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Linear relationship of normal and tangential contact stiffness with load

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Measurements with digital image correlation of normal and tangential contact stiffness for ground Ti-6Al-4V interfaces suggest a linear relationship between normal contact stiffness and normal load and a linear relationship between tangential contact stiffness and tangential load. The normal contact stiffness is observed approximately to be inversely proportional to an equivalent surface roughness parameter, defined for two surfaces in contact. The ratio of the tangential contact stiffness to the normal contact stiffness at the start of tangential loading is seen to be given approximately by the Mindlin ratio. A simple empirical model is proposed to estimate both the normal and tangential contact stiffness at different loads for a ground Ti-6Al-4V interface, on the basis of the equivalent surface roughness and the coefficient of friction.

1. Introduction

Contact stiffness characterizes the load–displacement behaviour of a contact interface. Contact stiffness properties affect wear [1], heat transfer [2] and the electric conductance [3] between interfaces. In addition, contact stiffness influences the impact response of colliding structures [4], tyre noise in a tyre-road contact [5,6] and the dynamic behaviour of high precision machines tools [7,8]. Also significantly for industry, contact stiffness values are needed to calculate the natural frequencies and the dynamic response of assemblies such as bladed-disks in turbines [9,10], for vibration modelling. More fundamentally, the relationship between tangential contact stiffness and tangential load provide the load–displacement relationship for an interface prior to sliding, and may be relevant in the study of fundamental properties of static and sliding friction, such as with rate and state models [11,12].

There are several models of normal contact stiffness that propose it has a linear relationship with normal load [13–17]. Of these studies, Shi & Polycarpou [13] and Medina *et al.* [14] modelled interfaces with discrete asperity contacts, following the approach of Greenwood & Williamson [18]. The other studies that found this relationship modelled the interfaces as having fractal geometry. This includes Akarapu *et al.* [15] who used molecular dynamics modelling, Persson [17] who used a theoretical approach, and Campaña *et al.* [16] who used both a theoretical model (following the Persson approach) and Green's function molecular dynamics (GFMD). A study, by Pohrt & Popov [19] with boundary element method (BEM) proposed a sublinear power law between normal contact stiffness and normal load. Similar power law relationships have been proposed by other studies [20–22]. BEM modelling by Pastewka *et al.* [23] suggested the sublinear relationship is due to a 'finite size effect'.

There are relatively fewer published theoretical models of tangential contact stiffness. Campaña *et al.* [16] looked at tangential contact stiffness at low tangential loads (when there is no slip) and found the ratio of tangential contact stiffness to normal contact stiffness (for isotropic elastic interfaces) is given by the Mindlin ratio, $2(1 - \nu)/(2 - \nu)$. This relationship was also derived by Medina *et al.* [14], following a Greenwood and Williamson approach. In effect, this is the ratio when the interface is fully stuck [14]. Paggi *et al.* [24] investigated tangential contact stiffness with further application of tangential load assuming partial slip contact conditions at the contact points. According to this study, there is a linear relationship between tangential load and tangential contact stiffness when the contact is modelled with a Greenwood and Williamson approach. When the contact surfaces are modelled as fractal they reported a power-relationship, which depends on the Hurst exponent of the fractal geometry.

However, experimental support for these models of both normal and tangential contact stiffness is modest. Experimental measurement of contact stiffness is usually done by one of three methods. The first is by sending ultrasound pulses and analysing the proportion of the waves reflected/transmitted at the interface, pioneered by Kendall & Tabor [25]. The second method known as the contact resonance method is by measuring the effect of the contact on the vibration response of a system [13]. The third method is to measure directly the relative displacement of points on either side of the interface, the contact displacement, at different loads. The slope of the normal load and normal contact displacement curve provides the normal contact stiffness while the slope of the tangential load and the tangential contact displacement curve provides the tangential contact stiffness [12,26–29]. The load is measured with a load cell and the contact displacement can be measured by different means, such as with a laser nanosensor [27] or digital image correlation (DIC) [26].

Shi & Polycarpou [13] with the contact resonance method observed the normal contact stiffness increased with normal load and is related to the interface roughness. Wang *et al.* [5] obtained experimental measurement consistent with Persson's theory [17] for the normal load relationship with mean interface separation, for rubber pressed on asphalt. Campaña *et al.* [16] note several studies with ultrasound measured the (initial) tangential contact stiffness to normal contact stiffness ratio to be around half the Mindlin ratio, i.e. half the value predicted by various models. Berthoud & Baumberger [12] observed the tangential contact stiffness values of polymer glass interfaces and aluminium alloy interfaces were proportional to the normal load and inversely related to a characteristic length parameter, which they posited may be related to interface topology. The measurements were however limited for relatively low tangential loading.

Due to the small displacements (submicrometre) involved, particularly for metal interfaces, contact stiffness measurements are prone to significant errors due to external compliances affecting the displacement measurement system. More recently, DIC has shown promise in measuring displacements very near to the contact interface accurately without the measurements being affected by external compliances [28,29]. Furthermore, unlike the ultrasound method and other methods, DIC can measure the change in the contact stiffness of a given interface with load. Mulvihill *et al.* [28] compared the measurements with DIC and with the ultrasound technique for the same interfaces and argue that the ultrasound technique in effect measured the unloading stiffness when the interfaces were being loaded. However, no serious attempt has been made to

construct a model of contact stiffness (tangential or normal) from the measurements made with DIC or to relate the measurements with theoretical models. This present study finds an empirical relationship for the normal and tangential contact stiffness of Ti-6Al-4V interfaces, measured with DIC. The measured normal and tangential contact stiffness values are related to normal and tangential loading (applied quasi-statically) and surface texture parameters. Ti-6Al-4V is an important alloy used in the aerospace industry (such as in turbine engines) [30].

