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A Robust Impedance Controller Design for Series Elastic Actuators using the Singular Perturbation Theory

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Abstract

Impedance control is capable of further flexibly adjusting the driving-point impedance of series elastic actuators (SEAs) in addition to impedance reduction by the elastic element. This enhances safety and compliance during interaction between humans and robots, in comparison with rigid robots under impedance control or SEAs under position control. In this study, we propose an impedance controller for SEA systems that is developed based on the singular perturbation (SP) theory and time-delay estimation (TDE) technique. The SP theory allows for alleviating the burden of the requirement for states to be measured. The TDE technique is effective in compensating for system dynamics and uncertainties involved in system behaviors with minute computation loads. Employing both a simulation study and experimental study, we demonstrate the efficacy of the proposed control created from the combination of the SP theory and TDE technique. The effect of the proposed impedance control on reducing the driving-point impedance of interacting SEAs is examined.

I. INTRODUCTION

In human-robot interaction, impedance matching between two parties could promote maximum exchange of power and information [1]. It also guarantees safety to humans interacting with robots. Perhaps, rigid robots are not suitable for interaction with humans in

that regard. In series elastic actuators (SEAs), compliance is purposefully introduced between an actuator and an end-effector, and masks the high impedance of the actuator from humans or environments [2]–[8]. SEAs are able to maintain low driving-point impedance at high frequencies, while maintaining the ability to shape the impedance within their operational bandwidth.

Insofar as the SEA design is a hardware approach, impedance control is a software approach for reducing the driving-point impedance of robots. Impedance controllers regulate a dynamic relationship between the desired trajectory and the interaction force [9]. The driving-point impedance of the controlled robot can be lowered based on the user's purpose, up to the actuator's controllable bandwidth. Recently, SEA systems as well as rigid robotic systems have had the benefit of impedance control [4], [10]–[14].

In comparison to rigid robot control, we face two major challenges in controlling SEA systems. These two issues result from the fact that SEAs are a non-collocated system in that the end-effector is controlled by the deformation of the elastic structure and the deformation is controlled by the actuator [15]. These systems that involve two independent masses have more than twice the number of parameters characterizing system behaviors in comparison with rigid robotic systems. In particular, the property of the elastic element dominantly influences the behaviors of SEA systems. Since SEAs are controlled with only one control command, they typically require high-order states to be measured and fed back for control. More sensors or numerical differentiation using measurable states can be used, but such approaches expose certain limitations including an increase in implementation cost and noise amplification. Too, for satisfactory control, the parameters governing system behaviors might require to be well identified. Even in use of PID-type controllers [4], [10], [16], [17], properties of the elastic element need to be known.

The singular perturbation (SP) theory is capable of dealing with the system order issue. It is widely accepted that the order of a system with elasticity included in it can be reduced using the theory [18]. According to the theory, SEA systems that include both slow variable (link position) and fast variable (elastic force) can be separated into two subsystems that are controlled separately based on these variables. The so-called slow system which is governed by the slow variable behaves like a rigid system so any control strategy designed for a rigid robot manipulator is applicable to the slow system. The other subsystem—fast system—is also a second-order system. No high-order derivatives need to be fed back to the control architecture. Researchers have proposed various controllers with the singular perturbation approach yielding satisfactory control performance [19]–[24].

Regarding the dynamics identification issue, the time-delay estimation (TDE) technique can be adopted. This technique has been regarded as an attractive tool in the control area enabling “simple but robust” control schemes [25]–[31]. When an input is imposed on a system, the output reflects system dynamics and disturbances. The TDE technique utilizes this fact; the technique uses the command (input) and its accompanying system state (output) to quantitatively estimate the system dynamics and disturbances, which otherwise are difficult to identify precisely. Thus controllers with TDE incorporated do not require

precise knowledge of the target system. In particular, the TDE technique eliminates the need to identify elasticity placed between an actuator and a robot link.

Here, we propose a robust impedance controller for SEAs developed based on the SP theory and TDE technique. As mentioned before, several impedance control algorithms for SEAs have been proposed [4], [10]–[12], [14]. While some focus on driving-point impedance at low frequencies only by modulating the apparent stiffness of the end-effector (e.g., [4], [10]), others take driving-point impedance at high frequencies into consideration through system dynamics compensation (e.g., [11], [12], [14]). These impedance controllers replace the system dynamics with the desired dynamics shaped by impedance components including inertia, damping and stiffness. Driving-point impedance, in particular, at high frequencies, is further reduced, since the inertia of the end-effector is replaced with a lower virtual inertia [11]. Impedance control through dynamics compensation is considered beneficial in terms of impedance rendering and tracking performance over a wide range of frequencies. However, these control schemes that involve dynamics compensation require tuning of many control gains. Also, at least nominal values of system parameters need to be identified. These restrictions hinder practical applications.

