Published in final edited form as:

Early Child Res Q. 2019 ; 47: 487–495. doi:10.1016/j.ecresq.2018.07.006.

Dimensionality of Preschoolers' Informal Mathematical Abilities

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Abstract

Recent research examining children's early mathematical abilities has focused primarily on number and operations (e.g., counting, addition) with considerably less attention directed to the role of other possible dimensions of early mathematical abilities, such as, measurement, geometry, and patterning. The current study examined the dimensionality of informal mathematical abilities by conducting categorical confirmatory factor analysis (CCFA) using data from a large sample of preschool children from low-income families ($N=1630$; Mean age = 4.46 years, $SD = .37$) using the Child Math Assessment (CMA; Klein & Starkey, 2004). The best fitting model consisted of four factors of Number and Operations, Measurement, Geometry, and Patterning, with the Number and Operations factor explaining common variance in three first-order factors of Numbering, Operations, and Relations. These findings support the view that informal mathematical knowledge is a multi-dimensional construct comprised of each of these separable dimensions. Additionally, a Multiple Indicators Multiple Causes model was used to determine if mathematical ability differed for male and female preschoolers on each of the four factors or on each of the 35 items of the CMA. Results showed no differences for mathematical abilities between males and females at this age. Future research and curricular implications are discussed.

Keywords

mathematics; dimensions; preschool; assessment

Dimensionality of Preschoolers' Informal Mathematical Abilities

Preschool children possess informal mathematical knowledge and abilities and these early competencies prior to school instruction provide a foundation for formal mathematical achievement (Bisanz, Sherman, Rasmussen, & Ho, 2005; Ginsburg, Klein, & Starkey, 1998;

Purpura, Baroody, & Lonigan, 2013). There is, however, variability in children's informal mathematical development with some preschoolers considered at-risk for poor academic achievement in math (e.g., Dyson, Jordan, & Glutting, 2011; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Klein & Starkey, 2004). Children who acquire mathematical knowledge slowly or who have difficulty grasping concepts are unlikely to catch-up to their same-age classmates without intensive and sustained intervention (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Morgan, Farkas, & Wu, 2009). In recent years, a growing body of research has focused on the development of assessment tools to identify and monitor early mathematical development and interventions to support young children's individual instructional needs in early mathematics (Barnes et al., 2016; Clements, 2007; Clements & Sarama, 2008; Ginsburg, Lee, & Boyd, 2008; Klein, Starkey, Clements, Sarama, & Iyer, 2008; Purpura & Lonigan, 2015; Sophian, 2002). However, many of these studies have focused on young children's knowledge of number and operations (e.g., counting, addition), sometimes referred to as numeracy, as a foundational ability in early mathematics (e.g., Jordan et al., 2009; Purpura & Lonigan, 2013; Toll & Van Luit, 2013). Considerably less research has focused on other possible domains of mathematical abilities that may fit within a broader conceptualization of preschool informal mathematical knowledge, such as, patterning, geometry, and measurement. Furthermore, little is known about mathematical achievement between boys and girls during preschool. In the current study, the dimensionality of informal mathematical abilities was investigated to determine if there were separable dimensions of mathematics at the preschool level, and, if so, what they were and whether they differed for boys and girls.

There is an assumption underlying early childhood mathematics curricular standards that informal mathematical knowledge is multi-dimensional. The National Research Council's (2009) report on mathematics learning in early childhood recommends three key strands of mathematics instructional content in early childhood classrooms--number and operations, geometry, and measurement. The National Council of Teachers of Mathematics (NCTM; 2006) also recommends these same three strands of early mathematical abilities as essential *Prekindergarten Curriculum Focal Points* for instruction as part of a coherent mathematics curriculum from prekindergarten to eighth grade. Looking beyond the prekindergarten year, the Common Core State Standards (CCSS; Council of Chief State School Officers, 2010), widely adopted throughout the United States, identifies expectations for children's knowledge and skills upon graduation from high school, and accordingly, establishes content standards for kindergarten to twelfth grade that align with these expectations. The CCSS for kindergarten mathematics instruction include number and operations (e.g., counting and cardinality, operations and algebraic thinking), measurement (e.g., weight, length, size), and geometry (e.g., identify, describe, and compose shapes). These curricular expectations of mathematics, along with patterning, are also included in kindergarten curricula internationally, such as, Australia (e.g., Australia Curriculum Assessment and Reporting Authority, 2013) and Canada (e.g., Ontario Ministry of Education: The Kindergarten Program, 2006). Furthermore, this assumption that informal mathematical knowledge is multi-dimensional also applies to curricula developed and evaluated in intervention studies (e.g., Clements & Sarama, 2008; Klein et al., 2008; Presser, Clements, Ginsburg, & Ertle, 2015; Starkey, Klein, & Wakeley, 2004).

