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System identification of proportional solenoid valve dynamics

Bakir Hajdarevic^{1,2}, Jacob Herrmann^{1,2}, Andrea Fonseca da Cruz⁴, David W. Kaczka^{1,2,3,*}

¹Department of Biomedical Engineering, University of Iowa, Iowa City, IA 52242 USA

²Department of Anesthesia, University of Iowa, Iowa City, IA 52242 USA

³Department of Radiology, University of Iowa, Iowa City, IA 52242 USA

⁴Pulmonary Division, Universidade de Sao Paulo, Sao Paulo, SP, Brazil

Abstract

We present a system identification technique for the characterisation of the linearity and dynamic response of a PSOL valve and its corresponding electronic control unit (ECU) using bandlimited white noise, as well as pseudo random “non-sum non-difference” (NSND) waveforms consisting of mutually prime frequencies to mitigate the effects of nonlinear distortions. The parameters of several transfer function models were simultaneously estimated from the voltage-flow frequency response using a nonlinear gradient descent technique. Candidate transfer function models were assessed using the mean squared residual (MSR) criterion and the corrected Akaike information criterion (AICc). The MSR yielded a transfer function consisting of 10 poles and 9 zeros, while the AICc yielded a simpler transfer function consisting of 5 poles and 3 zeros. Monte Carlo analysis demonstrated fragile stability for the MSR-selected model with respect to varying parameter values within estimated uncertainties, yet a robust stability for the AICc-selected model.

Keywords

proportional solenoid valve; mean squared residuals; Akaike information criterion; system identification; transfer functions; model optimization; pneumatic systems

1 Introduction

Proportional solenoid (PSOL) valves are critical mechanical components that regulate pneumatic and hydraulic processes in many industrial and medical systems (Hardwick, 1984; Lua et al., 2001; Tehrani, 2008; Vaughan, and Gamble, 1996). Flow through the valve is typically regulated by a spool inside a cylinder, resulting in partial or complete obstruction of an orifice between a pressurized gas source and its outlet. Position of the spool is controlled using an electrical current applied to a surrounding solenoid coil, generating an electromagnetic force on the spool in opposition to a spring recoil force. The spool position, and thus corresponding obstruction to flow, will vary in proportion to the electrical current applied to the solenoid (Hardwick, 1984; Vaughan, and Gamble, 1996). As flow delivery devices, PSOL valves have advantages of low power consumption, a range of static input

*Corresponding Author david-kaczka@uiowa.edu.

voltage for which the output steady flow is nearly linearly dependent, and fast dynamic response time (i.e., ~20 msec) (Gölcuk, and Guler, 2017; Lua, et al., 2001).

Theoretical approaches for predicting the dynamic response of a PSOL valve often rely on lumped element representations of its electromechanical components, whose ensemble behaviour may be simulated in the time- or frequency-domains (Barth et al., 2003; Taghizadeh et al., 2009; Vaughan, and Gamble, 1996). Such linear approximations may involve deriving the state space representation of the valve dynamics, whereupon feedback and stability can be assessed over varying design parameters (Barth, et al., 2003). The electromagnetic components of a PSOL valve may also be modelled, yielding a predicted frequency response that may be compared to experimental measurements (Taghizadeh, et al., 2009). However various sources of nonlinearities may influence the measured voltage-flow relationship of the valve, most commonly manifesting as harmonic distortions in which output flow energy occurs at non-input frequencies (Zhang et al., 1995).

The most straightforward experimental approach for quantifying the frequency response of a physical system such as a PSOL relies on excitation with a sequence of discrete sinusoids at various frequencies, and measuring the corresponding output amplitudes and relative phase shifts (Johnson, 2000). However such a method can be tedious, especially for applications requiring many frequencies to appropriately describe the dynamic response over a large bandwidth. The use of broadband excitation signals containing several simultaneous frequencies, such as uniform-amplitude ‘white’ noise, may expedite the acquisition process, albeit at the cost of enhancing nonlinear interactions and distortions of the measured output signal (Suki, and Lutchen, 1992). Restricting the harmonic content of the input signal to carefully selected frequencies may mitigate the effects of spectral interaction and harmonic distortion (Kaczka, and Lutchen, 2004; Suki, and Lutchen, 1992). For example, so-called ‘non-sum non-difference’ (NSND) signals provide ideal conditions for estimating the linear relationship between input and output signals in weakly nonlinear systems (Suki, and Lutchen, 1992). Despite expeditious data acquisition and mitigation of nonlinear effects, the use of broadband pseudorandom signals remains underutilized in frequency-domain identification.

The identification of an appropriate transfer function to characterise any physical system requires experimentally acquired data over a sufficient bandwidth, a cohort of candidate models capable of predicting input-output relationships, various model-fitting criteria and metrics, as well as methodology for validating the suitability of each proposed model (Ljung, 1999; Pintelon, and Schoukens, 2008). In this study, we propose a novel and systematic approach for identifying an optimal linear transfer function describing the dynamic response of a PSOL valve. Our approach relies on carefully selected broadband excitations of the system to mitigate the influence of nonlinear distortions on the input-output relationship of the valve. Such excitations may allow for a robust description of the system dynamics with a linear transfer function. Our study builds upon previous work demonstrating the feasibility of using a PSOL valve as a component of a servo-controlled oscillator for respiratory impedance measurements and high-frequency ventilation (Kaczka, and Lutchen, 2004). However, this previous work was limited by *a priori* assumptions of the transfer function description of the PSOL valve. Thus in the current work, emphasis is

placed on the generalisability of linear descriptions and parametric characterisations of the frequency response of the PSOL valve. These optimization criteria are then compared to assess the best candidate model to describe the PSOL dynamics.