In this study, we first go over the basic expectation concerning advantages of dynamics compensation under impedance control of SEAs using simplified singular-perturbed models, in terms of operational bandwidth and driving-point impedance. Impedance control that involves system dynamics compensation is expected to exhibit wider bandwidth but lower impedance over frequencies, in comparison with control which does not cancel out system dynamics. Inspired by the observed benefits, an impedance controller for SEA systems is developed that is incorporated with the TDE technique capable of successfully replacing system dynamics with the desired dynamics. Singular perturbation eliminates the requirement for high-order states. In what follows, an experimental study demonstrates that the proposed control is advantageous in operational bandwidth and driving-point impedance. In addition, it is superior in tracking performance, in comparison with an impedance controller involving no dynamics compensation.

II. IMPEDANCE OF REDUCED-ORDER SEAs

A. System order reduced by the SP theory

Fig. 1 presents a typical SEA system. With the elasticity between the link and rotor modeled as a linear torsional spring with stiffness k , the SEA system can be expressed, which is structurally similar to flexible-joint systems [32], as follows:

$$\begin{aligned} I\ddot{q}_1 + mgl\sin q_1 &= k(q_2 - q_1) + f_1 + d_1, \\ B\ddot{q}_2 + k(q_2 - q_1) &= f_2 + d_2 + u, \end{aligned} \quad (1)$$

where q_1 , q_2 denote the angles of the link and rotor, respectively; I , m the moment of inertia and mass of the link, respectively; g the gravitational constant; l the distance between the joint and the center of mass of the link; B the inertia of the rotor; f_1 , f_2 friction forces; d_1 , d_2 bounded disturbances that include sensor noise, communication noise or/and external impulse; u control input.

We attempt to reduce the order of the SEA system, which is of 4th order, by employing the SP theory [33]. The SP theory is basically a system decomposition approach for separating a high-order system into a slow dynamics model (steady-state system) and fast dynamics model (boundary-layer system). A controller is separately designed for each model, and the overall system is controlled through the operations of the separated controllers. Typically, the theory is applied to a system that can be expressed as, with a small value of ϵ :

$$\begin{aligned}\dot{\mathbf{x}} &= f_1(\mathbf{x}, \mathbf{z}, \epsilon, t), \\ \epsilon \dot{\mathbf{z}} &= f_2(\mathbf{x}, \mathbf{z}, \epsilon, t).\end{aligned}\quad (2)$$

To apply the SP theory to the SEA system, we define the torque produced by the elastic element as state $z (\triangleq k(q_2 - q_1))$, and double differentiate this state with respect to time, assuming that the stiffness of the elastic element is constant:

$$\ddot{z} = k(\ddot{q}_2 - \ddot{q}_1) = k((-z + f_2 + d_2 + u)/B - (z + f_1 + d_1 - mgl\sin q_1)/I). \quad (3)$$

With an additional introduction of $k \triangleq \frac{k_1}{\epsilon^2}$, (3) can be rearranged as

$$\epsilon^2 \ddot{z} = -k_1(1/I + 1/B)z + k_1(f_2 + d_2 + u)/B - k_1(f_1 + d_1 - mgl\sin q_1)/I. \quad (4)$$

From (4), we see that the SEA system (1) can be represented in SP form, with the vectors $\mathbf{x}$ and $\mathbf{z}$ denoting $[q_1 \quad \dot{q}_1]^T$, $[z \quad \dot{z}]^T$, respectively.

Now, we split the SEA system into two coupled subsystems by setting ϵ as 0: a steady-state system and a boundary-layer system. This setting leads to elimination the dynamics of the spring from the overall system dynamics in the steady-state system, and the boundary-layer system deals with the dynamics of the spring.

1. Quasi-steady-state system—When ϵ is set to 0, the quasi steady state of z can be obtained as

$$h(\mathbf{x}, t) \triangleq z|_{\epsilon=0} = (I(f_2 + d_2 + u) - B(f_1 + d_1 - mgl\sin q_1))/(I + B). \quad (5)$$

Substituting (5) to the link part of the SEA system yields the steady-state system:

$$(I + B)\ddot{q}_1 + mgl\sin q_1 = u|_{\epsilon \rightarrow 0} + f_1 + d_1 + f_2 + d_2. \quad (6)$$

