Published in final edited form as:

Sens Imaging. 2018 ; 19(1): . doi:10.1007/s11220-018-0200-4.

Comparison Study of Regularizations in Spectral Computed Tomography Reconstruction

Morteza Salehjahromi¹, Yanbo Zhang¹, Hengyong Yu¹

¹Department of Electrical and Computer Engineering, University of Massachusetts Lowell, Lowell, MA 01854, USA

Abstract

The energy-resolving photon-counting detectors in spectral computed tomography (CT) can acquire projections of an object in different energy channels. In other words, they are able to reliably distinguish the received photon energies. These detectors lead to the emerging spectral CT, which is also called multi-energy CT, energy-selective CT, color CT, etc. Spectral CT can provide additional information in comparison with the conventional CT in which energy integrating detectors are used to acquire polychromatic projections of an object being investigated. The measurements obtained by X-ray CT detectors are noisy in reality, especially in spectral CT where the photon number is low in each energy channel. Therefore, some regularization should be applied to obtain a better image quality for this ill-posed problem in spectral CT image reconstruction. Quadratic-based regularizations are not often satisfactory as they blur the edges in the reconstructed images. As a result, different edge-preserving regularization methods have been adopted for reconstructing high quality images in the last decade. In this work, we numerically evaluate the performance of different regularizers in spectral CT, including total variation, non-local means and anisotropic diffusion. The goal is to provide some practical guidance to accurately reconstruct the attenuation distribution in each energy channel of the spectral CT data.

Keywords

Spectral CT; Iterative reconstruction; Regularization; Computed tomography

1 Introduction

In the computed tomography (CT) field, there are two main groups of X-ray detectors: energy integrating and energy discriminating detectors. The conventional CT makes use of energy integrating detectors to measure projection data from a wide energy range. The projection data can be used to estimate the linear attenuation coefficients of the object being investigated. On the other hand, in a spectral CT system, detections in different energy levels with narrower energy ranges could be achieved by utilizing energy discriminating detectors (also called photon counting detectors) [1]. In other words, instead of having one projection in the conventional CT, multiple projections are produced in spectral CT with narrower energy ranges for a fixed view. Each one of these multi-energy projections can be used to

make an image which is called the channel image. By looking at narrower energy ranges, we are able to make a better distinction between different materials compared with the conventional CT.

The energy discriminating detectors possess a capability to count the received photon number and also assign them to several different energy levels/channels by combining appropriate image post-processing techniques. By utilizing the same system matrix for each energy channel, we are able to use the corresponding projections to reconstruct the energy-dependent attenuation coefficients. These projections in different energy ranges are not independent. In fact, they are highly correlated. Because the channel images are reconstructed from these correlated projections, the channel images are also correlated, and one could take advantage of the correlation of projections using the corresponding correlation in channel images. As a result, one has to consider all the projections simultaneously to gain the potential advantages of this energy-dependent information.

The real measurements obtained from an X-ray CT scanner are noisy, and the spectral CT image reconstruction sometime is also an ill-posed problem. A regularizer is usually applied to obtain a better image quality with less noise. In other words, a satisfactory solution could be achieved for this inverse problem by utilizing regularizations. Quadratic regularizers trend to smooth the edges, and they result in loss of important image features. Therefore, quadratic regularization is not often satisfactory as it penalizes its large argument severely and makes the filtered image blurry. Several different non-quadratic regularizations have been developed to overcome the aforementioned drawback. In 2002, Yu et al. [2] proposed a piecewise penalty function (Huber function) which increases more slowly than the quadratic function for larger arguments. Since then, Huber penalty has also been used as an edge preserving regularization in medical image processing applications [3, 4]. Perona and Malik proposed an Anisotropic diffusion (AnDi) regularization where a weighted Gaussian filtering is applied by computing the gradient vector at each pixel [5]. The AnDi has been used for denoising in the medical image processing field [6, 7]. In 2011, Yang et al. used weighted AnDi as an edge preserving filter to reduce noise in CT images. In 2005, Buades et al. proposed the non-local means (NLM) algorithm for image denoising which was widely used in image and medical image denoising later. In 2011, Huang et al. [8] used the NLM regularization in iterative CT reconstruction. They first reconstructed an intermediate image by the projection onto convex sets (POCS) method, and then they applied the NLM regularization in each iteration step. The edge-preserving denoising total variation (TV) was first proposed by Rudin, Osher and Fatemi for image denoising in 1992 [9]. The TV based regularization has been widely used for medical image denoising in the last decade (see [10-12]). In 2012, Xu et al. [13] applied the TV regularization in spectral CT. They reconstructed each channel independently using a statistical iteration reconstruction (SIR) method and then applied the TV regularization to reduce noise. In 2011, Correia et al. [14] compared different regularizations such as Huber, Tukey [15] and AnDi using the peak signal-to-noise ratio (PSNR) metric in optical tomography.

To reconstruct the original attenuation coefficients of a 2D object, we model the imaging object as an unknown 3D data array $\mathbf{x}$ where its first and second dimensions are spatial and the third dimension shows different energy levels. Because the 2D images in various energy

levels are obtained from the same object, they are highly correlated. This energy dependent information could be extracted by recovering a low-rank matrix of the 3D data array, and the low rank solution can be reached by a nuclear norm minimization [16]. This minimization can also be solved by a singular value thresholding algorithm. Particularly, three different matrices can be obtained by arranging all the elements of the 3D data into 2D data array. Then, the low rank minimization could be applied to these matrices in terms of nuclear norm. Finally, we can obtain the low rank 3D data array by rearranging each matrices into the 3D data array and average them out [17].