2 Methods

Experimental Apparatus

All experimental measurements were obtained using the ASCO Posiflow™ PSOL valve SD8202G027V, which has a maximum orifice diameter of 7 mm and a maximum steady unidirectional flow rate of 4.7 L sec^{-1} for a 10 PSI pressure differential. To control core positioning, ASCO's Electronic Control Unit (ECU) Model 8908A001 converted a 0 to 10 VDC control signal to a 24 V pulse-width-modulated (PWM) coil excitation signal at a frequency of 300 Hz. A 24 VDC power supply (Power-One HC24–2.4-A, Camarillo, CA) was used to power the ECU. The entire experimental apparatus is schematized in Figure 1. An air regulator (Marsh Bellofram Type 70, 960–131-010, Newell, WV) stepped down a standard 50 PSI wall source pressure to a 10 PSI driving pressure for the PSOL. A digital-to-analog (D/A) converter (National Instruments USB-6356, Austin, TX) generated excitation signals to the ECU-PSOL system. The output of the D/A converter was filtered using an 8-pole low-pass Butterworth filter (Frequency Devices 858L8B-2, Haverhill, MA) with a cutoff frequency of 40 Hz. The filtered excitation signal was presented as the input control voltage to the ECU. Output flow from the PSOL valve was measured with a pneumotachograph (Hans Rudolph 4700A, Shawnee, KS) coupled to a 0–2 cmH_2O differential pressure transducer (Celesco LCVR-0002, Canoga Park, CA). The linearity of the pneumotachograph and associated transducer instrumentation was confirmed over the flow range from 0 to 8 L s^{-1} (see Figure A-1 in Appendix). Both the input excitation voltage signal, v , and electrically transduced output flow waveform, $\dot{V}$, were low pass filtered at a cutoff frequency of 40 Hz (Frequency Devices 858L8B-2) and sampled by an analog-to-digital converter (National Instruments USB-6356) at 100 Hz.

Steady State Linearity and Hysteresis of the ECU-PSOL System

The voltage-flow linearity and hysteresis of the combined ECU-PSOL system was determined by analysis of output flow during ascending and descending input voltage excitations (Kaczka, and Lutchen, 2004). The ascending loop of the voltage-flow curve was measured using increasing voltage applied in a step-wise manner from 0 to 10 V in 0.5 V increments, allowing 2 seconds at each step for the system to reach steady-state. The descending loop was measured using decreasing voltage from 10 to 0 V in 0.5 V decrements. Hysteresis was calculated using the difference between the ascending and descending voltage-flow curves at the 5 V operating point (Clegg et al., 2003; Say, and Jones, 2003). For all subsequent dynamic measurements of the ECU-PSOL transfer function, voltage excitation amplitudes were constrained to the most linear region of the system's steady-state voltage-flow relationship.

Dynamic Performance of the Open Loop ECU-PSOL System

Excitation of a physical system with uniform-amplitude white noise should theoretically provide an efficient estimation of the frequency response of the system with high resolution

(Bruce, 2001; Michaelson et al., 1975; Zhang, et al., 1995). For an idealized linear system, only the frequencies present in the input driving signal with nonzero energy will be appearing in the system's output response (Bruce, 2001; Suki, and Lutchen, 1992). However, all physical systems exhibit nonlinear behavior to some degree. Under dynamic conditions, such nonlinearity may manifest itself as energy in the output system response at frequencies other than those presented by the input excitation. Examples of such nonlinear interactions include harmonic distortion and crosstalk (Suki, and Lutchen, 1992), for which energy appears at integer multiples or linear combinations (i.e., sums or differences) of the input frequencies. To mitigate the effects of nonlinear distortions on the apparent frequency response of the open loop ECU-PSOL, pseudorandom 'non-sum non-difference' (NSND) waveforms consisting of multiple simultaneous sinusoids at pre-selected frequencies were used as input driving signals to the system (Suki, and Lutchen, 1992). These NSND waveforms restrict the frequency content to harmonics which are neither multiples of each other, nor equal to the sum or difference of any other harmonics in the series. The specified order L of the NSND waveform implies that none of its frequencies can be obtained as the sum or difference of up to L different component frequencies (Suki, and Lutchen, 1992; Zhang, et al., 1995).