This system is slower than the boundary-layer system. We can design the control input $u|_{\epsilon \rightarrow 0}$ for this slow system in the following subsection, naming it u_s :

$$(I + B)\ddot{q}_1 + mgl\sin q_1 = u_s + f_1 + d_1 + f_2 + d_2. \quad (7)$$

2. Boundary-layer system—The boundary-layer system is an error system between the value of state z and that of its quasi steady state h . Introducing an error state ($y \triangleq z - h$) and the fast time scale $\sigma = t/\epsilon$, we derive the following from (4):

$$\begin{aligned} \frac{d^2 y}{d\sigma^2} + \frac{d^2 h}{d\sigma^2} &= -k_1(1/I + 1/B)(y + h) + k_1(f_2 + d_2 + u)/B \\ &\quad - k_1(f_1 + d_1 - mgl\sin q_1)/I. \end{aligned} \quad (8)$$

By setting ϵ as 0, the variables x and t become stationary; h is constant at the fast time scale σ . Neglecting the high derivative of h and using (5), we have the boundary-layer system as follows:

$$\frac{d^2 y}{d\sigma^2} = -k_1(1/I + 1/B)y + k_1 u_f / B, \quad (9)$$

where $u_f = u - u_s$.

Remark: Each control input, u_s and u_f works for the quasi-steady-state system and boundary-layer system, respectively. The control input u is the summation of the two control inputs applied to the two systems:

$$u = u_s + u_f. \quad (10)$$

B. Closed-loop bandwidth and impedance

We analyze the advantage of impedance control of SEAs involving dynamics compensation in terms of operational bandwidth and driving-point impedance over impedance control of rigid robots and position control of SEAs.

Control system—Schematic descriptions of the three control systems with/without an elastic element are displayed in Fig. 2. In the SEA, the end-effector M is separated from the rotor B by a spring while the end-effector and rotor form one mass in the rigid body. Friction is imposed on the end-effector. We rewrite the equations of motion of the simplified SEA for convenience:

$$\begin{aligned} M\ddot{x}_l &= k(x_m - x_l) + F_e, \\ B\ddot{x}_m + k(x_m - x_l) &= u, \end{aligned} \quad (11)$$

where F_e denotes the external force applied to the end-effector.

The sampled-data system environment is considered; continuous systems are controlled using a digital computer. Two limitations are taken into account: the limited bandwidth of

the actuator and a time delay from zero-order hold (ZOH), as depicted in Fig. 3. The delay of half the sample time T results from the digital computer using A/D and D/A converters [34]. The two factors can be established, for simplification, as

$$D(s) \triangleq \frac{\omega^2}{s^2 + 2\omega s + \omega^2} e^{-\frac{T}{2}s}, \quad (12)$$

where ω denotes the bandwidth of the motor.

The objective of impedance control is to eliminate the system (end-effector) dynamics and inject dynamics formed by the desired impedance, characterized by M_d , b_d and k_d . Position control pushes the end-effector to the desired position. We calculate the bandwidth and driving-point impedance of each control system, assuming that the exact models are identified.

For the quasi-steady-state system of the SEA that governs the dynamics of the end-effector (refer to (7)), we can design a simple PD controller for position control:

$$u_{s,PD} = d(\dot{x}_d - \dot{x}_l) + p(x_d - x_l), \quad (13)$$

where d and p are control gains.

Meanwhile, the impedance controller that replaces the system dynamics with the desired dynamics can be developed as

$$u_{s,Imp} = (M + B)\ddot{x}_l + M_d(\ddot{x}_d - \ddot{x}_l) + b_d(\dot{x}_d - \dot{x}_l) + k_d(x_d - x_l). \quad (14)$$

For the boundary-layer system, we design a simple control law as follows [23], [35]:

$$u_f = k_v(\dot{x}_m - \dot{x}_l), \quad (15)$$

where k_v is a control gain.

An assumption that the derivative of the manifold h with respect to σ is neglected and substituting (15) into (9) lead to

$$\frac{d^2 y}{d\sigma^2} + k_1 k_v / B \frac{dy}{d\sigma} + k_1 (1/I + 1/B) y = 0. \quad (16)$$

This error dynamics converge to 0 exponentially. Given that the boundary-layer system is time-invariant, the control input (15) is sufficient to ensure exponential stability [35].

Closed-loop performance—To investigate the closed-loop bandwidth with the control inputs (13), (14) and (15) injected into the simplified system (11), the closed-loop forward transfer function relating the end-effector position x_l to its desired position x_d can be obtained in Laplace domain. These transfer functions are presented in (17) and (18) for

position control and impedance control, respectively. Meanwhile, we obtain the driving-point impedances defined as the external force F_e over the velocity $\dot{x}_l$, are presented in (19) and (20), respectively.

