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Analogous Lubberts effect in photon counting detectors

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Abstract

For a light burst generated by an x-ray-scintillator interaction, the quasi-isotropic spreading of optical quanta determines that both the light collection efficiency and the spread function of the light burst depend on the depth (z) of interaction: the closer the initial position of light burst to the light sensor, the less light loss and spatial resolution degradation a detector system encounters. For scintillator-based indirect-conversion detectors, the variation of the x-ray conversion gain and the MTF along the z direction contributes to a significant fraction of DQE loss and is known as the Lubberts effect. Understanding the physics behinds the effect and developing solutions to reduce the effect have led to tremendous improvements in the performance of indirect-conversion detectors. In contrast, for direct-conversion photon counting detectors (PCDs), since no optical quanta generation or transport is involved, there is no classical Lubberts effect. However, the transport properties of the generated charge carriers in semiconductors do depend on their travel distance. As a result, the performance of direct conversion detectors may depend on z . This dependence can be considered as a counterpart of the classical Lubberts effect. The purpose of this work is to analyze the contributions of a variety of physical mechanisms that may contribute to a Lubberts-like effect in direct-conversion PCDs. The results show that the Lubberts-like effect is primarily caused by the depth-dependent charge trapping and recombination likelihood. Based upon the physics analysis, PCD electrode design can be optimized to minimize the Lubberts-like effect and the associated DQE loss.

Keywords

Lubberts effect; photon counting detector; detective quantum efficiency

1. INTRODUCTION

For a light burst generated by the interaction of x-rays and granular scintillators, the quasi-isotropic spreading of optical quanta determines that both the light collection efficiency and the spread function of the light burst depend on the depth (z) of interaction: the closer the initial position of light burst to the light sensor (e.g. photodiode), the less light loss and spatial resolution degradation a detector system encounters. For scintillator-based indirect-conversion x-ray detectors, the variation of the x-ray conversion gain and the MTF along the z direction contributes to a significant fraction of DQE loss and is known as the Lubberts effect.¹ Understanding the physics behinds the Lubberts effect and developing solutions

(such as using columnar scintillators) to reduce the effect have led to tremendous improvements in the performance of indirect-conversion detectors.²⁻⁷

In contrast, for semiconductor-based direct-conversion photon counting detectors (PCDs), since no optical quanta generation or transport is involved in the image formation chain, there is no classical Lubberts effect. However, the transport properties of the generated charge carriers in semiconductors do depend on the travel distance. For example, electrons and holes are more likely to be trapped if they travel a longer distance in the semiconductor. As a result, the performance of direct conversion detectors may depend on z . This possible dependence can be considered as a counterpart of the Lubberts effect in in-direct conversion detectors.

The purpose of this work is to analyze the contributions of a variety of physical mechanisms that may contribute to the Lubberts-like effect in direct-conversion PCDs. Based upon the physics analysis, corresponding design to minimize the Lubberts-like effect can be developed.

2. METHODS

2.1. Revisiting the Lubberts effect in indirect conversion detectors

The Lubberts fraction L is defined as²

$$\begin{aligned} L(\mathbf{k}) &= \left[\frac{M_1^2(\mathbf{0})}{M_0(\mathbf{0})M_2(\mathbf{0})} \right]^{-1} \cdot \frac{M_1^2(\mathbf{k})}{M_0(\mathbf{k})M_2(\mathbf{k})} \\ &= \frac{1}{I_0} \cdot \frac{M_1^2(\mathbf{k})}{M_0(\mathbf{k})M_2(\mathbf{k})} \end{aligned} \quad (1)$$

where $\mathbf{k}$ denotes the two-dimensional spatial frequency vector defined in the detector plane, and the M^{th} Lubberts moment, M_i ($i \in \{0, 1, 2\}$), is defined as

$$M_i(\mathbf{k}) = \int_0^Z dz \langle n(z) \rangle \langle g^i(z) \rangle [\text{MTF}(\mathbf{k}, z)]^i. \quad (2)$$

In Equation (2), Z denotes the total thickness of the x-ray conversion layer, $\langle n(z) \rangle$ denotes the expected number of x-ray photons per unit area interacting with the conversion layer at depth z , $\text{MTF}(\mathbf{k}, z)$ denotes the depth-dependent modulation transfer function of the detector system, and $\langle g^i(z) \rangle$ denotes the i^{th} moment of the depth-dependent x-ray conversion gain. For scintillator-based indirect conversion detectors, the conversion gain is the number of optical quanta generated and collected per each interacted x-ray photon.

