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Computational Study of the Effect of Homogeneous and Heterogeneous Bubbly Flows on Bulk Gas–Liquid Heat Transfer

A numerical investigation is performed on buoyancy-driven homogeneous and heterogeneous bubbly flows to compare the bulk gas–liquid heat transfer effectiveness for Prandtl (Pr) numbers 0.2–20 and void fractions $\langle \alpha_g \rangle$ 0.3–0.5. For this purpose, transient two-fluid model simulations of bubbles rising in a stagnant pool of liquid are conducted in a rectangular box by applying periodic boundary conditions to all the sides. The temperature difference (ΔT) between gas and liquid phase is averaged over the rectangular box and monitored with respect to time, the heat transfer rate is studied based on the time at which the ΔT tends to zero. The results of numerical study show that at low Pr numbers, faster decay of ΔT is observed for homogeneous flow of bubbles indicating higher heat transfer rate in comparison with the heterogeneous flow of bubbles for the same void fraction. On the contrary, for high Pr numbers, higher heat transfer rate is observed in heterogeneous flow compared to the homogeneous. The comparison of heat transfer behavior between different void fractions for heterogeneous flow show that, for low Pr numbers higher heat transfer rate is achieved for void fraction 0.4 in comparison with void fraction 0.5. And for high Pr numbers, higher heat transfer is observed for void fraction 0.5 in comparison with void fraction 0.4. [DOI: 10.1115/1.4047806]

1 Introduction

Two-phase buoyancy-driven bubbly flow heat transfer rates are 100 times larger than its single phase counterparts [1]. The ability of bubbly flows to provide high heat transfer rates can be attributed to high bubble–liquid contact area and buoyancy-driven bubble agitation of the liquid enabling convective transport of heat. Thus, bubbly flow-driven heat exchange processes or equipment are common in chemical engineering industries, especially in bubble columns. The study of heat transfer in bubbly flows is essential in controlling the heat transfer processes and designing the heat transfer equipment encountered in these industries. There are typically two types of heat transfer scenarios encountered in these equipment. First, the heat transfer from the walls or immersed solid (heating or cooling coils, pipes) to the bulk gas–liquid flow, which is mainly controlled by the agitation caused by the bubbles in the liquid near the walls or close to the immersed solid. Second, the heat transfer at the bulk or bulk gas–liquid heat transfer controlled mainly by gas–liquid contact area and the agitation caused by the bubbles in the liquid. Note that this study is solely focused on the second scenario. However, the heat transfer studies of buoyancy-driven bubbly flows conducted for both these scenarios by several researchers are discussed below.

1.1 Wall or Immersed Solid to Gas–Liquid Heat Transfer.

Konsetov [2] carried out a theoretical analysis on heat transfer from an immersed cooling coil or wall to the bubbling layer. From his analysis, he found that the heat transfer coefficient highly depends on the turbulent fluctuations due to bubble agitation in the liquid and the physicochemical properties of the medium. He deduced a heat transfer coefficient for wall to gas–liquid flow by incorporating the effect of bubble agitation. He accounted for bubble agitation by calculating the amount of total kinetic energy of bubble rise converted into turbulent kinetic energy through an

energy balance approach. Nishikawa et al. [3] performed experimental investigation on heat transfer from wall or immersed cooling coil to the bubbling liquid for Newtonian and non-Newtonian liquids. They concluded that the heat transfer in non-Newtonian liquid is lower compared to Newtonian liquid due to higher strain rate near the wall leading to the dissipation of bubble agitation compared to Newtonian liquid. They developed heat transfer correlations based on their experiments to incorporate the effect of both non-Newtonian and Newtonian fluids based on apparent viscosity method. Hikita et al. [4] performed experimental investigation to determine the influence of liquid properties on the heat transfer rate between the gas–liquid flow and the wall. They carried out experiments using air and various liquids of different physical properties. Based on their experiments, they developed a new dimensionless correlation for heat transfer coefficient for wall to the gas–liquid heat transfer. Verma [5] conducted experimental investigation to study the effect of superficial gas velocity injected from a sparger on wall to gas–liquid heat transfer. They found that although the bubble agitation effect increases with superficial gas velocity, the heat transfer rate diminished due to the trapping of liquid between the bubble and the wall by the rising bubbles. The heat transfer through the trapped liquid film is predominantly due to conduction rather than bubble agitated convection; hence, more the bubbles passing near the wall, greater the area of wall surface affected by the immobile liquid film. They proposed a new correlation for heat transfer coefficient from wall to gas–liquid to account for this inhibition of heat transfer rate by correcting the correlation developed by Deckwer [1]. Avdeev et al. [6] conducted experimental investigation of heat transfer from gas–liquid flow to an immersed cooling surface to study the effect of pressure on gas–liquid to wall heat transfer. Their study suggests that the heat transfer correlation developed by Deckwer [1] at standard atmospheric pressure condition under predicts the heat transfer to the surface by 30%, hence a modified correlation is proposed in their work, which considers the effect of pressure on the heat transfer. Tow and Lienhard [7] carried out experimental investigation of heat transfer from an immersed cylindrical cooling coil to the gas–liquid flow to study the effect of the placement of heater coil for maximum heat transfer

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performance. They developed a semitheoretical correlation for heat transfer coefficient which depends on the height of the cylinder placement, superficial gas velocity of injection, and liquid properties.