- Bandwidth: Position control

$$\frac{X_l(s)}{X_d(s)} = \frac{D(s)(ds + p)}{\frac{M}{k}Bs^4 - \frac{M}{k}D(s)k_v s^3 + (M + B - D(s)d)s + D(s)p}. \quad (17)$$

- Bandwidth: Impedance control

$$\begin{aligned} \frac{X_l(s)}{X_d(s)} &= \frac{D(s)(M_d s^2 + b_d s + k_d)}{\frac{M}{k}Bs^4 - \frac{M}{k}D(s)k_v s^3 + (M + B - D(s)(M + B - M_d))s^2 + D(s)b_d s + D(s)k_d}. \end{aligned} \quad (18)$$

- Driving-point impedance: Position control

$$\frac{F_e(s)}{sX_l(s)} = \frac{M Bs^4 - M D(s)k_v s^3 + k(M + B)s^2 - kD(s)ds + kD(s)p}{Bs^3 - D(s)k_v s^2 + ks} \quad (19)$$

- Driving-point impedance: Impedance control

$$\begin{aligned} \frac{F_e(s)}{sX_l(s)} &= \frac{M Bs^4 - M D(s)k_v s^3 + (k(M + B) - kD(s)(M + B - M_d))s^2 + kD(s)b_d s + kD(s)k_d}{Bs^3 - D(s)k_v s^2 + ks} \end{aligned} \quad (20)$$

For a comparative study, we consider rigid robots under impedance control with the control input designed as $(-M - B + M_d)\ddot{x}_l + b_d\dot{x}_l + k_d x_l$. The closed-loop transfer function of the end-effector position x_l versus the desired position x_d can be obtained as

$$\begin{aligned} \frac{X_l(s)}{X_d(s)} &= \frac{D(s)(M_d s^2 + b_d s + k_d)}{M + B - D(s)(M + B - M_d)s^2 + D(s)b_d s + D(s)k_d}, \end{aligned} \quad (21)$$

while driving-point impedance of rigid robots can be obtained as

$$\begin{aligned} \frac{F_e(s)}{sX_l(s)} &= (M + B)s + D(s)\left(-M - B - M_d\right)s + b_d + \frac{k_d}{s}, \end{aligned} \quad (22)$$

Fig. 4 presents numerical results regarding the closed-loop bandwidth and driving-point impedance for three cases: impedance control of a rigid robot, position control of an SEA and impedance control of an SEA. The numerical results are based on all control gains set to the same value, respectively, emphasizing on driving-point impedance at high and low frequencies. The required parameters are adopted based on [11], [36] and summarized in TABLE I. The actuator bandwidth is set low as 500 Hz arbitrarily for the simulation study. The impedance parameters are selected as $M_d = 0.1$ (50% of the driven part inertia), $b_d = 0.05$ and $k_d = 0.02$. For position control, the gains d, p are set as 0.05 and 0.02, respectively. The gain k_v is chosen as 20.

As shown in Fig. 4(a), the operational bandwidth of SEA impedance control is wider than that of SEA position control. The bandwidth of SEA position control is bounded around $\omega = \sqrt{p/(M+B)}$, whereas the bandwidth of SEA impedance control is limited to $\omega = \sqrt{k_d/M_d}$, which is nearly the same as that of rigid impedance control. These results indicate that the inertia replacement by impedance control is beneficial in increasing the operational bandwidth.

Up to the operational bandwidth of each case, the programmed stiffness is apparent at low frequencies. The trajectories of the driving-point impedances of each case overlap on the asymptote of the rendered stiffness; k_d for the two impedance control cases and p for the position control case. In these cases in which the programmed stiffness is greater than the stiffness of the elastic element, the lower one appears [4]. Meanwhile, beyond the operational bandwidth, the driving-point impedance approaches to the asymptote of the desired impedance M_d for the two impedance control cases. In contrast, the impedance of SEA position control remains above the asymptote of the end-effector inertia M over the middle range of frequencies. The reduced driving-point impedance suggests the advantageous use of impedance control on SEA systems. The range where the driving-point impedance of SEA impedance control remains near the asymptote of the desired impedance M_d is narrower than rigid impedance control. The main reason for this is the physical elastic element between the two masses; the stiffer elastic element leads to the wider range clinging to the asymptote of the desired impedance M_d . As predicted from a study [4], SEA systems shows lighter impedance than rigid systems at high frequencies beyond the motor limit defined in 12; while the rigid system approaches the asymptote of the summed impedance $M+B$, the driving-point impedance of SEA systems saturates to the end-effector inertia M , as seen in Fig. 4(b).

III. CONTROLLER DEVELOPMENT WITH TDE

A. Quasi-steady-state system

The objective toward the steady-state system (7) is to design the control input u_s that causes state q_1 to track a desired trajectory q_{1d} . We develop a controller using the TDE technique that eliminates the need of a model for the system.