Despite this prevailing assumption of multi-dimensionality of preschoolers' informal mathematical abilities, the extent to which number and operations, geometry, measurement, and patterning constitute separable dimensions of informal mathematical abilities is unclear. On the one hand, informal mathematical knowledge might be a unidimensional construct and, as such, represents a single underlying cognitive ability. If this were the case, any mathematical task (e.g., counting) should measure preschool children's general mathematical ability and any curricular activity (e.g., counting practice) should support children's general mathematical ability. On the other hand, if informal mathematical knowledge is a multi-dimensional construct, then specific tasks would be needed to measure discrete constructs and specific curricular activities related to each construct would be required to promote children's development in those areas. Accordingly, it is necessary to understand the dimensional structure of preschoolers' informal mathematical abilities to ensure that curricular activities align with mathematical development for preschool children (Clements, Fuson, & Sarama, 2017) and that measures assess a broad range of early mathematical abilities to identify young children who may require more intensive and individualized instruction in mathematics (Barnes et al., 2016).

Research results demonstrate that each of the four instructional strands of informal mathematical abilities emerge prior to formal instruction and these early competencies are associated with their mathematical achievement (Clements et al., 2017; Nguyen et al., 2016). The findings from multiple studies provide compelling support for ensuring that young children acquire a foundation in number and operations during early childhood (Jordan et al., 2009; Purpura et al., 2013; Purpura, Hume, Sims, & Lonigan, 2011; Purpura & Lonigan, 2013). Jordan et al. (2009) reported that kindergarten children's level of number competence predicted rate of growth in mathematics achievement between first and third grades. Moreover, children's rate of growth in number competence predicted their level of mathematics achievement at the end of third grade. Nguyen et al. (2016) reported that prekindergarten children's counting competencies were predictive of fifth grade mathematics achievement; specifically, more advanced counting abilities (e.g., counting with cardinality, counting forward and backward from a given number) explained the variance in children's fifth grade achievement outcomes even after controlling for the three other math strands of interest in this study (i.e., geometry, patterning, measurement). Taken together, these results indicate the importance of early number and operations during the preschool years prior to formal instruction.

In addition to number and operations, young children develop knowledge of geometry, such as recognizing and analyzing two- and three-dimensional shapes and reasoning about spatial orientation, during the preschool years. Clements, Wilson and Sarama (2004) identified a developmental progression for geometric understanding from prekindergarten to second grade. It begins with knowledge about physical shapes, then involves mental imagery about shapes and constructs, and finally entails explicit geometric knowledge. Furthermore, these early geometric abilities have been associated with more general mathematics achievement in school (for a review, see National Research Council, 2009). As such, geometric abilities that emerge in preschool are important precursors of children's later mathematical learning.

Preschool children also demonstrate knowledge of informal measurement, including the direct comparison of objects on physical dimensions (e.g., length) and the use of a non-standard unit to measure objects. Conceptual skills that underlie or are associated with children's knowledge of non-standard units of measurement include partitioning (i.e., mentally separating an object into same-sized units), unit iteration (i.e., mentally recognizing the length of a small item fitting within the length of a longer one), and conservation (i.e., understanding that if an object is moved or rotated it is still the same length). Informal measurement abilities have also been the focus of curricula activities developed for preschool math interventions (Clements & Stephan, 2004; Klein & Starkey, 2004; Sophian, 2004). For example, curricula activities may help children learn to directly compare the relative length of two objects (e.g., which pencil is longer?) or to use a non-standard unit of measurement (e.g., which string is about the same length as these cubes?). Sophian (2004) developed a preschool math intervention that focused primarily on measurement concepts, and reported a significant difference in the treatment group's performance on a broad assessment of early mathematical abilities as compared to the control group.

Finally, research has established that patterning, particularly the ability to duplicate and extend patterns, emerges during the preschool years (Rittle-Johnson, Fyfe, Loehr, & Miller, 2015; Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013). Patterning involves recognizing a relationship that can be generalized over a set of elements, such as a repeating pattern ABAB as seen in red star, blue star, red star, blue star or a growing pattern of a 2, 4, 6 structure). Initially, young children's pattern knowledge is based on the concrete properties of objects comprising a pattern, such as color or shape, but older children can use pre-algebraic reasoning and symbolic language to describe the pattern relationship. There is both correlational and causal evidence of the association between children's early patterning abilities and later mathematical achievement. Longitudinal studies have indicated that preschool patterning ability is a unique predictor of math achievement in fifth grade, even after controlling for demographics, cognitive ability, literacy and other mathematics skills (Fyfe & Rittle-Johnson, 2016; Rittle-Johnson, Fyfe, Hofer, & Farran, 2016). Furthermore, preschool children who received an intervention to learn repeating color and shape patterns significantly outperformed the control group across a variety of patterning tasks, both immediately following the intervention and one year later (Papic, Mulligan, & Mitchelmore, 2011).

