Multiple dipole source localization of EEG measurements using particle filter with partial stratified resampling

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Abstract

Tracking and detection of neural activity has numerous applications in the medical research field. By considering neural sources, it can be monitored by electroencephalography (EEG). In this paper, we focus primarily on developing advanced signal processing methods for locating neural sources. Due to its high performance in state estimation and tracking, particle filter was used to locate neural sources. However, particle degeneracy limits the performance of particle filters in the most utmost situations. A few resampling methods were subsequently proposed to ease this issue. These resampling methods, however, take on heavy computational costs. In this article, we aim to investigate the Partial Stratified Resampling algorithm which is time-efficient that can be used to locate neural sources and compare them to conventional resampling algorithms. This work is aimed at reflecting on the capabilities of various resampling algorithms and estimating the performance of locating neural sources. Simulated data and real EEG data are used to conduct evaluation and comparison experiments.

Keywords EEG · Particle filter · Resampling · Localization · Inverse problems

1 Introduction

Electroencephalography (EEG) is a non-invasive neurological investigative tool used to quantify electrical fields produced by neural development in the cerebrum. EEG-based estimates are used to locate neural action and prompt new medicines, treat stroke, epilepsy, and diseases of Parkinson. Neural source localization has been used to distinguish sources of seizure activity in the brain, since the position of a seizure between the basal cortex, temporal cortex and lateral cortex can be separated by a neural dipole moment [10]. This must be accomplished if neural sources can be accurately classified from estimates. This issue is referred to the EEG inverse problem [35]. Moreover, since current dipole models have been used effectively to determine the electrical current appropriation of the cerebrum, the inverse problem can be used to approximate the factors from EEG

observations of localized current dipole cerebral sources [40]. Despite the fact that EEG yields high temporal resolution when observing cerebral activity, high spatial resolution is also required for predominant diagnostics. As a result, different sensors are placed over the whole scalp and advanced filtering techniques are used to find an ideal solution to the inverse problem [3, 17].

To know how the cerebrum works, we should first see how the scalp produces EEG observations. The primary sources of EEG voltages measured at the scalp derive from the current movements of many of the adjacent neurons that occur simultaneously in the same direction. In an activated area of the cerebrum, the aggregate electrical current is displayed regularly as a numerical current dipole with a sufficient dipole moment in that area. Furthermore, many of these present dipoles speak to current streams of similar orientation and can be replaced by an equivalent current dipole [26].

Currently, two major research areas exist in modeling neural generators. The first modeling technique involves utilizing imaging models, which clarify the information utilizing a thick arrangement of current dipoles circulated at settled areas. The second method is a parametric approach that takes advantage of the fact that these dense sets of current dipoles can be supplanted utilizing one equal current

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dipole [25]. Although the imaging-based techniques can create a thorough map of the cerebrum's neuronal activity, the parametric methodology provides a direct mapping of the EEG measurements to few parameters [29]. By using the parametric approach, the equivalent current models can provide more intuitive interpretations that explain the electrical activity in the cerebrum and can be fostered in emerging technologies, such as BCI systems.

An important challenge of the parametric methodology is the estimation of the areas of the equivalent dipole sources in 3D volume of the cerebrum utilizing EEG estimations recorded from the scalp. Most of the previous work [1, 13, 16, 27, 28, 36, 39] in EEG source localization is based on two assumptions (1) the quantity of dipoles is fixed (2) the locations of the sources are fixed in time.

In general, however, the quantity and positions of neural dipoles vary dynamically over time. In [35], the quantity of neural dipoles and their positions are dynamically evaluated and updated at every step described from the measured data. The unknown source positions and moments can be taken as the hidden state function and EEG data is taken as the state space model's measurement function. The optimal estimation of the hidden state in a Bayesian context depends on the state's posterior density function (pdf) given in the observations [9].

The Particle Filter (PF) is a sequential Monte Carlo procedure which uses Sequential Importance sampling to approximate the state's posterior distribution at each step [2]. Particle filter uses particle arrangement to sample the framework's state-space. These particles are then weighted utilizing the measurement model to give a measure of the state posterior density.

These weights fit in with the improvement of the system activities with respect to time. Particle weights are repetitively updated whenever new measured information is available to anticipate the framework's future condition. The particle filter is suffering from particle degeneration after a few iterations where many particles will have small weights. This implies that with a very small number of iterations, most particles will have irrelevant weights and, thus, the arbitrary measure they determine is undependable. Several resampling algorithms are suggested in the particle filter system to overcome particle degeneration. Resampling is considered to be a bottleneck in implementations due to the increasing complexity. An advanced resampling algorithm is proposed to deal with such difficulties.

