Published in final edited form as:

IEEE Trans Med Imaging. 2020 October ; 39(10): 2996–3007. doi:10.1109/TMI.2020.2983414.

Spectral CT Reconstruction via Low-rank Representation and Region-specific Texture Preserving Markov Random Field Regularization

Yongyi Shi,

Department of Radiology, State University of New York at Stony Brook, Stony Brook, NY 11794, USA, and is also with the Institute of Image Processing and Pattern Recognition, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, China.

Yongfeng Gao,

Department of Radiology, State University of New York at Stony Brook, Stony Brook, NY 11794, USA.

Yanbo Zhang,

PingAn Technology, US Research Lab, Palo Alto, CA 94306, USA.

Junqi Sun,

Department of Radiology, State University of New York at Stony Brook, Stony Brook, NY 11794, USA, and is also with the Department of Radiology, Yuebei People's Hospital, Shaoguan, Guangdong, 710049, China.

Xuanqin Mou,

Institute of Image Processing and Pattern Recognition, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, China

Zhengrong Liang [Fellow, IEEE]

Departments of Radiology, Electrical and Computer Engineering, Computer Science and Biomedical Engineering, State University of New York at Stony Brook, Stony Brook, NY 11794, USA

Abstract

Photon-counting spectral computed tomography (CT) is capable of material characterization and can improve diagnostic performance over traditional clinical CT. However, it suffers from photon count starving for each individual energy channel which may cause severe artifacts in the reconstructed images. Furthermore, since the images in different energy channels describe the same object, there are high correlations among different channels. To make full use of the inter-channel correlations and minimize the count starving effect while maintaining clinically meaningful texture information, this paper combines a region-specific texture model with a low-rank correlation descriptor as an *a priori* regularization to explore a superior texture preserving Bayesian reconstruction of spectral CT. Specifically, the inter-channel correlations are characterized by the low-rank representation, and the inner-channel regional textures are modeled

by a texture preserving Markov random field. In other words, this paper integrates the spectral and spatial information into a unified Bayesian reconstruction framework. The widely-used Split-Bregman algorithm is employed to minimize the objective function because of the non-differentiable property of the low-rank representation. To evaluate the tissue texture preserving performance of the proposed method for each channel, three references are built for comparison: one is the traditional CT image from energy integration detection. The second one is spectral images from dual-energy CT. The third one is individual channels images from custom-made photon-counting spectral CT. As expected, the proposed method produced promising results in terms of not only preserving texture features but also suppressing image noise in each channel, comparing to existing methods of total variation (TV), low-rank TV and tensor dictionary learning, by both visual inspection and quantitative indexes of root mean square error, peak signal to noise ratio, structural similarity and feature similarity.

Keywords

spectral CT reconstruction; low-rank representation; texture preservation

I. INTRODUCTION

THE spectral computed tomography (CT) can significantly improve diagnostic performance over traditional CT techniques because of its capabilities for tissue characterization [1], differential diagnosis [2] and material decomposition [3], etc. One special category of spectral CT techniques is dual-energy CT (DECT). For DECT category, there are mainly three clinically and commercially available types, such as dual-source CT (e.g. Somatom Definition, Siemens) [4], fast kVp switching technique (e.g. Discovery CT750, GE) [5], and dual-layer sandwich detector technique (e.g. Brilliance-64, Philips) [6]. These available DECT systems have only two energy channels and may suffer beam-hardening effect. They also have limited spectral resolution [7, 8]. Recently, photon-counting detectors (PCDs) [9] have been developed for spectral CT, which have multiple energy thresholds/bins (i.e. potentially more than two) and energy discriminating capabilities. Hence, PCDs have capable for energy discrimination and can eliminate electronic background noise [10], which has certain superiority compared with energy integral detector of clinical DECT [11]. Benefit from the PCDs techniques, photon-counting spectral CT (PCCT) is capable for poly material decomposition and can improve signal-to-noise ratio (SNR) in the reconstructed images. However, PCCT may be subjected to photon count starving for individual energy channel. The image quality reconstructed directly by conventional filtered back-projection (FBP) method can be degraded dramatically due to the count starving effect or excessive quantum noise. Therefore, developing a superior reconstruction method to achieve high image quality for each energy channel is extremely important to translate PCCT toward clinical utility, when reconstructing all energy channels jointly.

In previous decades, iterative reconstruction methods, as an alternative to the FBP reconstruction, have shown the potential to improve the image quality of traditional CT. These approaches combine the statistical noise properties of the measurements and the image spatial priors into the reconstruction [12, 13]. Recently, several spatial priors such as

total variation (TV) [14], structure tensor TV [15], tight frame (TF) [16] and dictionary learning [17–19], have been developed for PCCT reconstruction. However, all the aforementioned methods only regard the observations in the inner-channel spatial domain, where the priors are designed solely to address the excessive quantum noise in each individual energy channel, and neglect the inter-channel correlation among the individual energy channels. Obviously, the images in the individual energy channels share the same object structure, i.e. there is a strong synergy among these individual energy channels. Hence, consideration of the inter-channel correlation is desired.

The low-rank prior information is one of the major constraints to encourage the inter-channel correlation among the individual energy channels. Up to now, several methods incorporate the low-rank representation into PCCT reconstruction. In our opinion, all these efforts can be divided into two categories. One category is the patch-based methods which extracts three-dimensional patches from the multichannel image and then considers the sparsity and low-rank property in the small patches, such as patch-based low-rank penalty [20], nonlocal low-rank and sparse matrix decomposition [21], tensor-based dictionary learning [22, 23] and nonlocal low-rank cube-based tensor factorization [24]. The other category applies the low-rank representation to take advantage of the global spectral information and then combine some spatial priors to make use of the spatial information. For example, Gao et al. proposed a PRISM (prior rank, intensity, and sparsity model) for PCCT reconstruction [25]. Thereafter, Liang et al. improve the PRISM by introducing a new conception of tensor and unfolding operation [26]. Chu et al. combined the TV and low rank regularization to further improve the quality of the spectral images [27]. However, the central theme of these developed methods is based on the assumption that the image spatial prior knowledge shall be piece-wise smooth. Consequently, the reconstructed images may sacrifice the tissue region image texture characteristics, which have been shown as a useful imaging marker for many clinical tasks [28, 29]. Hence, it is necessary to investigate superior spatial prior to preserve the texture features, in addition to the inter-channel correlation, for PCCT reconstruction.

As an extensive used spatial prior, the Markov random field (MRF), has been reported proficient in preserving edge sharpness while suppressing noise in traditional CT. Since the simple spatially-invariant MRF did not explicitly consider the non-homogeneity for CT images, more recently, we proposed a texture preserving MRF prior (TP) to enhance the low dose CT (LDCT) image via learning region-specific textures, such as muscle, fat, bone and lung, from the same patient's previous full dose CT (FDCT) scans [30, 31]. On the basis of that, we further investigated the feasibility of learning the region-specific textures from a FDCT database [32], which remove the constraint of using the FDCT scans from the same patient.

In this paper, we combine both the low-rank representation and the TP to investigate a superior texture preserving method for PCCT reconstruction. Especially in the chest CT, there are abundant texture features in the lung area. The reconstruction should preserve not only the edges but also the textures inside the tissue regions. We refer our proposed method as low-rank representation and texture preserving MRF prior (LRTP). Firstly, to make use of the synergy among the individual energy channel, the inter-channel correlation is modeled

by low-rank representation technique. Since it is known that the ground-truth spectral images in inter-channel have high correlation and can be viewed as lying in several low-dimensional subspaces, low-rank representation as an excellent subspace recovery technique can be effective to restore the ground-truth images from its corrupted version. On the other hand, the inner-channel spatial information of the target corrupted images is characterized by TP, which is convex and can well preserve the edges and texture features. Since both of low-rank representation and TP can improve the image quality for PCCT reconstruction in some extent, the proposed method combines the inter-channel correlation and the inner-channel spatial information into a unified Bayesian reconstruction framework. The superiority of this combination will be discussed in section IIIC.