Dimensionality of Informal Mathematical Abilities

Although previous studies have investigated relations between individual instructional strands of informal mathematical abilities during the preschool years and children's formal math achievement, few studies have examined the dimensionality of preschoolers' math knowledge. Purpura and Lonigan (2013) conducted confirmatory factor analysis (CFA) to examine the underlying structure of Number and Operations with a large sample of preschool children and reported that a three-factor model consisting of Numbering, Relations, and Arithmetic Operations fit the data best. This finding provides empirical support for instructional activities that promote discrete aspects of Number and Operations. However, it does not show how Number and Operations fit within a broader model of

informal mathematical abilities nor does it provide support for the recommended curricular strands of mathematical instruction (e.g., NCTM; 2006).

Clements, Sarama, and Liu (2008) developed and revised the Research-based Early Math Assessment (REMA) using a Rasch model. They determined that the items of this measure conceptualized two broad domains of early mathematical ability, number and operations as one factor and geometric/spatial competencies as a second factor. This multi-dimensional two-factor model two underlying constructs of mathematical ability and provides support for treating instructional activities that promote competencies in geometry, patterning, and measurement as a single construct. LeFevre et al. (2010) used path analysis to examine cognitive precursor skills (i.e., linguistic ability, quantitative knowledge, and spatial attention) and early numeracy abilities in preschool children as predictors of formal mathematical abilities two years later. They reported that linguistic abilities (i.e., elision, vocabulary) predicted linguistic mathematical abilities (e.g., number naming) but not non-linguistic abilities (e.g., subitizing--a rapid, perceptual process that can form numerical representations of small sets of items without counting them). This finding suggests that, for preschool children, informal mathematical abilities may be characterized as a multi-dimensional two-factor model of linguistic and non-linguistic math abilities.

Finally, Robinson, Abbott, Berninger, and Busse (1996) used CFA to determine the factor structure of informal mathematical ability with preschool children who were identified as having advanced or precocious mathematical abilities. The authors reported that a three-factor model of mathematical competencies (i.e., Quantitative, Verbal, and Spatial Abilities) fit the data best. Collectively, all these studies support multi-dimensional models of informal mathematical abilities, but they differ in the number of dimensions proposed and the underlying theoretical models. More importantly, no studies have yet examined the four-dimensional model that aligns with the key instructional strands in mathematics for preschool children.

Current Study

The primary purpose of this study was to determine the dimensionality of preschool children's informal mathematical abilities. To do this, we used categorical confirmatory factor analysis (CCFA) on responses to the 35 items on the Child Math Assessment (CMA; Klein & Starkey, 2004; Starkey & Klein, 2012) from a large sample of preschool children from low-income families to examine the absolute and relative fit of four primary models. A one-factor model in which all responses were assumed to be the result of a common underlying Informal Mathematics Skill factor served as the base model. In line with the results of studies that have linked performance on the four possible dimensions of informal mathematics abilities to formal mathematics abilities of older children (e.g., Clements, 2004; Rittle-Johnson et al., 2013), we evaluated a proposed four-factor model consisting of Number and Operations, Geometry, Measurement, and Patterning factors. Two two-factor models were also evaluated. First, in line with results of Clements et al. (2008), we evaluated a model that included Number and Operations as a single Numeracy factor and Geometry, Measurement, and Patterning as a single Visual-Spatial factor. Because measurement items on the CMA assess children's knowledge of relative weight, length, and size, skills that rely

on linguistic knowledge (e.g., knowing the word *longer*). We also evaluated a model that included number and operations combined with measurement as a single Linguistic Math factor and geometry and patterning as a single Non-linguistic Math factor in-line with the findings of LeFevre et al. (2010).

A secondary purpose of this study was to determine if mathematical abilities in preschool, as measured by the CMA, identified differences in male and female performance on latent variables of informal mathematical abilities. To do this analysis, Multiple Indicator Multiple Causes (MIMIC) structural equation models (SEM) were examined. Studies have reported an achievement gap for mathematical abilities favoring males over females (Denton & West, 2002; Husain & Millimet, 2009; LoGerfo, Nichols, & Chaplin, 2006) and that this gap begins to appear before first grade (Penner & Paret, 2008; Robinson & Lubienski, 2011). In a study of 9,927 gifted middle school children, Benbow and Stanley (1980) reported a significant difference in male and female performance on the Mathematics subtest of the Scholastic Aptitude Test (Daniel, 2010). More recently, studies have reported that males perform better than females on tests of mathematical skills as early as kindergarten (Penner & Paret, 2008; Robinson & Lubienski, 2011), and this performance difference widens by third grade (Denton & West, 2002; Husain & Millimet, 2009; LoGerfo et al., 2006). However, it is unknown whether differences in mathematical performance for males and females might begin prior to kindergarten and whether differences might be attributable to relative strengths and weaknesses for specific dimensions of mathematical ability. Spelke (2005) conducted a meta-analytic review of behavioral and neuroimaging studies and identified small-to-negligible differences showing that females tend to perform better on tests of verbal fluency, calculation, and memory for spatial locations of objects, whereas males tend to perform better on tests of verbal analogies, mathematical word problems, and memory for visual-spatial information. Based on these findings and on-going concerns that females are underrepresented in professions in many fields related to science, technology, engineering, and math (Hyde & Linn, 2006; National Science Foundation, 2015), the current study evaluated whether male and female preschool children performed differently in their mathematical ability, as measured by the items on the Child Math Assessment.

