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MEASUREMENT OF THE HYDRAULIC CONDUCTIVITY OF THE VITREOUS HUMOR

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Abstract

The hydraulic conductivity of the vitreous humor has been measured for the bovine eye. The experiment was carried out by placing it within upright cylindrical chamber, open at both ends, and letting its liquid content drain out of the bottom opening by gravity, through a 20 μ m nylon mesh filter. Additional negative pressure was provided at the exit by a hanging drainage tube. The diminishing vitreous volume was measured in terms of the height in the chamber and recorded as a function of time. The reduction in the vitreous liquid content also caused the hydraulic conductivity to reduce and this parameter was quantified on the basis of previously-developed theories of fibrous porous media that have been very well established. A theoretical model with a fully analytical expression for the vitreous volume undergoing drainage was developed and used as a least-squares best fit to deliver the initial hydraulic conductivity value of $K_0/\mu = (7.8 \pm 3.1) \times 10^{-12} \text{ m}^2 \text{ (Pa-s)}$. The measurements were made with the hyaloid membrane intact and therefore represents an effective conductivity for the entire system, including possible variations within the vitreous.

1. INTRODUCTION

While intravitreal drug delivery for eye diseases is now an established therapy, the corresponding problems associated with accurately assessing actual delivery rates to, for example, the retina are in need of considerable scientific attention. A promising approach is to develop predictive models that can be used to determine the transport rates of selected

intravitreally injected drugs through the vitreous humor towards the retina. As such, for the mathematical modeling of ocular fluid dynamics and mass transfer, it is essential that accurate measurements of the biophysical and biochemical material properties of various ocular tissues be obtained. In this regard, the hydraulic resistance (or permeability for that matter) of the vitreous humor to water is a basic property that needs thorough examination and careful measurement.

The vitreous humor is a gel-like material that is 99% water in a meshwork of collagen and hyaluronic acid (Mains & Wilson, 2013; Sharif-Kashani, et al., 2011; Nickerson, et al., 2008), encased by the hyaloid membrane. This structure characterizes the vitreous as a porous medium. While solid porous media are generally not difficult to experiment with, the vitreous humor presents many challenges since it is a very delicate material and needs to be treated as such. A cut vitreous quickly drains its liquid content while a whole vitreous with the hyaloid membrane intact, separated from the globe, is sustainable in structure but still prone to collapse when subjected to hydraulic pressure. The hyaloid membrane is porous and water permeates through it.

The vitreous can be treated as a fully saturated porous medium. The gel-like behavior emanates from the fibrous structure itself. It should be noted that the liquid when separated from the fiber appears almost Newtonian based on what we have seen draining out. When the fiber is separated from the vitreous, what remains is not a gel anymore. The fibers may be nanosized in diameter but are long-chain molecules of hyaluronic acid that occupy the space between collagen fibrils (Figure 1). This type of structure is observed through electron microscopy images shown in Figure 2. In a live healthy young eye, the structure can be considered intact since the whole vitreous is attached to the eyeball. While collagen fibers have a general front to back orientation (Sebag & Balazs, 1989), the hyaluronic acid appears to be more random. With the latter being on a much finer scale of effective pore size, it is likely to be the dominant form of resistance, and the orientation may therefore have little effect.

While the fiber volume fraction is approximately just 0.1% of the whole vitreous, there is resistance to the flow through the network. We can show by scaling arguments backed by rigorous theory that volume fraction alone is not what determines the resistance. Regarding this issue, we refer to Higdon & Ford (1996) who carried out a detailed analytical development based on three-dimensional fibrous lattices. For a fiber of radius a and volume fraction ϕ , in the dilute limit, the conductance coefficient K has the asymptotic behavior,

$$K \sim \frac{-\ln \phi}{\phi} a^2 \text{ or } \frac{1}{K} \sim -\frac{\phi}{a^2 \ln \phi}. \quad (1)$$

Several authors (Jackson & James, 1986; Drummond & Tahir, 1984) have theoretically derived such characterization. It can be inferred from this relationship that holding the fiber volume-fraction ϕ constant while making the fiber radius a smaller increases resistance. From a purely geometrical viewpoint, to have constant fiber volume fraction and to reduce the fiber radius, the fibers need to get closer. On the one hand thinner fibers will reduce resistance but on the other, getting them closer increases resistance even more since the latter

competing mechanism overwhelms the overall resistance as we see from the asymptotic result in Equation (1) above.