For the interfaces tested, it is deduced there is a linear relationship between normal load and normal contact stiffness and a linear relationship between tangential load and tangential contact stiffness. Interface roughness, characterized by an equivalent roughness parameter for two surfaces in contact, is seen inversely related to normal contact stiffness, consistent with theoretical models. The initial tangential contact stiffness, at fully stuck state, is observed to be proportional to the normal contact stiffness. The constant of proportionality is near the Mindlin ratio, consistent with theoretical models. The linear decrease in the tangential contact stiffness with further application of tangential load is a novel observation and can be used to predict the coefficient of friction of the interface.

2. Definition of contact stiffness

In this study, contact displacement is defined as the additional displacement of two orthogonal points on either side of a contact interface due to the presence of a rough interface. The differential of the normal load with respect to the normal contact displacement gives the normal contact stiffness and the differential of the tangential load with respect to the tangential contact displacement provides the tangential contact stiffness.

In effect, as per this definition, the contact compliance (the reciprocal of contact stiffness) is the additional compliance due to the presence of the contact. It can be obtained by subtracting the bulk compliance (the compliance of the two bodies if they were fused) from the total compliance. From figure 1, if K_{tn} is the total normal stiffness and K_{bn} is the normal bulk stiffness (i.e. the stiffness of the two bodies if they were fused) then the normal contact stiffness, K_n , is therefore given by

$$\frac{1}{K_n} = \frac{1}{K_{tn}} - \frac{1}{K_{bn}}, \quad (2.1)$$

and the tangential contact stiffness, K_t , is given by

$$\frac{1}{K_t} = \frac{1}{K_{tt}} - \frac{1}{K_{bt}}, \quad (2.2)$$

where K_{tt} is the total tangential stiffness and K_{bt} is the tangential bulk stiffness.

It should be noted that the definition of contact displacement used in this study differs from that in other studies. Mulvihill *et al.* [28] measured contact stiffness as the differential of load with respect to the contact displacement defined as the relative displacement of two points on either side of the contact interface. They noted the distance between the chosen points affected the measurement, particularly at high loads. This was due to bulk deformation of material between the two points and the definition adopted in this current study reflects the need to ensure the contact displacement measured is unaffected by bulk deformation.

Furthermore, contact models by Persson [17] and others [5,16] define normal contact stiffness as the differential of normal load with respect to the mean interfacial separation (or mean gap) between the surfaces in contact. However, laboratory measurement of the mean gap between surfaces for two rough surfaces is in many cases impractical. Where measurements have been conducted [5], one of the surfaces is nominally smooth and the other is assumed relatively rigid. Wang *et al.* [5], for instance, measured the normal contact stiffness, as per this definition, for rubber pressed against asphalt. It was assumed the rubber contact surface was perfectly smooth (or sufficiently so), and the asphalt surface profile (obtained from profilometry) did not to change during loading (the mean surface plane is assumed unaffected). The change in mean gap at given loads is found straightforwardly from the displacement of the rubber contact surface [5].

Figure 1. For two bodies in contact and loaded by a normal load, K_{tn} is the total normal stiffness of the two bodies and K_{bn} is the normal stiffness of the two bodies if they were fused. When the bodies are subject to normal and tangential loading, K_{tt} is the total tangential stiffness of the two bodies and K_{bt} is the tangential stiffness of the two bodies if they were fused. (Online version in colour.)

The definition of contact stiffness in this current study is more practical. The contact stiffness as defined here can be measured more straightforwardly in a laboratory setting when there are two rough surfaces in contact, since it can be computed from the displacement field near the contact interface, without asperity level measurements. Some authors such as Shi & Polycarpou [13] do not clearly distinguish between bulk and contact stiffness. For instance, Shi & Polycarpou [13] define the contact stiffness for a smooth Hertzian contact as the differential of the relative approach with normal loading. In many cases, particularly with low loads and certain geometries, the effect of bulk deformation may be negligible, and therefore comparability of results is dependent on the loading and geometry. Asperity models of the Greenwood and Williamson kind also tend to define contact displacement as the change in the mean interface gap [13,14]. The contact displacement values from these models should be comparable to the values from the current study, since these models assume the bulk of the material is in practice rigid, and the relative approach is only due to asperity deformation.

3. Contact stiffness measurement procedure

A new rig similar in design to that used in previous studies [26,28] was used in these tests. In this set-up, two flat and rounded pads are in contact with a specimen and are loaded as shown in figure 2. The pad contact surface (the flat portion of the flat and rounded surface) is of width 5 mm (in the direction through the paper in figure 2) and length 4 mm, giving a nominal contact area, A , of 20 mm². The specimen has the same width as the pad (5 mm). The specimen contact surface is 50 mm long and the specimen thickness (in the horizontal direction in figure 2) is 10 mm. Figure 3 shows the experimental set-up. To measure normal contact stiffness, normal load, P , is applied in quasi-static increments onto the pads, by means of a hand-operated hydraulic actuator. For each increment an image of the centre of one contact interface, between one pad and the specimen, is taken. The image has a resolution 1280 by 1024 pixels. Length of each pixel is calibrated in each test and is around 1 μ m. Approximately 1 mm at the centre of the 4mm long interface is therefore photographed. To measure tangential contact stiffness, once the maximum normal load is applied ($P = P_{tan}$), tangential load, $2Q$, is applied to the specimen by a hydraulic single-actuator test machine in quasi-static increments. The set-up ensures the pads are restrained from moving. For each increment of tangential load, an image of the contact interface is also taken. The tangential loading is increased to a maximum value, $2Q_{max}$, and decreased to a minimum value, $-2Q_{max}$,

Figure 2. Specimen in contact with two pads and subject to normal load (P) and tangential load ($2Q$). (Online version in colour.)

cyclically (up to five cycles). The tangential load is then brought to zero and the normal load is incrementally reduced. Images are now taken at each normal unloading increment to measure the unloading normal contact stiffness.