The remainder of the paper is organized as follows. The proposed LRTP method is described in Section 2. Section 3 presents the experimental setting and results both on the traditional CT image and the spectral CT images. Section 4 discusses the advantage obtain from low-rank presentation and TP, and then the conclusion is drawn in the final paper.

II. METHODS

We first describe the low-rank matrix recovery in PCCT images. We then describe the solution for the inverse problem of PCCT using the proposed LRTP algorithm.

A. Low-rank Matrix Recovery for PCCT Imaging

Based on the observation that images in different energy channels describe the same object, rich similarity redundancy information exists among those different channels, which leads to the low-rank matrix characteristics of the spectral images. We can use the concept of tensor to build the connections among images in different energy channels. A tensor with N dimensions is defined as $\mathcal{X} \in \mathbb{R}^{L_1 \times L_2 \times \dots \times L_N}$, each element is denoted as $x_{l_1 l_2 \dots l_N}$, and $1 \leq l_i \leq L_i$. Since it is a nondeterministic polynomial (NP-hard) problem to directly compute the rank of a high-order tensor, we can calculate the tensor in our familiar way using the “unfold” operator to turn a tensor into a matrix. We define the “unfold” operator along the i -th dimension as follows:

$$\text{unfold}_i(\mathcal{X}): = \mathcal{X}_{(i)} \in \mathbb{R}^{L_i \times (L_1 \dots L_{i-1} L_{i+1} \dots L_N)} \quad (1).$$

Conversely, we can recover the tensor using a fold operator, which is represented as:

$$\text{fold}_i(\mathcal{X}_{(i)}): = \mathcal{X} \quad (2).$$

By turning the tensor into matrixes, we can make the calculation easily. For example, the Frobenius norm of a tensor can be calculated by the following equation:

$$\|\mathcal{X}\|_F = \left(\sum |x_{l_1 l_2 \dots l_N}|^2 \right)^{1/2} = \|\mathcal{X}_{(i)}\|_F \quad (3).$$

where $\|\cdot\|_F$ is the Frobenius norm. Since the rank of a tensor $\mathcal{X}$ is a nonconvex function, directly solving the low-rank problem is a difficult task. A common approach is to relax it using the trace norm, which converts it to a convex optimization problem [33]. The rank is then approximated as the combination of trace norms of all matrices unfolded along each dimension:

$$\text{Rank}(\mathcal{X}) = \|\mathcal{X}\|_{tr} = \sum_{i=1}^N \alpha_i \|\mathcal{X}_{(i)}\|_{tr} \quad (4),$$

where N is the number of image dimensionality, $\{\alpha_i\}_{i=1}^N$ are weights parameters to combine unfolded matrices along each dimension in rank computation, which satisfying $\alpha_i \geq 0$ and $\sum_{i=1}^N \alpha_i = 1$, $\|\mathcal{X}_{(i)}\|_{tr}$ is the trace norm of $\mathcal{X}_{(i)}$ with the definition as follows:

$$\|\mathcal{X}_{(i)}\|_{tr} = \sum_j \sigma_j(\mathcal{X}_{(i)}) \quad (5),$$

where $\sigma_j(\mathcal{X}_{(i)})$ is the j -th largest singular value of $\mathcal{X}_{(i)}$. Hence, we can solve the trace norm of $\mathcal{X}_{(i)}$ with the singular value decomposition (SVD) method [34].

In this paper, the spectral CT images can be considered as a spectral sequence of traditional CT images:

$$\mathcal{X} = \{\mathbf{X}_k, 1 \leq k \leq S\} \quad (6),$$

where the tensor $\mathcal{X}$ denotes the PCCT images, $\mathbf{X}_k = \{x_{i,j,k}, 1 \leq i \leq N_x, 1 \leq j \leq N_y\}$ corresponds to one of the energy channel images, where N_x and N_y are the image width and height, respectively, S is the number of energy channels. The PCCT images is a three-dimension tensor, where two dimensions describe the coordinates in the inner-channel image, an extra dimension refers to the energy channel. Then, the tensor $\mathcal{X}$ can be turned into three matrixes using the unfolding operator as: $\mathcal{X}_{(1)} \in \mathbb{R}^{N_x \times (N_y \times S)}$, $\mathcal{X}_{(2)} \in \mathbb{R}^{N_y \times (N_x \times S)}$, $\mathcal{X}_{(3)} \in \mathbb{R}^{S \times (N_x \times N_y)}$. Here we only use low-rank to constrain the correlation among different channel images. Hence, we assumed $\alpha_1 = \alpha_2 = 0$, $\alpha_3 = 1$ in Eq. (4).

To evaluate whether PCCT images can be sufficiently characterized using their low-rank approximation, we simulated five-channel PCCT images with an array size of $512 \times 512 \times 5$ from the well-known Mayo Clinic dataset which acquired by a 2D cylindrical detector and a helical cone-beam trajectory. More details will be provided in Section IIIA. The three-dimension (3D) tensor was first turn into a $5 \times (512 \times 512)$ matrix using the unfolding operator. We then performed SVD on this matrix. As shown in Fig. 1, most of the singular values being close to zero. We remove the small singular values and only use the top two singular values to recover the images. The recovered images and absolute difference images are shown in Fig. 1. It can be observed that there is no visual difference between the original reconstructed images and recovered images. Weeny absolute difference also implies that the

high similarity or low-rank characteristic of the PCCT images. Relative root-mean-square error (RRMSE) was also employed to quantitative evaluate the quality of low-rank representation:

$$\text{RRMSE} = \sqrt{\frac{\sum_{p=1}^P (x(p) - x_{ref}(p))^2}{\sum_{p=1}^P (x_{ref}(p))^2}} \quad (7),$$

where $x(p)$ and $x_{ref}(p)$ represent the voxel value of recovered and original reconstructed images at pixel p , respectively. P is the total number of voxels in the images. The results show that, by using the top two singular value, the reconstructed image has small RRMSE (0.06% and 0.08% for first channel and last channel, respectively), which means that most of the information can be recovered by the top two singular values. The ranks are relatively low in comparison to the total number of channels, suggesting spectral images could be represented using their low-rank approximation.

B. PCCT Images Reconstruction Framework

In PCCT, the measurement of projection at energy channel k can be expressed as follows:

$$\mathbf{Y}_k = \mathbf{A}\mathbf{X}_k \quad (8),$$

where $\mathbf{A}$ represents the system matrix which depends on the projection geometry, and its element is determined as the intersection lengths of a ray path with the associated voxel on the path. The PCCT measurements at all energy channels can be given as follows:

$$\mathbf{y} = \{\mathbf{Y}_k = \mathbf{A}\mathbf{X}_k, k \leq S\} \quad (9).$$

In traditional CT image reconstruction methods, we deal with the projection of each channel separately. In this paper, we propose to consider the connections among different energy channels. We can treat the image tensor $\mathcal{X}$ as low-rank tensor to describe the global spectral information, which already been proven in section IIA. Hence, we incorporate the low-rank and spatial prior information for Bayesian reconstruction. The basic object function can be expressed as follows:

$$\text{argmin}_{\mathcal{X}} \|\mathbf{A}\mathcal{X}_{(3)}^T - \mathcal{Y}_{(3)}^T\|_{\Lambda}^2 + \lambda \|\mathcal{X}\|_{lr} + \beta U(\mathcal{X}) \quad (10),$$

where the first term refers to the fidelity part, where the projections are the product of the origin images and system matrix. The second term treats the tensor $\mathcal{X}$ as a low-rank tensor and use the trace norm $\|\mathcal{X}\|_{lr}$, as a constraint. The third term $U(\mathcal{X})$ represents spatial prior information. Notations λ and β are hyper parameters for the second term and the third term to balance the strength between the prior and fidelity. Since the noise is statistically independent among all the measurement data elements, Λ is a weight matrix for its corresponding datum. A mathematical derivation for Λ in each energy channel is given in [35].