With the extracted vitreous being a very delicate transparent material in a gelatinous state, it challenging to bias the orientation in the apparatus for any type of anisotropy measurement. Our measurement is therefore an average over the vitreous volume and any effect of orientation is a higher order effect. Additionally, it is difficult at this stage to measure the resistance in different regions of the vitreous, but an overall average measurement is a start. It is also difficult to vary the form and structure of the vitreous to measure such effects, except that with the current set of experiments, the drainage increases the fiber partial density and this effect is incorporated in the measurement. In the broader context of transport in porous media, the form coefficient is also of concern. However, it should be noted that the physiological flow velocities in the vitreous are of the order 1 mm/hr (~2.5 mm/hr in our experiments), and for such a low magnitude, the contribution of the form drag to the overall resistance is negligible. With such weak inertial effects, we have not carried out any measurements of the form coefficient since this effect is inconsequential to actual flow in the vitreous. Furthermore, the question of inertial effects has been addressed by (Vafai, 1984) who has systematically illustrated the role of inertia as higher order in terms of the porosity.

A hyaluronic acid based fibrous porous medium has been experimented upon by Jackson & James (1986), and our formulation relies on their development of a universal formula for the hydraulic conductivity as a function of fiber volume fraction. Since these are not particles, but long fibers (see Figures 1 and 2), the collapse compresses the fibrous structure. All this happens while a very large liquid content is still maintained.

Previous measurements of the hydraulic permeability of the vitreous humor include the work of Brubaker & Riley (1972) that has been further analyzed by Fatt (1975). Subsequently, measurements were carried out by Xu, et al. (2000) whose experimental technique was based on mechanically squeezing the vitreous humor placed in a cylinder with a porous piston. With an established relationship between the compressive force and the hydraulic conductivity, the measurement of the former on a vitreous being squeezed is used to obtain the latter. We have taken the approach where we let the vitreous to drain under negative hydrostatic pressure. For moderate compression levels, a simple model for the variable permeability calculation can be used. While variable porosity has been examined by (Vafai, 1984), the context in the current investigation is due to the depletion of the liquid volume, and we are able to account for the variation using the well-tested relationship given by Jackson & James (1986) that gives the permeability as a function of the fiber volume fraction. The relationship is particularly reliable for low volume fraction of the fiber, and we are able to depend on it because even after depleting half the vitreal liquid volume, we are able to maintain less than 0.5 % volume fraction of the fiber. The works of Jackson & James (1982, 1986) are particularly relevant since they include experimentation with hyaluronic acid which is a component of the fibrous matter of the vitreous humor. The approach taken in the present investigation has the slight advantage that it does not require the use of the mechanical device to squeeze the vitreous and eliminates one potential source of error.

Furthermore, the current work provides an additional measurement in an area where data are skimpy.

2. METHODS AND MATERIALS

For the purpose of carrying out the hydraulic conductivity measurement of the vitreous humor, we used fresh bovine eyes (Manning Beef, Pico Rivera, CA). As mentioned earlier, the vitreous is very delicate gel-like porous material, and we carefully extract this from the eye while keeping intact the hyaloid membrane encasing it. We use a specially-designed chamber for this experiment consisting of a cylindrical vessel into which the whole vitreous is carefully placed. At the bottom of the vessel, a 20 μm nylon filter is used to support the vitreous while its liquid content is allowed to drain through it, as shown in Figure 3. Additional negative hydrostatic pressure is applied by a drainage tube pre-loaded with saline. The top of the cell is shown open. However, in the picture shown in Figure 4, the top has a cap with an air hole. As the liquid drains while retaining the whole fibrous content of the vitreous, the permeability of the vitreous gradually decreases. We are applying the theory of fibrous porous media (Jackson & James, 1986) to account for the hydraulic conductivity variation as liquid is withdrawn from the vitreous. The drainage tube empties into a vessel set below the bottom level of the vitreous, and provides negative pressure there, giving a sufficient pressure gradient to have substantial amount of vitreous liquid drain out in a matter of 4–7 hours. The process is slow and easily recordable.