Let us re-express (7), introducing a constant $\bar{\beta}$ and a term H , as follows:

$$\bar{\beta}\ddot{q}_1 + H = u_s, \quad (23)$$

where

$$H \triangleq (I + B)\ddot{q}_1 - \bar{\beta}\ddot{q}_1 + mgl\sin q_1 + Bf_1 + Bd_1 - f_2 - d_2. \quad (24)$$

The term H contains the system dynamics and all uncertainties. It is reasonable to construct an assumption that the term h is piece-wise continuous since the SEA system is generally composed of continuous components (we assume that the friction effective on the system is piece-wise continuous). The current value of the term H can be estimated with the previous-step control input and its corresponding measured output as follows:

$$H(t) \approx H(t - L), \quad (25)$$

$$\hat{H}(t) \triangleq H(t - L) = -\bar{\beta}\ddot{q}_1(t - L) + u_s(t - L). \quad (26)$$

The accuracy of TDE can be improved by reducing the sampling period L . This can be achieved by increasing the sampling frequency. Utilizing this strategy, the control law is constructed as

$$u_s(t) = \hat{H}(t) + \bar{\beta}v = -\bar{\beta}\ddot{q}_1(t - L) + u_s(t - L) + \bar{\beta}v. \quad (27)$$

The new control input, denoted as v , determines driving-point impedance by shaping the relationship between the error dynamics of the system and the external torque τ_e . We can design the new input v as

$$v = \ddot{q}_{1d}(t) + I_d^{-1}(b_d\dot{e}(t) + k_d e(t) + \tau_e(t)), \quad (28)$$

where $e(t) \triangleq q_{1d}(t) - q_1(t)$, and again I_d , b_d and k_d are impedance parameters.

A combination of (7), (27) and (28) produces

$$I_d\ddot{e}(t) + k_d\dot{e}(t) + k_p e(t) + \tau_e = I_d\varepsilon, \quad (29)$$

where $\varepsilon \triangleq \bar{\beta}^{-1}(H(t) - H(t - L))$.

As long as the sampling interval L is sufficiently small so ε is negligible, the control input (27) stabilizes the steady-state system with a unique solution. A proof is presented in an Appendix.

The final control law covering u_s and u_f is expressed as

$$u(t) = u_s(t - L) + \bar{\beta}[\ddot{q}_{1d}(t) - \ddot{q}_1(t - L) + I_d^{-1}(b_d \dot{e}(t) + k_d e(t) + \tau_e(t))] + k_v(\dot{q}_2(t) - \dot{q}_1(t)). \quad (30)$$

B. Stability analysis

The stability of the closed-loop system can be analyzed based on Tychonov's theorem [33]. According to this theorem, bounded stability of the SEA system is guaranteed if the convergence of the quasi-steady-state system is guaranteed and if the boundary-layer system is exponentially stable.

Tychonov's theorem: If the quasi-steady-state system has a unique solution $\bar{q}_1$ defined on the interval $t \in [0, t_1]$, and the boundary-layer system is uniformly exponentially stable in $(t, \bar{q}_1)$, then there exists ϵ^* such that $\forall \epsilon \in [0, \epsilon^*]$, the solution of the system satisfies:

$$z(t) = \bar{z}(t) + \eta(\sigma) + O(\epsilon), \quad (31)$$

$$q_1 = \bar{q}_1 + O(\epsilon), \quad (32)$$

where $\eta = z - \bar{z}$.

Since the boundary-layer system is exponentially stable with control input $u_r(15)$, it is concluded that the angle q_1 remains within a neighborhood $O(\epsilon)$ if the control input u_s guarantees the boundness of the closed-loop quasi-steady-state system. The stability of the closed-loop quasi-steady-state system with control input u_s is guaranteed (see the Appendix). Therefore, we conclude that the tracking error of the overall closed-loop system is bounded as long as external disturbances are bounded. The gain $\bar{\beta}$ determines stability as well as tracking performance. The criterion of selecting this gain is described in the Appendix.

C. Measurement noise

The proposed control, especially for the steady-state system, requires the position state and its derivatives to be measured. For practical reasons, all required states are difficult to measure using sensors. One way to deal with this issue is to use numerical differentiation and reduce its noise effect through use of a relatively small value of $\bar{\beta}$, which results in the same effect as a 1st-order digital LPF.

The proposed control input for the steady-state system (27) can be written by introducing $Q(\triangleq v - \ddot{q}_1(t - L))$ as

$$u(t) = u(t - L) + \bar{\beta}Q. \quad (33)$$

When a 1st-order LPF is injected into the control law, Eq. (33) can be rewritten as

$$u(t) = u^F(t - L) + \bar{p}Q, \quad (34)$$

where u^F is the filtered control input.