In this paper, we will numerically evaluate the performance of spectral CT iterative reconstruction algorithms with different spatial regularizations (e.g. TV, AnDi, and NLM) with/without low rank regularization. The goal is to provide some guidance for practical applications. The rest of this paper is organized as follows. In Sect. 2, spectral CT reconstruction, low rank and three representative regularizations are briefly described. Section 3 describes the experiment and the parameters in the numerical simulations. Section 4 shows the simulation and comparison analysis results of the aforementioned regularizers, as well as the results of preclinical application of a mouse study. Finally, we discuss some related issues and make a conclusion in the last section.

2 Low Rank and Regularization Methods

In this section we briefly describe the spectral CT reconstruction problem with low rank and other spatial domain regularizations. We also provide a short description of three representative regularizations that will be used in our comparison study.

2.1 Minimization Problem and Low Rank

Suppose that $X = \{X_{i,j}\}$ is a 2D intensity function to represent an image where i and j are spatial coordinates and $X_{i,j}$ indicates the pixel value at (i,j) . The spectral image $\mathbf{x}$ can be expressed as a 3D matrix defined as

$$\mathbf{x} = \{X_C, C : \text{the index of energy channel}\}, X_C = \{X_{i,j,C}\}. \quad (1)$$

The 3D data $\mathbf{x}$ is a cube-like array where its frontal slices X_C are the images in different channel energies. The projection $\mathcal{Y}$ of $\mathbf{x}$ is a 3D data array, which is obtained as

$$\mathcal{Y} = \{Y_C, C : \text{the index of energy channel}\}, Y_C = \mathbf{A} X_C, \quad (2)$$

where Y_C is the projection in the C -th energy channel and $\mathbf{A} = \{A_{i,j}\} \in \mathbb{R}^{(M \times N)}$ is the system matrix. N is the number of pixels in the channel image $X_C \in \mathbb{R}^{(N \times 1)}$, and M is the total number of measurements which is the product between the view angle number and the detector bin number. It should be noted that, for convenience, we use the same sign X_C for its matrix and vector forms where we need to re-arrange the matrix image $X_C \in \mathbb{R}^{(\sqrt{N} \times \sqrt{N})}$ into its vector form $X_C \in \mathbb{R}^{(N \times 1)}$ for the model equations in (2), (3) and the rest of this paper. $A_{i,j}$ defines the contribution of the j -th pixel to the i -th measurement.

The optimization problem, with the parameters λ_1 for the low rank regularization due to the correlation along the spectral dimension and λ_2 for image prior regularization in the spatial domain, can be expressed as

$$\operatorname{argmin}_{\mathbf{X}} \frac{1}{2} \|\mathbf{A}(\mathbf{X}) - \mathcal{Y}\|_W^2 + \lambda_1 \|\mathbf{X}\|_* + \lambda_2 \mathbf{R}(\mathbf{X}), \quad (3)$$

where the subscript W indicates the weighted least square, $\mathbf{R}(\mathbf{X})$ is a prior information regularization term, $\mathbf{A}(\mathbf{X})$ is the 3D data array obtained by putting the projections of $\mathbf{X}_C$ in the frontal matrices, and $\|\mathbf{X}\|_*$ is the nuclear norm of 3D data array $\mathbf{X}$ defined as

$$\|\mathbf{X}\|_* = \sum_{k=1}^3 \mu_k \|X_{(k)}\|_*. \quad (4)$$

In Eq. (4), the matrix $X_{(k)}$ is the mode- k unfolding of 3D data array $\mathbf{X}$, and the value of μ_k determines the importance of mode- k unfolding matrix. Since the reconstructed 2D images in various energy levels are highly correlated, the mode-3 unfolding is considered to be low rank and its role should be the most important. Therefore, here we only consider the nuclear norm of mode-3 unfolding $\|X_{(3)}\|_*$ by setting $\mu_1 = \mu_2 = 0$. The mode-3 unfolding matrix can be obtained by converting the frontal matrices of $\mathbf{X}$ to row vectors and placing them as the rows of the matrix $X_{(3)} \in \mathbb{R}^{(N_C \times N)}$ [18] where N_C is the number of channel energies.

We make use of the alternating minimization scheme (AMS) [19] to reconstruct the 3D image $\mathbf{X}$ in Eq. (3). The AMS could approximately solve Eq. (3), and it converges to a stable solution as the iteration increases which will be demonstrated by the convergence curve in Sect. 4. The fidelity term and the low-rank minimization procedure are performed in four steps as follows:

Step I The intermediate image for each channel is respectively obtained by the ordered-subset simultaneous algebraic reconstruction techniques (OS-SART [20]) to form the 3D data image $\mathbf{X}$ for S nonempty subsets.

$$\mathbf{X}_C^{(new)} = \mathbf{X}_C^{(old)} + \lambda_{SART} \mathbf{V}^{-1} \mathbf{A}_{[n]}^T \mathbf{W}_{[n]} (\mathbf{Y}_{C[n]} - \mathbf{A}_{[n]} \mathbf{X}_C^{(old)}), \quad (5)$$

where $\mathbf{W} \in \mathbb{R}^{(M \times M)}$ and $\mathbf{V} \in \mathbb{R}^{(N \times N)}$ are diagonal matrices with the diagonal elements $V_{jj} = \mu_j A_{jj}$ and $W_{ii} = 1/\mu_i A_{ii}$, respectively, $[n] = n \bmod (S) + 1$ for $n = 0$, and the iteration starts from $n = zS$ to $n = (z+1)S$. The subscript $[n]$ in vector $\mathbf{Y}_C$ only leaves the measurements corresponding to each subset and makes the other measurements equal to zero (Similar for the coefficients in matrices $\mathbf{A}$ and $\mathbf{W}$). Finally, each channel image $\mathbf{X}_C$ is obtained by using an iterative process over all the S subsets.

Step II The mode-3 unfolding matrix $X_{(3)}$ is obtained from the 3D data image $\mathbf{X}$.