While the vitreous is 99% water, the liquid content is 99.9% since there are dissolved salts and nutrients, typically 0.9% of the liquid. This leaves a fiber content of 0.1% (Stay, et al., 2003). Therefore, even after draining nearly half the liquid, the liquid content is reduced to just about 99.8% of the remaining vitreous since the 0.1% fiber content approximately doubles to about 0.2%.

The process can be seen visually in the schematic of the drainage progression shown in Figure 5 which is not to scale. The fact that the fiber content stays very small is an important point since it means the dilute limit where the theory (Jackson & James, 1982, 1986) is the strongest is still valid.

During the drainage process, the matrix mesh size reduces, decreasing the Darcy coefficient K . A theoretical model for the drainage system is developed based on previous theories of permeability properties fibrous porous media (Jackson & James, 1986). Specifically, the permeability variation with liquid volume reduction is incorporated into the differential equation for the vitreous volume as a function of time. An important aspect of this system is that it is one-dimensional from a modeling standpoint and affords considerable mathematical simplicity. Furthermore, it does not require any sophisticated microscale stress measurement instrumentation.

3. MATHEMATICAL MODEL

For the experiment, the vitreous is considered to be a porous medium consisting of a meshwork of hyaluronic acid and collagen fibers. The mathematical description of transport processes in bioporous media is available in various books and classical papers (Vafai, 2011;

Ai & Vafai, 2006; Mukundakrishnan & Ayyaswamy, 2011; Shafahi & Vafai, 2011; Khanafer & Vafai, 2006; Khanafer & Vafai, 2005). Mass-diffusion aspects have been detailed in our investigations on the diffusion coefficient measurement (Penkova et. al., 2014; Rattanakijsumton et. al, 2018) and more recently, a comprehensive review of in various studies on this subject have been provided (Penkova et. al, 2019). In earlier studies, the vitreous has been modeled as a porous medium with flow description given by Darcy's equation (Fatt, 1975; Balachandran, 2010; Missel, 2012; Zhang, et al., 2018). The drainage from the vitreous is dependent on the porosity of the vitreous, i.e., the Darcy coefficient K in

$$\nabla p = -\frac{\mu}{K}\mathbf{u} - \rho g \mathbf{e}_z, \quad (2)$$

where p is the pressure, μ is the viscosity of the liquid part of the vitreous, and $\mathbf{u}$ is the velocity distribution. This is force-balance equation includes a downward body-force term $-\rho g \mathbf{e}_z$. The pressure field satisfies Laplace's equation,

$$\nabla^2 p = 0. \quad (3)$$

As mentioned earlier, we define ϕ as the volume fraction of the fiber in the vitreous. This is close to 0.1% for a fully hydrated vitreous, and gradually increases as the liquid is drained. Based on the work of Jackson & James (1986), we use the relationship,

$$\frac{K}{a^2} = \frac{20}{3\phi}(-\ln\phi - 0.931), \quad (4)$$

where a is the fiber radius. There are several other developments in relation to fibrous porous media, and variations of Equation (4) have been reported (see e.g., Drummond & Tahir (1984); Higdon & Ford (1996)). A comprehensive summary of many of these works is available in Jackson & James (1986). For hyaluronic acid, the fiber radius has been reported as 0.49 nm (Jackson & James, 1982). However, we do not need this value since we are presently concerned only with relative changes in ϕ as drainage progresses. If we define K_0 and ϕ_0 as the initial (pre-drainage) values of the corresponding parameters in Equation (4), then we have the relationship

$$\frac{K}{K_0} = \frac{\phi_0}{\phi} \left(\frac{\ln\phi + 0.931}{\ln\phi_0 + 0.931} \right). \quad (5)$$