C. Texture Preserving MRF Prior

In Section IIB the term $U(\mathcal{X})$ in Eq. (10) represents spatial prior information on the reconstructed images. Various assumptions about the spatial prior information lead to different reconstruction algorithms. One of the extensive used spatial priors is MRF model, which provide a simple and often effective way to model the spatial dependencies in images. We extend the MRF model to PCCT images, $U(\mathcal{X})$ can be expressed as:

$$U(\mathcal{X}) = \sum_k \sum_j \sum_{m \in \Omega_j} w_{jmk} \phi(x_{jk} - x_{mk}) \quad (11),$$

where index k denote the energy channel, index j runs over all the pixels in each channel image, Ω_j represents a small fixed neighborhood MRF window of the j -th pixel in the channel image, ϕ denotes a potential function (e.g. quadratic function), and w_{jmk} is the weighting coefficient which represents the correlation between pixel m and pixel j in the k -th channel.

We developed a region-specific TP in our previous work of [30, 31]. We also demonstrated the clinical impact of learning the region-specific TP from a FDCT database for LDCT image reconstruction in [32]. Here, we extend the region-specific TP to PCCT images. In general, ϕ in Eq. (11) is typically chosen as a quadratic function. The penalty term $U(\mathcal{X})$ can be written as:

$$U(\mathcal{X}) = \sum_{r=1}^R \sum_k \sum_{j \in \text{Region}(r)} \sum_{m \in \Omega_j} w_{r,jmk}^{FD} (x_{r,jk} - x_{r,mk})^2 \quad (12),$$

where R is the number of different tissue regions. The index j runs over all the pixels in a specific tissue region. Notation $\{w_{r,jmk}^{FD}\}_{m \in \Omega_j} = \mathbf{w}_r^{FD}$ represents a set of MRF coefficients of tissue region r (each tissue region in each channel has a specific set of MRF coefficients) predicting from full-dose images, which is equivalent to solve the following optimization problem:

$$\begin{aligned} \mathbf{w}_r^{FD} &= \underset{\mathbf{w}_r}{\text{argmin}} \sum_{j \in \text{Region}(r)} \left(x_j^{FD} - \mathbf{w}_r^T \mathbf{x}_{\Omega_j}^{FD} \right)^2 \\ &= \left[\sum_{j \in \text{Region}(r)} \left(\mathbf{x}_{\Omega_j}^{FD} \left(\mathbf{x}_{\Omega_j}^{FD} \right)^T \right) \right]^{-1} \\ &\quad \times \left[\sum_{j \in \text{Region}(r)} \left(\mathbf{x}_{\Omega_j}^{FD} x_j^{FD} \right) \right] \end{aligned} \quad (13),$$

where vector $\mathbf{x}^{FD}$ is the full-dose image, T denotes the transpose operate. Moreover, the sum of the predicted MRF coefficients for each tissue region should equal to one.

In this study, the number of tissue types is setting as $R = 4$ to represent lung, fat, muscle and bone for chest CT imaging. The reconstructed FDCT images were segmented using our previously reported vector quantization (VQ) algorithm [36]. Fig. 2 shows the tissue masks of the chest imaging from the full-dose images of Fig. 1. Different channels shared the same

tissue masks in the PCCT images because the spectral data was obtained from the same object.

Given the segmented masks of the four tissue types from the FDCT images of Fig. 2, we apply Eq. (13) to calculate the corresponding MRF coefficients from different energy channels. Hence, the MRF coefficients from different energy channels should be different. The MRF window size is set to 7×7 , which has been proved to be optimal in our previous work [30, 31]. Fig. 3 illustrates the predicted MRF coefficients set of the four tissue regions in the first channel and the last channel, respectively. We can observe that MRF coefficients in the same tissue region among different channels have strong similarity, which agrees with our hypothesis that strong correlation exists among energy channels. However, there still have some differences. For example, in the fat plot of Fig. 3, the red region of the first channel is larger than that of the last channel. This is also expected because the attenuation coefficient differs under different X-ray energy. Since the attenuation coefficient of lung tissue (most is air) varies smaller than the soft tissue through the CT X-ray energy, the MRF coefficient of fat is expected to change larger than that of lung among different channels. This expectation is clearly observed in Fig. 3.

D. The Proposed LRTP Algorithm and Optimization

Due to the trace normal constraint is not differentiable, Eq. (10) cannot be solved directly by the conventional gradient descent methods. Here, we use the split-Bregman method [37] to solve the problem. Firstly, we introduce an auxiliary variable $\mathcal{D}$ to represent $\mathcal{X}$. The new objective function is as follows

$$\begin{aligned} \arg \min_{\mathcal{X}, \mathcal{D}} & \|\mathbf{A}\mathcal{X}_{(3)}^T - \mathcal{Y}_{(3)}^T\|_{\Lambda}^2 + \lambda \|\mathcal{D}\|_{tr} + \beta U(\mathcal{X}), \\ \text{subject to } & \mathcal{X} = \mathcal{D} \end{aligned} \quad (14).$$

The augmented Lagrangian of the above objective function is written below, where μ is the Lagrangian parameters:

$$\arg \min_{\mathcal{X}, \mathcal{D}} \|\mathbf{A}\mathcal{X}_{(3)}^T - \mathcal{Y}_{(3)}^T\|_{\Lambda}^2 + \lambda \|\mathcal{D}\|_{tr} + \mu \|\mathcal{D} - \mathcal{X}\|_F^2 + \beta U(\mathcal{X}) \quad (15).$$

We further introduce another variable $\mathcal{V}$ to represent the different between $\mathcal{X}$ and $\mathcal{D}$:

$$\arg \min_{\mathcal{X}, \mathcal{D}, \mathcal{V}} \|\mathbf{A}\mathcal{X}_{(3)}^T - \mathcal{Y}_{(3)}^T\|_{\Lambda}^2 + \lambda \|\mathcal{D}\|_{tr} + \mu \|\mathcal{D} - \mathcal{X} - \mathcal{V}\|_F^2 + \beta U(\mathcal{X}) \quad (16).$$

Then we break Eq. (16) into three sub-problems below that could be solved by iteratively updating the variables. In the below, ι denotes the current iteration step.

- 1) Subproblem 1: Update $\mathcal{X}^{\iota+1}$ by minimizing:

$$\begin{aligned} \mathcal{X}^{t+1} = \operatorname{argmin}_{\mathcal{X}} & \|A\mathcal{X}_{(3)}^T - \mathcal{Y}_{(3)}^T\|_{\Lambda}^2 + \beta U(\mathcal{X}) \\ & + \mu \|\mathcal{D}^t - \mathcal{X} - \mathcal{V}^t\|_F^2 \end{aligned} \quad (17).$$

This subproblem can be solved by the PWLS method, which performance well in our previously work [31].

- 2) Subproblem 2: Update $\mathcal{D}^{t+1}$ by minimizing:

$$\mathcal{D}^{t+1} = \operatorname{argmin}_{\mathcal{D}} \lambda \|\mathcal{D}\|_{tr} + \mu \|\mathcal{D} - \mathcal{X}^{t+1} - \mathcal{V}^t\|_F^2 \quad (18).$$

This subproblem is a normal problem of trace norm, which could be solved by SVD shrink method [34].

- 3) Subproblem 3: Update $\mathcal{V}^{t+1}$ by:

$$\mathcal{V}^{t+1} = \mathcal{V}^t + \mathcal{X}^{t+1} - \mathcal{D}^{t+1} \quad (19),$$

which can be solved directly.

In summary, the workflow of the proposed algorithm can be described as follows:

WORKFLOW FOR THE PROPOSED LRTP ALGORITHM

Texture learning and region segmentation

Segment the original reconstructed image into R regions.

Learn the region-specific texture from each region.

Segment the full energy data reconstructed image into R regions.

For each region:

Image Reconstruction

Initialize $\mathcal{X}$;

Set parameters λ, μ, β .

While stop criterion is not met:

Setp1: Update $\mathcal{X}$ by Eq. (17);

Step2: Update $\mathcal{D}$ by Eq. (18);

Step3: Update $\mathcal{V}$ by Eq. (19);

End until the stop criterion is satisfied.