The ECU-PSOL system was excited using band-limited white noise over a bandwidth of 0.01 to 40 Hz, as well as three different 4th order NSND waveforms (Table 1). Three different NSND-4 waveforms were defined, consisting of either 8, 9, or 10 discrete frequency components with uniform amplitudes and random phases. The frequencies in each of the three waveforms were multiples of 0.0976562 Hz, yielding 22 unique frequencies in combination, over a range from 0.195 to 37.4 Hz. The NSND waveforms were initially developed to characterize the frequency response of the respiratory system using a single waveform (Suki, and Lutchen, 1992). However, for this application the use of three different NSND waveforms allowed us to increase the total number of frequencies used in the parameter estimation of the transfer function models as described below. The observed transfer function of the open loop ECU-PSOL system (H_{obs}), consisting of real and imaginary components, was computed from the sampled input voltage and output flow waveforms for the band-limited white noise and the three NSND-4 waveforms using a Welch overlapped-window averaging periodogram technique with a 10.24-second rectangular window and 80% overlap (Welch, 1967). After removing the first 1024 points in the data record (10.24 s) to minimize the influence of transient responses, six overlapping windows were used to calculate transfer function values. The H_{obs} was determined by combining the transfer function values from all three NSND-4 waveforms. Redundant transfer function values (i.e., measured at the same frequency in multiple waveforms), were averaged. Coherence values (γ^2) at each frequency were computed for all waveforms using the appropriate auto- and cross-power spectra (Bendat, and Piersol, 2010). To quantify the nonlinear harmonic distortion present in the output $\dot{V}$ waveforms of the open loop ECU-PSOL system, the dynamic nonlinear harmonic distortion index (k_d) was computed for each of the three NSND-4 excitations (Zhang, et al., 1995):

$$k_d = 100\% \cdot \sqrt{\frac{P_{NI}}{P_{tot}}} \quad (1)$$

where P_{tot} is the total spectral power in $\dot{V}$ (i.e., sum of squared magnitudes in the frequency domain) and P_{NI} is the power in $\dot{V}$ at non-input (i.e., non-NSND) frequencies.

Parametric Transfer Function Model

The dynamic relationship between flow and voltage was characterized by a parametric transfer function model H_{fit} , consisting of polynomials of varying order in the numerator and denominator:

$$H_{\text{fit}}(s) = \Gamma \frac{\prod_{m=1}^M (s - \alpha_m)}{\prod_{n=1}^N (s - \beta_n)} \quad (2)$$

where s denotes the complex angular frequency (i.e., $s = j2\pi f$, where f is expressed in units of Hz and $j = -1$), while Γ , α_m , and β_n denote the gain, zeros, and poles of $H_{\text{fit}}(s)$, respectively. The numbers of zeros and poles in the model are denoted by M and N , respectively. Alternatively, Equation 2 may be expressed in polynomial form as:

$$H_{\text{fit}}(s) = \frac{\sum_{m=0}^M A_m s^m}{s^N + \sum_{n=0}^{N-1} B_n s^n} \quad (3)$$

Where A_m and B_n denote transfer function polynomial coefficients. For each combination of M and N , the parameters A_m and B_n were estimated using a nonlinear gradient search as described below.

Model Optimization

For each combination of M and N , the transfer function parameters A_m and B_n of Equation 3 were obtained using the MATLAB function `tfit` (MATLAB v9.0, The MathWorks Inc., Natick, MA). This function receives as inputs the measurement frequencies, measured transfer function values (H_{obs}), desired number of poles and number of zeroes, and other optional parameters related to model initialization and optimization. The output of `tfit` is a set of parameter estimates for a model of the observed transfer function, of the form in Equation 3. The general procedure for transfer function estimation using this MATLAB routine is shown in Figure 2. Briefly, the initial values of the transfer function parameters of Equation 3 were selected prior to numerical optimization using: 1) the Instrumental Variable method (Garnier et al., 2003; Young, 1985); 2) the State Variable Filter method (Garnier, et al., 2003; Ljung, 1999); 3) the Generalized Poisson Moment method (Bastogne et al., 1997; Garnier, et al., 2003); and 4) the Subspace Estimation Method (Garnier, et al., 2003). Initial parameter values were selected from the initialization algorithm yielding the smallest prediction error between the actual and model-predicted frequency responses. The transfer function parameters were then iteratively adjusted to determine the optimal transfer function parameters for specified values of M and N . Multiple combinations of M and N were defined, with each ranging from 0 to 10. A multistep numerical optimization using Sanathanan-Koerner (S-K) iteration first identified an approximate solution for the model parameters, which was then refined by a nonlinear gradient descent technique (Drma et al., 2015; Sanathanan, and Koerner, 1963). As part of the estimation algorithm, a bilinear

transformation was applied to the transfer function prior to numerical optimization, and then inverted once an optimized model was returned from the refinement process. To ensure stability of the estimated transfer function model, the optimization was constrained such that all poles were located in the left half of the complex plane (i.e., with negative real parts) (Goodwin et al., 2001). Additional information about this implementation can be found in the MATLAB documentation for the function `ttest` (MathWorks, 2017).

The estimated uncertainty for each transfer function parameter of Equation 3 was estimated by the square root of the corresponding diagonal element of the associated covariance matrix (Kaczka et al., 1995). Robustness of transfer function stability to parameter uncertainty was assessed by Monte Carlo simulation, using 10^7 randomly generated transfer functions with parameter values drawn from normal distributions. The mean and standard deviation of each normal distribution was given by the corresponding parameter estimate and uncertainty. Robustness of stability was quantified by the fraction of randomly generated transfer functions exhibiting strictly negative parts of all poles.