Normal load is logged by measuring the oil pressure applied to pads via a piston of nominal area of 6.5 cm^2 . This is done with a Druck DPI 104 pressure gauge (with stated accuracy of $\pm 0.035\text{ MPa}$ [31]). During tangential loading, the normal load pressure is kept within 1% of the required value. Furthermore, the tangential load applied is measured with a load cell calibrated to be accurate within 1%. DIC with DaVis StrainMaster software (v. 7.2 and 8.3) was used to calculate the displacements at points on both the pad and the specimen for each image corresponding to each load increment. Figure 4 shows an example image of the contact interface between the pad and specimen. A mask is defined over the interface, so displacements are not computed at points next to the interface. Typically, DIC measurements for points right next to the interface are subject to excess noise (the image subsets used in the correlation process will overlap both the pad and specimen regions for these points). In DaVis StrainMaster settings, we specified the cross-correlation method with multiple passes, with ‘window sizes’ (image subset sizes) which correspond to displacement resolutions of the order of 0.01 pixels or better.¹ See [32] for more information about the software. Displacements are initially computed in terms of pixels. These are later multiplied by the pixel length (in micrometre) to provide the physical displacements (the pixel length is determined before each test during calibration). No special patterning of the surface of the pads or specimen were required since the machining marks are sufficient for the DIC process at the optical magnification used.

At each load increment, the DIC displacement values (in both normal and tangential directions) for points on the pad and the specimen with the same ‘ x ’ coordinate are averaged (x is the axis in the direction of normal loading). Figure 5 shows the tangential contact displacement computed for an example tangential load increment. Two lines are fitted onto the displacement versus the x coordinate data, one for the points on the pad and another for points on the specimen. The mask region contains the actual interface. The step change in the displacement at the centre of this region provides the contact displacement value for this frame. It is measured by extrapolating the distance between the fitted lines at the centre. The value of contact displacement thus found is unaffected by the bulk deformation. It is a measure of the additional relative displacement of two points on either side of the interface due to the presence of a rough interface. The slope of the normal load versus normal contact displacement gives the normal contact stiffness, which is found to vary with normal load. Similarly, the slope of the tangential load and tangential contact displacement plot provides the tangential contact stiffness as a function of tangential load.

¹Initial ‘window sizes’ varied from 128×128 pixels to 512×512 pixels. Final ‘window sizes’ varied from 32×32 pixels to 256×256 pixels.

Figure 3. (a) Front view of the rig. (1) A cast-iron block that houses the pads. (2) Hand-operated hydraulic actuators that apply normal load onto the pads. (3) Load cell to measure tangential load. (4) Hydraulic single actuator that applies the tangential load. (b) Close-up of the rig. (1) The specimen. (2) The pads. (c) Side-view showing the camera and lens set-up at the back of the test rig. (1) Questar lens. (2) The cast-iron block. (3) A light source. (4) The camera. (Online version in colour.)

The main disadvantage of this measurement process is that displacements are measured at the outer surfaces of the components, rather than at the centre of the contact area. Furthermore, the measurement may be prone to error due to features such as rounded corners on each body at the interface. The experimental interfaces examined are denoted R1, R2, R3, S1, S2, S3, S4 and S5.

Figure 4. Example image taken of the area near the contact interface. A mask is defined at the centre over the contact area. Pad is on the left and the specimen is on the right. (Online version in colour.)

Figure 5. Contact displacement is found as the gap between the lines that fit the mean displacement data. (Online version in colour.)

R1, R2 and R3 each involved a set of pads in contact with a ground ‘rough’ specimen [with areal root mean square (RMS) roughness S_q of $1.422\ \mu\text{m}$], at different locations. S1, S2 and S3 involved a similar set of pads in contact with a ground ‘smooth’ specimen (S_q of $0.687\ \mu\text{m}$) at different locations. Interfaces S4 and S5 involved sets of pads in contact with different specimens ground to a similar finish as the ‘smooth’ specimen used in the tests with S1, S2 and S3 interfaces.² All the pads were ground to the same specification as the ‘smooth’ specimen. The interfaces were subject to different maximum normal loads, P_{tan} , so the tangential contact stiffness is measured at different normal loads. The maximum nominal normal contact pressure applied was between 200

²The areal roughness values of the specimens and pads used in the S4 and S5 interfaces were not measured.

and 600 MPa. Relatively high loads were used in order to produce relatively large displacements. However, the normal pressure applied is within the range of loading commonly used in fretting fatigue tests and is typical of the normal pressure found in aircraft engine blade roots during operation [33,34].

The areal RMS roughness, S_q , of the pads and the specimens in the R1, R2, R3, S1, S2 and S3 test interfaces was measured before testing, using an optical profilometer (Alicona), using the focus variation method. A cut-off wavelength of 800 μm and a sample area of 2 mm^2 were used to calculate S_q . To characterize the roughness of two surfaces in contact for each interface, we define an equivalent surface for two surfaces in contact. The equivalent surface is defined such that the probability distribution function of the gap between the equivalent surface and a flat surface is the same as that between the two test surfaces. Assuming both surfaces are isotropic and the surface height variation from the mean lines are normally distributed, it is straightforward to show that the roughness of the equivalent surface, S_q^e , is given by $S_q^e = \sqrt{S_{q1}^2 + S_{q2}^2}$, where S_{q1} and S_{q2} are the areal RMS roughness of the two surfaces (i.e. the pad surface and the specimen surface). A similar parameter was derived by Nayak to characterize the roughness of two surfaces in contact in a theoretical model [35]. The results of the contact stiffness measurements are compared with measured values of this parameter for six experimental interfaces.