III. EXPERIMENTS AND RESULTS

To evaluate the performance of the proposed method, one traditional CT image was used as our phantom instead of the analytical phantom which usually not includes texture information to simulate the PCCT images. Then, the real clinical data from one DECT scanner were used to perform the evaluation. The real data scanned by custom-made PCCT were also used to perform the evaluation. For comparison study, some well-established

methods were employed, including the simultaneous algebraic reconstruction technique (SART) [38], total variation minimization (TV) [14], low-rank representation and total variation regularization (LRTV) [27] as well as tensor dictionary learning (TDL) [22]. We demonstrate the superiority of our proposed method from both image visual inspection and quantitatively assessment. For quantitatively assessment, the root-mean-square error (RMSE) and peak-signal-to-noise ratio (PSNR) were used. Additionally, we employed another two image quality measurements named structural similarity (SSIM) index and feature similarity (FSIM) index [39], which measure the similarity between two images and have been widely reported to be consistent with the quality perception of the human visual system (HVS). Both SSIM and FSIM are bounded from 0 to 1. The closer to 1 means the better recovery. To compute SSIM and FSIM, the dynamic range of each image was scale to [0 255] in advance.

A. Real Clinical Phantom Study

A reconstructed image from Mayo Clinic full-dose image was employed as the clinical phantom to simulate the spectral CT images [40]. Due to different materials having different spectral attenuation curves, we first segmented the image into different materials, such as bone, muscle, fat, and lung tissues. Then, the attenuation coefficients of the four tissues at each energy could be calculated using the linear scaling method according to the spectral attenuation curves provided by the National Institute of Standards and Technology (NIST) [41].

In this clinical phantom study, equidistant fan-beam geometry was assumed. There were 672 detector bins of a 0.8 mm width per bin. 1160 projection views were collected over a full scan range. The distance from source to rotation center was 595 mm, the distance from source to detector was 1085.6 mm. A polychromatic 140 kVp x-ray source was assumed whose normalized spectrum is given in Fig. 4 [42]. The spectrum was divided into five energy channels from low to high energy: (55, 65] keV, (65, 75] keV, (75, 85] keV, (85, 95] keV, (95, 105] keV [43]. Similar with [20], five virtual-monochromatic images (VMI) (60, 70, 80, 90, and 100 keV) were used to simulate the multi-channel projection with a forward projection operation. The total emitted photons were assumed as 1×10^5 . In addition, quarter dose (2.5×10^4 photons) also employed to simulate a higher noise projection. Then the photon counts for each energy channel can be calculated according to the normalized X-ray spectrum in Fig. 4. Poisson noise is superimposed to the measurement to consider the statistical property. From Fig. 4, we can observe that the area under the spectrum curve in (55, 65] keV is larger than (95, 105] keV, which means that low energy channel contains more photon counts than the high energy channel. This may affect the noise level of the reconstructed images. We will discuss this issue in details in the results analysis.

Once we simulated the five projections, we reconstructed all the images with methods mentioned above. All the compared algorithms were implemented in MATLAB with Intel Core (TM) i7-7820HK CPU, 32 GB RAM and a GPU card (NVIDIA GTX 1080 with max-Q design) notebook computer platform. To update the fidelity term efficiently, the time-consuming task (forward/backward projection) were accelerated by GPU. Table I summarizes the required time in one iteration for all algorithms. Due to the inner loop for

sparse coding in TDL regularization, TDL requires longer time than TV and LRTV. Our proposed LRTP regularization, which is implemented by GPU, only takes one convolution using the calculated MRF coefficient with the to-be-reconstructed image. Although the convolutional kernels (MRF coefficients) in different tissue types and energy channels are different, all the convolutions could be parallel calculated by GPU. Hence the costs were remarkably reduced. The final reconstructed images were a $512 \times 512 \times 5$ tensor with in-plane resolution of $0.75 \text{ mm} \times 0.75 \text{ mm}$. In the following subsection, we will evaluate the image quality qualitatively and quantitatively.

1) Visual Inspection—The representative chest CT reconstructed images from 1×10^5 and 2.5×10^4 photons are shown in Fig. 5 and Fig. 6 accordingly, where the reference images are reconstructed from noise-free projections by the SART method. In Fig. 5, it can be observed that without any prior knowledge, there are strongest noise and artifacts in the SART results. It's worth noting that the noise level in different energy channels are difference. Even though the photons absorbed heavier in the low energy channel, low photon counts to be the dominant factors leading the noise of the high energy channel is more serious than that of the low energy channel. In the TV results, the reconstructed images contain less noise, the tissue structures and edges were blurred to some extent. It is difficult to distinguish the shape of bony structures and image edges in the high energy channel, which indicated by the arrow “1” and “2” in Fig. 5(c2). Moreover, there are serve block artifacts labelled by arrow “3” in Fig. 5(c2), which is more serious in the high energy channel due to the lower photon counts. LRTV method apply the correlation among different channels which can reduce the blocky effect to some extent and improve the image quality. But it still hardly to recover the bony structures and image edges in location “1” and “2” in Fig. 5(d2) as a result of the constant assumption of TV regularization. In comparison, the TDL method and the proposed LRTP method can provide more accurate bony structures and image edges than TV and LRTV methods. In Fig. 6, it can be seen that the images reconstructed by SART method contain severe noise, especially in Fig. 6(b2), which represent the high energy channel. TV and LRTV methods can reduce the noise in some extent. But the block artifact emerged in Fig. 6(c2) and Fig. 6(d2). TDL and the proposed LRTP methods can achieve favorable image quality even at a higher noise level.

For more clear comparison, the lung regions of interest (ROIs) which have abundant texture features, are extracted from the red box in Fig. 5 and Fig. 6. The magnified ROIs of images reconstructed by different methods and their corresponding absolute difference images to the ground truth are also shown in Fig. 7. The SART reconstructed images contain strong noise artifacts and some texture features are drowned out by noise. The strong random noise can be clearly seen in the absolute difference images. TV and LRTV methods significantly reduced the noise but most of the small texture features, such as the blood vessels were smoothened. The corrupted texture can be clearly observed in their absolute difference images. TDL method could identify these texture features, and the details were retained without blurring. The proposed LRTP method yielded very clear images and the smallest difference with the reference image comparing with other methods.

2) Quantitative Evaluation—For quantitative evaluation, the ROIs in the same location of Fig. 7 were chosen on five channel images. The results are summarized in TABLE II. The proposed LRTP had the lowest RMSE and the highest PSNR for all energy channels. The SSIM and FSIM also employed to measure the structure similarity and feature similarity between the references and reconstructed images. In TABLE II, the proposed LRTP method obtains the greatest SSIM and FSIM values than other competing methods except in the 5th channel. As for the 5th channel, TDL method has a greater SSIM value than other algorithms for 1×10^5 photons. However, our proposed LRTP method performs better in other evaluation indexes and has a smallest error with the reference image. Generally, LRTP method has the best image quality in terms of quantitative assessment.

B. Real Clinic Dual-Energy CT Data Study

We further evaluated the proposed LRTP method on the real clinical DECT data. A patient who revealed suspected pulmonary tumor appearing as nodules was scanned using a dual-energy high-definition CT scanner (GE Discovery CT750 HD) of the chest in gemstone spectral imaging (GSI) mode. The VMI at 80 and 140 kVp with the thickness of 2.5 mm and the spacing of 0.75 mm were transferred to an AW 4.4 workstation for analysis. The VMI were generated at 10 keV monochromatic energy level increments from 60 to 100 keV, resulting in one set of 5 images, which represents the VMI for each channel in Fig. 4. A representative slice which truncated by the field of view (FOV) was specified from the image volume to evaluate our methodology. The scanning geometry and the Poisson noise were set same as the first patient.