Method

Participants

The sample for this study included 1,630 preschool children recruited from 174 preschool centers that served primarily children from low-income families (e.g., Head Start, state-funded prekindergarten centers). These centers were located in four geographic regions, Florida ($n = 455$; 58 preschools), Texas ($n = 431$; 53 preschools), California ($n = 367$; 36 preschools), and Kentucky ($n = 377$; 27 preschools) with 4 to 11 children per class. The children ranged in age from 2.89 to 5.54 years (Mean = 4.46 years, $SD = .37$). The sample included 820 females and 800 males (10 children had missing data related to sex), and there were no significant differences in the ages of the males and females in this sample, $t(1594) = .190$, $p = .850$. Additionally, the children were from diverse racial and ethnic backgrounds (i.e., 38% were white, 33% were black/African American, 21% were Hispanic/Latino, 7% were other, and 1% of the sample was unreported).

Measure: Child Math Assessment (CMA)

The CMA was designed to assess a broad set of mathematical concepts and skills that develop during early childhood, including number, arithmetic operations, geometry and space, measurement, and patterning. The CMA is an untimed test that includes 9 tasks with between two and six items per task. The CMA is administered individually to children in one session that takes approximately 20 minutes. All tasks begin at the first item for all children, and the items generally increase in difficulty within each task. Items are scored as correct or incorrect with no partial scores awarded. Ceiling differs across tasks. The 9 tasks are presented here in the order they were administered to the children.

Object counting (OC1–5).—This task includes five items that assess children’s ability to count an array of objects, ranging in set sizes from 3 to 30, and state the cardinality of each set. Linear arrays of objects are presented to children and they are asked to count each object and state how many there are. A stop rule is activated if the child responds incorrectly on two consecutive items.

One-set addition and subtraction (OS1–4).—This task includes four items that assess children’s ability to solve simple addition and subtraction problems in which the final set of objects is hidden from view. Children are shown an initial set of objects (bunnies) that are then dropped into an opaque container (the “rabbit” hole). Children then observe objects being added to or removed from the container, and they are asked “How many bunnies are in the rabbit hole altogether?” or “How many bunnies are left in the rabbit hole?” All four items are administered. This task is comprised of four items that assess children’s ability to solve simple addition and subtraction problems in which the final set of objects is hidden from view. Children are shown an initial set of objects (bunnies) that are then dropped into an opaque container (the “rabbit” hole). Children then observe objects being added to or removed from the container, and asked “How many bunnies are in the rabbit hole altogether?” or “How many bunnies are left in the rabbit hole?” All four items are administered.

Geometric reasoning (GR1–3).—This task includes three items that assess children’s ability to reason about pairs of congruent triangles, one of which that has undergone a geometric transformation (slide, rotation, or flip). Each item involves presenting a picture card with a pair of right-angled scalene triangles in different spatial orientations. Children are then asked to identify the corresponding sides of the two triangles, by “touching the side of your (the second) triangle that is the same as my (the first) side.” This is repeated for each of the three sides. If the child does not touch two out of three sides (correctly or incorrectly) on the first item, a stop rule is activated and the task is discontinued.

Construction of equivalent sets (CS1–2).—This task includes two items that assess children’s ability to construct a set with the same number of objects as a model set. Children are shown the model set of objects, and asked to construct a set of the same number using a basket of similar objects. All items are administered.

Two-set addition and subtraction (TS1–4).—This task includes four items that assess children’s ability to determine the effect of an addition or subtraction to one of two hidden sets of objects (marbles in an opaque container). Children watch as two initially equal or unequal sets of marbles (e.g., 4 vs. 4 or 4 vs. 6) are dropped into containers. Children are then asked whether the initial sets “have the same number of marbles,” or if “one container has more?” If incorrect, the item ends. If correct, one marble is either added to or taken from one set. Children are then asked whether the final sets “have the same number of marbles,” or if “one container has more?” A stop rule is activated after two incorrect responses.

Direct measurement (DM1–6).—This task includes six items that assess children’s ability to compare two items based on weight, capacity, or length. Children are asked to determine which of two items weighs more, holds more, or is longer. Children are shown a toy dog, placed on the center of a large mat. The objects for comparison are introduced and then each placed on opposing sides of the mat. Children are asked to select the object that is heavier, bigger, or longer. All items are administered.