4. Results

The normal load and normal contact displacement relationship for the R2 interface, with a maximum normal load of 8 kN, is plotted on the left in figure 6 for both normal loading and unloading steps. A model of the following form was seen to fit the data well for all the tested interfaces

$$P = \alpha_n e^{\beta_n u_c} + P_0, \quad (4.1)$$

where α_n , β_n and P_0 are fitting parameters.³ This implies a linear relationship between normal load and normal contact stiffness, given by

$$K_n = \beta_n (P - P_0). \quad (4.2)$$

Plots on the right of figure 6 provide the normal contact stiffness relationship with normal load. Since it is reasonable to assume for a given interface pressure, the contact stiffness will be at least roughly proportional to the nominal contact area, contact stiffness values are normalized with respect to the nominal contact area, A . This is so the results can be easily compared with those obtained in other studies such as [28]. Results are presented here with margin of error bounds, calculated on the basis of uncertainties of the fitted parameters and the load measurement (see appendix A for details). This model fits the normal contact displacement relationship with normal load very well. The adjusted root mean square goodness of fit (GOF) is above 0.99 for most of the experimental interfaces, as shown in table 1. While it is reasonable to expect the normal contact stiffness will tend to zero at zero normal load, as predicted by theoretical models [14,16], the P_0 parameter has a non-zero value. This possibly indicates that some initial load is required in order to overcome a small degree of misalignment between the surfaces.

For some of the interfaces investigated, significant increases in the normal contact stiffness, K_n , were noted between the normal loading and unloading steps of the experiment. The normal unloading step occurs after the normal loading step and five tangential loading cycles. Therefore it is reasonable to expect the normal contact stiffness measured in the normal unloading step to differ from that recorded during the normal loading step, as the interface texture has been modified. Table 2 provides details on the loading applied during the tangential loading steps. During the normal loading step and the repeated cyclic tangential loading, some asperities are likely to have become blunted, making the interface smoother. Table 3 shows the areal roughness of six pad surfaces, measured before and after the tests. It is seen the areal roughness of all six

³Note u_c will involve a zero-offset that affects the α_n parameter but not the contact stiffness. We take u_c as positive when the interfaces move towards each other.

Figure 6. Plots of the normal contact displacement and normal load relationship and the normal contact stiffness and normal load relationship. Plots are for the R2 interface. Top figures are for the loading step and the bottom figures are for the unloading step. (Online version in colour.)

pad surfaces decreased after the tests. (Areal roughness of the S4 pad surface was not measured). This reduction in roughness may account for the increase in normal contact stiffness observed in the normal unloading step, since as per theoretical models [14,16] the normal contact stiffness is inversely related to surface roughness. Permanently blunted asperities may result in a reduced increase in the interface gap during unloading.

The model of Medina *et al.* [14] predicts that the parameter β_n in equation (4.2) should be given by $\beta_n = (\sigma_{\text{rms}})^{-1}$, where σ_{rms} is the standard deviation of the asperity heights of an equivalent surface in contact with a smooth surface. (Campana *et al.* [16] provide a similar expression, in which β_n is in the order of the reciprocal of the RMS surface roughness of the equivalent surface). The equivalent surface roughness parameter, S_q^e , was measured for the six interfaces that were characterized before testing. As discussed above, this is an estimation of the areal RMS roughness of an equivalent surface for the two rough surfaces in contact. As shown in table 4, the product $\beta_n S_q^e$ for these interfaces is close to 1 when β_n values from the normal loading steps are used. This suggests the normal contact stiffness measurements support the Medina model in two respects: (i) the parameter varies approximately inversely with roughness, as predicted by the model and (ii) the magnitude of the parameter also seems to fit with the model predictions. (This is assuming the areal RMS roughness of the equivalent surface is broadly similar to the standard deviation of the asperity heights of the equivalent surface.)

The margin of error values for the results shown in table 1 are generally lower for rough interfaces (R1, R2 and R3) than for the interfaces with smoother profiles (S1, S2 and S3) with lower S_q^e . This is to be expected, since there is a significantly larger change in the normal contact displacement values with normal load in the tests with the rough interfaces (5–12 μm compared with 2–3 μm for the smoother interfaces).

Table 1. Data obtained from the normal contact stiffness measurement. Normal contact stiffness was not measured for the S5 interface.

interface	GOF	P_{tan}/A (kN mm ⁻²)	K_n/A (kN mm ⁻³) at P_{tan}	$\delta K_n/A$ (kN mm ⁻³) at P_{tan} , margin of error	β_n (μm^{-1})	$\delta\tilde{\beta}_n$ (μm^{-1}), fit margin of error	$\tilde{P}_0$ (kN), fit margin of error
loading							
R1	1.000	0.200	94.923	5.558	0.503	0.014	0.225 0.039
R2	0.997	0.400	156.987	16.072	0.403	0.025	0.212 0.190
R3	0.996	0.600	367.343	42.948	0.692	0.051	1.382 0.291
S1	0.998	0.200	152.505	23.427	1.015	0.092	0.994 0.122
S2	0.992	0.400	575.263	123.065	1.457	0.197	0.096 0.509
S3	0.999	0.600	504.917	63.068	0.770	0.056	−1.115 0.509
S4	0.986	0.205	109.998	25.931	0.674	0.111	0.829 0.165
unloading							
R1	0.999	0.200	101.429	11.124	0.536	0.035	0.216 0.097
R2	0.993	0.400	205.145	34.958	0.587	0.069	1.001 0.251
R3	0.996	0.600	405.257	62.130	0.789	0.083	1.715 0.336
S1	0.999	0.200	199.915	41.371	1.389	0.173	1.120 0.171
S2	0.990	0.400	728.181	282.793	1.862	0.463	0.172 0.984
S3	0.997	0.600	1140.553	125.448	2.218	0.155	1.706 0.246
S4	0.981	0.205	224.252	54.520	1.422	0.251	0.940 0.142

Table 2. Loads applied in the tangential load steps and the mean values of K_{Turn}/A and f obtained. Steps with large uncertainty of K_{Turn} value (greater than 75%) are not used to calculate the mean values. GOF are for the steps used. Interfaces R1, R2, R3, S1, S2, S3 and S4 were subject to 10 tangential load steps (five loading and five unloading steps) before the tangential load was brought to zero. For S1 interface tangential contact stiffness was measured only for five of the steps. Interface S5 was subject to nine tangential load steps (five loading and four unloading steps) before tangential load was brought to zero.

