^{opt} , and (c) optimal interval $(\phi_1 - \phi_0)^{opt}$; for $\bar{T}_w = 391.0$ K and the variation of spanwise wavelength $\lambda(\xi)$ is based on the local divergence of inviscid streamlines.

Figure 9.

Effect of wall temperature on (a) optimal mean energy gain G_{mean}^{opt} and corresponding (b) optimal wavelength λ_0^{opt} and (c) optimal interval $(\phi_1 - \phi_0)^{opt}$, with wavelength definition based on streamlines and for unit Reynolds number $Re' = 4.4 \times 10^6/\text{m}$.

Figure 10.

Effect of wall temperature on (a) optimal outlet energy gain G_{out}^{opt} and corresponding (b) optimal wavelength λ_0^{opt} and (c) optimal interval $(\phi_1 - \phi_0)^{opt}$, with wavelength definition based on streamlines and for unit Reynolds number $Re' = 4.4 \times 10^6/\text{m}$

Figure 11.

(a) Transition onset parameter A_{tr} divided by peak-to-valley roughness height k/R , calculated with selected wall temperatures and streamline-based wavelength variation. (b) Best fit power-law exponent of wall-to-edge temperature ratio for selected variations in spanwise disturbance wavelength. The unit Reynolds number is set to $Re' = 4.4 \times 10^6/\text{m}$.

Figure 12.

Comparison of transition onset locations measured in experiments (PANT, Reda, Hollis, and ACE) with PANT empirical correlations and present semi-empirical correlation based on transient growth. The light-blue area represents a confidence level of $\pm 25\%$ with respect to the present correlation.