In Equation (1), a normalization factor I_0 is used so that L is always one at the zero frequency. As a matter of fact, removing this normalization factor would allow the frequency-dependent DQE of the detector system to be directly related to the Lubberts moments as follows:

$$\begin{aligned} \text{DQE}(\mathbf{k}) &= \frac{\int_0^Z dz \langle n(z) \rangle}{\langle n_{\text{in}} \rangle} \cdot \frac{\left[\int_0^Z dz \langle n(z) \rangle \langle g(z) \rangle \text{MTF}(\mathbf{k}, z) \right]^2}{\int_0^Z dz \langle n(z) \rangle \int_0^T dz \langle n(z) \rangle \langle g^2(z) \rangle \text{MTF}^2(\mathbf{k}, z)} \\ &= \varepsilon \cdot \frac{M_1^2(\mathbf{k})}{M_0(\mathbf{k})M_2(\mathbf{k})}, \end{aligned} \quad (3)$$

where $\langle n_{\text{in}} \rangle$ denotes the expected number of input x-ray photons per unit area, and $\varepsilon = 1 - \exp(-\mu T)$ is the x-ray absorption efficiency of the scintillator layer. It can be proven that $\text{DQE}(\mathbf{k}) \leq 1$ using the following Cauchy-Schwarz inequality:

$$\left[\int dz \langle n(z) \rangle \langle g(z) \rangle \text{MTF}(\mathbf{k}, z) \right]^2 \leq \int dz \langle n(z) \rangle \cdot \int dz \langle n(z) \rangle \langle g(z) \rangle^2 \text{MTF}^2(\mathbf{k}, z) \quad (4)$$

and the fact that $\langle g(z) \rangle^2 = \langle g^2(z) \rangle - \sigma_{g(z)}^2 \leq \langle g^2(z) \rangle$. The condition under which the inequality in Equation (4) becomes an equality is that

$$\sqrt{\langle n(z) \rangle} \langle g(z) \rangle \text{MTF}(\mathbf{k}, z) = \text{constant} \cdot \sqrt{\langle n(z) \rangle} \quad (5)$$

and

$$\sigma_{g(z)}^2 = 0. \quad (6)$$

In other words, DQE is maximized when both $\langle g \rangle$ and MTF are independent of z , and the variance of g is zero. Under these ideal conditions, the DQE of the detector system is

$$\text{DQE}(\mathbf{k}) = \varepsilon \frac{\langle g \rangle^2 \text{MTF}^2(\mathbf{k}) \left[\int dz \langle n(z) \rangle \right]^2}{\langle g \rangle^2 \text{MTF}^2(\mathbf{k}) \int dz \langle n(z) \rangle \int dz \langle n(z) \rangle} = \varepsilon. \quad (7)$$

Unfortunately for indirect conversion detectors, reducing the variations of $\langle g \rangle$ and MTF along z usually comes with a degradation in ε , because the most straightforward strategy to suppress these variations is to reduce the thickness of the scintillator layer. The remainder of this work discusses whether such a tradeoff (between absorption efficiency and Lubberts effect) can be relaxed or reduced for direct conversion PCDs.

2.2. Lubberts-like effect in direct conversion photon counting detectors

For direct conversion PCDs, the second Lubbert moment (M_2) needs to be modified as follows

$$M_2(E_t) \approx \int_0^Z dz \langle n(z) \rangle \langle m^2(z, E_t) \rangle, \quad (8)$$

where E_t denotes the energy threshold of PCD and m denotes the so-called multiplicity that quantifies the number of pixels triggered (i.e., registering counts) for each interacted x-ray photon. Note that there is no frequency-dependent term in Equation (8) since the noise of

PCDs is white or pseudo-white except under the condition of severe pulse pileup or charge sharing.⁸ In comparison, the first Lubberts moment M_1 of the PCD is given by

$$M_1(\mathbf{k}, E_t) = \int_0^T dz \langle n(z) \rangle \langle m(z, E_t) \rangle \text{MTF}(\mathbf{k}, z, E_t). \quad (9)$$

Accordingly, the DQE of the PCD is given by

$$\text{DQE}(\mathbf{k}, E_t) = \epsilon \frac{[\int dz \langle n(z) \rangle \langle m(z, E_t) \rangle \text{MTF}(\mathbf{k}, z, E_t)]^2}{\int dz \langle n(z) \rangle \cdot \int dz \langle n(z) \rangle \langle m^2(z, E_t) \rangle}. \quad (10)$$

In an ideal case where both m and the MTF are independent of z , the DQE formula reduces itself to:

$$\text{DQE}(\mathbf{k}, E_t) = \epsilon \frac{\langle m(E_t) \rangle^2}{\langle m^2(E_t) \rangle} \text{MTF}^2(\mathbf{k}, E_t). \quad (11)$$

This is consistent with the zero-frequency DQE formula for PCDs proposed by Michel *et al.*,⁹ who demonstrated that the zero-frequency DQE of PCDs is given by

$$\text{DQE}(\mathbf{k} = \mathbf{0}, E_t) = \epsilon \frac{\langle m(E_t) \rangle^2}{\langle m^2(E_t) \rangle}. \quad (12)$$

In reality, both the MTF and m of a PCD system may depend on z due to the following reasons:

(1) Depth-dependent x-ray escape fraction: for x-ray interactions occurring closer to the top or bottom surface of the photoconductor, their descendant photons (K-fluorescence and scattered photons) have a higher likelihood to escape the photoconductor without depositing their energies. Figure 1 plots the escape fraction of 50 keV x-rays in 2 mm-thick CdTe. The escape fraction varies from 14% near the surface ($z = 0.1$ mm) to 0% at the central depth ($z = 1$ mm).