We now consider the drainage problem as one-dimensional in the cylindrical region shown in Figure 3. In the vitreous region, following Equation (2), the drainage velocity $u_0(t)$ at the exit is given by

$$\frac{dp}{dz} + \rho g = \frac{\mu}{K} u_0(t), \quad (6)$$

assuming uniform velocity within the liquid part of the vitreous (see illustration in Figure 6). Here ρg is the downward body force in the same direction as the flow. In Equation (6), the

sign on the velocity has changed from what is given in Equation (2). This is because we define the downward velocity as positive opposite to the direction of the z coordinate. Let us now define the height of the vitreous as $H(t)$, and the area of cross section of the cylinder as A . The volume of the fiber part of the vitreous can be expressed as

$$V_s = H_0 A \phi_0, \quad (7)$$

which stays constant throughout the experiment. Here, $H_0 = H(0)$ is the initial height. The volume fraction $\phi(t)$ can therefore be written as

$$\phi(t) = \frac{H_0 A \phi_0}{H(t) A} = \frac{H_0 \phi_0}{H(t)}, \quad (8)$$

where A is the cross-sectional area of the cylinder. This expression, when used in Equation (5), leads to

$$\frac{K(t)}{K_0} = \frac{H(t)}{H_0} \left(\frac{\ln\left(\frac{H_0 \phi_0}{H(t)}\right) + 0.931}{\ln \phi_0 + 0.931} \right) \quad (9)$$

The pressure gradient in the vitreous can be expressed as

$$\frac{dp}{dz} = \frac{\rho g h_0 - p_m - p_v}{H(t)}, \quad (10)$$

where $\rho g h_0$ is the hydrostatic negative pressure provided by the hanging drainage tube, p_m represents the pressure jump due to the hydraulic resistance of the 20 μm filter, and p_v is used to represent part of the driving pressure that is taken up as compressive force on the viscous gel. While the simple hydraulic resistance of microfilter in a fully liquid medium can be quantified by Stokes flow through a perforated membrane (see e.g., Tio & Sadhal (1994)), with a porous medium on one side the resistance is quite like a constriction-type as with thermal contact problems, and depends on K . Since the differential equation (3) for pressure is the same as that for steady temperature in a solid, we can, by analogy, rely on the thermal-contact results of Tio & Sadhal (1992) who gave in terms of currently-used parameters

$$p_m = \frac{\psi \sqrt{\pi} u_0(t) \mu d}{4 K \kappa}, \quad (11)$$

where κ stands for the porous area fraction of the filter ($\kappa = 0.14$ as per manufacturer specification), d is the filter pore size (20 μm), $u_0(t)$ is the drainage velocity, and ψ is given by

$$\psi = 1 - 1.4009\kappa^{\frac{1}{2}} + 0.34427\kappa^{\frac{3}{2}} + 0.074098\kappa^{\frac{5}{2}} - 0.023705\kappa^{\frac{6}{2}} + 0.036598\kappa^{\frac{7}{2}} + \dots \quad (12)$$

This effect for the type of filter we are using turns out to be quite insignificant. However, we shall keep this term, and drop it after demonstrating its insignificance. We have adapted the results for a square array of circular pores to represent the square meshwork where the pore size d is the area-equivalent side of a square.