Simulation-based study of wall or immersed solid to gas–liquid flow heat transfer is very rare compared to experimental studies; to the best of our knowledge, Larimi and Ramiar [8] studied the influence of single bubble rising in a vertical channel on the heat transfer from the wall to the liquid. They performed multiphase volume of fluid simulations of bubble train rising in a 2D vertical channel for various bubble diameters, bubble numbers, and shapes. They reported significant augmentation of heat transfer from wall to liquid due bubble agitation caused in liquid near the wall.

1.2 Bulk Gas–Liquid Heat Transfer. Kumar et al. [9] performed experimental investigation to study the effect of gas distributor designs on bulk gas–liquid heat transfer. They concluded that heat transfer rates were considerably higher when perforated distributor used in comparison to porous distributor because the larger bubbles produced by the perforated distributor caused enhanced bubble agitation in the liquid compared to smaller bubbles by porous distributor, thereby enhancing heat transfer. Wu et al. [10] performed experimental investigation on a bubble column to study the impact of superficial gas velocity and operating pressure on the heat transfer. According to their experiments, bulk gas–liquid heat transfer rate increased with increase in superficial gas velocity because the tendency of coalescence increases with increase in superficial gas velocity leading to larger bubbles with enhanced bubble agitation as observed also in Ref. [9]. However, with increase in operating pressure, they observed reduction in heat transfer rate due to the absence of coalescence or larger bubbles. Gandhi and Joshi [11] developed a correlation for heat transfer coefficient for bulk gas–liquid heat transfer using statistical method called support vector regression. The correlation incorporated the effect of various operating parameters, sparger geometry, operating temperature, pressure, superficial gas velocity, etc. They concluded that the proposed support vector regression-based correlation showed improved predictions of heat transfer coefficient over other traditional empirical correlations in the literature. Jhawar and Prakash [12] performed experimental investigation on a bubble column to study the effect of column diameters on heat transfer. They observed an increase in the heat transfer rate with increasing column diameter, which they attributed toward the increase in rise velocity of bubbles causing larger bubble agitation promoting heat transfer. Abdulmohsin et al. [13] carried out experiments on a bubble column to study the effect of superficial gas velocity on both bulk gas–liquid heat transfer and wall to gas–liquid heat transfer. They observed that the bulk gas–liquid heat transfer increased with superficial gas velocity and reached a plateau. The increased bulk heat transfer rate with superficial gas velocity was attributed toward the improved mixing of liquid caused at high superficial gas velocity. Moreover, they observed that the heat transfer rate in the bulk was significantly higher than that at the wall at higher superficial gas velocity because the bubbles started to move close to the center of the column than close to the walls leading to greater bubble agitation at the center than at the walls. Zizka et al. [14] performed experimental investigation on bulk gas–liquid heat transfer for two different bubble injection patterns SC and CC3 for various superficial gas injection velocities. Low temperature gas was injected in a pool of hot liquid using SC and CC3 patterns, and heat transfer rates were measured. They observed higher transfer rate for SC pattern for low superficial gas velocities compared to CC3 pattern and vice versa for high superficial gas velocities.

Literature review on bubbly flow heat transfer shows a lack of simulation studies on bulk gas–liquid heat transfer based on homogeneous and heterogeneous flow and its dependence on gas void fraction and Prandtl numbers. Our study in Ref. [15] shows

that the homogeneous flow sustains for lower global gas or bubble void fractions and as the void fraction is increased to a certain value, buoyancy-induced instabilities appear in the flow leading to the formation of bubble clusters or swarms, consequently transitioning from homogeneous to heterogeneous bubbly flow. The homogeneous flow has bubbles rising in a uniform fashion (refer to Fig. 1) with little to no liquid mixing due to bubble agitation. On the contrary, heterogeneous flow consists of large or meso-scale liquid-phase eddies due to buoyancy-induced instabilities with bubble swarms rising in a chaotic fashion. The turbulent flow behavior in both these flows is studied and the difference in the characteristics is established in Ref. [15]. The effectiveness of bulk gas–liquid heat transfer in these flow scenarios will depend on two factors: (1) The effective contact area of bubble and liquid and (2) liquid mixing caused due to bubble agitation. The homogeneous flow with little to no mixing due to bubbles offers low heat transfer rate in terms of bubble agitation; however, in terms of contact area, it can offer better performance than heterogeneous flow. The heterogeneous flow, although having poor performance in terms of gas–liquid contact area, may offer better heat transfer rate due to the presence of complex eddies caused by bubble swarms. Therefore, in this work, we conduct numerical studies to investigate the difference in the heat transfer effectiveness in both these flow scenarios using the widely used [17–19] two-fluid model. Even though the two-fluid model is widely used, the flow predictions by two-fluid model are plagued by the nonhyperbolicity of the partial differential equations [20]. In this study, we adopt a two-fluid model with a bubble dispersion model discussed in Ref. [20] to resolve the nonhyperbolicity.