1) Reconstruction Results—In terms of the spectral CT images generated from DECT, Fig. 8 and Fig. 9 shows the reconstructed images from 1×10^5 and 2.5×10^4 photons which represents the VMI in high noise level and low noise level, respectively. In this case, more abundant texture features, such as fibrosis and the edge of lung nodule, required to recovery due to the pulmonary disease of the patient. In Fig. 8, it can be observed that most of image edges and texture features were buried in the noise without any prior, especially in Fig. 8(b2) which represents the high energy channel. TV provides better image reconstruction for whole images but some finer structures and texture features were blurring, which was indicated by arrows “4” and “5” in Fig. 8(c2). The block artifacts also emerged in the bony structure region, which was labelled by arrow “6” in Fig. 8(c2). LRTV improves the image quality based on TV method, especially in terms of block artifacts suppression. However, the finer image details and texture features are still lost. TDL and the proposed LRTP method achieve the similar performance in terms of artifacts reduction and texture preservation in the whole image visual inspection. In Fig. 9, it can be seen that the image quality further decreased due to high noise level in Fig. 9(b1) and Fig. 9(b2). TV and LRTV methods can improve the image quality. However, the block artifacts are more clearly in Fig. 9(c2) and Fig. 9(d2). TDL and the proposed LRTP methods can achieve favorable image quality even at a higher noise level.

The close-up views in the red box from Fig. 8 and Fig. 9 are shows in Fig. 10, the absolute difference image also presented. Both of the magnified images and the weakest absolute different images demonstrated that our proposed LRTP method achieves the best

performance on noise reduction and texture preservation. Although TDL provide a very similar results to LRTP, the difference can be clearly seen in the absolute difference images. LRTV and TV method perform well in noise suppression, but the texture feature in ROI were blurring. It's worth mentioning that the noise in the low energy channel is very weak while it's more serious in the high energy channel. One common regularization parameter was employed for all the channels. We can observe from Fig. 10, in order to optimize the overall image quality, the edge was blurred to some extent in the low energy channel images for all the methods, where the TDL method is the most serious.

2) Quantitative Evaluation—Quantitative evaluation in the five ROIs from different energy channels are presented in the TABLE III. The quantitative results provide a similar trend to visual inspection. TDL method have made considerable progress to LRTV method followed by TV method in all the indexes. For the reconstructed images at 2.5×10^4 photons, TDL method achieves the best performance in the 5th channel. However, the close-up views in Fig. 10 show that some tiny textures were over smooth by TDL method while our proposed LRTP method preserves the textures information well. All the competing methods do not perform well in the low energy channel, which is also consistent with visual inspection. In generally, LRTP method achieves the best results, while offering a great reduction in running time.

C. Preclinical Animal Lung Study

The proposed LRTP method was also evaluated using preclinical projections from an animal lung. A custom-made micro-CT system with one flat-panel photon-counting detector was employed to generate the real PCCT data. We extract the central slice and emerge the neighboring detector bins to form fan-beam geometry with 512 detector bins. The distance from source to rotation center was 380 mm, the distance from source to detector was 440 mm. Over a full scan range, 360 projections were uniformly collected to cover a FOV of 43.92 mm in radius. Three energy channels were used to collect projections with the source being operated at 120 kVp and 100 μ A. The channels of real datasets were different from that for the aforementioned numerical simulation. The three energy channels from low to high energy were set to (60, 120] keV, (80, 120] keV and (100, 120] keV, respectively. Hence, the low energy channel includes almost all the photons, while the high energy channel took the fewest photons. We set $R=1$ in this preclinical animal lung study. The reconstructed images were a $512 \times 512 \times 3$ tensor with in-plane resolution of $150 \mu\text{m} \times 150 \mu\text{m}$.

In terms of the preclinical PCCT data, Fig. 11 shows the reconstructed images and the corresponding magnified ROIs. It can be observed that the images reconstructed by SART contain severe noise artifacts. Especially the reconstructed image at the high energy channel, structures were indiscernible even in the magnified ROI. TV method could effectively suppressed noise. The structures were clearly observed in the magnified ROI. However, the block artifacts were emerged in the image at the high energy channel, while the images were over smoothed at the low energy channel. Compared with TV method, LRTV method can improve the image quality to some extent. But both of TV and LRTV methods were presented the same disadvantages due to the defect of TV operation. TDL method performed

better than LRTV method in noise suppression and contrast enhancement. However, the image details were distorted because some noises were enhanced as tiny textures. Hence, it's difficult to distinguish the authenticity of tiny textures in the magnified ROIs. By comparison, our proposed LRTP method were well preserve the tiny textures while suppress noise, which is more suitable for clinical diagnosis.

D. The Superiority of Combination

In this paper, we proposed a novel PCCT reconstruction algorithm, which combines both the low-rank or LR representation and the texture prior or TP under the Bayesian framework. This combination considers the structure similarity correlation among different energy channels and also preserves the texture information of the specific tissue regions while smoothing the noise in each individual energy channel. To fully understand the benefit of the two priors, the TP and the LR representation were employed in an isolated way to show their gains. The reconstructed images for the real clinical phantom study in Section IIIA are shown in Fig. 12. All the aforementioned methods can improve the image quality in some extent. However, when we only use TP method (without LR representation), i.e. only one common regularization parameter was employed for different energy channels, it is hard to obtain optimal results simultaneously for all energy-channel images because of the different noise level in different energy channel. We can observe that small amount of noise is still contained in the high energy-channel image while the low energy channel image looks well, see Fig. 12(b1) and Fig. 12(b2). On the other hand, when we only use LR representation method (without TP), the noise is suppressed because there is strong correlation in PCCT images but weak correlation between the noise of each channels. We can observe that there are still contain small amount of noise in both high energy channel image and low energy channel image, but the noise level is close to each other, see Fig. 12(c1) and Fig. 12(c2). The PSNR indexes of ROIs were also given in the image for quantitative evaluation. Our proposed LRTP method best preserves textures and achieves the highest PSNR values in all the energy channels, which proved that our proposed method not only takes advantages from the LR representation, but also takes advantages from the TP to preserve the tissue textures.

IV. DISCUSSIONS AND CONCLUSION

The hyper parameters selection plays an important role in iterative CT image reconstruction. There are three main tuning parameters for LRTP method: μ , λ and β . In this paper, μ controls the strength of LR constraint and λ controls the number of singular values used in the PCCT images. As described in Section IIA, most of the information in the first patient can be recovered by the top two singular values, which was also found to apply to the second patient. Hence, we choose the same parameters as $\mu = 0.01$, $\lambda = 0.1$ for the LR representation. The smoothing parameter β controls the tradeoff between the noise and smoothness. For real Clinical Phantom study, we empirically select the $\beta = 400$ for 1×10^5 photons and $\beta = 1000$ for 2.5×10^4 photons. For real Clinic Dual-Energy CT Data Study, we set the $\beta = 300$ for 1×10^5 photons and $\beta = 800$ for 2.5×10^4 photons. For the preclinical animal lung study, the system geometry was different with clinical CT. Hence, the parameters were set as $\mu = 0.025$, $\lambda = 0.1$, $\beta = 2000$. The reconstruction stopped after 110

iterations for all the aforementioned study. To impartially compare image quality, the hyper-parameters are empirically optimized using RMSE. For example, a common TV regularization parameter was selected for all the channels. Then, we optimized the LRTV method by employing the same TV regularization parameter and chose the best low-rank parameter. On the other hand, the default parameters of TDL implementation in [22] were used.

In conclusion, it is a viable solution to PCCT reconstruction by combining both the low-rank representation and the texture-preserving prior. This combination brings together inner-channels correlation and inter-channel specific texture information for effective recovery of the PCCT images. Experimental results indicate that the proposed method is able to remedy the noise artifacts and preserve the image edges and texture features from both physical phantom and clinical dual-energy CT data. Quantitative evaluation shows that proposed method outperforms other competing methods such as TV, LRTV and TDL. Additionally, the time consuming of the proposed LRTP is also economical compared with the competing methods. There are still two remaining issues to address. In this study, we use one real clinical CT phantom, one clinical dataset from dual energy CT and one preclinical animal lung dataset from custom-made PCCT system to perform our study. We did not consider scatter and beam-hardening effects, which can degrade the performance. Even though our results were closer to the reference image in both visual inspection and quantitative evaluation, which means more accuracy for material decomposition and clinical diagnosis, there is no proof that this is true for the aforementioned applications. Further evaluation in material decomposition and clinical diagnosis are one interest of our future work. In Fig. 3, MRF model coefficients in the same tissue region among different channels are observed to have strong similarity. How to utilize this similarity to further improve our model is another research interest of our future work.