Shape recognition (SR1–4).—This task includes four items that assess children’s ability to recognize exemplars of basic two-dimensional shapes (circles, rectangles, triangles, and squares). For each item, children are presented with a picture board containing multiple two-dimensional shapes. Each board consists of three or four exemplars of the target shape, several distractor shapes, and a shape that is similar, but does not satisfy all geometric properties of the target shape (e.g., an oval on the circle board). Children are then asked to point to all of the target shapes. A stop rule is activated if the child points to all shapes on the board on two consecutive items.

Pattern duplication (PD1–4).—This task includes four items that assess children’s ability to replicate a simple repeating pattern of objects (e.g., A-B-A-B, AA-BB-AA-BB). For two items, children are shown a model pattern, provided with a source set consisting of more than the minimum number of identical objects, and instructed to use the objects to “make the same pattern.” For the remaining two items, children are shown a model pattern, but provided with a source set of objects differing in color and shape prior to being instructed to “make the same kind of pattern.” A stop rule is activated if the child responds incorrectly on two consecutive items.

Division (DV1–3).—This task includes three items that assess children’s understanding of division of small sets, both with and without a remainder. Children are asked to share a homogeneous set of objects (treats) between two or three recipients (toy friends). All items are administered.

Procedure

Once parental consent was received, a research assistant, trained in the administration of the CMA, met with each child individually in a quiet area of the child’s school to administer the CMA. Children’s correct responses were scored as 1 and incorrect responses were scored as 0. Analyses were conducted on item-level data from the CMA using Mplus 7.4 (Muthén & Muthén, 2013). All items were specified as unordered categorical variables. Four models

were evaluated using CCFA to test model fit and the robust weighted least squares mean and variance adjusted (WLSMV) was the estimator. Simulation studies have shown the WLSMV estimator is superior to the maximum likelihood estimator for categorical data both in terms of model rejection rates and appropriate estimation of factor loadings (Beauducel & Herzberg, 2006). The DIFFTEST function in Mplus was used to compare nested models. Statistically significant p -values indicated that the less restricted model resulted in significant improvement of fit. A sandwich estimator was used to account for the nesting of children in preschool classrooms (Asparouhov, 2005). We used a weighted root-mean square residual (WRMR), a fit index calculated for categorical data, with a score close to 1.0 considered to be a good model fit (Yu, 2002). Further, Yu recommended these additional values for indicators of good model fit: the confirmatory fit index (CFI) .96, and root mean square error of approximation (RMSEA) .05 with CFI performing relatively better than Tucker–Lewis index (TLI) and RMSEA. Based on these recommendations, emphasis was given to WRMR and CFI as fit indices in these analyses.

Results

Table 1 includes summary data of the frequency of the children's correct responses to each item within each proposed construct or strand of mathematical ability. There was no set of items that displayed floor or ceiling effects in the data. Zero-order correlations of the children's scores across the 35 CMA items are shown in Table S1 in the on-line supplementary materials.

A priori hypotheses for possible test-item form influences for items within task (e.g., the four Shape Recognition items, SR1–4) were considered, and residuals for those items were allowed to correlate in all models. The following residuals were allowed to correlate for within-task items: (a) the three items of the Division task, (b) items 1 and 2 (i.e., weight), items 3 and 4 (i.e., size), and items 5 and 6 (i.e., length) of the Direct Measurement task, (c) the four items of the Shape Recognition task, (d) items 1 and 3 of the Pattern Duplication task (i.e., both items following the ABAB pattern) and items 2 and 4 of the same task (i.e., both items following the AABBAABB pattern), and (e) the first two items of the two-Set Addition and Subtraction task. Although form influences were considered possible for other items within tasks, these were the ones hypothesized to be related due to item form.

Tests of model fit for the four theoretical models are shown in Table 2. CCFA indicated that the one-factor model produced inadequate model fit. The two two-factor models were compared to Model 1. Model 2, the two-factor model of Number and Operations and a second factor that included items related to geometry, measurement, and patterning yielded significantly better fit than did Model 1. Model 3, the two factor model of Number and Operations and Measurement as a Linguistic Math factor and Geometry and Patterning as a second Visuospatial Math factor also yielded significantly better fit than did Model 1; however, Model 3 was not a better fitting model than Model 2 based on fit indices. Given that these two two-factor models were not nested, they could not be compared to each other using a chi-square difference test. The four-factor model, Model 4, yielded significantly better fit than did either of the two-factor models.

Post-hoc Examination of Discrete Dimensions for Number and Operations

Based on the finding that the items from the CMA data fit a four-factor model of Number and Operations, Geometry, Measurement, and Patterning best, additional analyses were conducted to determine whether Purpura and Lonigan's (2013) three-factor model could be fit to the Number and Operations factor using the CMA item-level data. First, using only the Number and Operations items from the CMA, the same five models tested by Purpura and Lonigan (i.e., a one-factor model of informal math, three two-factor models (i.e., Numbering + [Operations and Relations], [Numbering and Operations] + Relations, [Numbering and Relations] + Operations), and a three-factor model (i.e., Numbering + Operations + Relations) were considered. These analyses (see Table 3) replicated the finding of Purpura and Lonigan (2013), identifying that the three-factor model of Numbering, Operations, and Relations fit the data best, $\chi^2(126) = 439.97$, $p < .001$, CFI = .97; TLI = .97; RMSEA = .04; WRMR = 1.57.