Step III The low rank matrix is obtained by using the singular value shrinkage operator for $\tau = 0$.

$$D_{\tau}(X_{(3)}) = U \Sigma_{\tau} V^T, \quad (6)$$

where the unitary matrices $U \in \mathbb{R}^{(N_C \times N_C)}$, $V \in \mathbb{R}^{(N \times N)}$ and the diagonal matrix $\Sigma_{\tau} \in \mathbb{R}^{(N_C \times N)}$ are obtained as

$$X_{(3)} = U \Sigma V^T, \quad (7)$$

$$\Sigma_{\tau} = \text{diag}(\max\{\sigma_m - \tau, 0\}). \quad (8)$$

Equation (7) is the singular value decomposition for matrix $X_{(3)}$ and σ_m is the corresponding m -th singular value.

Step IV The low rank 3D data image is obtained by rearranging $D_{\tau}(X_{(3)})$ into 3D data image (“fold” operation) [17, 21]. The full procedure of image reconstruction for comparing different regularizations is summarized as the following pseudocode:

Pseudocode of the full procedure of image reconstruction

1. Input: A , $\mathcal{Y}$, λ_1 , λ_2 , τ , spatial domain regularization parameters;
 2. Performing the FBP to reconstruct each channel image;
 3. Computing the relative noise level of each channel image;
 4. Initialization: $\mathbf{x}^0 = \mathbf{0}$ ($X_C = \mathbf{0}$ for $C = 1, \dots, N_C$);
For $t = 0, \dots, T_t$;
 5. Reconstructing all the channel images $X_C^{t+\frac{1}{3}}$ independently by Eq. (5) using an iterative process over all the S subsets;
 6.
 - a) Building the mode-3 unfolding matrix $X_{(3)}^{t+\frac{1}{3}}$;
 - b) Performing low rank regularization to obtain $D_{\tau}\left(X_{(3)}^{t+\frac{1}{3}}\right)$ using Eq. (6);
 - c) Separating each channel image $X_C^{t+\frac{1}{3}}$ from $D_{\tau}\left(X_{(3)}^{t+\frac{1}{3}}\right)$ (fold operation);
 7. Adopting the corresponding spatial regularization on each channel image $X_C^{t+\frac{1}{3}}$ independently based on their relative noise level computed in step 3 and obtaining $\mathbf{x}_C^{t+1}$;
- End

In the above pseudocode, t indicates the iteration number, $\mathbf{x}^t$ is the result of $\mathbf{x}$ after t iterations, and $\mathbf{x}^{t+\frac{1}{3}}$ and $\mathbf{x}^{t+\frac{2}{3}}$ are the intermediate results between t and $(t+1)$ iterations. For details of different spatial regularizers, please see the next subsection.

2.2 Spatial Regularization

The image prior information regularization procedure is adopted after the minimization of fidelity term and low rank reconstruction. Because the low rank prior information has been applied for the correlation between different channels, this spatial regularization is applied to each 2D channel image independently.

2.2.1 Total Variation Regularization—The goal in this regularization is to reduce the total variation (TV) of an image. The TV of a 2D image $\mathbf{X}$ can be defined as

$$TV(\mathbf{X}) = \nabla \mathbf{X}_1 = \sum_{i,j} \sqrt{(X_{i+1,j} - X_{i,j})^2 + (X_{i,j+1} - X_{i,j})^2}. \quad (9)$$

This regularization actually is the L_1 norm of gradient of $\mathbf{X}$. In order to apply the TV regularization, one has to take the derivative of Eq. (9) with respect to $\mathbf{X}$. In practical applications, a small positive constant value, [22] ϵ^2 is added to make Eq. (9) differentiable when the gradient of $\mathbf{X}$ is $\mathbf{0}$. Therefore, we have

$$TV(\mathbf{X}) \cong \sum_{i,j} \sqrt{(X_{i+1,j} - X_{i,j})^2 + (X_{i,j+1} - X_{i,j})^2 + \epsilon^2}. \quad (10)$$

The final regularized image is obtained by minimizing the following cost function

$$T_{TV}(\mathbf{X}) = \frac{1}{2} \|\mathbf{X} - \hat{\mathbf{X}}\|^2 + TV(\mathbf{X}) \quad (11)$$

where $\hat{\mathbf{X}}$ is the noisy image obtained by low rank regularization described in previous subsection. This minimization can be iteratively solved by the gradient descent algorithm as

$$\mathbf{X}^{L+1} = \mathbf{X}^L - q_{L,TV} \cdot \nabla T_{TV}(\mathbf{X}^L), \quad (12)$$

where $q_{L,TV}$ is the step size at every iteration and $\nabla T_{TV}(\mathbf{X}^L)$ is the gradient of image at iteration L .

2.2.2 Anisotropic Diffusion Regularization—This regularization is a generalized form of Gaussian filtering. In this filter, Gaussian blurring of a point is minimal along the image edges and is maximal along the corresponding orthogonal directions. The directions can be determined by obtaining the gradient vector at each point. This regularization could be performed in the following three steps [6, 23].