Acknowledgments

This work was partially supported by the NIH/NCI Grant CA206171. Y. Shi is also supported partially by China Scholarship Council (No. 201806280297). Y. Zhang and X. Mou are supported by the National Key Research, Development Program of China (No. 2016YFA0202003), National Natural Science Foundation of China (NSFC) (No. 61571359).

REFERENCES

- [1]. Boll DT, Patil NA, Paulson EK, Merkle EM, Nelson RC, Schindera ST et al., "Focal cystic high-attenuation lesions: characterization in renal phantom by using photon-counting spectral CT—improved differentiation of lesion composition," *Radiology*, vol. 254, no. 1, pp. 270–276, 2009.
- [2]. Sun X, Shao X, Chen H, "The value of energy spectral CT in the differential diagnosis between benign and malignant soft tissue masses of the musculoskeletal system," *European journal of radiology*, vol. 84, no. 6, pp. 1105–1108, 2015. [PubMed: 25819070]
- [3]. Long Y, Fessler JA, "Multi-material decomposition using statistical image reconstruction for spectral CT," *IEEE transactions on medical imaging*, vol. 33, no. 8, pp. 1614–1626, 2014. [PubMed: 24801550]
- [4]. Flohr TG, McCollough CH, Bruder H, Petersilka M, Gruber K, Süß C et al., "First performance evaluation of a dual-source CT (DSCT) system," *European radiology*, vol. 16, no. 2, pp. 256–268, 2006. [PubMed: 16341833]
- [5]. Matsumoto K, Jinzaki M, Tanami Y, Ueno A, Yamada M, Kuribayashi S, "Virtual monochromatic spectral imaging with fast kilovoltage switching: improved image quality as compared with that

- obtained with conventional 120-kVp CT,” *Radiology*, vol. 259, no. 1, pp. 257–262, 2011. [PubMed: 21330561]
- [6]. Sones RA, Barnes GT, “Noise correlations in images acquired simultaneously with a dual-energy sandwich detector,” *Medical physics*, vol. 16, no. 6, pp. 858–861, 1989. [PubMed: 2685530]
- [7]. Maaß C, Baer M, Kachelrieß M, “Image-based dual energy CT using optimized precorrection functions: A practical new approach of material decomposition in image domain,” *Medical physics*, vol. 36, no. 8, pp. 3818–3829, 2009. [PubMed: 19746815]
- [8]. Liu X, Yu L, Primak AN, McCollough CH, “Quantitative imaging of element composition and mass fraction using dual-energy CT: Three-material decomposition,” *Medical physics*, vol. 36, no. 5, pp. 1602–1609, 2009. [PubMed: 19544776]
- [9]. Willemink MJ, Persson M, Pourmorteza A, Pelc NJ, Fleischmann D, “Photon-counting CT: technical principles and clinical prospects,” *Radiology*, vol. 289, no. 2, pp. 293–312, 2018. [PubMed: 30179101]
- [10]. Taguchi K, Iwanczyk JS, “Vision 20/20: Single photon counting x-ray detectors in medical imaging,” *Medical physics*, vol. 40, no. 10, p. 100901, 2013. [PubMed: 24089889]
- [11]. McCollough CH, Leng S, Yu L, Fletcher JG, “Dual-and multi-energy CT: principles, technical approaches, and clinical applications,” *Radiology*, vol. 276, no. 3, pp. 637–653, 2015. [PubMed: 26302388]
- [12]. Elbakri IA, Fessler JA, “Statistical image reconstruction for polyenergetic X-ray computed tomography,” *IEEE Transactions on medical imaging*, vol. 21, no. 2, pp. 89–99, 2002. [PubMed: 11929108]
- [13]. Xu Q, Mou X, Wang G, Sieren J, Hoffman EA, Yu H, “Statistical interior tomography,” *IEEE transactions on medical imaging*, vol. 30, no. 5, pp. 1116–1128, 2011. [PubMed: 21233044]
- [14]. Xu Q, Yu H, Bennett J, He P, Zainon R, Doesburg et R et al., “Image reconstruction for hybrid true-color micro-CT,” *IEEE transactions on biomedical engineering*, vol. 59, no. 6, pp. 1711–1719, 2012. [PubMed: 22481806]
- [15]. Gong C, Han C, Gan G, Deng Z, Zhou Y, Yi et J et al., “Low-dose dynamic myocardial perfusion CT image reconstruction using pre-contrast normal-dose CT scan induced structure tensor total variation regularization,” *Physics in Medicine & Biology*, vol. 62, no. 7, pp. 2612–2635, 2017. [PubMed: 28140366]
- [16]. Zhao B, Gao H, Ding H, Molloy S, “Tight-frame based iterative image reconstruction for spectral breast CT,” *Medical physics*, vol. 40, no. 3, pp. 8217–8229, 2013.
- [17]. Xu Q, Yu H, Mou X, Zhang L, Hsieh J, Wang G, “Low-dose X-ray CT reconstruction via dictionary learning,” *IEEE transactions on medical imaging*, vol. 31, no. 9, pp. 1682–1697, 2012. [PubMed: 22542666]
- [18]. Zhao B, Ding H, Lu Y, Wang G, Zhao J, Molloy S, “Dual-dictionary learning-based iterative image reconstruction for spectral computed tomography application,” *Physics in Medicine & Biology*, vol. 57, no. 24, pp. 8217–8229, 2012. [PubMed: 23192234]
- [19]. Bai T, Yan H, Jia X, Jiang S, Wang G, Mou X, “Z-index parameterization for volumetric CT image reconstruction via 3-D dictionary learning,” *IEEE transactions on medical imaging*, vol. 36, no. 12, pp. 2466–2478, 2017. [PubMed: 28981411]
- [20]. Niu S, Yu G, Ma J, Wang J, “Nonlocal low-rank and sparse matrix decomposition for spectral CT reconstruction,” *Inverse problems*, vol. 34, no. 2, p. 024003, 2018. [PubMed: 30294061]
- [21]. Kim K, Ye JC, Worstell W, Ouyang J, Rakvongthai Y, El Fakhri G et al., “Sparse-view spectral CT reconstruction using spectral patch-based low-rank penalty,” *IEEE transactions on medical imaging*, vol. 34, no. 3, pp. 748–760, 2014. [PubMed: 25532170]
- [22]. Zhang Y, Mou X, Wang G, Yu H, “Tensor-based dictionary learning for spectral CT reconstruction,” *IEEE transactions on medical imaging*, vol. 36, no. 1, pp. 142–154, 2016. [PubMed: 27541628]
- [23]. Wu W, Zhang Y, Wang Q, Liu F, Chen P, Yu H, “Low-dose spectral CT reconstruction using image gradient ℓ_0 -norm and tensor dictionary,” *Applied Mathematical Modelling*, vol. 63, pp. 538–557, 2018. [PubMed: 32773921]