The compression pressure p_v is expressed in the form of the relationship,

$$\beta = -\left(\frac{1}{V}\right)\frac{\partial V}{\partial p_v} = -\left(\frac{1}{H}\right)\frac{dH}{dp_v}, \quad (13)$$

where β is the compressibility of the fiber when separated from the liquid portion of the vitreous. With a little manipulation, assuming constant compressibility, this integrates to

$$p_v = \frac{1}{\beta} \ln\left(\frac{H_0}{H}\right). \quad (14)$$

Using this along with Equation (11) in Equation (10) and further applying Equation (6), together with

$$u_0(t) = -H'(t), \quad (15)$$

results in

$$\frac{\rho g h_0 - \frac{1}{\beta} \ln\left(\frac{H_0}{H}\right) - \frac{\psi \sqrt{\pi} \mu d}{4K\kappa}}{H(t)} + \rho g = \frac{\mu}{K} u_0(t) = -\frac{\mu}{K} \frac{dH(t)}{dt}, \quad (16)$$

which may be written as

$$\frac{\rho g [h_0 + H(t)] - \frac{\psi \sqrt{\pi} \mu d}{4K\kappa}}{H(t)} = -\frac{\mu}{K} \frac{dH(t)}{dt} + \frac{1}{\beta H(t)} \ln\left(\frac{H_0}{H(t)}\right), \quad (17)$$

or after some manipulation,

$$\rho g [h_0 + H(t)] = -\frac{dH(t)}{dt} \left[\frac{\mu}{K} \left(H(t) + \frac{\psi \sqrt{\pi} d}{4\kappa} \right) \right] + \frac{1}{\beta} \ln\left(\frac{H_0}{H}\right). \quad (18)$$

Here $\rho g H(t)$ represents the hydrostatic head within the cylinder. While this is small compared with the full applied pressure $\rho g h_0$, we shall include it here. At this stage, it can be seen that with $d = 20 \mu\text{m}$, $\kappa = 0.14$, and $\psi \approx 0.5$, filter pressure jump is quite small, and may be dropped for the present case. Additionally, in Equation (18), for the first term on the right-hand side to stay positive for all time, we need to have β to be large enough so that $\rho g [h_0 + H(t)] > (1/\beta) \ln(H_0/H)$. Otherwise, there would be equilibrium between the hydrostatic pressure force and the compressive resistance. However, we have seen experimentally that even with moderate hydrostatic pressures, drainage continues, indicating that the compressibility β is quite large and its effect may be ignored.

Now, making use of Equation (9) in Equation (18), and taking $\rho g h_0 \beta \gg 1$, it is found that

$$-\frac{dH(t)}{dt} \left[\frac{\mu H_0}{K_0} \left(\frac{\ln \phi_0 + 0.931}{\ln \left(\frac{H_0}{H(t)} \right) + \ln \phi_0 + 0.931} \right) \right] = \rho g [h_0 + H(t)]. \quad (19)$$

If we define

$$\xi = \ln \left(\frac{H_0}{H(t)} \right), \text{ and } b = -(\ln \phi_0 + 0.931), \quad (20)$$

Equation (19), with this change of variables, can be written as

$$\left[\frac{H_0^2 \mu}{K_0} \left(\frac{b}{b - \xi} \right) e^{-\xi} \right] \frac{d\xi}{dt} = \rho g [h_0 + H_0 e^{-\xi}]. \quad (21)$$

This may be rearranged in the form

$$\rho g h_0 \frac{dt}{d\xi} = \left(\frac{\mu H_0^2}{K_0} \right) \frac{b e^{-\xi}}{(b - \xi) [1 + q e^{-\xi}]}, \quad (22)$$

where $q = (H_0/h_0) \cong 0.025$. Noting that $q e^{-\xi} \ll 1$, Equation (22) can be further expressed as

$$\rho g h_0 \frac{dt}{d\xi} = \left(\frac{\mu H_0^2}{K_0} \right) \frac{b e^{-\xi}}{(b - \xi)} \left[\sum_{n=0}^{\infty} (-q)^n e^{-n\xi} \right], \quad (23)$$

and integrated exactly to give

$$\left(\frac{\mu}{K_0} \right) H_0^2 b \left[\sum_{n=1}^{\infty} (-q)^{n-1} e^{-nb} [\text{Ei}(nb) - \text{Ei}(nb - n\xi)] \right] = \rho g h_0 t, \quad (24)$$