Model Comparison

Various combinations of M and N in Equations 2 and 3 were used to describe the ECU-PSOL dynamic responses. Only stable and causal transfer function models were considered for comparison. These criteria mandated that real parts of all poles were negative, and that $M < N$ (Flower, and Parr, 2003; Goodwin, et al., 2001; Johnson, 2000). For each combination of M and N , the residual error R_k between the actual and model-predicted voltage-flow response at each frequency was computed as:

$$R_k = H_{\text{obs}}(s_k) - H_{\text{fit}}(s_k), k = 1, 2, \dots, K \quad (4)$$

where the subscript k indexes each discrete complex angular frequency, and K denotes the number of frequencies used in the estimation. The mean squared residual (MSR) for each model was then defined as:

$$\text{MSR} = \frac{1}{2K} \sum_{k=1}^K \text{Re}\{R_k\}^2 + \text{Im}\{R_k\}^2 \quad (5)$$

where $\text{Re}\{\}$ and $\text{Im}\{\}$ denote the real and imaginary components of the enclosed arguments, respectively. The MSR quantifies ‘goodness-of-fit’ according to the difference between the observed and model-predicted frequency responses over the bandwidth of interest (Draper, and Smith, 1998). However, the MSR criterion favors models with greater numbers of parameters, which allows for prediction of additional variability in the observed data, albeit at the expense of increased model complexity (Bermejo, and Cabestany, 2001). The Akaike Information Criterion (AIC) is an alternative metric that accounts for both reductions in the MSR and increases in the number of model parameters (Akaike, 1974):

$$\text{AIC} = 2K \ln(\text{MSR}) + 2(M + N + 1) \quad (6)$$

The value of the AIC will thus decrease with reductions in MSR, but will increase with additional model parameters. Model selection based on the lowest AIC thus results in a

compromise between the MSR and model complexity. For a small sample size (i.e., a small number of frequencies), there is a greater likelihood that AIC, as estimated according to Equation 6, will favor overly- complex models (Caby et al., 2000; Claeskens, and Hjort, 2008; Giraud, 2014). In such circumstances, the corrected AIC (AICc) may be used to apply a greater penalty for additional model parameters (Burnham, and Anderson, 2002; Cavanaugh, 1997):

$$\text{AICc} = \text{AIC} + 2(M + N) \left(\frac{M + N + 1}{2K - (M + N + 1)} \right) \quad (7)$$

Thus as K becomes large in Equation 7, the extra penalty term will tend towards zero, and the AICc will converge to the AIC (Burnham, and Anderson, 2004).

3 Results

Figure 3 shows the steady-state voltage-flow curves for both increasing and decreasing control voltages. Exciting the ECU-PSOL over the control voltage range of 0–10 V produces minimum and maximum flows of 0 and 4.70 L/s, respectively. The calculated hysteresis for the open loop ECU-PSOL system was approximately 18%, which was larger than the corresponding value for the different ECU-PSOL system used in our previous study (Kaczka, and Lutchen, 2004). Both the increasing and decreasing voltage limbs were sigmoidal but demonstrated approximately linear operation for input voltages between 2 to 8 V, with a slope of $0.5632 \text{ L s}^{-1} \text{ V}^{-1}$. Thus, all subsequent dynamic measurements were confined to the control voltage range of 2.5 to 7.5 V, as indicated by the gray area in Figure 3.

Figure 4 illustrates the differences of using an example 4th order NSND waveform (NSND-4) in comparison to band-limited white noise, both in time- and frequency-domain representations. In the time-domain, both signals appear as “noisy” waveforms. However, in the frequency- domain, the NSND-4 signal expresses concentrated signal energy at fewer frequencies over the same bandwidth.

Figure 5 shows γ^2 vs. frequency for the uniform-amplitude white noise, as well as all three NSND-4 waveforms. By contrast, γ^2 values for uniform amplitude white noise were as low as 0.6324 over the frequency range 0.1 to 40 Hz, averaging $\gamma^2 = 0.9889 \pm 0.0001$. Values of γ^2 were greater than 0.9994 for all NSND-4 frequencies, suggesting minimal influence of nonlinear distortions on the spectral estimates of H_{obs} at those frequencies (Maki, 1986). The k_d computed for the output flow for each of the three waveforms was as (NSND 4–1) 12.43%, (NSND 4–2) 7.00%, and (NSND 4–3) 11.13%, yielding an average k_d of 10.2%. Figure 6 shows the results of model comparison using MSR and AICc for varying number of poles and zeros. The asterisks in this figure denote the best transfer function selected by either AICc or MSR. The transfer function minimizing MSR consisted of 10 poles and 9 zeros, whereas the transfer function minimizing AICc consisted of only 5 poles and 3 zeros, achieving comparable quality of fit with fewer parameters.