- [24]. Wu W, Liu F, Zhang Y, Wang Q, Yu H, "Non-local Low-rank Cube-based Tensor Factorization for Spectral CT Reconstruction," *IEEE transactions on medical imaging*, vol. 38, no. 4, pp. 1079–1093, 2019. [PubMed: 30371357]
- [25]. Gao H, Yu H, Osher S, Wang G, "Multi-energy CT based on a prior rank, intensity and sparsity model (PRISM)," *Inverse problems*, vol. 27, no. 11, p. 115012, 2011. [PubMed: 22223929]
- [26]. Li L, Chen Z, Wang G, Chu J, Gao H, "A tensor PRISM algorithm for multi-energy CT reconstruction and comparative studies," *Journal of X-ray science and technology*, vol. 22, no. 2, pp. 147–163, 2014. [PubMed: 24699344]
- [27]. Chu J, Cong W, Li L, Wang G, "Combination of current integrating/photon-counting detector modules for spectral CT," *Physics in Medicine & Biology*, vol. 58, no. 19, pp. 7009–7024, 2013. [PubMed: 24043342]
- [28]. Aerts HJ, Velazquez ER, Leijenaar RT, Parmar C, Grossmann P, Carvalho et S et al., "Decoding tumour phenotype by noninvasive imaging using a quantitative radiomics approach," *Nature communications*, vol. 5, p. 4006, 2014.
- [29]. McCollough CH, Yu L, Kofler JM, Leng S, Zhang Y, Li et Z et al., "Degradation of CT low-contrast spatial resolution due to the use of iterative reconstruction and reduced dose levels," *Radiology*, vol. 276, no. 2, pp. 499–506, 2015. [PubMed: 25811326]
- [30]. Zhang H, Han H, Wang J, Ma J, Liu Y, Moore et W et al., "Deriving Adaptive MRF Coefficients from Previous Normal-dose CT Scan for Low-dose Image Reconstruction via Penalized Weighted Least-squares Minimization," *Medical Physics*, vol. 41, no. 4, p. 041916, 2014. [PubMed: 24694147]
- [31]. Zhang H, Han H, Liang Z, Hu Y, Liu Y, Moore et W et al., "Extracting information from previous full-dose CT scan for knowledge-based Bayesian reconstruction of current low-dose CT images," *IEEE transactions on medical imaging*, vol. 35, no. 3, pp. 860–870, 2015. [PubMed: 26561284]
- [32]. Gao Y, Liang Z, Moore WH, Zhang H, Pomeroy MJ, Ferretti et JA et al., "A feasibility study of extracting tissue textures from a previous full-dose CT database as prior knowledge for Bayesian reconstruction of current low-dose CT images," *IEEE transactions on medical imaging*, vol. 38, no. 8, pp. 1981–1992, 2019. [PubMed: 30605098]
- [33]. Shi F, Cheng J, Wang L, Yap PT, Shen D, "LRTV: MR image super-resolution with low-rank and total variation regularizations," *IEEE transactions on medical imaging*, vol. 34, no. 12, pp. 2459–2466, 2015. [PubMed: 26641727]
- [34]. Cai JF, Candès EJ, Shen Z, "A singular value thresholding algorithm for matrix completion," *SIAM Journal on Optimization*, vol. 20, no. 4, pp. 1956–1982, 2010.
- [35]. Ma J, Liang Z, Fan Y, Liu Y, Huang J, Chen et W et al., "Variance analysis of x-ray CT sinograms in the presence of electronic noise background," *Medical Physics*, vol. 39, pp. 4051–4065, 2012. [PubMed: 22830738]
- [36]. Han H, Li L, Han F, Song B, Moore W, Liang Z, "Fast and Adaptive Detection of Pulmonary Nodules in Thoracic CT Images Using a Hierarchical Vector Quantization Scheme," *IEEE Transactions on Information Technology in BioMedicine*, vol. 19, no. 2, pp. 648–659, 2015.
- [37]. Goldstein T, Osher S, "The split Bregman method for L1-regularized problems," *SIAM Journal on Imaging Sciences*, vol. 2, no. 2, pp. 323–343, 2009.
- [38]. Andersen AH, Kak AC, "Simultaneous algebraic reconstruction technique (SART): a superior implementation of the ART algorithm," *Ultrasonic imaging*, vol. 6, no. 1, pp. 81–94, 1984. [PubMed: 6548059]
- [39]. Zhang L, Zhang L, Mou X, Zhang D, "FSIM: A feature similarity index for image quality assessment," *IEEE transactions on Image Processing*, vol. 20, no. 8, pp. 2378–2386, 2011. [PubMed: 21292594]
- [40]. AAPM, "Low dose CT grand challenge," 2017 [Online]. Available: <http://www.aapm.org/GrandChallenge/LowDoseCT/>
- [41]. [Online]. Available: <http://spectrumgui.sourceforge.net/>.
- [42]. [Online]. Available: <http://www.nist.gov/pml/>.
- [43]. Leng S, Yu L, Wang J, Fletcher JG, Mistretta CA, McCollough CH, "Noise reduction in spectral CT: Reducing dose and breaking the trade-off between image noise and energy bin selection," *Medical physics*, vol. 38, no. 9, pp. 4946–4957, 2011. [PubMed: 21978039]

Fig. 1:

Low-rank approximation of PCCT images with five channels. From left to right, the top row shows the original reconstructed PCCT images and singular-value from SVD. The second row shows the original reconstructed first channel image and the last channel image, respectively. The third row shows the corresponding recovered images by using the top two singular values. The display window is $[0, 0.03] \text{ mm}^{-1}$. The bottom row shows the corresponding absolute difference images between the original reconstructed images and the recovered images. The display window is $[0, 0.0001] \text{ mm}^{-1}$.

Fig. 2:
Segmented masks of the chest CT image. The upper left, upper right, lower left, and lower right corresponds to the mask of lung, fat, muscle and bone, respectively.

Fig. 3:

The segmented images and the predicted MRF coefficients for the four tissue regions with a 7×7 MRF window size. The first and second columns are the segmented images at the first channel and last channel, respectively. The third and fourth columns are the 7×7 MRF coefficients for the first channel and last channel, respectively. From top to bottom are for lung, fat, muscle and bone, respectively.

Fig. 4:
X-ray normalized spectrum of 140 kV for 5 energy channels.

Fig. 5:

The reconstructed images at 1×10^5 photons. (a1) - (f1) are the reference image and the images reconstructed by using the SART, TV, LRTV, TDL and LRTP methods at 60 keV (low energy channel). (a2) - (f2) are the corresponding images at 100 keV (high energy channel). The display windows for 60 keV and 100 keV are $[0, 0.035] \text{ mm}^{-1}$ and $[0, 0.03] \text{ mm}^{-1}$, respectively.

Fig. 6:

The reconstructed images at 2.5×10^4 photons. (a1) - (f1) are the reference image and the images reconstructed by using the SART, TV, LRTV, TDL and LRTP methods at 60 keV (low energy channel). (a2) - (f2) are the corresponding images at 100 keV (high energy channel). The display windows for 60 keV and 100 keV are $[0, 0.035] \text{ mm}^{-1}$ and $[0, 0.03] \text{ mm}^{-1}$, respectively.

Fig. 7: close-up views. From left to right are the reference images reconstructed from full-dose projections and the low-dose images reconstructed by SART, TV, LRTV, TDL and LRTP, respectively. From first to fourth rows are the magnified lung ROI indicated in Fig. 5(a1) - (f1), Fig. 5(a2) - (f2), Fig. 6(a1) - (f1) and Fig. 6(a2) - (f2), respectively. The display window is $[0, 0.025] \text{ mm}^{-1}$. The next four rows are the corresponding absolute difference images relative to the reference images.

Fig. 8:

The reconstructed images at 1×10^5 photons. (a1) - (f1) are the reference image and the images reconstructed by using the SART, TV, LRTV, TDL and LRTP methods at 60 keV (low energy channel). (a2) - (f2) are the corresponding images at 100 keV (high energy channel). The display windows for 60 keV and 100 keV are $[0, 0.035] \text{ mm}^{-1}$ and $[0, 0.03] \text{ mm}^{-1}$, respectively.

Fig. 9:

The reconstructed images at 2.5×10^4 photons. (a1) - (f1) are the reference image and the images reconstructed by using the SART, TV, LRTV, TDL and LRTP methods at 60 keV (low energy channel). (a2) - (f2) are the corresponding images at 100 keV (high energy channel). The display windows for 60 keV and 100 keV are $[0, 0.035] \text{ mm}^{-1}$ and $[0, 0.03] \text{ mm}^{-1}$, respectively.

Fig. 10: close-up views. From left to right are the reference images reconstructed from full-dose projections and the low-dose images reconstructed by SART, TV, LRTV, TDL and LRTP, respectively. From first to fourth rows are the magnified lung ROI indicated in Fig. 8(a1) - (f1), Fig. 8(a2) - (f2), Fig. 9(a1) - (f1) and Fig. 9(a2) - (f2), respectively. The display window is $[0, 0.025] \text{ mm}^{-1}$. The next four rows are the corresponding absolute difference images relative to the reference images.