Step 1 Computing eight nearest-neighbor differences and the following C_x quantities:

$$\begin{aligned}
\nabla_N X_{i,j} &\equiv X_{i-1,j} - X_{i,j}, & C_N &\equiv g(|\nabla_N X_{i,j}|), \\
\nabla_S X_{i,j} &\equiv X_{i+1,j} - X_{i,j}, & C_S &\equiv g(|\nabla_S X_{i,j}|), \\
\nabla_E X_{i,j} &\equiv X_{i,j+1} - X_{i,j}, & C_E &\equiv g(|\nabla_E X_{i,j}|), \\
\nabla_W X_{i,j} &\equiv X_{i,j-1} - X_{i,j}, & C_W &\equiv g(|\nabla_W X_{i,j}|), \\
\nabla_{NE} X_{i,j} &\equiv X_{i-1,j+1} - X_{i,j}, & C_{NE} &\equiv g(|\nabla_{NE} X_{i,j}|), \\
\nabla_{NW} X_{i,j} &\equiv X_{i-1,j-1} - X_{i,j}, & C_{NW} &\equiv g(|\nabla_{NW} X_{i,j}|), \\
\nabla_{SE} X_{i,j} &\equiv X_{i+1,j+1} - X_{i,j}, & C_{SE} &\equiv g(|\nabla_{SE} X_{i,j}|), \\
\nabla_{SW} X_{i,j} &\equiv X_{i+1,j-1} - X_{i,j}, & C_{SW} &\equiv g(|\nabla_{SW} X_{i,j}|),
\end{aligned} \tag{13}$$

Two representative functions were proposed for $g(\cdot)$ [5]. The first one is $g(w) = \exp(-(w/K)^2)$ and the second one is $g(w) = 1/(1 + (w/K)^2)$, where K is a constant which can be empirically or adaptively selected by estimating the noise level. Since the two functions would give fairly similar results, we would only apply the first one in our implementation.

Step II Updating the value of $X_{i,j}$

$$X_{i,j}^{L+1} = X_{i,j}^L + \lambda_{AnDi} \left(\Delta_1 + \frac{\Delta_2}{2} \right), \tag{14}$$

where $0 < \lambda_{AnDi} < 1/6$ and the parameters Δ_1 and Δ_2 are defined as

$$\Delta_1 = C_N \cdot \nabla_N X_{i,j} + C_S \cdot \nabla_S X_{i,j} + C_E \cdot \nabla_E X_{i,j} + C_W \cdot \nabla_W, \tag{15}$$

$$\Delta_2 = C_{NE} \cdot \nabla_{NE} X_{i,j} + C_{NW} \cdot \nabla_{NW} X_{i,j} + C_{SE} \cdot \nabla_{SE} X_{i,j} + C_{SW} \cdot \nabla_{SW} X_{i,j}. \tag{16}$$

The value of $\nabla_x X_{i,j}$ is the nearest-neighbor difference along the x direction and C_x is the weight for the contribution of this difference. In noisier images, the contribution of neighboring pixels should be greater and we know that higher value for K leads to a higher value in function g . As a result, the higher value should be chosen for K in noisier images.

2.2.3 Non-local Means Regularization—Suppose that $\mathcal{N}_{i,j}$ indicates a fixed square matrix from the image centered at pixel (i,j) . The non-local means regularization determines the value of each pixel (i,j) by calculating a weighted average of every pixel (p,q) in the image with higher weights for similar patches between $\mathcal{N}_{i,j}$ and $\mathcal{N}_{p,q}$. Searching for similar patches could be narrowed in a “search window” instead of the whole image due to the computational limitation of this regularization. This regularization is defined as [24]

$$X_{i,j}^{NLM} = \frac{1}{Z_{i,j}} \sum_{pq} e^{\frac{-[\mathcal{N}_{i,j} - \mathcal{N}_{p,q}]^2 \cdot * G_\alpha}{h_{NLM}^2}} \cdot X_{p,q}, \tag{17}$$

where “ $\cdot$ ” defines the element-wise multiplication and G_α is a Gaussian kernel with standard deviation of α . The exponential function represents the similarity between two patches of $\mathcal{N}_{i,j}$ and $\mathcal{N}_{p,q}$. The value $Z_{i,j}$ is the normalization factor and is defined as

$$Z_{i,j} = \sum_{pq} e^{-\frac{(\mathcal{N}_{i,j} - \mathcal{N}_{p,q})^2 \cdot * G_{\alpha}}{h_{NLM}^2}}. \quad (18)$$

The parameter h_{NLM} is the filtering parameter which indicates the growth rate of the exponential function as the Euclidean distance between two patches $\mathcal{N}_{i,j}$ and $\mathcal{N}_{p,q}$ decreases.

3 Experiment

In our experiment, both numerically simulated and real preclinical mouse datasets are utilized to evaluate the performance of the regularizations. The initial guess is set to zero at the beginning and we use the same regularization parameters for all the channels. For the NLM regularization, the searching window and the similarity window are set to 17×17 and 7×7 , respectively. The parameter ϵ^2 in TV regularization is set to 10^{-6} . The noise levels in different channel images are rather different. Therefore, first, the channel images are reconstructed by FBP. Second, a filter-based noise estimation method [25] is used to estimate the standard deviation σ_C of noise in each channel image to select the parameters in spatial regularizations. In the NLM regularization, the parameter $h_{NLM} = h\sigma_C^N$ where σ_C^N is the normalized standard deviation of noise in channel image C so that $\sigma_C^N = 1$ for the noisiest channel image. The same strategy is utilized in TV and AnDi where $q_{L,TV} = q_L\sigma_C^N$ and $\lambda_{AnDi} = \lambda\sigma_C^N$.

3.1 Numerical Simulation

In the numerical simulations, 512 equally spaced detector bins of width 0.1 mm in fan-beam geometry are assumed. 640 projections are equi-angularly generated over one full scan. The distance from the source to the object center and detectors are 132 and 180 mm, respectively. A 50 kVp X-ray spectrum (stepped with 1 keV) generated by SpectrumGUI software is considered [26]. Those parameters are used to mimic a flat-panel photon counting spectral micro-CT.