(2) Depth-dependent MTF: without suffering spatial spreading of optical quanta, the MTF of PCDs has a much more relaxed dependence on z . The remaining mild variation of MTF along z shown in Fig. 1 is primarily attributed to charge diffusion and transport of K-fluorescence x-rays: at deeper z , electrons and holes have a longer drift time, allowing a slightly longer lateral diffusion length. Similarly, at deeper z , a smaller fraction of K-fluorescence escape, which may increase the contribution of their isotropic spreading to spatial resolution loss. Using Monte-Carlo simulation, the dependence of MTF on z was evaluated for our CdTe PCD. As shown in Figure 2, the variation of MTF along the z direction was not significant.

(3) Effects of weighting field and charge trapping: If there is no charge trapping, based on the Shockley-Ramo theorem,^{10,11} the total charge Q induced on an electrode by the motion of N_{eh} electron-hole pairs in the photoconductor is given by

$$Q = eN_{eh}\{\phi_0(z) - \phi_0(0) + [\phi_0(Z) - \phi_0(z)]\} = eN_{eh}[\phi_0(Z) - \phi_0(0)] = eN_{eh}, \quad (13)$$

where ϕ_0 denotes the weighting potential of the electrode. In this ideal case, Q is independent of z

In reality, electrons and holes have finite lifetimes of τ_e and τ_h before being trapped in the photoconductor such as CdTe. In this case, the induced charge is reduced to¹²

$$Q = eN_{eh}\left\{[\phi_0(z) - \phi_0(0)]\frac{\tau_h}{t_h(z)}(1 - e^{-t_h(z)/\tau_h}) + [\phi_0(Z) - \phi_0(z)]\frac{\tau_e}{t_e(z)}(1 - e^{-t_e(z)/\tau_e})\right\}, \quad (14)$$

where $t_e(z)$ and $t_h(z)$ are the drifting times for electrons and holes created at depth z :

$$t_h(z) = \frac{z}{\mu_h V_{\text{bias}} / Z}, \quad t_e(z) = \frac{Z - z}{\mu_e V_{\text{bias}} / Z}. \quad (15)$$

In Equation (15), μ_e and μ_h are the electron mobility and hole mobility, respectively. As shown in Equation (14), the charge collected in an electrode depends on z since the likelihood of charge trapping depends on the drift time, which is a function of interaction depth. Q also depends on the weighting potential ϕ_0 .

2.3. Optimization of PCD design to minimize Lubberts-like loss and maximize DQE

For a given material and x-ray energy, we cannot control the x-ray escape fraction and charge lifetime. However, there are factors such as the electrode design that can be optimized to reduce the Lubberts loss. To provide an example, we compared three electrode designs shown in Fig. 3 in terms of their Lubberts fractions. For design a, the front anode is pixelated; for design b, the pixelated cathode is at the bottom; design c is similar to design b, except that a Frisch grid⁷ is introduced to alter the weighting field and suppress the contribution of holes.

Figure 4 shows the DQE results calculated under the condition of mean x-ray energy = 63 keV, CdTe thickness = 2 mm, pixel size = 0.1 mm, energy threshold = 15 keV, electronic noise = 1000 e, and electric field = 1×10^4 V/m. The optimal electrode design depends on the semiconductor material. For CdTe, putting the pixelated anode in the front or in the back does not generate any major difference in DQE, and it is undesirable to use the backend pixelated cathode design due to the smaller drift velocity of holes. The zero-frequency DQE values for designs a, b, and c are 0.94, 0.92, and 0.24, respectively. For GaAs, it is more desirable to put the pixelated cathode in the back surface. For this design, the weighting potential of each charge collecting cathode is almost zero near the front surface of the detector, where the majority of x-rays are absorbed to generate electron-hole pairs.

According to Equation 14, if $\phi \approx 0$ between $[0, z]$, the contribution of the short-life electrons in GaAs to the PCD output become negligible, which means the variation of multiplicity over the z direction can be suppressed, and thus the Lubberts-like effect can be mitigated.

4. CONCLUSIONS

Compared with scintillator-based detectors, direct conversion PCDs suffer less Lubberts loss due to the absence of light spreading and significantly relaxed dependence of MTF on z , adding another benefit to the PCD technology. The remaining small but non-negligible Lubberts-like effect is primarily caused by the variation of charge trapping likelihood over the z direction. For a given photoconductive material and readout chip, this work shows that there is still room for further improving the DQE of the PCD system by optimizing the design of electrodes attached to the photoconductor.

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Figure 1.

(a) Locations of x-ray interactions in 2 mm-thick CdTe generated from a Geant4-based Monte-Carlo simulation. The input beam energy is 63 keV. The transport of K-fluorescence x-rays was considered. (b) Fractions of scattered and K-fluorescence x-rays escaped from the CdTe layer.

Figure 2.
Depth-dependent MTF.

Figure 3.

Three electrode designs were compared. X-rays enter the depletion region from the top surface.

Figure 4.

(left) Frequency-dependent DQE calculated using Equation (10). (middle) Comparison of weighting potentials for the four electrode designs shown in Fig. 3. (right) Principle of using the bottom electrode design to suppress variation of PCD response over the depth direction.