The values of the best-fit transfer function coefficients and uncertainties are shown in Table 2. Note that the model selected by MSR exhibits a wide range of parameter uncertainties (4

to 118%) compared to the model selected by AICc (24 to 55%). Out of 10^7 Monte Carlo simulations using transfer functions with parameter values randomly sampled from normal distributions, transfer function stability was found in only 0.5% of models sampled using the MSR-selected model parameter estimates and uncertainties. By contrast, 97.5% of transfer functions sampled using the AICc-selected model parameter estimates and uncertainties were stable. Figure 7 shows the resulting frequency response from exciting the open loop ECU-PSOL system with the three distinct NSND waveforms, along with that obtained from the white noise simulations. Also shown in this figure are the corresponding predictions from best-fit transfer function models (i.e., MSR and AICc). The models selected by MSR and AICc are qualitatively similar, with multiple appreciable discrepancies between 8 to 40 Hz, emphasized in Figure 8, wherein the MSR-selected model exhibits deviations from the otherwise smooth curve between measured NSND points.

4 Discussion

In this study, we have presented a method for estimating the linear dynamic transfer function of a proportional solenoid valve system. To minimize the influence of nonlinearities on measurements of the voltage-flow relationship, broadband excitation was applied using NSND input signals with oscillations in absolute voltage limited to the linear operating range of the valve. We used a nonlinear gradient descent algorithm suited for transfer function estimation using 22 NSND frequencies, while also applying a constraint that all poles had to be on the left side of the real and imaginary plane. Only strictly proper transfer function models were considered when using both the MSR and AICc to compare various transfer function candidate models of the voltage-flow frequency response. Based on the minimum MSR, a 10-pole, 9-zero transfer function was the most appropriate model to describe the PSOL dynamics. However, based on the AICc criterion, a 5-pole, 3-zero transfer function was the most appropriate model to characterize the PSOL dynamic response.

Quantifying Nonlinearities

The performance of the ECU-PSOL system revealed both steady and dynamic nonlinearities in its voltage-flow relationship, with a hysteresis of about 18%, and an average k_d of 10.2%. The observed k_d may be attributed to nonlinearities in the ECU-PSOL and/or turbulent gas flow phenomena (Kaczka, and Lutchen, 2004). The hysteresis of the PSOL valve is most likely due to the magnetic flux density and magnetic field intensity, and losses in the iron core (Clegg, et al., 2003; Say, and Jones, 2003; Vaughan, and Gamble, 1996). During magnetization a flux density produced by a given magnetic field intensity depends on the previous magnetization history thus creating the effect of hysteresis. The area of the hysteresis loop is a measure of the energy dissipation across the range of increasing and decreasing input voltages (Say, and Jones, 2003), to changing cycles of magnetic flux density (Clegg, et al., 2003). In proportional solenoid valves, factors such as the electromagnetic force generated by the solenoid, friction of the solenoid plunger, and the pressure drop across valve openings will impact the positioning accuracy of the spool and overall hysteresis of the valve (Canuto, and Acuña-Bravo, 2013). In addition to a higher flow coefficient, differences in these factors may explain why the valve used in this study

exhibited greater hysteresis compared to the valve used in our previous study (Fisher, 2017; Kaczka, and Lutchen, 2004).

Due to the sigmoid steady-state relationship between voltage and flow, broadband NSND excitations were restricted to an absolute voltage range of 2.5 to 7.5 V, to mitigate the effects of saturation nonlinearities (Suki, and Lutchen, 1992). The benefit of using NSND excitations for linear systems identification, as compared to uniform-amplitude noise, is apparent in Figure 7. The frequency response measured during excitation with uniform-amplitude noise was erratic from one frequency to the next, whereas the frequency responses during excitation with any of the three NSND-4 waveforms varied smoothly over the measured bandwidth and were consistent across the waveforms. Despite these efforts to obtain a linear voltage-flow frequency response from the ECU-PSOL system, the computed harmonic distortion index k_d indicated that approximately 10.2% of the total output signal energy occurred at frequencies not present in the input signal.

Model Comparison

Model optimization was subjected to a stability constraint, such that all poles were required to have negative real parts (Flower, and Parr, 2003; Goodwin, et al., 2001). Furthermore, only strictly proper transfer functions were considered during model selection, requiring models to contain fewer zeroes than poles, in order to exclude nonphysical systems with potentially unbounded output responses at frequencies increasing towards infinity (Johnson, 2000). Model comparison revealed different trends in MSR and AICc for varying model complexity. The rate of improvement in MSR with increasing number of poles and/or zeroes slows greatly for $N > 3$ or $M > 0$. As a result, the benefit of additional poles or zeroes decreases in this region of the parameter space. Thus, the AICc exhibits a local minimum at $N = 5$ and $M = 3$, where after increasing model complexity produces only minor reductions in MSR. The best fit transfer function based on the MSR criterion consisted of 10 poles and 9 zeros but exhibited undesirable irregularities in the frequency response shown in Figure 8. These nonsmooth irregularities may be attributed to overfitting of the measured transfer function values. No such irregularities were exhibited by the model selected by AICc.

Differences in parameter uncertainties between the models selected by MSR and AICc may reflect the robustness of model estimation to variations in the measured data. The MSR-selected model exhibits a wide range of parameter uncertainties (4 to 118%), suggesting that parameter estimation was extremely sensitive to changes in some transfer function coefficients with low uncertainties, yet almost entirely unaffected by changes in other coefficients. In contrast, the AICc-selected model exhibits an internally consistent range of parameter uncertainties (24 to 55%), indicating greater robustness of fit and less variability in parameter sensitivity. Monte Carlo simulations demonstrated that transfer function stability was robust to parameter uncertainty for the AICc-selected model (97.5% of simulations), but not for the MSR-selected model (0.5% of simulations).