Four non-overlapping channels are assumed in the spectrum: 15–23, 24–28, 29–33 and 34–40 keV. A mouse thorax phantom with 1.2% iodine contrast agent is employed (see Fig. 1). The size of the reconstructed images is $512 \times 512 \times 4$ where the area of each pixel is $0.060 \text{ mm} \times 0.060 \text{ mm}$. Moreover, we assume the following raw measurements based on a Poisson noise model.

$$g_{z,C} \sim \text{Poisson}\left\{U_C B e^{-[AX_C]_z}\right\} \quad z = 1, \dots, Z, \quad C = 1, \dots, N_C, \quad (19)$$

where Z is the number of ray measurements, N_C is 4 in our experiment, B is the number of photons emitted along each X-ray path, U_C is the proportion of the photon number in each channel to the total emitted photon numbers which is 25.3, 25.9, 24.3 and 24.5% for the four channels in the experiment as depicted in Fig. 2, $A \in \mathbb{R}^{(640 \times 512) \times (512 \times 512)}$ is the system matrix and $[AX_C]_z$ indicates the line integral of the attenuation. In our experiment, the

system matrix is computed using the distance-driven projection and backprojection method [27]. Two different values of parameter B ($B = 1 \times 10^4, 5 \times 10^3$) in Eq. (19) have been selected to simulate low-dose and ultra-low dose projections. It should be noted that a smaller B corresponds to a lower signal to noise ratio (SNR) which results in higher noise level in projections. In this study, the maximum number of iteration is 20 and the number of subsets used in the OS-SART algorithm is also 20. The normalized standard deviation of noise are computed as follows: $[\sigma_1^N \sigma_2^N \sigma_3^N \sigma_4^N] = [1.000.750.700.66]$. The parameter K in AnDi is set to 0.3 and 0.5 for the first and second simulations ($B = 1 \times 10^4, 5 \times 10^3$), respectively.

3.2 Preclinical Mouse Study

In this subsection, preclinical projections from a mouse study are employed to compare and evaluate different regularizations. The projections are obtained by scanning a mouse on a MARS micro-CT scanner with a Medipix MXR CdTe layer detector [28]. The x-ray source is Source-Ray SB-120-400 whose minimum focal spot size is 75 μm . The geometric parameters are listed in Table 1. Eight channel projections are collected at 120 kVp and 175 mA. In this study, each energy channel receives photons whose energies are higher than a given energy threshold, and the energy threshold increases with the increment of the energy channel index. Therefore, the first channel has the lowest energy threshold and all the photons are counted. The last channel, on the other hand, collects the fewest photons. In order to suppress ring artifacts on the measured projections, a Wavelet-Fourier filtering method is applied before reconstruction (see Fig. 3) [29]. The reconstructed CT image is 3D of $400 \times 400 \times 8$ covering an area of $20.00 \times 20.00 \text{ mm}^2$. Here, the maximum number of iteration is 20, the number of subsets used in the OS-SART algorithm is 15, and the normalized standard deviation of noise are computed as follows:

$[\sigma_1^N \sigma_2^N \sigma_3^N \sigma_4^N \sigma_5^N \sigma_6^N \sigma_7^N \sigma_8^N] = [0.770.740.900.930.951.000.960.95]$. The parameter τ is set to 0.1 for the LR regularizations and the parameter K in AnDi is set to 0.5. Here, the system matrix A is as $A \in \mathbb{R}^{(371 \times 512) \times (400 \times 400)}$.

4 Result and Analysis

4.1 Numerical Simulation

In this subsection, experimental results and analysis are presented. Three image evaluation methods will be employed for the comparison study. First, we use the relative root mean square error (rRMSE) which is defined as

$$rRMSE(X^r, X^o) = \frac{\sqrt{\frac{1}{N_p} \sum_{i,j} (X_{i,j}^r - X_{i,j}^o)^2}}{\text{mean}(X^r)}, \quad (20)$$

where N_p is number of pixels, X^r is the reference image reconstructed by FBP from noise-free projections and X^o is the reconstructed image. A smaller rRMSE implies that the recovered image is relatively closer to the true image. However, smaller rRMSE does not guarantee that the reconstructed image possesses higher visual quality. Therefore, the structural similarities (SSIM) index has been adopted for comparison. In each spatial regularization, the 3D image obtained with the smallest mean of rRMSE among the four

channels was selected as the representative reconstructed image for that group. The structural similarity index between image X^r and image X^o is given by:

$$SSIM(X^r, X^o) = \left(\frac{2\mu_{X^r}\mu_{X^o} + C_1}{\mu_{X^r}^2 + \mu_{X^o}^2 + C_1} \right) \left(\frac{2\sigma_{X^r}\sigma_{X^o} + C_2}{\sigma_{X^r}^2 + \sigma_{X^o}^2 + C_2} \right) \left(\frac{2\sigma_{X^rX^o} + C_3}{\sigma_{X^r}\sigma_{X^o} + C_3} \right), \quad (21)$$

where μ_{X^r} , μ_{X^o} are the mean of the images X^r and X^o , σ_{X^r} , σ_{X^o} and $\sigma_{X^rX^o}$ are the standard deviations, and cross-covariance of images X^r and X^o , and C_1 , C_2 and C_3 are regularization constants. The SSIM index is in the range of [0, 1] and a higher index reveals higher similarity between two images in terms of luminance, contrast and structural components. Finally, visual quality of the reconstructed images will be displayed as the third assessment. In the following subsections, we present the comparison study results with the aforementioned regularizations at two different dose levels.

4.1.1 Low-Dose Projections ($B = 1 \times 10^4$)—In Fig. 4, we provide the procedure for selecting the parameter for each regularization. The value corresponds to the blue line with the square marker is the selected parameter for each regularization. This value provides the smallest mean of rRMSE for the reconstructed channel images (which does not necessarily provide the smallest rRMSE for all the channels). The spatial parameters in TV-LR, NLM-LR and AnDi-LR regularizations are selected based on their optimized value in their corresponding non-LR regularizations. For example, in the TV-LR regularization, the q_L is selected based on its selection in TV regularization which is 4×10^{-4} .