where $\text{Ei}(\cdot)$ represents the exponential integral. Here, with $\phi_0 \approx 0.001$, we have $b \approx 6$, while ξ has a maximum of about $\xi_{\max} \approx \ln 3 = 1$. With these ranges of corresponding values, we do not cross the singularity on the right side of Equation (23). Additionally, we note that with increasing n in the series, the term $q^{n-1} e^{-nb}$ becomes extremely small and reaffirms the premise that the body force within the vitreous body for this experiment is extremely weak. Restoring (24) back to the measured variables, we obtain

$$\left(\frac{\mu}{K_0} \right) H_0^2 b F[H(t)] = \rho g h_0 t, \quad (25)$$

where

$$F[H(t)] = \sum_{n=1}^{\infty} (-q)^{n-1} e^{-nb} \left[\text{Ei}(nb) - \text{Ei}\left(nb - n \ln\left(\frac{H_0}{H(t)}\right)\right) \right]. \quad (26)$$

Here, the initial vitreous height H_0 is known from the measurement while K_0 is an unknown that we are interested in. In the numerical calculations, we use $\phi_0 = 0.001$ available from past studies (Stay, et al., 2003). While exact values may be difficult to determine, the results are quite insensitive to 25% variations in ϕ_0 around this value.

4. RESULTS AND DISCUSSION

For this set of experiments, measurements with 64 fresh bovine eyes were carried out, and in each case 20–30 time-points at 15-minute intervals were obtained for the vitreous height $H(t)$ versus t . These data were recorded as the vitreous drained under negative pressure at the bottom. This history was compared with the theoretical result given in Equation (24).

Different negative pressure values were applied, and the measured data for $H(t)$ was fed into the left-hand side of Equation (24) and plotted against time (Figure 7). A least-squares best fit for $F[H(t)]$ versus t was made while floating K_0/μ . However, the experimental data appeared to have a steep slope at early times and small $\xi = \ln(H_0/H(t))$, and does not show a good fit. Therefore, some adjustment to the formulation is needed for this trend. To adjust for this effect for small ξ , we adopt the following formula in place of Equation (21):

$$\left[\frac{H_0^2 \mu}{K_0} \left(\frac{b}{b - \xi} \right) \frac{(1 + m\xi)e^{-\xi}}{(1 + qe^{-\xi})} \right] \frac{d\xi}{dt} = \rho g h_0. \quad (27)$$

The linear term $m\xi$ is introduced on the basis of the measured behavior of ξ versus t as seen in experimental points in Figure 7. Such behavior can be approximated by adding a linear term in the expression for $dt/d\xi$ which, upon integration, would have a parabolic characterization for small ξ . Integration of Equation (27) yields

$$\left(\frac{\mu}{K_0} \right) H_0^2 b \left\{ (1 + mb) \sum_{n=1}^{\infty} (-q)^{n-1} e^{-nb} [\text{Ei}(nb) - \text{Ei}(nb - n\xi)] + \frac{m}{q} \ln\left(\frac{1 + qe^{-\xi}}{1 + q}\right) \right\} = \rho g h_0 t. \quad (28)$$

Equation (28) was least-squares fitted (least L_2 norm) with the experimental data keeping μ/K_0 as an unknown. At the same time the value of m was adjusted to provide the best fit as shown in Figures 8 and 9. For each experiment the value of K_0/μ was obtained by the least-squares fit. The average over all the experiments gave the resulting value for the hydraulic conductivity as $K_0/\mu = (7.9 \pm 3.1) \times 10^{-12} \text{ m}^2/(\text{Pa}\cdot\text{s})$ with $m = 7.23$. The uncertainty is the standard deviation over the different experiments. This result compares well with the results of Fatt (1975) but is somewhat lower than the results of Xu et al. (2000) who gave $K_0/\mu = (8.4 \pm 4.5) \times 10^{-11} \text{ m}^2/(\text{Pa}\cdot\text{s})$. The error bars were based on allowing a 1-mm visual accuracy in the measurement of $H(t)$ and then adjusted for ξ which is the parameter for the vertical axis.