The numerical study is conducted by performing transient highly resolved two-fluid model simulations as performed in Ref. [15] for bubbles rising in a rectangular box with periodic boundary conditions for five different Prandtl numbers (0.2–20) and three different gas fractions (0.3, 0.4, and 0.5). The gas–liquid temperature difference (ΔT) response from the simulation is averaged over the rectangular box and monitored with respect to time. The heat transfer behavior difference in the flows is assessed based on the time at which the ΔT decays to zero for each case. The results from the simulations are assessed and the underlying physics is discussed.

The remainder of the paper is organized as follows: In Sec. 2, two-fluid model with interphase closures used in the study is discussed. In Sec. 3, Simulation setup and the parameters used in the simulation are discussed. In Sec. 4, how temperature evolution for homogeneous bubbly flow is calculated by simplifying the two-phase energy equation is discussed. In Sec. 5, temperature difference evolution from computational fluid dynamics (CFD) simulation is compared with the analytical solution for homogeneous flow. In Sec. 6, results obtained from the simulations for various Prandtl numbers and void fractions are discussed. In Sec. 7, conclusions are derived.

2 Two-Fluid Model for Bubbly Flow

Two-fluid model is widely used to predict bubbly flows due to its low computational cost. Two-fluid model is obtained by ensemble averaging the instantaneous conservation equations [21], as a consequence of averaging, the momentum and energy equations have unclosed terms appearing as interphase force terms drag ($\mathbf{M}_D$), virtual mass ($\mathbf{M}_V$), lift ($\mathbf{M}_L$), and bubble dispersion ($\mathbf{M}_B$) in the momentum Eq. (2) and heat exchange term ($\mathbf{Q}_I$) in the energy Eq. (3). The predictive capability of the two-fluid model is affected by unphysical fluctuations due to lack of hyperbolicity [20]; therefore, a bubble dispersion term ($\mathbf{M}_B$) is added to the momentum along with other interphase force terms used in the literature.

Continuity

$$\frac{\partial \rho_\phi \alpha_\phi}{\partial t} + \nabla \cdot \rho_\phi \alpha_\phi \mathbf{u}_\phi = 0 \quad (1)$$

Fig. 1 Bubble clustering (right) can cause different bulk heat transfer behavior compared to homogeneous bubbles (left), picture (top) adapted from Ref. [16]

Momentum

$$\frac{\partial \rho_\phi \alpha_\phi \mathbf{u}_\phi}{\partial t} + \nabla \cdot \rho_\phi \alpha_\phi \mathbf{u}_\phi \mathbf{u}_\phi = -\alpha_\phi \nabla p_1 + \nabla \cdot \boldsymbol{\sigma}_\phi + \rho_\phi \alpha_\phi \mathbf{g} + \mathbb{I}_\phi (\mathbf{M}_D + \mathbf{M}_V + \mathbf{M}_L + \mathbf{M}_B) \quad (2)$$

Energy

$$\frac{\partial \rho_\phi c_{p\phi} \alpha_\phi T_\phi}{\partial t} + \nabla \cdot \rho_\phi c_{p\phi} \alpha_\phi \mathbf{u}_\phi T_\phi = \nabla \cdot \mathbf{q}_\phi + \mathbb{I}_\phi \mathbf{Q}_1 \quad (3)$$

where the viscous stress tensor is modeled by

$$\boldsymbol{\sigma}_\phi = \alpha_\phi \mu_\phi \left[\nabla \mathbf{u}_\phi + (\nabla \mathbf{u}_\phi)^T - \frac{2}{3} (\nabla \cdot \mathbf{u}_\phi) \mathbf{I} \right] \quad (4)$$

Conductive heat flux is modeled by

$$\mathbf{q}_\phi = \alpha_\phi k_\phi \nabla T_\phi \quad (5)$$

and $\phi = g, l$ corresponds to gas and liquid, respectively. The function $\mathbb{I}_\phi$ is 1 for $\phi = l$ and -1 for $\phi = g$. In these equations, α_ϕ is the phase volume fraction and $\mathbf{u}_\phi$ is the phase velocity. The phase density ρ_ϕ is assumed to be constant (refer to Table 2). The effect of pseudo-turbulent fluctuations in the liquid phase, which is predominantly found in homogeneous flow, mainly caused due to the wakes of the bubbles is accounted as an additional viscosity μ_{PT} [22], which is widely used in two-fluid models to predict bubbly flows

$$\mu_{\text{leff}} = \mu_l + \mu_{PT} \quad (6)$$

where $\mu_{PT} = 0.6 \rho_l \alpha_g d_b |u_g - u_l|$

And note that, as discussed in Ref. [15], the large-scale buoyancy-driven turbulent eddies found in heterogeneous flow are resolved by the two-fluid model.