The reconstructed images in the first and last channels are shown in Fig. 5 and their corresponding zoomed-in bone and lung ROIs are displayed in Figs. 6 and 7. From Figs. 6 and 7, one can see that the TV, TV-LR, NLM and NLM-LR regularizations give high quality images in the first channel. Moreover, the fine structure between the bony structures (indicated by red arrows in Fig. 6) is only reconstructed in the NLM and NLM-LR regularizations. This shows the capability of NLM regularization in preserving the structural regions. Moreover, in terms of denoising in soft tissue regions, by looking at Fig. 7 it is observed that the NLM-LR, TV-LR and NLM provide clearer regions. This fine structure is not visible in the last channel image except the one in the NLM-LR. The advantage of using LR regularization is more clear in noisier channels such as the forth channel in our experiment.

The quantitative analysis results of rRMSE and SSIM are provided in Fig. 8. The images reconstructed by the FBP are not listed as their SSIM and rRMSE indices are far worse than those of the iteratively reconstructed images. The reconstructed images have the highest SSIM in the order of NLM-LR, NLM, TV-LR, TV, AnDi-LR and AnDi. This order, however, in terms of better rRMSE is NLM-LR, TV-LR, NLM, TV, AnDi-LR and AnDi. Although the reconstructed images using AnDi and AnDi-LR regularization have high quality in terms of rRMSE. However, these images are blurry with the lowest SSIM.

4.1.2 Ultra-Low-Dose Projections ($B = 5 \times 10^3$)—Similar to the previous subsection, the procedure for selecting the parameters for each regularization is provided in Fig. 9 and the blue line with the square marker is the selected parameter for each regularization. We

can see the blue line does not provide the smallest rRMSE for all the channels such as in AnDi and TV-LR. However, as we mentioned earlier the blue line provides the smallest mean of rRMSE.

The reconstructed images in the first and last channels are shown in Fig. 10 and their counterparts of zoomed-in bone and lung ROIs are displayed in Figs. 11 and 12. From these figures, we can see that the NLM and NLM-LR regularizations would reconstruct better images than the TV and TV-LR in the first and last channels. For the last channel, the reconstructed images from the NLM-LR have the best quality followed by the NLM and TV-LR. The AnDi-LR provides a good bony structural detail in the last channel. However, the soft tissue regions (Fig. 12) reconstructed by this regularization is noisier than the TV based regularizations (TV and TV-LR) and the NLM based ones (NLM and NLM-LR). The fine bony structures are only well reconstructed by the NLM and NLM-LR regularizations. Moreover, in terms of denoising in soft tissue regions, by looking at Fig. 12 it is observed that the NLM-LR, NLM and TV-LR provide clearer regions. This observation is consistent with the previous subsection.

Quantitative analysis results of the rRMSE and SSIM are provided in Fig. 13. Similar to the previous subsection, the images reconstructed by the FBP are not listed as their SSIM and rRMSE indices are far worse than those of the iteratively reconstructed images. The reconstructed images have the highest SSIM in the order of NLM-LR, NLM, TV-LR, TV, AnDi-LR and AnDi. This is the same order as we observed in the previous subsection. In terms of rRMSE index, the order is as NLM-LR, NLM, TV-LR, TV, AnDi-LR and AnDi. The reconstructed images using the AnDi and AnDi-LR regularization have higher quality in terms of rRMSE in the first three channels. However, these images are blurry and their corresponding SSIM indices are also lower than those regularizations based on the TV and NLM.

In the above two different noise level scenarios, although the AnDi based regularizations provide comparable images in terms of rRMSE index, the visual assessment and SSIM index provide relative worse performance for these regularizations. This indicates that the rRMSE could not always be a reliable index. The quality difference between the AnDi-based and other regularizations is clearer in noisier channel images. This is because in noisier images, the global image properties should be used for denoising, and it is not sufficient to use only the neighbor pixels. A representative example of the AMS convergence curve is plotted in Fig. 14 for the NLM-LR regularization. We can see that the error decreases monotonically and finally it keeps stable for the last iterations. There is a similar trend for other regularizations. In terms of rRMSE and SSIM indices, the competition among the three regularizations is intense in less noisy channels such as the first channel. However, as the noise level increases in the last channel images, the NLM based regularizations deliver better performance compared to other regularizations. On the other hand, the performance of AnDi based regularization decreases significantly compared to the TV and NLM based regularizations.

In summary, the NLM-LR followed by the TV-LR regularizations deliver the best performance as they can better suppress the noise. The use of LR is necessary to obtain the

inter channel correlation. We can see the decrease of rRMSE and at the same time the increase of SSIM index after using LR. In term of parameter selection in the regularizations we utilized, a greater value for parameter h should be selected when the image is noisier, and a small value for this parameter produces blocky structures in the reconstructed image and make the image too blocky and unacceptable. For the AnDi and TV based regularization, however, choosing smaller value for the parameters does not make the image unacceptable. Figure 15 shows this phenomenon where red and green regions indicate the unacceptable and acceptable values for regularizations, respectively. The range of acceptable value for h is also small which is shown in the figure. This makes the selection of NLM parameter harder compared to the TV and AnDi.

4.2 Comparison Studies of Different Regularizations with the Preclinical Mouse Study

In this subsection, preclinical projections from a mouse study are employed to compare different regularizations. Figure 16 shows the reconstructed images in three representative channels (first, fourth and eighth) and the results of material decomposition. It can be seen that the three noisiest images are from FBP, AnDi and TV regularization in order. Moreover, the effect of LR regularization is clear in the reconstructed images in these three channels. It also can be seen that the NLM-LR and TV-LR deliver better performance in denoising channel images. The material decomposition results show the same pattern in terms of denoising of the channel images where the best images are from the NLM-LR/TV-LR followed by the NLM.