Designing a 1st-order LPF with the cutoff frequency of λ as

$$u^F(t) = \lambda L(1 + \lambda L)^{-1}u(t) + (1 + \lambda L)^{-1}u^F(t - L), \quad (35)$$

and substituting Eq. (34) into (35) leads to

$$u^F(t) = u^F(t - L) + \bar{b}Q, \quad (36)$$

where $\bar{b} = \lambda L(1 + \lambda L)^{-1}\bar{p}$.

From the fact that Eqs. (33) and (36) are in the same form, it is concluded that lowering the value of $\bar{p}$ plays the same role as a 1st-order digital LPF.

IV. EXPERIMENTAL VALIDATION

Here we validate the benefits of the proposed impedance control for SEA systems that carries out dynamics compensation in terms of operational bandwidth and driving-point impedance. For a comparative study, an impedance controller is selected that involves no dynamics compensation [4]. The target SEA system is a 1 degree-of-freedom hand rehabilitation system for extending and flexing the metacarpophalangeal joints (MCP) of the hand at various velocities (Fig. 5). In the previous section, the numerical analyses showed that impedance control produces wider operational bandwidth while providing lower driving-point impedance, in comparison with the control that involves no dynamics compensation. Also, it is expected that the proposed control that is incorporated with TDE is superior in tracking performance, noting that TDE is efficient in compensating for system dynamics and all kinds of uncertainties.

A. Settings

Fig. 5 displays the target SEA system that consists of a hand holder, a torsional spring and a motor. The hand holder is mounted to support and fixate the hand on it. The torsional spring (Lee Spring, LTL112H10M) of stiffness 0.0255 Nm/degree is included between the motor and the hand holder. The motor (LS Mecapion, APM-SA01ACN-8) embeds an encoder of a resolution of 2048 pulses per revolution and is connected with a harmonic drive of a gear ratio of 50:1. The hand holder weighs 270 g in total. A custom ACM motor drive (ADVANCED Motion Controls) is used to convert the motor command. A potentiometer (P3 America, STC12 R10K) is located inside the spring to measure its deformation. The absolute position of the hand holder is obtained using both the encoder and the potentiometer. The velocity and acceleration of the holder are computed through numerical differentiation. The external torque that is fed back to the control input is estimated with the deformation of the spring multiplied by its stiffness, assumed that the spring behaves in a linear manner. Data processing is conducted in a LabVIEW environment. The control signal is limited to ± 10 V. The sampling time is set as 1000 Hz.

B. Procedure

To investigate driving-point impedance (and accordingly the operational bandwidth), the hand holder under impedance control is manually shaken at various speeds increasing from 0. First, we investigate how successfully the proposed impedance control is capable of rendering the designed impedance, in particular, at low frequencies and high frequencies. The effects of values of the impedance parameters I_d , k_d on driving-point impedance are evaluated. We evaluate the effect of the desired inertia I_d for the proposed control on driving-point impedance beyond the operational bandwidth. The stiffness k_d is selected as 0.001, 0.005 and 0.01, while the desired inertia I_d is chosen as 0.0001 and 0.001. The parameter b_d is tuned to a value ranging from 0.0005 to 0.0085. Frequency-domain analyses are carried out using the estimated torque and angular velocity of the hand holder.

Secondly, the tracking performance of the proposed impedance control with motions of three frequencies is investigated, in comparison with the Vallery control that was proposed in [4], under the same designed impedance at low frequencies. The desired trajectories are designed as $30\sin(t)$, $30\sin(2t)$ and $30\sin(5t)$ degrees. For the proposed control, the impedance parameters are selected as $I_d = 0.0001$, $b_d = 0.001$ and $k_d = 0.005$. For the counterpart, the desired stiffness gain is set the same as the one for the proposed control ($P = 0.005$). The control gains for the controller are tuned as $K_o = 850$, $I_o = 85$, $K_j = 0.1$, and $I_j = 20$. These values are thought to provide the best performance.

Throughout the experimental study, the control gain k_v of the control in $u_f(15)$ is fixed at 0.001. The value of the gain in a certain range is found to not affect the difference in performance between cases. The gain $\tilde{\beta}$ is tuned to 3.5×10^{-5} for the best performance of each case.

C. Results

Fig. 6 displays a frequency response of driving-point impedance under the proposed impedance control. First of all, we observe that the impedances of the controlled system converge to the asymptotes of the desired impedance below the operational bandwidth ($k_d = 0.001$, 0.005 and 0.01). As another view of impedance at low frequencies, Fig. 7 displays plots of how far the hand holder retreats in response to an interaction force imposed by the user. Slopes of the curves of the interaction torque versus the angle of the hand holder under the proposed control converge to the desired stiffness of each case, which is comparable with the results under the Vallery control.