interface	GOF	P_{tan}/A (kN mm ⁻²)	Q_{max}/A (kN mm ⁻²)	K_{Turn}/A (kN mm ⁻³)	$\delta K_{\text{Turn}}/A$ (kN mm ⁻³) margin of error	f	δf margin of error	no. of steps used
R1	0.993 to 0.997	0.200	0.025	68.077	9.043	0.313	0.094	9/10
R2	0.982 to 0.998	0.400	0.050	137.029	13.112	0.211	0.042	10/10
R3	0.969 to 0.994	0.600	0.075	338.578	30.231	0.179	0.032	10/10
S1	0.965 to 0.975	0.200	0.025	214.722	53.348	0.140	0.043	3/5
S2	0.981 to 0.996	0.400	0.050	443.078	103.220	0.303	0.097	5/10
S3	0.978 to 0.998	0.600	0.075	470.754	48.715	0.231	0.041	7/10
S4	0.979 to 0.993	0.205	0.025	108.168	10.926	0.144	0.036	10/10
S5	0.857 to 0.971	0.300	0.050	327.767	62.367	0.165	0.049	8/9

Turning to the case of tangential loading, the tangential load and tangential contact displacement relationship, for the R2 interface at a normal load of 8 kN, is given in figure 7. A model of the following form was found to fit the experimental data well:

$$Q = \alpha_t e^{\beta_t v_c} + Q_s. \quad (4.3)$$

Table 3. Roughness of the pad surface before and after the tests for different interfaces.

interface	$S_q(\mu\text{m})$ before test	$S_q(\mu\text{m})$ after test	percent reduction
R1	1.460	1.334	9%
R2	0.822	0.785	4%
R3	0.814	0.742	9%
S1	0.623	0.489	21%
S2	0.760	0.631	17%
S3	0.821	0.704	14%

Table 4. The equivalent roughness parameter, S_q^e , for the interfaces and the $\beta_n S_q^e$ value. The β_n values from the normal loading steps are used. S_q^e is computed from the areal roughness measurements of the pad and specimen surfaces before testing.

interface	equivalent S_q^e (μm)	$\beta_n S_q^e$
R1	2.038	1.03
R2	1.642	0.66
R3	1.638	1.13
S1	0.928	0.94
S2	1.025	1.49
S3	1.070	0.82

Here, α_t , β_t and Q_s are fitting parameters.⁴ A separate curve is fitted to each load step. A load step is defined as a series of load increments while the loading is increased or decreased monotonically (each tangential loading cycle therefore consists of two load steps: a loading step and an unloading step). The above equation implies a linear relationship between tangential contact stiffness and tangential load given by

$$K_t = \beta_t(Q - Q_s), \quad (4.4)$$

where β_t is negative for the loading case and positive for the unloading case. Hence, the stiffness reduces as the load is increased during loading and also reduces when the load is reduced during unloading. This is consistent with the need for the tangential contact stiffness to reduce to zero as slip is approached. There is, of course, a step change in stiffness at each of the two load reversal points. The relationship between tangential contact stiffness and tangential load for the R2 interface is plotted in figure 8 for the first two loading and unloading steps. It will be seen that the relationship is slightly different for the first loading step compared with the other steps. This behaviour was observed also for the other tested interfaces. Figure 9 shows the initial tangential contact stiffness, K_{Turn} , for each load step for the R2 interface. (K_{Turn} is the tangential contact stiffness at the start of each load step, i.e. before the first load increment for the first loading step and at $Q = -Q_{\text{max}}$ for subsequent loading steps and at $Q = Q_{\text{max}}$ for the subsequent unloading steps). Unlike the normal contact stiffness which is observed to increase in the unloading phase, no significant change in the relationship between tangential contact stiffness and tangential load is seen for the tested interfaces after the first loading step, with the exception of the S5 interface. S5 was subject to larger $Q_{\text{max}}/P_{\text{tan}}$ than the other interfaces, and the interface was taken to near sliding in each step. Uniquely for this interface, a pattern of a gradually increasing K_{Turn} with each load step was observed (the increase was however generally within the error margins).

⁴Note v_c will involve a zero-offset that affects the α_t parameter but not the contact stiffness.

Figure 7. Plot of the tangential load and tangential contact displacement relationship with fitted curves for the R2 interface subject to a normal load of $P_{\text{tan}} = 8$ kN. (Online version in colour.)

Figure 8. Plots of the tangential load and tangential contact stiffness relationship obtained for the R2 interface at normal load $P_{\text{tan}} = 8$ kN. (Online version in colour.)

A key implication of equation (4.4) is that Q_s is the tangential load at which the tangential contact stiffness approaches zero, i.e. the load at slip. So for loading steps

$$Q_s = fP_{\text{tan}},$$

where f is the coefficient of friction (for unloading steps $Q_s = -fP_{\text{tan}}$). This implies f can be estimated by

$$f = \left| \frac{Q_s}{P_{\text{tan}}} \right|.$$

Figure 9. The tangential contact stiffness at the beginning of different load steps for the R2 interface with normal load of $P_{tan} = 8$ kN.

Figure 10. Coefficient of friction deduced from the loading and unloading steps for the R2 interface.

Figure 10 shows the coefficient of friction values found in this manner for the R2 interface. The values are all around 0.2, irrespective of whether the data is taken from the loading or the unloading steps. This value is in the expected range for ground Ti-6Al-4V interfaces during the first few cycles of contact.