Figure 17 shows the magnified ROI in three channels containing bony structures indicated in Fig. 16 and the corresponding zoomed-in material decomposition. The FBP provides the noisiest channel images. The TV regularization could suppress noise to some extent. However, the bony structures are not clear. Adding the LR regularization could address this problem. Similar results are observed in the NLM and NLM-LR and AnDi and AnDi-LR. For example, some artifacts (indicated by red arrows) in higher energy channel can be introduced only using the NLM regularization, and the bone edges are better preserved by adding the LR regularization. The reconstructed images by the AnDi and AnDi-LR are a little bit noisier than the counterparts of TV-LR and NLM-LR. Moreover, by looking at the material decomposition images, we see that the NLM-LR and TV-LR could better preserve the bone structures. For example, by looking at the region indicated by a blue arrow, the NLM-LR and to some extent the TV-LR are the only ones which could preserve the continuity of bone structures.

5 Discussion and Conclusion

In this work, we implement and numerically evaluate three representative regularizations and make a performance comparison among them for spectral CT. In the numerical simulations, the NLM-LR along with TV-LR regularization give better performance than other regularizations while the NLM-LR is the best in terms of rRMSE and SSIM indices. This regularization provides images with the lowest noise while preserving the main structures. The AnDi based regularization produces blurrier images in general. In the preclinical mouse study, similar to numerical simulations the qualities of images

reconstructed by the NLM-LR and TV-LR are better than the ones reconstructed by the AnDi-LR and others. It should be noted that the conclusions are drawn based on one real preclinical data set and a set of numerical simulations with two different noise levels. However, the same regularization could have slightly different effects on different data sets. As a result, it is suggested that more preclinical data sets would be utilized to reach more accurate conclusions where visual assessment, SSIM and rRMSE indices could be more distinctive for different regularizations.

From Figs. 8 and 13, we understand that in the less noisy channel images, the AnDi based regularizations which only use the neighbor pixels can compete with more complicated regularizations. For the noisier channel images, however, more image features or properties (utilizing similar patches in the NLM and the total variation of an image in TV) should be used for denoising. This suggests that someone should also consider noise level of the channel images for spectral CT denoising. Moreover, the use of LR is beneficial for utilizing the inter channel correlation and demonstrating a better performance. In numerical cases, the decrease of rRMSE and the increase of SSIM index can be seen after using the LR. Moreover, in the preclinical mouse study, the advantage of using LR can be observed visually which is consistent with the results of numerical cases.

In terms of the difficulty in finding the optimal parameter in each regularization, finding the parameter h in the NLM regularization is not as easy as the step sizes in the TV or AnDi. The AnDi regularization is the second hardest one as it has two main parameters (step size λ and K) to be optimized. Finally, the NLM based regularizations are the most time consuming one. Since the block matching part in NLM regularization is computationally costly. A short summary of the comparison is provided in Table 2.

Acknowledgements

This work was supported in part by the NIH/NIBIB U01 Grant EB017140.

References

1. Heismann BJ, Schmidt BT, Flohr T (2012). Spectral computed tomography. Bellingham, WA: SPIE.
2. Yu DF, & Fessler JA (2002). Edge-preserving tomographic reconstruction with nonlocal regularization. *IEEE Transactions on Medical Imaging*, 21(2), 159–173. [PubMed: 11929103]
3. Wang J, Li T, & Xing L (2009). Iterative image reconstruction for CBCT using edge-preserving prior. *Medical Physics*, 36(1), 252–260. [PubMed: 19235393]
4. Elbakri IA, & Fessler JA (2002). Statistical image reconstruction for polyenergetic X-ray computed tomography. *IEEE Transactions on Medical Imaging*, 21(2), 89–99. [PubMed: 11929108]
5. Perona P, & Malik J (1990). Scale-space and edge detection using anisotropic diffusion. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 12(7), 629–639.
6. Gerig G, Kubler O, Kikinis R, et al. (1992). Nonlinear anisotropic filtering of MRI data. *IEEE Transactions on Medical Imaging*, 11(2), 221–232. [PubMed: 18218376]
7. Carballido-Gamio J, Belongie SJ, & Majumdar S (2004). Normalized cuts in 3-D for spinal MRI segmentation. *IEEE Transactions on Medical Imaging*, 23(1), 36–44. [PubMed: 14719685]
8. Huang J, Ma J, Liu N, et al. (2011). Sparse angular CT reconstruction using non-local means based iterative-correction POCS. *Computers in Biology and Medicine*, 41(4), 195–205. [PubMed: 21334607]