Similarly, a pseudo-turbulent conductivity model is added to account for the pseudo-turbulent transport of heat. The conductivity model is extended from pseudo-turbulent viscosity model using a turbulent Prandtl number as shown below:

$$k_{\text{leff}} = k_l + \frac{c_{pl} \mu_{PT}}{Pr_T} \quad (7)$$

where $Pr_T = 0.7$

The pseudo-turbulent conductivity is modeled based on a similar approach followed to model turbulent thermal diffusivity for single phase flows with a turbulent Prandtl number $Pr_T = 0.7$.

The interphase closures for the two-fluid model given in Table 1 are discussed in Sec. 2.1.

2.1 Interphase Models

2.1.1 Drag Force $\mathbf{M}_D$. This work involves the flow of bubbles in high Re_b inertial flow; hence, an inertial drag law [23–25] is adopted as given in Table 1. The drag coefficient C_D is often modeled as a function of Re_b [24,26] developed for particles. However, in bubbly flows, bubbles have a tendency to deform based on size and the flow properties; hence, correlations that account for the effect of deformation on the drag have to be adopted. Therefore, drag correlation developed by Tomiyama [27] for bubbles is adopted in this work.

2.1.2 Virtual-Mass Force $\mathbf{M}_V$. The virtual-mass force accounts for the interphase interactions resulting from the acceleration of displaced fluid in the neighborhood of a bubble. The model form given in Ref. [28] is adopted in this work as given in Table 1. The model coefficient $C_{VM} = 0.5$ is deduced

Table 1 Interphase models

Interphase	Model
$\mathbf{M}_D$	$\alpha_g \rho_l C_D \mathbf{u}_g - \mathbf{u}_l (\mathbf{u}_g - \mathbf{u}_l)$ $C_D = \max \left\{ \min \left[\frac{24}{\text{Re}_b} (1 + 0.15 \text{Re}_b^{0.687}), \frac{48}{\text{Re}_b} \right], \frac{8}{3} \frac{\text{Eo}}{\text{Eo} + 4} \right\}$ $\text{Eo} = \frac{g(\rho_l - \rho_g) d_b^2}{\sigma}$
$\mathbf{M}_V$	$\alpha_g C_{VM} \rho_l \left(\frac{D\mathbf{u}_g}{Dt} - \frac{D\mathbf{u}_l}{Dt} \right)$ $C_{VM} = 0.5$
$\mathbf{M}_L$	$\alpha_g \rho_l C_L (\mathbf{u}_g - \mathbf{u}_l) \times \nabla \times \mathbf{u}_l$ $C_L = \begin{cases} \min[0.288 \tanh(0.121 \text{Re}), f(\text{Eo})] & \text{Eo} \leq 4 \\ f(\text{Eo}), & 4 \leq \text{Eo} \leq 10.7 \end{cases}$ $f(\text{Eo}) = 0.00105 \text{Eo}^3 - 0.00159 \text{Eo}^2 - 0.0204 \text{Eo} + 0.474$
$\mathbf{M}_B$	$-\alpha_g \rho_l C_D \mathbf{u}_g - \mathbf{u}_l \frac{\delta_{\text{dis}}}{\alpha_g \alpha_l} \nabla \alpha_g$ $\delta_{\text{dis}} = C_{\text{dis}} d_p \mathbf{u}_g - \mathbf{u}_l \sqrt{\alpha_g \alpha_l}$
$\mathbf{Q}_I$	$K(T_g - T_l)$ $K = \frac{6\alpha_g k_l \text{Nu}}{d_b^2}$ $\text{Nu} = (7 - 10\alpha_l + 5\alpha_l^2)(1 + 0.7 \text{Re}_b^{0.2} \text{Pr}^{0.33}) + (1.33 - 2.4\alpha_l + 1.2\alpha_l^2) \text{Re}_b^{0.7} \text{Pr}^{0.33}$

Table 2 Physical parameters used in simulations

Property	Value
box height	0.7 m
box width	0.16 m
d_b	4 mm
ρ_g	1.2 kg/m ³
μ_g	1.983×10^{-3} Pa·s
ρ_l	1000 kg/m ³
μ_l	10^{-3} Pa·s
σ	0.072 N/m
g	9.81 m/s ²
k_g	$0.0026 \text{ W m}^{-1} \text{ K}^{-1}$
c_{pg}	$1007 \text{ J kg}^{-1} \text{ K}^{-1}$
c_{pl}	$4195 \text{ J kg}^{-1} \text{ K}^{-1}$

theoretically for spherical rigid particles in potential flow by Maxey and Riley [29]. This model coefficient is used in majority of the works [30–32] to successfully predict bubbly flows; hence, this value is used in this study.