However, we need to keep in mind that our experiments were conducted at room temperature (20°C) while Xu, et al., (2000) did it at 37°C. Here, the effect of liquid viscosity plays a role. Comparison of saline viscosity values at different temperatures shows that if we conducted the experiment at 37°C, we can expect approximately 50% higher value [$K_0/\mu = (1.2 \pm 0.47) \times 10^{-11} \text{ m}^2/(\text{Pa}\cdot\text{s})$]. Undoubtedly, the analytical approach to the vitreous drainage problem in the current development is different from Xu, et al. (2000), in that, we have employed the Jackson & James (1986) formula [Equation (4) here] for K . This relationship between K and ϕ is an aggregate of several sets of experiments and lends itself to a high degree of confidence. Furthermore, the current experimental work is for the bovine vitreous where the hyaloid membrane is kept intact, and the measurement is a representation of the effective average resistance for the entire vitreous body. The hyaloid membrane offers additional resistance that has not been measured as yet and further experimentation is needed. The numerical result obtained here provides useful information for mathematical modeling of transport within the vitreous and forms an important component for a comprehensive mathematical model for the eye.

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Figure 1:
Vitreous humor structure consisting of collagen and hyaluronic acid. Adapted from Nickerson, et al. (2008) with permission

Figure 2:

Micrograph images of the vitreous.

(A) Human vitreous humor detailing a single collagen fibril and a network of hyaluronan. Bar 200 nm. Adapted from Brewton & Mayne (1992) with permission.

(B) Bovine calf vitreous fibrils rotary shadowed from 0.05 M ammonium bicarbonate-glycerol. The fibrils diameter is reported as 28 nm. Adapted from Seery & Davison (1991) with permission.

Figure 3:
 Schematic of the drainage cell. The drainage tube provides the negative hydrostatic pressure.

Figure 4:
A single cell for water drainage from the vitreous.

Figure 5.

The fiber part is trapped above the porous membrane and the liquid part (saline and liquid vitreous) goes through the membrane. Liquid content partially drained so fiber volume fraction increases. The fiber is shown with the a highly exaggerated scale for the purpose of illustration. Vitreous images adapted from Nickerson, et al. (2008) with permission.

Figure 6:

Illustration of spatially uniform liquid percolation velocity $u_0(t)$ in the vitreous. The top surface recedes downwards with decreasing $H(t)$ and pushes the liquid through the filter at the same speed (average) as the recession rate.

Figure 7:

Plot of experimental data for ξ versus t indicating the steep trend at early time. The continuous line is the least-squares fitted curve based on Equation (24), and the blue band is the standard deviation over the different experiments.

Figure 8:
Plot of ξ versus t after adjusting Equation (21) with (27) to obtain the best fit. The error bars are omitted for clarity.

Figure 9:
Plot in Figure 8 with error bars.

Table 1:

Nomenclature

		dimension
a	fiber radius	m
A	cylinder cross-section area	m ²
b	$-(\ln \phi_0 + 0.931)$, Equation (20)	-
d	filter pore size	m
$Ei()$	exponential integral	-
$F(\xi)$	function defined in Equation (26)	-
g	gravitational acceleration	m ² /s
h_0	negative hydrostatic head	Pa
$H(t)$	height of the vitreous body	m
H_0	initial height $H(0)$	m
K	Darcy coefficient	m ²
K_0	initial value of K	m ²
m	parameter in Equation (25)	-
p	pressure	Pa
p_m	pressure jump at filter	Pa
p_v	compressive force pressure	Pa
q	ratio (H_0/h_0) in Equations (21)–(22)	-
t	time	s
u	fluid velocity	m/s
u_0	drainage speed	m/s
V	volume	m ³
V_s	fiber volume	m ³
z	vertical coordinate	m
β	fiber compressibility	Pa ⁻¹
K	filter open area fraction	-
μ	liquid viscosity	kg/(m-s)
ξ	$\ln[//_0/H(t)]$, Equation (20)	-
ρ	liquid density	kg/m ³
ϕ	fiber volume fraction	-
ψ	filter resistance parameter	-