The model described in equation (4.3) was used to fit the results from all the tested interfaces. Table 2 shows for each test the normal load applied, P_{tan}/A , the maximum tangential load applied, Q_{max}/A , the mean f and the mean tangential contact stiffness at beginning of a load step, K_{Turn} . Note K_{Turn} is the tangential contact stiffness when the asperities can be assumed fully stuck [36, §7.4]. Measurements from some load steps that involved significant uncertainty (greater than 75%) in the computed K_{Turn} value were not used in calculating the mean values of f or K_{Turn} . It should be noted that signs of ratcheting were observed in the test data for S2 and S3 interfaces for some load steps. However, measurements from majority of the load steps support the proposed model with adjusted root mean square goodness of fit generally greater than 0.97.

Figure 11 shows the mean coefficient of friction measured for the different interfaces, that vary from 0.15 to 0.3. Figure 12 shows the ratio K_{Turn}/K_n for different interfaces, using the mean K_{Turn} and loading K_n values. Table 5 shows this ratio when the value of K_n is derived from the normal loading step, normal unloading step and the mean of the two values. Despite the different normal loads and surface roughness values for the different interfaces, the measured K_{Turn}/K_n appears unaffected. This is consistent with models that assume or predict this ratio to be independent of normal loading and surface roughness [14,16]. This also indicates the measured K_{Turn} values are proportional to the normal load and inversely related to the surface roughness, like the measured

Figure 11. Mean coefficient of friction derived for each interface. (Online version in colour.)

Figure 12. K_{Turn}/K_n ratio for each interface at respective P_{tan} . Mean K_{Turn} and the loading K_n values are used. (Online version in colour.)

Table 5. K_{Turn}/K_n ratio for each interface at respective P_{tan} . Mean K_{Turn} is used.

interface	with mean K_n		with loading K_n		with unloading K_n	
	K_{Turn}/K_n	margin of error	K_{Turn}/K_n	margin of error	K_{Turn}/K_n	margin of error
R1	0.693	0.137	0.717	0.137	0.671	0.163
R2	0.757	0.154	0.873	0.173	0.668	0.178
R3	0.876	0.165	0.922	0.190	0.835	0.203
S1	1.219	0.468	1.408	0.566	1.074	0.489
S2	0.680	0.320	0.770	0.344	0.608	0.378
S3	0.572	0.109	0.932	0.213	0.413	0.088
S4	0.647	0.183	0.983	0.331	0.482	0.166

K_n . The theoretical value for the K_{Turn}/K_n ratio according to these models is given by the Mindlin ratio, $2(1 - \nu)/(2 - \nu)$ [14,16]. Taking the Poisson's ratio for Ti-6Al-4V as 0.31 [37], the Mindlin ratio for the tested interfaces is 0.817. The measured K_{Turn}/K_n ratios (particularly when loading K_n is used) are for most interfaces near this value. It should be noted, K_{Turn} values for the first load step for some of the interfaces were however lower than the values for subsequent steps, possibly due to some initial plasticity.

5. Discussion and conclusion

The results presented here show a linear relationship between normal contact stiffness and normal load and a linear relationship between tangential contact stiffness and tangential load for the

Ti-6Al-4V interfaces tested. The normal contact stiffness is also seen to be inversely related to the interface roughness, characterized by the equivalent root mean square roughness of both surfaces.

The results from the normal contact stiffness measurements are important, since they validate several models of normal contact stiffness, particularly the simple model proposed by Medina *et al.* [14]. Furthermore, the linear decrease of tangential contact stiffness with tangential load is a novel observation. Taken together with the results for normal contact stiffness, for the first time normal and tangential contact stiffness values for these interfaces may be estimated straightforwardly for a range of normal and tangential loads on the basis of three parameters: β_n , K_{Turn}/K_n and f .

This result will provide a significant improvement in the accuracy of modelling contact interfaces, particularly for example, in the simulation of the vibration behaviour of assemblies. The results imply that for Ti-6Al-4V ground interfaces, β_n can be satisfactorily estimated to be equal to $1/S_q^e$. Furthermore, it is implied the Mindlin ratio may be a suitable estimate of the K_{Turn}/K_n parameter. In addition, the coefficient of friction is a standard parameter and there are numerous resources to assist in the selection of a suitable value.

Taking $\beta_n \approx 1/S_q^e$ and neglecting the P_0 term in equation (4.2), we get an expression for K_n similar to that provided by the Medina model [14]

$$K_n = \frac{P}{S_q^e}. \quad (5.1)$$

Also, equation (4.4) for tangential loading steps can be written as

$$K_t = K_{\text{Turn}} \left(\frac{fP_{\text{tan}} - Q}{fP_{\text{tan}} - Q_{\text{Turn}}} \right), \quad (5.2)$$

where Q_{Turn} is the starting load of the load step (the tangential load at full stick condition) and P_{tan} is the normal load. Assuming K_{Turn}/K_n is given by the Mindlin ratio and using equation (5.1), we obtain

$$K_t = \frac{2(1-\nu)}{2-\nu} \frac{P_{\text{tan}}}{S_q^e} \left(\frac{fP_{\text{tan}} - Q}{fP_{\text{tan}} - Q_{\text{Turn}}} \right). \quad (5.3)$$

Consistent with Medina *et al.* [14] and the obtained results, the above expression posits the tangential contact stiffness is proportional to normal load and is inversely related to interface roughness. Furthermore, consistent with experimental observations, the tangential contact stiffness reduces with tangential load linearly, reaching zero at sliding.

Our experimental results have provided an important step forward in the understanding of contact behaviour under normal and tangential loading. Our work has concentrated on the first few cycles of loading, and the results will, therefore, be appropriate in cases where surface modification during service is relatively mild. This may well be the case for engineering joints that are essentially static, but subjected to vibration (e.g. flange joints). However, other interfaces may experience surface texture changes after many tangential loading cycles due to plasticity, wear and oxidation. Future investigation of contact stiffness properties should therefore examine the effect of cyclic loading on both normal and tangential contact stiffness. In addition, investigation of different surfaces and different loading conditions is required to extend the applicability of the key findings presented here to a wider range of conditions.