2.1.3 Lift Force $\mathbf{M}_L$. The lift force accounts for the motion of the bubbles due to local velocity gradients of the liquid phase. We adopt the lift force formulation (see Table 1) given in Ref. [33], which is used in many works [19,31,34,35] to predict bubbly flow behavior. The lift coefficient (C_L) in these works was assumed to be constant 0.5. However, the experimental investigation of Tomiyama et al. [36] suggests that lift coefficient may deviate from 0.5 based on the fluid properties and bubble size; hence, in this study, lift coefficient (see Table 1) modeled in Ref. [36] is employed.

2.1.4 Bubble Dispersion $\mathbf{M}_B$. Bubble dispersion accounts for the hydrodynamic interactions between bubbles. In addition to the microscale physics accounted by the bubble dispersion, the term also corrects the nonhyperbolic behavior of the two-fluid model as discussed in Ref. [20], thereby removing nonphysical fluctuations in the flow, which would corrupt turbulent flow characteristics. A

simple bubble-dispersion model (see Table 1) is discussed in Ref. [37] and $C_{\text{dis}} = 1.3$ is deduced in Ref. [20]. The coefficient in front of the volume fraction gradient can be considered as a model for the pseudo-turbulent pressure generated by bubble–bubble interactions, which are not resolved in the two-fluid model.

2.1.5 Interphase Heat Transfer $\mathbf{Q}_I$. The interphase heat exchange between gas and liquid phase in the two-fluid model is modeled as a function of temperature difference between the phases and the heat transfer coefficient. The heat transfer coefficient is modeled using the correlation by Ranz and Marshall [38] in many works [39–43] in order to successfully predict the gas–liquid heat transfer in bubbly flows. Advanced models for swarm heat transfer coefficient are scarce, Gunn [44] developed a correlation for heat transfer coefficient with an eye on particle cluster heat transfer. This correlation is used in the sophisticated multiphase code MFIx [45] to predict two-phase heat transfer using two-fluid model. Moreover, a recent study on the effect of particle clusters on the gas-particle heat transfer is conducted by Guo and Capecelatro [46] using this correlation. We adopt this correlation to predict heat transfer exchange coefficient as given in Table 1.

Note that the bubble size change due to coalescence and breakup is not modeled in this study, it requires coupling and solving population balance equation with two-fluid model, which is numerically challenging and computationally expensive; therefore, this study could be considered as first step toward understanding bubbly swarm heat transfer.

3 Simulation Setup

The simulation is performed using the finite volume-based open-source code OPENFOAM [47]. The two-fluid model equations were solved using a second order in space and time scheme. The capabilities of reacting TwoPhaseEulerFoam base solver are extended to simulate the bubbly swarm heat transfer by adding interphase exchange terms to momentum and heat transport equations.

Fig. 2 Computational domain

The computational domain (Fig. 2) used to study the gas–liquid heat transfer is a rectangular three-dimensional box with the dimensions specified in Table 2. The z direction is the vertical direction in which the bubble flows against the gravity due to buoyancy force, and x, y directions are horizontal. Periodic boundary conditions are applied on all sides of the box in order to eliminate the influence of wall on the flow and heat transfer characteristics. However, the absence of wall leads to the continuous fall of liquid in the direction of gravity (z -direction) due to its weight, which is undesirable for the numerical study. Therefore, a time-dependent mean pressure gradient term is added to the liquid phase momentum to balance the weight of the liquid to achieve a scenario where bubbles are rising in a stagnant pool of liquid. The influence of the periodic boundary condition on the turbulent flow characteristics is eliminated by choosing a domain size based on the velocity pair correlation function [15].

The properties of both the phases and other simulation parameters are given in Table 2. The gas–liquid heat transfer study is performed for initial void fractions as given in Table 3, where the flow falls in the heterogeneous flow for both of those void fractions as noted in Ref. [15]. The initial gas or void fraction (0.4 or 0.5) at $t = 0$ is patched or distributed uniformly into the rectangular box using subroutine “SETFIELDS” in OPENFOAM [47] at $t = 0$ as shown in Fig. 3. Furthermore, the study is performed for four different liquid Prandtl numbers as given in the Table 3. The Prandtl numbers are varied by changing liquid thermal conductivity k_l .

The flow is allowed to evolve at a constant gas and liquid temperature of 25 C till the flow is fully developed or becomes statistically stationary (refer to Ref. [15]), then a temperature of 225 C is introduced into the liquid phase in the entire computational box. Then the volume-averaged temperature difference ΔT between the gas and the liquid phase over the computational box is monitored with respect to time for different void fractions and Prandtl numbers till ΔT reaches nearly zero. The ΔT time evolution for homogeneous flow with the same void fractions and Prandtl numbers is deduced analytically in Sec. 4.