Fig. 11:

The reconstructed image of the animal lung. Left and right columns are the reconstructed image from (100, 120] keV and (60, 120] keV, respectively. The center columns are the corresponding magnified ROIs. From top to bottom are the images reconstructed by SART, TV, LRTV, TDL and the proposed LRTP methods. The display window is [0, 0.03].

Fig. 12:

From left to right are the reference images and the images reconstructed by TP, low-rank, and LRTP methods. From top to bottom are the images at 60 and 100 keV (represent low energy channel and high energy channel), and the display widow is $[0, 0.035]$ and $[0, 0.03]$ mm^{-1} , respectively. The close-up views in the red box are shown in the upper right corner.

TABLE I:

COMPUTATIONAL COSTS OF ALL RECONSTRUCTION METHODS.

Methods	SART	TV	LRTV	TDL	L RTP
Data term	1.2s	1.2s	1.2s	1.2s	1.2s
Regularizer	0s	0.38s	0.41s	9.0s	0.1s

TABLE II:RMSE (10^{-4}), PSNR, SSIM, FSIM INDEX FOR ROI IMAGES.

Metrics/ Photons	Methods	Reconstructed images (Channel Number)				
		1st	2nd	3rd	4th	5th
RMSE/ 1×10^5	SART	8.5458	9.6824	10.8828	11.3202	11.8741
	TV	6.3497	6.1092	6.0243	5.8540	5.9213
	LRTV	6.3153	6.0745	5.9184	5.7840	5.7968
	TDL	6.1146	5.3494	4.8438	4.6099	4.5055
	L RTP	4.2614	3.8496	3.4938	3.5401	4.4229
PSNR/ 1×10^5	SART	29.987	28.414	26.857	26.188	25.433
	TV	32.567	32.414	31.993	31.916	31.477
	LRTV	32.614	32.463	32.147	32.021	31.662
	TDL	32.895	33.567	33.888	33.992	33.850
	L RTP	36.031	36.425	36.726	36.285	34.011
SSIM/ 1×10^5	SART	0.8867	0.8548	0.8273	0.8090	0.7892
	TV	0.9532	0.9544	0.9571	0.9581	0.9563
	LRTV	0.9531	0.9551	0.9575	0.9584	0.9576
	TDL	0.9621	0.9669	0.9702	0.9721	0.9733
	L RTP	0.9801	0.9836	0.9862	0.9845	0.9704
FSIM/ 1×10^5	SART	0.9483	0.9361	0.9191	0.9119	0.9025
	TV	0.9577	0.9577	0.9589	0.9580	0.9584
	LRTV	0.9578	0.9582	0.9596	0.9584	0.9591
	TDL	0.9745	0.9774	0.9794	0.9801	0.9808
	L RTP	0.9894	0.9908	0.9920	0.9911	0.9840
RMSE/ 2.5×10^4	SART	16.8566	18.1488	20.0961	20.8274	21.6946
	TV	7.8624	7.8410	8.4368	8.2620	8.6810
	LRTV	7.7616	7.6820	8.2031	8.0393	8.4015
	TDL	6.3761	6.1628	6.2894	6.2045	6.5700
	L RTP	6.1857	5.8945	5.6697	5.6463	6.3723
PSNR/ 2.5×10^4	SART	24.0871	22.9568	21.5296	20.8934	20.1987
	TV	30.7114	30.2464	29.0683	28.9243	28.1543
	LRTV	30.8234	30.4243	29.3123	29.1617	28.4386
	TDL	32.5314	32.3382	31.6197	31.4120	30.5745
	L RTP	32.7948	32.7249	32.5206	32.2308	30.8398
SSIM/ 2.5×10^4	SART	0.6811	0.6532	0.5915	0.5739	0.5500
	TV	0.9359	0.9391	0.9253	0.9268	0.9222
	LRTV	0.9373	0.9411	0.9293	0.9305	0.9269
	TDL	0.9513	0.9528	0.9442	0.9410	0.9333
	L RTP	0.9551	0.9584	0.9607	0.9595	0.9415

Metrics/ Photons	Methods	Reconstructed images (Channel Number)				
		1st	2nd	3rd	4th	5th
$\text{FSIM}/2.5 \times 10^4$	SART	0.8588	0.8404	0.8176	0.8031	0.7895
	TV	0.9455	0.9448	0.9398	0.9404	0.9333
	LRTV	0.9460	0.9459	0.9417	0.9424	0.9357
	TDL	0.9744	0.9741	0.9713	0.9701	0.9660
	LRTP	0.9747	0.9753	0.9755	0.9747	0.9662

TABLE III:

RMSE (10^{-4}), PSNR, SSIM, FSIM INDEX FOR ROI IMAGES.

Metrics/ Photons	Methods	Reconstructed images (Channel Number)				
		1st	2nd	3rd	4th	5th
RMSE/ 1×10^5	SART	6.5373	7.2168	9.1134	9.8020	10.6390
	TV	6.3068	5.7223	5.9562	6.0101	5.9182
	LRTV	6.4144	5.5096	5.5894	5.4789	5.3151
	TDL	6.7663	5.4743	5.4770	5.2326	4.9444
	L RTP	4.7431	3.8528	3.8517	4.0398	4.2506
PSNR/ 1×10^5	SART	30.624	29.054	27.027	26.270	25.401
	TV	30.922	31.069	30.721	30.518	30.495
	LRTV	30.775	31.398	31.273	31.322	31.429
	TDL	30.311	31.453	31.449	31.721	32.057
	L RTP	33.397	34.505	34.507	33.969	33.370
SSIM/ 1×10^5	SART	0.9435	0.9222	0.8815	0.8666	0.8397
	TV	0.9473	0.9530	0.9513	0.9515	0.9538
	LRTV	0.9450	0.9556	0.9547	0.9571	0.9603
	TDL	0.9540	0.9676	0.9676	0.9708	0.9743
	L RTP	0.9779	0.9831	0.9830	0.9799	0.9753
FSIM/ 1×10^5	SART	0.9661	0.9594	0.9394	0.9252	0.9108
	TV	0.9615	0.9642	0.9638	0.9640	0.9662
	LRTV	0.9594	0.9657	0.9654	0.9676	0.9700
	TDL	0.9667	0.9751	0.9751	0.9772	0.9794
	L RTP	0.9855	0.9899	0.9899	0.9887	0.9869
RMSE/ 2.5×10^4	SART	12.8324	14.2141	17.7186	18.8545	20.6045
	TV	8.7402	8.0306	8.5267	8.2874	8.2825
	LRTV	8.7443	7.8449	8.2372	7.9818	7.9287
	TDL	7.2294	5.8861	5.9060	5.6709	5.3572
	L RTP	6.7290	5.4389	5.4108	5.5173	5.9670
PSNR/ 2.5×10^4	SART	24.7524	23.1660	21.2518	20.5880	19.6600
	TV	28.0881	28.1255	27.6048	27.7279	27.5760
	LRTV	28.0840	28.3287	27.9048	28.0542	27.9552
	TDL	29.7364	30.8239	30.7945	31.0233	31.3606
	L RTP	30.3595	31.5102	31.5552	31.2618	30.4241
SSIM/ 2.5×10^4	SART	0.8154	0.7584	0.6791	0.6490	0.6046
	TV	0.9070	0.9184	0.9136	0.9193	0.9208
	LRTV	0.9065	0.9206	0.9171	0.9228	0.9252
	TDL	0.9457	0.9612	0.9612	0.9644	0.9683
	L RTP	0.9524	0.9653	0.9658	0.9630	0.9517

Metrics/ Photons	Methods	Reconstructed images (Channel Number)				
		1st	2nd	3rd	4th	5th
$\text{FSIM}/2.5 \times 10^4$	SART	0.8995	0.8829	0.8337	0.8186	0.7897
	TV	0.9321	0.9370	0.9341	0.9356	0.9360
	LRTV	0.9311	0.9380	0.9360	0.9382	0.9394
	TDL	0.9646	0.9729	0.9728	0.9747	0.9769
	LRTP	0.9726	0.9794	0.9796	0.9783	0.9716