Next, we considered whether this three-factor model of Number and Operations would fit within the four-factor model to comprise a second-order model consisting of six first-order factors with the covariation between Numbering, Operations, and Relations explained by the second-order factor of Number and Operations. This second-order model provided a good fit to the data, $\chi^2(536) = 1159.27$, $p < .001$, CFI = .97; TLI = .96; RMSEA = .03; WRMR = 1.48, and fit the data significantly better than did the four-factor model, $\chi^2(3) = 395.68$, $p < .001$ (See Figure 1).

Differences Between Males and Females

Using the second-order model with correlated residuals as described above, a MIMIC model was used to test the effect of sex on items and factor means. To do this, the four factors (i.e., Number and Operations, Geometry, Patterning, Measurement) were regressed onto the covariate (i.e., sex: 1=female, 0=male). Additionally, each of the 35 indicators was regressed onto the covariate fixed at zero. A non-significant direct effect of the covariate on a latent factor indicates population homogeneity; that is, the factor means are not different at different levels of the covariate. A non-significant direct effect of the covariate on an item of a factor indicates that when the factor is held constant, the means of that indicator are not different at different levels of the covariate. The MIMIC model employs a single variance and covariance structure of both factors and indicator variables (or items) to allow for simultaneous analysis. Kline (1994) recommended evaluating the chi-square result in relation to the degrees of freedom having a ratio of chi-square to *df* less than 3:1 suggesting good model fit.

Before fitting the model to the data in this moderator analysis, the raw scores for males and females were compared using an independent samples *t*-test and the results showed that there was no significant difference in the mean total score for males ($M = 12.16$, $SD = .26$) and females ($M = 12.13$, $SD = .24$) for this measure, $t(1592) = -.07$, $p = .94$. Further, there were no significant differences for male and female children's responses on the 35 items on the CMA with the exception of Object Counting 3, $\chi^2(1) = 4.47$, $p = .03$, favoring the males, and Division 1, $\chi^2(1) = 5.15$, $p = .02$, favoring the females.

The results of this MIMIC model indicated that there were no significant differences for any of the four factors regressed on the covariate of sex, with estimates for Number and Operations of 0.02 ($p = .41$), estimates for Measurement of -0.01 ($p = .71$), estimates for Geometry of 0.01 ($p = .84$), and estimates for Patterning of -0.01 ($p = .67$). Model fit was good, $\chi^2(565) = 1177.01$, $p < .001$, with a CFI of .97, TLI of .96, RMSEA of .03, and WRMR 1.46. The ratio of chi-square to degrees of freedom was still less than 3:1 at 2.1:1. See Figure 1 for the factor structure of the CCFA regressed onto the covariate sex for the factor loadings and the correlations between the factors. To determine if the covariate of sex operated differentially at the item level, modification indices were examined. None of the pathways between sex and the 35 items were suggested to be relaxed in the modification indices, indicating that sex did not differentially affect the results.

Discussion

The goals of this study were twofold. The first goal was to determine the underlying structure of preschool children's informal mathematics skills using item-level data from a measure designed to assess the effects of a broad curriculum-based intervention for preschool mathematics. The second goal was to determine if there were differences in males' and females' performance on this underlying latent structure across both the factors and the items. Results of this study indicated that the model that fit the data best was a four-factor model of informal mathematical ability consisting of Number and Operations, Measurement, Geometry, and Patterning, with the Number and Operations factor consisting of three first-order factors of Numbering, Operations, and Relations (based on the model of Numeracy in Purpura and Lonigan, 2013). Furthermore, there were no differences between males and females in the latent means for the four factors. These results support a multidimensional model of informal mathematical ability. These findings have implications for both research and practice in early mathematics.

The factor structure of the children's responses to the 35 items from the CMA provides support for informal mathematical abilities as a multidimensional construct that includes four discrete cognitive strands of mathematics. Two previous studies have identified multidimensional models of informal mathematical abilities (Clements et al., 2008; Robinson et al., 1996). Robinson et al. (1996) used CFA to examine the factor structure of informal mathematics data for preschool children with precocious mathematical abilities and found that a three-factor model of mathematical competencies (i.e., quantitative, verbal, and spatial abilities) fit the data best. In the current study, examination of two two-factor models, one model that included a single Number and Operations factor that included quantitative and verbal abilities and one model that proposed a separable visuospatial abilities factor, indicated that although the models provided adequate fit to the data, the fit was not better than the four-factor model hypothesized in this study. Furthermore, Clements et al. (2008) reported that a Rasch analysis of informal math data from the REMA revealed two dimensions of mathematical abilities, number and operations as one dimension and geometry, measurement, and patterning as a second dimension. Note that this model was considered as one of the two-factor models (i.e., Numeracy and [Measurement + Geometry + Patterning]), but the four-factor model fit the data better than this model. Results of the current study established separable dimensions of informal mathematical abilities that align

with current instructional recommendations for mathematics in prekindergarten (NCTM, 2006; NRC, 2009) and support the addition of patterning as a discrete dimension of mathematics for children at this age.