9. Rudin LI, Osher S, & Fatemi E (1992). Nonlinear total variation based noise removal algorithms. *Physics D Nonlinear Phenomena*, 60(1–4), 259–268.
10. Sidky EY, & Pan X (2008). Image reconstruction in circular cone-beam computed tomography by constrained, total-variation minimization. *Physics in Medicine & Biology*, 53(17), 4777. [PubMed: 18701771]
11. Tang J, Nett BE, & Chen G-H (2009). Performance comparison between total variation (TV)-based compressed sensing and statistical iterative reconstruction algorithms. *Physics in Medicine & Biology*, 54(19), 5781. [PubMed: 19741274]
12. Ritschl L, Bergner F, Fleischmann C, et al. (2011). Improved total variation-based CT image reconstruction applied to clinical data. *Physics in Medicine & Biology*, 56(6), 1545. [PubMed: 21325707]
13. Xu Q, Yu H, Bennett J, et al. (2012). Image reconstruction for hybrid true-color micro-CT. *IEEE Transactions on Biomedical Engineering*, 59(6), 1711–1719. [PubMed: 22481806]
14. Correia T, Aguirre J, Sisniega A, et al. (2011). Split operator method for fluorescence diffuse optical tomography using anisotropic diffusion regularisation with prior anatomical information. *Biomed Optics Express*, 2(9), 2632–2648.
15. Black MJ, Sapiro G, Marimont DH, et al. (1998). Robust anisotropic diffusion. *IEEE Transactions on Image Processing*, 7(3), 421–432. [PubMed: 18276262]
16. Recht B, Fazel M, & Parrilo PA (2010). Guaranteed minimum-rank solutions of linear matrix equations via nuclear norm minimization. *SIAM Review*, 52(3), 471–501.
17. Chu J, Li L, Chen Z et al. (2012). Multi-energy CT reconstruction based on low rank and sparsity with the Split-Bregman method (MLRSS) (pp. 2411–2414).
18. Kolda TG, & Bader BW (2009). Tensor decompositions and applications. *SIAM Review*, 51(3), 455–500.
19. Beck A (2015). On the convergence of alternating minimization for convex programming with applications to iteratively reweighted least squares and decomposition schemes. *SIAM Journal on Optimization*, 25(1), 185–209.
20. Wang G, & Jiang M (2004). Ordered-subset simultaneous algebraic reconstruction techniques (OS-SART). *Journal of X-ray Science Technology*, 12(3), 169–177.
21. Semerci O, Hao N, Kilmer ME, et al. (2014). Tensor-based formulation and nuclear norm regularization for multienergy computed tomography. *IEEE Transactions on Image Processing*, 23(4), 1678–1693. [PubMed: 24808339]
22. Anagaw AY, & Sacchi MD (2012). Edge-preserving seismic imaging using the total variation method. *Journal of Geophysics and Engineering*, 9(2), 138.
23. Petrou M, & Petrou C (2010). *Image processing: The fundamentals*. Hoboken: Wiley.
24. Buades A, Coll B, & Morel JM (2005). A non-local algorithm for image denoising. In *Computer Vision and Pattern Recognition, 2005. CVPR 2005. IEEE Computer Society Conference (Vol. 2, pp. 60–65)*.
25. Tai SC, & Yang SM (2008). A fast method for image noise estimation using laplacian operator and adaptive edge detection. In *Communications, Control and Signal Processing, 2008. ISC-CSP 2008. 3rd International Symposium on IEEE (pp. 1077–1081)*.
26. “<https://sourceforge.net/projects/spectrumgui/>.”
27. De Man B, Basu S (2002) Distance-driven projection and backprojection 3:1477–1480.
28. Butler A, Anderson N, Tipples R, et al. (2008). Bio-medical X-ray imaging with spectroscopic pixel detectors. *Nucl Instrum Methods Phys Res Sect A Accel Spectrom Detect Assoc Equip*, 591(1), 141–146.
29. M nch B, Trtik P, Marone F, et al. (2009). Stripe and ring artifact removal with combined wavelet—Fourier filtering”. *Optics Express*, 17(10), 8567–8591. [PubMed: 19434191]

Fig. 1.
The mouse phantom (left) and the iodine contrast agent (right) used in the experiment

Fig. 2.
Four energy bins of the employed X-ray spectrum

Fig. 3.
Raw projections collected for a mouse in one channel (left) and the corresponding projections after preprocessing to suppress ring artifacts (right)

Fig. 4.

The rRMSE of reconstructed channel images with respect to different regularizations parameters. The blue line is the selected parameter for each regularization where it has the smallest mean of rRMSE among all the channels ($B = 1 \times 10^4$)

Fig. 5.

Visual assessments of the reconstructed images with $B = 1 \times 10^4$. The first and the fourth channels are shown in the top and bottom rows, respectively. The display windows is $[0, 3] \text{ cm}^{-1}$ for the first row and $[0 \ 1] \text{ cm}^{-1}$ for the second row

Fig. 6. Zoomed-in bone ROI indicated in Fig. 5 for the first channel (top row) and the fourth channel (bottom row)

Fig. 7.
Zoomed-in lung ROI indicated in Fig. 5 for the first channel (top row) and the forth channel (bottom row)

Fig. 8.

Quantitative analysis results for iterative reconstruction with different regularizations ($B = 1 \times 10^4$). The left plot is for rRMSE ($\times 10^{-4} \text{ cm}^{-1}$), and the right plot is for SSIM

Fig. 9.
Same as Fig. 3 but from projections with $B = 5 \times 10^3$

Fig. 10.

Same as Fig. 5 but from projections with $B = 5 \times 10^3$

Fig. 11.
Same as Fig. 6 but from projections with $B = 5 \times 10^3$

Fig. 12.

Same as Fig. 7 but from projections with $B = 5 \times 10^3$

Fig. 13.

Quantitative analysis results for iterative reconstruction with different regularizations ($B = 5 \times 10^3$). The left plot is for rRMSE ($\times 10^{-4} \text{ cm}^{-1}$), and the right plot is for SSIM

Fig. 14.
A representative convergence curve of the AMS reconstruction for the NLM-LR regularization

Fig. 15. The effect of choosing different regularization parameters on image (Parameters increase from left to right). **a** TV and AnDi parameters, **b** NLM parameter

Fig. 16.

Reconstructed images of a preclinical mouse study. The first, second and third column images are for the 1st, 4th and 8th channels, respectively (display window: $[0, 1] \text{ cm}^{-1}$). The last column images are the corresponding material decomposition results

Fig. 17. Magnified bone ROI indicated in Fig. 16. The columns have the same meaning as in Fig. 16

Table 1

System geometry of the preclinical mouse study

Parameter	Value
Source-object-distance	158 mm
Source-detector distance	255 mm
Detector bin spacing	0.11 mm
Number of detector bins	512
Number of views per rotation	371
Image data array	$400 \times 400 \times 8$

Table 2

Comparison of TV, NLM and AnDi

Method	TV	NLM	AnDi
Quality (rRMSE & SSIM)	Good	Best	Acceptable
Finding the optimal parameter(s)	Medium	Hard	Relatively hard
Computational cost	Low	High	Low