Data accessibility. Data provided on Dryad: <https://doi.org/10.5061/dryad.x0k6djhg7>.

Authors' contributions. K.S.P. designed and conducted the tests, analysed the data and wrote the first draft of the manuscript. R.J.P. assisted with the design and conduct of the tests, provided technical input and helped to revise the manuscript. D.N. supervised the project and helped to revise the manuscript. All authors have approved the final manuscript for publication and have agreed to be accountable for the work.

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Appendix A. Error estimation

Margin of error values are computed on the basis of the uncertainty in the load measurements and the least square error of the fit parameters, with 95% confidence. For quantities that are a function of the fitted parameters and/or the load measurement, the error is calculated using error propagation rules, that depend on whether the errors of the component variables are correlated or can be assumed independent. See [38] for details on error propagation rules.

Normal load, P , is found by measuring the pressure, p , of the oil that pushes a piston against a pad. $P = pA_p$, where A_p is the piston area (6.5 cm^2). The error of measured p can be taken as $\delta p = \delta p_{\text{gauge}} + \delta p_{\text{zero}}$, where δp_{gauge} is the (random) error in the pressure reading and δp_{zero} is the (systematic) zero error.

The stated error of the normal pressure gauge is 0.035 MPa. During normal loading/unloading steps, we add an extra 0.1 MPa uncertainty since the reading fluctuated during increments and logging was done manually (subject to human error). During tangential loading this fluctuation was kept with 1%. Therefore, for normal loading steps $\delta p_{\text{gauge}} = 0.135 \text{ MPa}$, and for tangential loading steps $\delta p_{\text{gauge}} = 0.035 \text{ MPa} + 0.01p_{\text{tan}}$ (p_{tan} is the oil-pressure during tangential loading). We take $\delta p_{\text{zero}} = 0.1 \text{ MPa}$ for all steps. We take $\delta A_p/A_p = 0.01$ (as estimated from the specification sheet).

Similarly, for tangential load measurement, $\delta Q = \delta Q_{\text{gauge}} + \delta Q_{\text{zero}}$. We take $\delta Q_{\text{gauge}} = 0.01Q$ (as per load cell calibration) and estimate $\delta Q_{\text{zero}} = 0.05 \text{ kN}$.

Normal and tangential contact displacement values are found by DIC in pixel-lengths. To obtain values in micrometre, these values are multiplied by C , the length of each pixel in μm , which is measured before each test. We estimate $\delta C/C = 0.02$.

(a) Normal contact stiffness

The expression $P = \alpha_n e^{\beta_n u_c} + P_0$ is equivalent to the following relationship between the measured pressure p and $\hat{u}_c$, the contact displacement in terms of pixel-lengths.

$$p = \alpha_{np} e^{\beta_{nc} \hat{u}_c} + p_0, \quad (\text{A } 1)$$

($\alpha_{np} = \alpha_n/A_p$, $\beta_{nc} = C\beta_n$ and $p_0 = P_0/A_p$).

The fit uncertainties, $\delta \tilde{p}_0$ and $\delta \tilde{\beta}_{nc}$, are due to the *random* errors of p and $\hat{u}_c$ data points. The systematic error component of measured p is δp_{zero} . The effect of δp_{zero} would be to shift the p versus $\hat{u}_c$ curve along the p -axis. This would not affect the β_{nc} parameter, therefore we take total error $\delta \beta_{nc} = \delta \tilde{\beta}_{nc} = C\tilde{\beta}_n$. The effect of δp_{zero} would be to shift p_0 by δp_{zero} . Therefore, total error $\delta p_0 = \delta p_{\text{zero}} + \delta \tilde{p}_0$. (Note $\delta \tilde{p}_0 = \delta \tilde{P}_0/A_p$).

Contact stiffness, K_n in units $\text{kN } \mu\text{m}^{-1}$, is given by $K_n = \beta_{nc}(p - p_0)(A_p/C)$. We do not assume quantities $\delta \beta_{nc}$ and $\delta(p - p_0)$ are independent, and derive

$$\frac{\delta K_n}{K_n} = \sqrt{\left(\frac{\delta \tilde{\beta}_n}{|\beta_n|} + \frac{\delta(p - p_0)}{|p - p_0|} \right)^2 + \left(\frac{\delta A_p}{A_p} \right)^2 + \left(\frac{\delta C}{C} \right)^2}. \quad (\text{A } 2)$$

For K_n at a measured load increment, p , we take $\delta(p - p_0) = \delta p_{\text{gauge}} + \delta \tilde{p}_0$ (the zero errors of p and p_0 cancel out). We did this to compute the margin of error for K_n/A at P_{tan} , shown in [table 1](#).

For [figure 6](#), we require K_n at the true load P (i.e. $\delta P = 0$). Note $K_n = [\beta_{nc}(P - P_0)]/C$. Since $\delta \beta_{nc}$ and δP_0 are independent of δC , we obtain

$$\frac{\delta K_n}{K_n} = \sqrt{\left(\frac{\delta \tilde{\beta}_n}{|\beta_n|} + \frac{\delta(P - P_0)}{|P - P_0|} \right)^2 + \left(\frac{\delta C}{C} \right)^2}. \quad (\text{A } 3)$$

Here, $\delta(P - P_0) = \delta P_0 = P_0 \sqrt{(\delta A_p/A_p)^2 + (\delta p_0/p_0)^2}$.

(b) Tangential contact stiffness

The expression $Q = \alpha_t e^{\beta_{tc} \hat{v}_c} + Q_s$ is equivalent to

$$Q = \alpha_t e^{\beta_{tc} \hat{v}_c} + Q_s, \quad (\text{A } 4)$$

where $\hat{v}_c$ is the tangential contact displacement in terms of pixel-length. ($\beta_{tc} = C\beta_t$). The fit uncertainties, $\delta\tilde{\beta}_{tc}$ and $\delta\tilde{Q}_s$, are due to the random error of Q and $\hat{v}_c$. The effect of δQ_{zero} is to shift the Q versus $\hat{v}_c$ curve along the Q -axis. Therefore, β_{tc} parameter is not affected and the total $\delta\beta_{tc} = \delta\tilde{\beta}_{tc} = C\tilde{\beta}_t$. δQ_{zero} shifts the Q_s parameter so $\delta Q_s = \delta\tilde{Q}_s + \delta Q_{\text{zero}}$.