Although higher-order transfer function models may capture more variability in the observed data, such variability may arise from random noise sources, nonlinearities, or other deterministic artifacts. For example, subtle differences in transfer function values among the different NSND waveforms are apparent in Figure 7. The combined frequency response H_{obs}

obtained from the three NSND-4 waveforms still contained minor fluctuations across frequencies, reflecting cross-measurement variability of the open loop ECU-PSOL system. Nonetheless, the benefits of reduced measurement time and enhanced spectral resolution for model fitting justify the combination of multiple NSND measurements. Furthermore, this cross-measurement variability was relatively minor in comparison to variability in the measured transfer function during uniform-amplitude white noise excitation. The use of AICc is justified to compromise between accurate system characterization and precise prediction of observed variability.

In comparison to modelling and identification approaches that are limited to linear transfer functions emanating from the *a priori* arrangement of lumped elements embodying energy storage and dissipation (Kabib et al., 2016), our technique is more mathematically generalizable, albeit perhaps devoid of physical description. Nonetheless, such generalizability allows for the characterization of system dynamics that may require additional poles and zeros to describe the frequency response of a physical ECU-PSOL system.

Limitations

Linear parametric transfer functions cannot characterize nonlinearities in the voltage-flow relationship produced by hysteresis and saturation. Therefore, the derived transfer function approximates the ECU-PSOL response over a limited range of input voltages that correspond to the linear portion of the sigmoidal curve shown in Figure 3. Our proposed method may be used to obtain piecewise linear approximations to the system response in nonlinear operating ranges, by applying small-amplitude excitation signals at multiple operating points. Multiple transfer functions could be beneficial for implementing a gain-scheduling control scheme in which various operating ranges are considered (Slotine, and Li, 2005).

In addition, inconsistencies in the general trend of decreasing MSR with increasing number of parameters may be attributed in part to the stability constraint. The method of ensuring stability by reflecting poles across the imaginary axis may result in substantial changes to the transfer function quality of fit, especially after optimizing the parameters without regard for stability. Additional methods for improving this system identification technique, subject to stability constraints, were not explored in this study.

Finally for the sake of simplicity, we did not use additional model criteria as part of our transfer function model selection. Furthermore, a common alternative criterion, the Bayesian Information Criterion (BIC), penalizes even greater than the AIC and will often select a less complex model (Burnham, and Anderson, 2002; Burnham, and Anderson, 2004). However, when the sample size is small, the BIC selected model will likely underfit the data (Burnham, and Anderson, 2002; Burnham, and Anderson, 2004). Thus, for our study using the BIC would not be beneficial for determining the best model. Future work may investigate how the results may change when the sample size of NSND frequencies increases by ten-fold.

5 Conclusion

In this study, we present a novel system identification technique to characterize the voltage-flow frequency response of a pulse-width modulated proportional solenoid valve and its electronic control unit. Our technique relies on the use of pseudorandom “non-sum-non-difference” excitations of the signal, along with generalized linear transfer function approximations based on mean squared residuals and the corrected Akaike Information Criterion. The proposed technique for transfer function estimation expedites the data acquisition process by utilizing broadband excitation signals to drive the system and enable measurement of its dynamic response at several frequencies simultaneously, while mitigating distortions from nonlinearities (i.e., harmonic distortion and hysteresis). The dynamic frequency response of the valve was adequately described using a linear transfer function with 10 poles and 9 zeros based on minimizing mean squared residuals, or 5 poles and 3 zeros based on minimizing AICc. Uncertainty analysis using Monte Carlo simulation demonstrated fragile stability for the transfer function based on mean squared residuals with respect to varying parameter values within estimated uncertainties, yet a robust stability for the transfer function based on AICc. Our proposed methodology for transfer function approximations may be used to simulate the behavior of an ECU-PSOL system in a closed loop configuration with adaptive or fuzzy control (Ben Halima Abid, and Chtourou, 2019; Habrache et al., 2019; Shuo et al., 2019; Wu et al., 2018), or extended to applications involving pneumatic or hydraulic processes.

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Appendix

Flow Transducer Linearity

The device used in this study to measure output flow rates from the proportional solenoid valve was the Hans Rudolph HR 4700A screen pneumotachograph coupled to a Celesco ± 2 cmH₂O differential pressure transducer. Although the calibrated flow range listed in the manufacturer-provided data sheet indicates linear operation of the device for flow rates up to ± 2.7 L s⁻¹, we investigated the ability of the flow measurement apparatus to maintain a linear flow-voltage relationship up to ± 6.0 L s⁻¹.

The pneumotach was connected to the outlet of a calibrated 3.0 L super-syringe. Transducer voltage was tuned to zero under zero-flow conditions. Transducer voltage output was low-

pass filtered using a 30 Hz cutoff frequency before being sampled at 800 Hz using an analog-to-digital converter. A series of approximately 400 manually driven pushes and pulls from the syringe were recorded, with push/pull durations varying between 0.4 to 4.0 seconds. Constant plunger speed was attempted within each push and pull of the syringe, resulting in average flow rates varying between 0.75 to 8.0 L s⁻¹ (Figure A-1).