The system impedances above the operational bandwidth approach the asymptotes of the designed impedance ($I_d = 0.0001$ and $I_d = 0.001$). Theoretically, the rendered inertia converges to the desired inertia at high frequencies within the actuator's bandwidth under the impedance control. This leads to the advantage of the proposed control in terms of impedance over high frequencies. As well, the operational bandwidth can be broaden under the proposed control. As predicted, the operational bandwidth varies depending on the impedance parameters I_d and k_d . A lower value of I_d produces wider operational bandwidth, given that the desired stiffness is fixed at a certain value. The advantage in operational bandwidth, in comparison with under impedance control that does not modulate the inertia

of the end-effector (i.e., the Vallery control), can be revealed by taking a look at tracking performance with motions of various frequencies.

A comparative study between the proposed control and Vallery control regarding tracking performance with motions of several frequencies is presented in Fig. 8. Generally, the proposed control exhibits superior tracking performance to the Vallery control in following motions of several frequencies, while there are no notable differences in the magnitude of control inputs. As the motion speed increases, the effects of the inertia and friction forces including Coulomb friction correspondingly increase. This leads to greater tracking error in the second ($30\sin(2t)$) and third rows ($30\sin(5t)$) in comparison with the first row ($30\sin(t)$). TABLE II presents root mean square (RMS) errors in tracking between the desired and actual trajectories under the proposed control and Vallery control. These results imply that the controlled system has broader operational bandwidth under the proposed control; the proposed control enables the system to follow high-frequency motions. The superiority is also due partially to the efficacy of the TDE technique in cancelling out system dynamics and uncertainties. The noise component in control inputs may be alleviated if the potentiometer is replaced with an encoder.

V. CONCLUSION

With the need of impedance control of SEA systems, we have developed a model-free and robust control strategy by incorporating the SP method and TDE technique. The SP approach reduces the number of states required to be fed back to the controller. The TDE technique compensates for system dynamics and various uncertainties with low computation load. The numerical and experimental studies demonstrated that the proposed impedance control shows wider operational bandwidth while providing lower driving-point impedance in comparison with position control and impedance control that does not conduct dynamics compensation. The proposed control shows excellent tracking performance even with low impedance, indicating that TDE is efficient in compensating for system dynamics and uncertainties.

Appendix

In this Appendix, we investigate the stability of the closed-loop quasi-steady-state system.

The quasi-steady-state system (7) can be simplified as

$$\beta\ddot{q}_1 + c = u_s, \quad (37)$$

where $\beta \triangleq I + B$, $c \triangleq \sim mgl\sin q_1 - f_1 - d_1 - f_2 - d_2$.

Eq. (37) can also be re-expressed with a constant $\bar{\beta}$, as

$$\bar{\beta}\ddot{q}_1 + H = u_s, \quad (38)$$

where

$$H \triangleq (\beta - \bar{\beta})\ddot{q}_1 + c. \quad (39)$$

The control input (27) is rewritten:

$$u_s(t) = \widehat{H}(t) + \bar{\beta}v(t) = H(t - L) + \bar{\beta}v(t). \quad (40)$$

Originally, the proposed computational model compensates for $H(t)$ with its previous-step value $H(t - L)$, but there exists a difference between $H(t)$ and $H(t - L)$. The estimation error ε is defined as

$$\varepsilon(t) \triangleq \bar{\beta}^{-1}(H(t) - H(t - L)). \quad (41)$$

This estimation error can be written in terms of the new control input v and the angular acceleration $\ddot{q}_1$ through a combination of Eqs. (38) and (40) as

$$\varepsilon(t) = v(t) - \ddot{q}_1(t). \quad (42)$$

From Eqs. (37), (39), (40), and (42), a relationship can be drawn as follows:

$$\begin{aligned} \beta\varepsilon(t) &= \beta(v(t) - \ddot{q}_1(t)) \\ &= \beta v(t) + c(t) - u_s(t) \\ &= \beta v(t) + c(t) - \bar{\beta}v(t) - H(t - L) \\ &= \beta v(t) + c(t) - \bar{\beta}v(t) - (\beta - \bar{\beta})\ddot{q}_1(t - L) - c(t - L) \\ &= (\beta - \bar{\beta})v(t) - (\beta - \bar{\beta})\ddot{q}_1(t - L) + c(t) - c(t - L) \\ &= (\beta - \bar{\beta})v(t) + (\beta - \bar{\beta})(\varepsilon(t - L) - v(t - L)) \\ &\quad + c(t) - c(t - L). \end{aligned} \quad (43)$$

Multiplying the inverse of β to the both hands of Eq. (43) leads

$$\varepsilon(t) = (1 - \beta^{-1}\bar{\beta})\varepsilon(t - L) + (1 - \beta^{-1}\bar{\beta})\zeta(t - L) + \beta^{-1}\eta(t - L), \quad (44)$$

where

$$\zeta(t - L) \triangleq v(t) - v(t - L), \quad (45)$$

$$\eta(t - L) \triangleq c(t) - c(t - L). \quad (46)$$

Eq. (44) is the closed-loop dynamics in terms of the estimation error ε .