Note that the grid convergence study of the OPENFOAM solver used in this study is already established in our previous work [15]; the turbulent Reynolds stress, mean bubble rise velocity, and probability distribution of gas volume fraction are analyzed for different grid resolutions. Based on the study, a grid resolution of $d_b/2$ is adopted and a time resolution of 1×10^{-7} s is used in this study.

4 Temperature Evolution for Homogeneous Flow

The homogeneous flow of bubbles in a periodic box can be considered to have no spatial gradients. The spatial gradient terms convection ($\nabla \cdot \rho_\phi c_{p\phi} \alpha_\phi \mathbf{u}_\phi T_\phi$) and conduction ($\nabla \cdot \mathbf{q}_\phi$) can be

Table 3 Void fractions and Prandtl numbers studied

$\langle \alpha_g \rangle$	Pr
0.4	0.2
	0.7
	2
	20
0.5	0.2
	0.7
	2
	20

Fig. 3 Instantaneous gas fraction of heterogeneous flow after 30 s of simulation time for initial gas fraction $\langle \alpha_g \rangle = 0.5$ and $\langle \alpha_g \rangle = 0.4$

eliminated from the Energy equation (Eq. (3)) for the homogeneous flow of bubbles and $\rho_\phi, c_{p\phi}, \alpha_\phi$ can be assumed to be constants. The energy equation for the homogeneous case can be rewritten as

Gas phase

$$\frac{dT_g}{dt} = \frac{K}{\rho_g c_{pg} \alpha_g} (T_l - T_g) \quad (8)$$

Liquid phase

$$\frac{dT_l}{dt} = \frac{K}{\rho_l c_{pl} \alpha_l} (T_g - T_l) \quad (9)$$

Equations (8) and (9) for the gas and the liquid phase, respectively, can be solved simultaneously to obtain the temperature evolution of gas or liquid as a function of time for homogeneous flow of bubbles. The ordinary differential equation (ODE) solver ODE45 of MATLAB [48] is used to solve the differential equations for gas and liquid temperature. The coefficient K in Eqs. (8) and (9) is calculated from the expression in Table 1 using the Nusselt number (Nu) correlation. In the Nu expression, Pr is known from the fluid and thermophysical properties and the bubble Reynolds number (Re_b) is estimated based on the rise velocity for homogeneous flow in Ref. [15] and liquid properties in Table 2. α_g and Pr are varied in these equations to understand the dependence of temperature decay in the homogeneous flow with respect to these

Fig. 4 ΔT evolution: CFD versus Analytical solution for $\langle \alpha_g \rangle = 0.3$

Fig. 5 Instantaneous gas fraction of homogeneous flow after 30 s of simulation time for initial gas fraction $\langle \alpha_g \rangle = 0.3$

Fig. 6 Instantaneous velocity vector

parameters. Note that, in all the heat transfer comparison study performed in the current work, the temperature-decay curve for homogeneous flow at void fractions 0.4 and 0.5 is obtained from the solution of Eqs. (8) and (9) and heterogeneous flow for the same void fractions are obtained from the simulations.

5 Validation: Computational Fluid Dynamics Versus Analytical Solution for Homogeneous Flow

The ΔT decay curve obtained from the simulation and analytical solution for homogeneous flow of bubbles at $\langle \alpha_g \rangle = 0.3$ is reported in Fig. 4. Note that analytical solution in Sec. 4 is only valid for homogeneous bubbly flow not for heterogeneous due to the assumption of zero spatial gradient. From our previous study, we concluded that the flow remains homogeneous for $\langle \alpha_g \rangle = 0.3$; therefore, the ΔT decay for $\langle \alpha_g \rangle = 0.3$ obtained from the analytical solution is compared with the two-fluid model simulation. The results show that the decay curve from the simulation is in good agreement with the analytical solution. The time at which the thermal equilibrium is attained is the same for the simulation and analytical solution.

6 Results and Discussion

6.1 Two-Fluid Model Prediction of Instantaneous Gas Fraction: homogeneous Versus Heterogeneous Bubbly Flow.

The instantaneous gas fraction contour for initial $\langle \alpha_g \rangle = 0.3$ after 30 s of simulation is reported in Fig. 5. The instantaneous gas fraction contours for initial $\langle \alpha_g \rangle = 0.5$ and 0.4 after 30 s of simulation is reported in Fig. 3 (please refer to alpha.gas colorbar in Fig. 5). Instantaneous gas fraction heterogeneity after 30 s indicates the formation of bubble swarms due to buoyancy instability. This bubble swarm flow leads to turbulence resulting in liquid agitation. Dense bubble swarms are observed for $\langle \alpha_g \rangle = 0.5$ compared to 0.4 leading to more agitation for $\langle \alpha_g \rangle = 0.5$. However, lesser area of the bubbles is in contact with the liquid for $\langle \alpha_g \rangle = 0.5$ compared to 0.4. Heat transfer in these flows will depend on both liquid mixing and the effective contact area of bubbles and liquid. The bulk heat transfer effectiveness of both these void fractions is compared among each other and also with its homogeneous counterparts for various Prandtl numbers in Secs. 6.4–6.7.