The tangential contact stiffness (in $\text{kN } \mu\text{m}^{-1}$) is given by $K_t = \beta_{tc}(Q - Q_s)/C$. Since δC is independent of $\delta\beta_{tc}$ and $\delta(Q - Q_s)$

$$\frac{\delta K_t}{K_t} = \sqrt{\left(\frac{\delta\tilde{\beta}_t}{|\beta_t|} + \frac{\delta(Q - Q_s)}{|Q - Q_s|} \right)^2 + \left(\frac{\delta C}{C} \right)^2}. \quad (\text{A } 5)$$

(At true value of Q (i.e. $\delta Q = 0$), $\delta(Q - Q_s) = \delta Q_s$).

K_{Turn} is K_t at Q_{Turn} , when the loading is reversed. So

$$\frac{\delta K_{\text{Turn}}}{|K_{\text{Turn}}|} = \sqrt{\left(\frac{\delta\tilde{\beta}_t}{|\beta_t|} + \frac{\delta(Q_{\text{Turn}} - Q_s)}{|Q_{\text{Turn}} - Q_s|} \right)^2 + \left(\frac{\delta C}{C} \right)^2}. \quad (\text{A } 6)$$

For the first loading step Q_{Turn} is the load before the first tangential load increment. For all other loading steps, we take Q_{Turn} as the largest tangential load increment in the previous loading step. Measured Q_{Turn} is therefore subject to gauge error of $\delta Q_{\text{gauge}} = 0.01 Q_{\text{Turn}}$ and a zero error of $\delta Q_{\text{zero}} = 0.05 \text{ kN}$. We add an additional uncertainty of 0.05 kN to the δQ_{Turn} , in case there was load fluctuation before load reversal. The zero error of Q_{Turn} and Q_s parameters cancel out for $Q_{\text{Turn}} - Q_s$ since it shifts both parameters by the same amount. Therefore, we take $\delta(Q_{\text{Turn}} - Q_s) = 0.01 Q_{\text{Turn}} + 0.05 \text{ kN} + \delta\tilde{Q}_s$.

Since $f = |Q_s/P_{\text{tan}}|$, (not assuming δQ_s and δP_{tan} are independent)

$$\frac{\delta f}{f} = \frac{\delta Q_s}{|Q_s|} + \frac{\delta P_{\text{tan}}}{P_{\text{tan}}}. \quad (\text{A } 7)$$

Here, $\delta P_{\text{tan}}/P_{\text{tan}} = \sqrt{(\delta A_p/A_p)^2 + (\delta p_{\text{tan}}/p_{\text{tan}})^2}$, with $\delta p_{\text{tan}} = \sqrt{\delta p_{\text{gauge}}^2 + \delta p_{\text{zero}}^2}$. (δp_{gauge} and δp_{zero} are independent).

(c) Mean values and K_{Turn}/K_n ratio

K_{TurnP} is the tangential contact stiffness at Q_{Turn} in units $\text{kN (pixel-length)}^{-1}$. ($K_{\text{TurnP}}/C = K_{\text{Turn}}$). δK_{TurnP} from each loading step can be assumed independent to compute the error of the mean $\bar{K}_{\text{TurnP}}$, for each test.⁵ The error of mean $\bar{K}_{\text{Turn}}$ (in $\text{kN } \mu\text{m}^{-1}$) is then given by

$$\delta \bar{K}_{\text{Turn}} = \bar{K}_{\text{Turn}} \sqrt{\left(\frac{\delta \bar{K}_{\text{TurnP}}}{\bar{K}_{\text{TurnP}}} \right)^2 + \left(\frac{\delta C}{C} \right)^2}.$$

Similarly, k_{nP} is the normal contact stiffness in units $\text{kN (pixel-length)}^{-1}$ divided by piston area. ($k_{nP}/C = K_n/A_p$). The error δk_{nP} at P_{tan} from the normal and loading and unloading steps can be assumed independent to compute the error of the mean $\bar{k}_{nP}$ for each interface.⁶ The error of mean

⁵For a tangential loading step:

$$\frac{\delta K_{\text{TurnP}}}{K_{\text{TurnP}}} = \frac{\delta\tilde{\beta}_t}{|\beta_t|} + \frac{\delta(Q_{\text{Turn}} - Q_s)}{(Q_{\text{Turn}} - Q_s)}.$$

⁶For a normal loading/unloading step:

$$\frac{\delta k_{nP}}{k_{nP}} = \frac{\delta\tilde{\beta}_n}{|\beta_n|} + \frac{\delta(p - p_0)}{(p - p_0)}.$$

$\bar{K}_n$ (in $\text{kN } \mu\text{m}^{-1}$) is then given by

$$\delta\bar{K}_n = \bar{K}_n \sqrt{\left(\frac{\delta\bar{K}_{nP}}{\bar{K}_{nP}}\right)^2 + \left(\frac{\delta A_p}{A_p}\right)^2 + \left(\frac{\delta C}{C}\right)^2}. \quad (\text{A } 8)$$

For the error of mean f , errors from each step are propagated not assuming independence. The error of the ratio $\bar{K}_{\text{Turn}}/K_n$ is calculated with

$$\frac{\delta(\bar{K}_{\text{Turn}}/K_n)}{\bar{K}_{\text{Turn}}/K_n} = \frac{\delta\bar{K}_{\text{Turn}}}{\bar{K}_{\text{Turn}}} + \frac{\delta K_n}{K_n}.$$

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