Based on the previous finding of a three-factor model of Number and Operations (Purpura & Lonigan, 2013), follow-up analyses were conducted to test whether the data from the CMA would first replicate that model to achieve good fit, and second, whether the Number and Operations factor within the four-factor model could be extended to include these three first-order factors (i.e., Numbering, Operations, Relations). The children's scores on the CMA items that were consistent with Number and Operations provided good fit for Purpura and Lonigan's model, supporting a three-factor structure of Number and Operations using item-level data from a different set of questions and a larger sample of preschool children. Furthermore, when these three sub-dimensions of Number and Operations were included as first-order factors of the Number and Operations factor, the results indicated that informal mathematical abilities are consistent with a complex structure of constructs.

To explore the association of sex on the four factors of preschool children's informal mathematical abilities, a MIMIC model was conducted that indicated that there were no significant differences across males and females in performance on any of the four factors or on any of the items of this measure. Although studies of older children have indicated that there are differences in mathematical abilities between males and females (e.g., Denton & West, 2002; Spelke, 2005), the results of this study indicated that this was not the case for this preschool sample at either the factor or the item level. Two longitudinal studies using data from the Early Childhood Longitudinal Study, Kindergarten Class of 1998–1999 (ECLS-K) examined achievement trends of males and females in mathematics beginning at kindergarten and reported that there were no statistically significant differences in male and female overall performance at the beginning of kindergarten but that an achievement gap began to emerge during kindergarten for children at the top of the achievement distribution, and that by first grade the males performed better than the females (Penner & Paret, 2008; Robinson & Lubienski, 2011). Fryer and Levitt (2009) explored a number of possible explanations for differences in male and female mathematics abilities for kindergarten children, including less investment by females in mathematics, low parental expectations, and tests that were biased toward males, but they found little support for any of these. The results of the current study indicate that, for informal mathematical abilities, there are no differences in performance between males and females across the four dimensions at the beginning of the preschool year. This suggests that the conditions that influence differences in math performance begin after the preschool year, or they emerge during the preschool year and are not evident in math performance at this early point. Efforts to identify possible influences that contribute to an achievement gap for mathematical abilities between males and females have provided some answers; however, further work is needed.

Implications

The results of this study reveal that preschool children's informal mathematical ability is best characterized as a multidimensional construct that aligns with recommended curricular strands of early childhood mathematical instruction. Preschool programs that include

activities and opportunities for learning related to each of the four strands of informal mathematical abilities will promote competencies in each respective strand. These separable dimensions represent distinct abilities, and, as such, children may possess advanced abilities in one dimension but not the others. Conversely, children may display deficits in one or more of these discrete dimensions and intervention may be more effective by identifying and isolating the specific area of early mathematics and providing explicit support for that area. Moreover, these separable dimensions of mathematical abilities may relate uniquely to formal mathematical abilities or to other abilities – such as advanced mathematical abilities in later grades. Further research is needed to examine these relations. Finally, the absence of an observable difference between males and females in informal mathematical skills at the beginning of the preschool year indicates that efforts to identify sources of a disparity in achievement between males and females for STEM subjects may need to look beyond the age of the children in this sample to the kindergarten year to identify when differences in early math abilities between males and females begin to emerge and how they contribute to later differences in math achievement.

Limitations

Despite the strengths of this study, such as the large sample size and hypothesis testing of theoretical models of informal mathematical ability for this measure, some limitations need to be considered. First, this study focused on item-level data from the single measure CMA and further research is needed using multiple measures of each domain to confirm this multi-dimensional model of informal mathematical abilities. This may prove difficult because there are few assessments of informal mathematical abilities that include items intended to measure geometry, measurement, and patterning. One measure, the REMA (Clements et al., 2008) does include items that align with factors in the current four-factor model and data from that measure could be fitted to this model. Second, the sample in this study included children from families of lower socio-economic status, a feature that may account for differences in findings for dimensionality and differences in performance between males and females that may benefit from replication in future studies.