Fig. 7 ΔT contours at Prandtl numbers 0.2, 0.7 and 2 for $\langle \alpha_g \rangle = 0.5$ at time $t = 0.06$ ms

6.2 Two-Fluid Model Predicted Instantaneous Velocity Vectors: Homogeneous Versus Heterogeneous. The Eulerian gas-phase velocity vectors for homogeneous and heterogeneous flow are reported in Fig. 6. It can be noted that the homogeneous flow velocity vectors in the absence of bubble swarms show a straight upward motion of the bubbles uniformly with no lateral mixing. However, the heterogeneous flow with the presence of buoyancy-induced instabilities causes the bubbles to move randomly causing agitation of the liquid. This difference in flow behavior in homogeneous and heterogeneous flow indicates different mode of convective transport of heat in these flows.

Fig. 8 ΔT evolution for different Prandtl numbers for $\alpha_g = 0.5$

6.3 Heterogeneous ΔT Contours for Different Pr Numbers. The contours of heterogeneous ΔT from the simulation for Pr numbers 0.2, 0.7, and 2 for void fraction 0.5 after time 0.06 ms are reported in Fig. 7. The heterogeneity in the ΔT for all Pr numbers due to the bubble swarm formation for void fraction 0.5 is evident from the contours. At Pr 0.2, small red patches of high ΔT can be observed; however, at Pr 0.7, larger red patches of high ΔT can be observed, which becomes more prominent as the Pr number increases to 2. As the Pr number increases, the liquid conductivity

(a)

(b)

(c)

(d)

Fig. 9 ΔT evolution at different Prandtl numbers for $\langle \alpha_g \rangle = 0.5$: (a) Pr—0.2, (b) Pr—0.7, (c) Pr—7, and (d) Pr—20

becomes smaller and the heat transfer between gas and liquid becomes less efficient leading to large regions of high ΔT patches.

6.4 Comparison of ΔT Evolution for Different Pr Numbers for Heterogeneous Flow. The evolution of ΔT for $\langle \alpha_g \rangle = 0.5$ for various Pr numbers is reported in Fig. 8. The temperature difference ΔT decays to zero faster for low Pr numbers 0.2 and 0.7; however, the temperature difference decay is delayed at high Pr numbers. Low Pr number corresponds to high thermal diffusion; hence, ΔT decays faster. At the limit high Pr numbers, the heat transfer is mainly controlled by convection rather than thermal diffusion leading to slower ΔT decay.

6.5 Comparison of ΔT Evolution in Homogeneous and Heterogeneous Flow. The ΔT decay for homogeneous and heterogeneous flow of bubbles for different Pr numbers is reported in Fig. 9. It is interesting to note that the ΔT decay for homogeneous flow of bubbles is faster compared to heterogeneous flow of bubbles for low Pr numbers; however, the behavior changes for high Pr numbers, the ΔT decay happens faster in heterogeneous flow of bubbles compared to homogeneous flow. At low Pr numbers, in the thermal diffusion dominated flow, the homogeneous flow of bubbles provides high heat transfer area compared to heterogeneous flow of bubbles leading to faster ΔT decay. At high Pr numbers, the heat transfer is primarily controlled by turbulent

Fig. 10 Comparison of ΔT evolution at different void fractions for Prandtl numbers 0.2 and 20: (a) Pr=0.2 and (b) Pr=20

Fig. 11 Comparison of heat transfer rates for different Prandtl numbers and void fractions: (a) heterogeneous versus homogeneous; void fraction 0.5 and (b) heterogeneous void fraction 0.5 versus 0.4

convection, homogeneous case is turbulence free. However, the heterogeneous flow is highly turbulent due to the bubble swarms agitating the liquid which improves mixing and therefore ΔT decay occurs faster compared to homogeneous flow.

6.6 Comparison of ΔT Evolution in Heterogeneous Flow for Different Void Fractions. The ΔT decay for heterogeneous flow of bubbles for different void fractions for low and high Pr number is reported in Fig. 10. It is interesting to note that the ΔT decay, for void fraction 0.4, is faster compared to void fraction 0.5 for low Pr numbers. The degree of separation of bubbles into bubble swarms is small for void fraction 0.4 compared to 0.5, leading to larger gas–liquid contact area, consequently larger gas–liquid heat transfer compared to void fraction 0.4. On the contrary, the ΔT decay for void fraction 0.5 is faster compared to void fraction 0.4 for high Pr numbers. As the Prandtl number increases, the heat transfer becomes more convection dominated, the liquid mixing or bubble agitation due to turbulent eddies in void fraction 0.5 is higher than 0.4; hence, higher heat transfer rate is observed for void fraction 0.5.