Individual pushes and pulls were delineated in the recorded data by zero-crossings of the flow transducer signal. The average transducer voltage during each syringe push/pull was compared to the average actual flow rate, as estimated by the delivered syringe volume, divided by the duration of the push/pull. Linear and quadratic models of the flow-voltage relationship were regressed over the range of positive flows (i.e., unidirectional) between 0.0 to 4.7 L s⁻¹, according to the average flow rate. Each regressed model was constrained with a zero y-intercept.

Figure A1.

Average flow rates and average transducer voltages during a series of 3.0 L syringe pushes (circles) and pulls (triangles) through pneumotach apparatus. Vertical dotted line indicates upper bound of manufacturer-specified calibrated flow range for the pneumotach. Solid and dashed lines correspond to linear and quadratic regression, respectively, over the range 0.75 to 4.7 L s⁻¹, with constrained zero intercept. Coefficients of determination (r^2) were 0.9992 for linear regression and 0.9998 for quadratic regression.

Biography

Biographical notes: Bakir Hajdarevic received the B.S.E. and M.S. degrees in biomedical engineering from the University of Iowa, IA, USA, in 2016 and 2018, respectively. He was a Graduate Research Assistant with the Department of Anesthesia, University of Iowa Hospitals and Clinics, Iowa City, IA.

Jacob Herrmann received the B.S. (*magna cum laude*) and M.S. degrees in biomedical engineering from Boston University, Boston, MA, USA, in 2012 and 2015, respectively, and the Ph.D. degree in biomedical engineering from the University of Iowa, Iowa City, IA, USA, in 2018. He is currently a post-doctoral fellow at Boston University, Boston, MA.

Andrea Fonseca da Cruz received the M.S. degree in biomedical engineering from the Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, Brazil, in 2006 and the Ph.D. degree in science from Universidade de São Paulo, São Paulo, SP, Brazil, in 2016. She was a visiting research scholar at the University of Iowa, Iowa City, IA, USA in 2015. She is currently a post-doctoral fellow at the University of Iowa Hospitals and Clinics, Iowa City, IA.

David W. Kaczka received the B.S. (*summa cum laude*), M.S., and Ph.D. degrees in biomedical engineering from Boston University College of Engineering in 1990, 1993, and 2000, respectively, and the M.D. degree from the Boston University School of Medicine in 2000. He is an Associate Professor of Anesthesia, Biomedical Engineering and Radiology at the University of Iowa, Iowa City, IA.

Nomenclature

A_m	numerator coefficient of parametric transfer function
AIC	Akaike Information Criterion
AICc	corrected AIC
BIC	Bayesian Information Criterion
B_n	denominator coefficient of parametric transfer function
ECU	electronic control unit
H_{obs}	observed transfer function
H_{fit}	transfer function fit
k_d	harmonic distortion index
K	total number of frequencies
M	number of zeros
MSR	mean squared residuals
N	number of poles

L	order of an NSND waveform
P_{NI}	flow power at non-NSND frequencies
PSOL	proportional solenoid valve
P_{tot}	total flow power in signal
R_k	residual between actual and model-predicted values of transfer function at discrete frequency k
SE	standard error
$\dot{V}$	Flow output from ECU-PSOL system
v	Voltage input to ECU-PSOL system
α_m	zero of transfer function
β_n	pole of transfer function
γ^2	coherence value
Γ	gain of transfer function fit
Im	imaginary part
Re	real part

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Figure 1.

Block diagram of the data acquisition set up. A: Bellofram Type 70 pressure regulator, B: ECU-PSOL valve, C: Hans Rudolph pneumotachometer, D: Celesco differential pressure transducer, CDM: carrier demodulator, LPF: 8th order Butterworth low-pass filter.

Figure 2.

Flow chart of the algorithm implemented to estimate optimal transfer function models for various numbers of poles (M) and zeros (N), using the MATLAB function `tfest`.

Figure 3.

Steady-state voltage-flow curves of combined ECU-PSOL system for increasing (gray arrowed line) and decreasing control voltages (black arrowed line). The hysteresis curve represents the result of a single cycle. The shaded area corresponds to the range of control voltages in which the system behavior is approximately linear.

Figure 4.

Time-domain (left panels) and frequency-domain (right panels) representations of the output flow from the ECU-PSOL system during white noise (a, b) and NSND-4 (c, d) excitations. Flow power spectra (b, d) were computed using a Welch periodogram technique with a window length of 10.24 s. Both signals were low-pass filtered using a 40 Hz cutoff frequency.

Figure 5.

Coherence spectra during excitation of the ECU-PSOL system with uniform-amplitude white noise and each of the three 4th order non-sum non-difference waveforms. Black bars correspond to the frequencies used for computing transfer function values, whereas gray bars represent all other unused frequencies, including those above the 40 Hz low-pass filter cutoff frequency.

Figure 6.