These three equations can be rewritten in discrete-time domain as follows in order:

$$\varepsilon(k) = (1 - \beta^{-1}\bar{\beta})\varepsilon(k - 1) + (1 - \beta^{-1}\bar{\beta})\zeta(k - 1) + \beta^{-1}\eta(k - 1), \quad (47)$$

where

$$\zeta(k-1) = v(k) - v(k-1), \quad (48)$$

$$\eta(k-1) = c(k) - c(k-1). \quad (49)$$

From the viewpoint of the estimation error ε , the terms η , ζ in Eq. (44) are regarded as forcing functions that are bounded in the case of a sufficiently small sampling period. If the magnitude of $(1 - \beta^{-1}\bar{\beta})$ is less than 1, ε is asymptotically bounded. The coefficient $(1 - \beta^{-1}\bar{\beta})$, in particular, the constant $\bar{\beta}$ determines how accurately dynamics estimation is achieved; its value affects the difference between H and $\hat{H}$, which eventually affect the tracking error [37].

Biography

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Fig. 1:

Typical series elastic actuator. An elastic element of stiffness k is placed between the link of inertia I and the rotor of inertia B . Motor command u is applied to the motor so that the rotor produces the deformation of the elastic element, which is expressed by q_1 and q_2 .

Fig. 2:

Three control systems with a rigid robot and a simplified SEA: (a) impedance control of a rigid robot, (b) position control of an SEA (c) impedance control of an SEA. Black dashed boxes indicate the actual mass that is felt by the environment, while blue solid boxes indicate the desired mass that is realized under control.

Fig. 3:

Sampled-data system: a continuous signal is measured from the target system every T seconds through an analog-digital converter (ADC). A control signal, which is processed in a computer, is imposed on the system through a digital-analog converter (DAC).

Fig. 4:

Bode plots of (a) the transfer function of the desired position x_d to the actual position x_l and (b) the transfer function (driving-point impedance) of the actual velocity $\dot{x}_l$ to the external force F_e for three cases: impedance control of a rigid robot, position control of an SEA and impedance control of an SEA.

Fig. 5:

Picture of an SEA system for hand stretching. A torsional spring is located in a frame with which the motor transfers power to the hand through the spring. A potentiometer is installed inside the spring to measure its deformation. The blue box contains the motor with a harmonic drive.

Fig. 6:

Experimental results—frequency response of driving-point impedance under the proposed control: (a) $I_d=0.0001$, $b_d=0.0005$ and $k_d=0.001$, (a) $I_d=0.0001$, $b_d=0.0005$ and $k_d=0.001$, (b) $I_d=0.001$, $b_d=0.002$ and $k_d=0.005$, (c) $I_d=0.0001$, $b_d=0.002$ and $k_d=0.005$, and (d) $I_d=0.001$, $b_d=0.0085$ and $k_d=0.01$. The red dash lines indicate the desired impedance. The black and purple dash lines indicate the asymptotes of the rendered stiffness and inertia of each value, respectively.

Fig. 7:

Experimental results—behaviors of the interaction torque versus the angle of the hand holder in low-frequency (about 1 Hz) motions under the proposed control and Vallery control. The desired stiffness is selected as 0.001, 0.005 and 0.01 (Nm/deg).

Fig. 8:

Experimental results—comparisons in tracking performance during steady state with motions of several frequencies. The desired trajectories are designed as $30\sin(t)$, $30\sin(2t)$ and $30\sin(5t)$ degrees. The desired impedance is set as $I_d = 0.0001$, $b_d = 0.001$ and $k_d = 0.005$ for the proposed control, and is set as $P = 0.005$ for the Vallery control.

TABLE I:

Parameters of an SEA system presented in [11], [36].

Parameter	Symbol	Value	Unit
Driven part inertia	M	0.2	kg
Driving part inertia	B	5.65	kg
Stiffness	k	72.9	Nm/rad
Sampling time	L	0.001	s

TABLE II:

Comparisons in tracking RMS error between the proposed control and Vallery control, based on the results presented in Fig. 8.

RMS error (deg)	$30\sin(t)$	$30\sin(2t)$	$30\sin(5t)$
Vallery control	4.917	9.153	16.331
Proposed control	0.679	1.659	1.682