6.7 Heat Transfer Rate in Homogeneous and Heterogeneous Case for Different Pr Numbers. Heat transfer rate from liquid to gas phase is approximated based on Eq. (10). The net heat transfer rate based on Eq. (10) is calculated for different Pr numbers in the heterogeneous and homogeneous flow and reported in Fig. 11(a). For $Pr < 2$ the heat transfer rate is significantly higher for the homogeneous flow of bubbles compared to the heterogeneous flow. For $Pr > 2$, the heat transfer rate is smaller for homogeneous flow of bubbles compared to heterogeneous flow; however, the difference in the heat transfer rate between homogeneous and heterogeneous flow for $Pr > 2$ is not as high as $Pr < 2$. The heat transfer rate in the heterogeneous flow for $\langle \alpha_g \rangle = 0.4$ and 0.5 for different Pr numbers is reported in Fig. 11(b). For low Pr numbers

(<2), the heat transfer rate for $\langle\alpha_g\rangle = 0.4$ is noticeably higher than $\langle\alpha_g\rangle = 0.4$

$$Q \sim \frac{m_g c_{pg} \Delta T}{\tau_{\Delta T \rightarrow 0}} \quad (10)$$

$$m_g = \alpha_g \rho_g V \quad (11)$$

where m_g is the mass of the gas inside the computational domain, α_g is 0.5, V is the volume of the computational domain, $\tau_{\Delta T \rightarrow 0}$ is the time scale for the ΔT to decay to 0 for different cases.

The degree of segregation of bubbles is smaller for 0.4 compared to 0.5, resulting in better bubble–liquid contact and higher heat transfer. For high Pr numbers (>2), since the transport is predominantly due to convection, higher heat transfer rate is observed for 0.5 compared 0.4 due to higher turbulent intensity.

7 Conclusions

A numerical experiment is conducted using two-fluid model to study the bulk gas–liquid heat transfer effectiveness of homogeneous and heterogeneous bubbly flows. Note that the bubble size change due to bubble coalescence or breakup effect is not incorporated in the current simulation framework due to numerical challenges. This study can be considered as first step toward understanding bubbly heat transfer due to swarms. High fidelity two-fluid model coupled with population balance accounting for bubble coalescence and breakup can be used in the future to better understand the heat transfer behavior.

The average temperature difference between gas and the liquid is monitored from the simulation with respect to time for various Prandtl numbers and void fractions. The average temperature difference decay for the homogeneous flow is computed by solving the transient energy equations in MATLAB. The results show that at low Pr numbers, faster decay is observed for homogeneous flow of bubbles indicating higher heat transfer in comparison with the heterogeneous flow for the same void fraction. The formation of bubble swarms in the heterogeneous flow reduces the effective surface area of heat transfer compared to homogeneous flow leading to lower heat transfer in the heterogeneous flow compared to homogeneous flow. On the contrary, for high Pr numbers, higher heat transfer is observed in heterogeneous flow compared to homogeneous flow. At high Pr numbers, heat transfer is mainly controlled by turbulent convection, which is very high in the heterogeneous flow compared to homogeneous flow; hence, the effective surface area, which played a major role in increased heat transfer for homogeneous flow at low Pr numbers, does not play much role in increased heat transfer for high Pr numbers.

The results of comparison of ΔT decay between different void fractions for heterogeneous flow of bubbles show that higher heat transfer is observed for void fraction 0.4 in comparison with void fraction 0.5 for low Pr numbers. This can be attributed toward the lower degree of segregation for void fraction 0.4 compared to 0.5 leading to higher gas–liquid contact area. However, higher heat transfer is observed for void fraction 0.5 for higher Pr numbers in comparison with void fraction 0.4. This increase in heat transfer can again be attributed to higher turbulent mixing of the flow at void fraction 0.5 compared to 0.4.

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Nomenclature

b = bubble phase
 B = bubble dispersion

C = momentum exchange coefficient
 c_p = specific heat capacity
 d = diameter
 D = drag
 Eo = Eotvos number
 g = gravity
 g = gas
 k = thermal conductivity
 K = heat exchange coefficient
 l = liquid
 L = lift
 m = mass
 M = interphase momentum exchange
 Nu = Nusselt number
 Pr = Prandtl number
 q = heat flux
 Q = interphase heat exchange
 Re = Reynolds number
 t = time
 T = temperature
 u = velocity
 V = virtual mass
 V = volume of the computational domain
 α = volume fraction
 μ = viscosity
 ρ = density
 σ = surface Tension
 ϕ = phase indicator
 ∇ = gradient
 $\langle \rangle$ = volume average

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