Transfer function assessment based on (a) MSR and (b) AICc for varying numbers of poles and zeros. Color indicates model goodness-of-fit assessed by AICc or MSR. All models were required to exhibit stability, i.e., containing no poles with positive real parts. Model comparison was restricted to exclude transfer functions containing equal or more zeros than poles, as indicated by the dotted lines and translucent region. Asterisks indicate the transfer functions minimizing MSR or AICc within the applied constraints.

Figure 7.

Frequency response of the ECU-PSOL excited by band-limited white noise (dots) and 4th order non-sum non-difference waveforms (circles: NSND 4-1, squares: NSND 4-2, and diamonds: NSND 4-3). The best fit transfer functions for MSR (solid black line) and AICc (dashed gray line) are also shown. Complex-valued frequency responses are reported as magnitude (a) and phase (b).

Figure 8.

Detailed views of transfer function model over-fitting occurring over limited frequency ranges for the MSR-selected model (solid line) compared to the AICc-selected model (dashed line). Refer to Figure 7 for an overall view of the entire frequency range (0.2 to 40 Hz) as well as symbol definitions. Complex-valued frequency responses are reported as magnitude (a, c, e) and phase (b, d, f).

Table 1

List of the fundamental frequencies (f_0) and the number sequences representing frequency numbers used to generate the NSND 4 signals.

Waveform	f_0 (Hz)	Frequency Numbers
NSND 4-1	0.0976562	2, 5, 19, 37, 83, 151, 179, 331
NSND 4-2	0.0976562	3, 5, 17, 41, 59, 89, 139, 167, 313
NSND 4-3	0.0976562	2, 7, 17, 37, 97, 137, 223, 317, 383

Table 2

Parameter values for polynomial coefficients in descending order for the best fit transfer functions using the model criterions of MSR and AICc and their associated uncertainties (i.e., standard errors, SE). The uncertainty measurements are reported as the square root of the diagonal of the covariance matrix and the percentage of each transfer function coefficient, respectively.

MSR		
Coefficients	Uncertainty ($\pm$ SE)	Uncertainty ($\pm$ %)
$A_9 = -1.502 \cdot 10^1$	$1.919 \cdot 10^0$	12.8
$A_8 = 4.518 \cdot 10^3$	$2.238 \cdot 10^2$	5.0
$A_7 = -5.056 \cdot 10^5$	$9.810 \cdot 10^4$	19.4
$A_6 = 2.305 \cdot 10^8$	$1.315 \cdot 10^7$	5.7
$A_5 = 1.975 \cdot 10^9$	$2.329 \cdot 10^9$	117.9
$A_4 = 2.252 \cdot 10^{12}$	$1.549 \cdot 10^{11}$	6.9
$A_3 = 6.021 \cdot 10^{13}$	$2.380 \cdot 10^{13}$	39.5
$A_2 = 5.356 \cdot 10^{15}$	$5.484 \cdot 10^{14}$	10.2
$A_1 = 1.410 \cdot 10^{17}$	$4.878 \cdot 10^{16}$	34.6
$A_0 = 2.149 \cdot 10^{18}$	$7.439 \cdot 10^{17}$	34.6
$B_9 = 1.810 \cdot 10^2$	$2.001 \cdot 10^1$	11.1
$B_8 = 6.422 \cdot 10^4$	$2.678 \cdot 10^3$	4.2
$B_7 = 8.922 \cdot 10^6$	$1.171 \cdot 10^6$	13.1
$B_6 = 1.154 \cdot 10^9$	$1.047 \cdot 10^8$	9.1
$B_5 = 9.686 \cdot 10^{10}$	$1.585 \cdot 10^{10}$	16.4
$B_4 = 7.371 \cdot 10^{12}$	$9.509 \cdot 10^{11}$	12.9
$B_3 = 3.253 \cdot 10^{14}$	$7.797 \cdot 10^{13}$	24.0
$B_2 = 1.409 \cdot 10^{16}$	$2.280 \cdot 10^{15}$	16.2
$B_1 = 3.342 \cdot 10^{17}$	$1.178 \cdot 10^{17}$	35.2
$B_0 = 3.369 \cdot 10^{18}$	$1.164 \cdot 10^{18}$	34.5
AICc		
Coefficients	Uncertainty ($\pm$ SE)	Uncertainty ($\pm$ %)
$A_3 = -5.738 \cdot 10^3$	$3.237 \cdot 10^3$	56.4
$A_2 = 2.327 \cdot 10^6$	$8.603 \cdot 10^5$	37.0
$A_1 = 1.864 \cdot 10^8$	$5.224 \cdot 10^7$	28.0
$A_0 = 2.919 \cdot 10^9$	$7.052 \cdot 10^8$	24.2
$B_4 = 5.618 \cdot 10^2$	$1.744 \cdot 10^2$	31.0
$B_3 = 1.248 \cdot 10^5$	$3.111 \cdot 10^4$	24.9

$B_2 = 1.016 \cdot 10^7$	$2.717 \cdot 10^6$	26.7
$B_1 = 4.519 \cdot 10^8$	$1.197 \cdot 10^8$	26.5
$B_0 = 4.564 \cdot 10^9$	$1.113 \cdot 10^9$	24.4

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