Conclusion

The findings of this study indicated that early childhood informal mathematical ability is a multi-dimensional construct at preschool age, based on the items of the CMA, and that there was no evidence of an achievement gap between males and females for these factors or for the items at this young age. Understanding that the structural nature of informal mathematical abilities extends beyond the model of three first-order factors of Number and Operations to include three additional dimensions of Measurement, Geometry, and Patterning abilities provides strong support for the inclusion of activities that support all four strands in early mathematics instruction, assessment, and intervention. Furthermore, these results provide a framework for future research in informal mathematical achievement, and, in-line with behavioral and neuro-imaging studies with older children (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999), these separable dimensions may be associated with other cognitive abilities that contribute to children's overall academic achievement.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

Acknowledgments

This research and report was supported by grants from Eunice Kennedy Shriver National Institute of Child Health and Human Development (HD048497, HD052120) to Florida State University, and the Institute of Education Sciences, United States Department of Education to Florida State University (R305F100027) and to WestEd (R305K050004). The views expressed herein are those of the authors and have not been reviewed or approved by the granting agency.

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Figure 1.

Categorical confirmatory factor analysis model of informal mathematical abilities using item-level data from the Child Math Assessment with a large sample of preschool children.

Table 1

Frequency of Correct Responses to Items Within Task and By Construct

Construct	Task	Item	Frequency Correct	Percentage Correct (per task)
<i>Number and Operations</i>				
Numbering				
	Object Counting ^a	OC1	1028	.64
		OC2	770	.48
		OC3	643	.40
		OC4	267	.17
		OC5	82	.05
Operations				
	One-Set Addition & Subtraction ^a	OS1	1041	.65
		OS2	413	.26
		OS3	929	.58
		OS4 ^b	494	.31
	Two-Set Addition & Subtraction	TSP1 ^b	258	.16
		TSP2 ^a	248	.16
		TSP3 ^c	110	.07
		TSP4 ^d	66	.04
	Division	DV1 ^e	565	.35
		DV2 ^e	480	.30
		DV3 ^f	39	.02
Relations				
	Construction of Equivalent Sets ^a	CS1	764	.48
		CS2	719	.45
<i>Measurement</i>				
	Direct Measurement ^g	DM1	535	.34
		DM2	563	.35
		DM3	1058	.67
		DM4	944	.59
		DM5	924	.58
		DM6	908	.57
<i>Geometry</i>				
	Geometric Reasoning ^a	GR1	634	.40
		GR2	402	.25
		GR3	573	.36
	Shape Recognition ^e	SR1	536	.34
		SR2	276	.17

Construct	Task	Item	Frequency Correct	Percentage Correct (per task)
		SR3	440	.28
		SR4	277	.17
	<i>Patterning</i>			
	Pattern Duplication ^f	PD1	522	.33
		PD2	433	.27
		PD3	178	.11
		PD4 ^h	88	.06

Note.

^a
n=1596;

^b
n=1595;

^c
n=1529;

^d
n=1490;

^e
n=1594;

^f
n=1593;

^g
n=1591;

^h
n=1592.

Table 2

Tests of Model Fit for the Factor Structure of Informal Mathematical Ability

Model	Factors	Description	χ^2	df	CFI	TLI	RMSEA	WRMR	Compare Models	χ^2 -diff	df
1	4	Num; Meas; Geo; Patt	1834.56***	539	0.93	0.93	0.039	2			
2	2	Num; (Meas + Geo + Patt)	2653.19***	544	0.89	0.88	0.049	2.5	1 vs. 2	428.26***	5
3	2	(Num + Meas); (Geo + Patt)	2520.46***	544	0.9	0.89	0.048	2.44	1 vs. 3	353.56***	5
1	1	Informal Math Ability	3181.21***	545	0.86	0.85	0.055	2.77	1 vs. 4	704.29***	6

Note. $N = 1595$. χ^2 and df are estimated by Mplus. χ^2 -diff is Chi-Square Test for Difference Testing using DIFFTEST in Mplus 7.4; CFI = Confirmatory Fit Index; TLI = Tucker-Lewis Index; RMSEA = Root Mean Square Error of Approximation; WRMR = Weighted Root-Mean-Square Residual; Num = Number and Operations; Meas = Measurement; Geo = Geometry; Patt = Patterning.

 $p < .001$.

Table 3

Tests of Model Fit for Factor Structure of Number and Operations

Model	Factors	Description	χ^2	df	CFI	TLI	RMSEA	WRMR	χ^2	df
1	3	Numb; Oper; Relat	439.97***	126	0.97	0.97	0.04	1.57		
2	2	Numb; [Oper + Relat]	895.69***	128	0.94	0.93	0.06	2.34	2 v 1	237.09***
3	2	[Numb + Oper]; Relat	765.02***	128	0.95	0.94	0.06	2.15	3 v 1	182.04***
4	2	[Numb + Relat]; Oper	1028.37***	128	0.93	0.91	0.07	2.53	4 v 1	293.22***
5	1	Numeracy	1366.03***	129	0.90	0.88	0.08	2.93	5 v 1	492.28***

Note. $N = 1595$. Numb = Numbering; Oper = Operations; Relat = Relations.

 $p < .001$.