The reason for this article is that it seeks to examine different methods of resampling for identifying neural sources. In this article, we mainly study the performance of time-efficient partial stratified resampling in the particle filter to estimate the location of neural sources from EEG data and conduct a comparative analysis of partial stratified resampling (PSR) with conventional resampling algorithms i.e.

multinomial, systematic and stratified resampling methods [4, 24]. Our goal is to verify whether and to what extent each method of resampling can use the synthetic data to locate dipole sources. Our study aims to provide comparative analysis of localizing neural source with particle filter using different methods of resampling to analyze real EEG data.

The paper's plan is the following. Section 2 provides a brief description of the EEG model and introduces the particle filter. It is followed by discussing the methods of multinomial, systematic, stratified and partial stratified resampling. Sections 3 and 4 consists of our results and discussion of the work, while the final remarks and bibliography are contained in part 5.

2 Methodology

2.1 EEG model

The localization of neural sources from the measured EEG data, which is called the inverse problem, is one of the most important issues for electrophysiology. An inverse problem solution can provide us with a model that maps generators to the recorded projections. The inverse problem is to determine the true functional tomography by identifying the source with minimum error. Nonetheless, the main challenge for this issue is that there is not enough information on generators in the measurements to make the problem inconsistent and therefore does not provide a perfect tomography. This is because various configurations of internal sources produce the same external electromagnetic field, and only a relatively small number of locations calculate these fields [33].

The brain cortex, which is the outer surface of the brain, contains 10 billion neurons. The most observed scalp activity occurs within this 1.5–4.5 mm thick part of the brain. The synchronous, synaptic simulation of a large number of neurons [41] guides the dipole current source onto the cortical surface. The measured EEG is simply the distribution of this current on various electrodes. In order to model the EEG data using the forward problem [30], it is important to quantify the relationship between the distribution of different primary dipoles and the data collected on electrodes. The brain sources need: (1) EEG data, (2) scalp electrode 3D position data, (3) Head model information on the electrical and geometrical properties of the head and (4) Source Model information on the location/orientation of dipole sources. The accurate measurements of electric fields in the brain was provided with realistic head models using spherical Head Model, Boundary Element Method (BEM) or Finite Element Methods (FEM)

[34]. The efficiency of EEG improves with the inverse solution by increasing accuracy in the definition of the head structure with related conductivity.

A primary current distribution is used to model the neural activity generating the electromagnetic field calculated by EEG that is approximated by the superposition of an unknown number of current dipole. In dipole theory, the pair (r, q) indicates a single dipole, where r is the integer variable that is the dipole position in a given discretized source space and q the dipole moment is the three-dimensional vector. A dipole moment is a measure of the two opposite electric charges separation. Moments of the dipole are a vector quantity. The magnitude is equal to the charge multiplied by the distance from the negative charge to the positive charge. This is illustrated in Fig. 1.

As the number of dipole (N) is unknown, the state-space of the unknown primary current j is defined as: $j = (r_1, q_1, \dots, r_N, q_N)$.

Primary current $j_t(r)$ at time t can be represented for N number of dipoles as

$$j_t(r) = \sum_{i=1}^N q_{t,i} \delta(r - r_{t,i}) \quad (1)$$

where r is the radius of the head model and $r_{t,i} = [r_{t,i}^x, r_{t,i}^y, r_{t,i}^z]^T$ and $q_{t,i} = [q_{t,i}^x, q_{t,i}^y, q_{t,i}^z]^T$ are location and moment vectors of the i th dipole of $i = 1, 2, \dots, N$ and at time $t = 1, 2, \dots, T$.

The i th dipole moment is defined as $q_{t,i} = o_{t,i} s_{t,i}$. Where $o_{t,i}$ is the 3-dimensional orientation vector and $s_{t,i}$ is the amplitude of the dipole.

Fig. 1 Dipolar position and moment coordinate system for the spherical head model. A dot shows a dipole position and a tail represents a dipole moment with an orientation and strength

Let z_t be the set of EEG measurements in which t is the time from $0, 1, 2, \dots, t$, taken from the scalp with the number of sensors n_s . Each $z_t = (z_t^1, \dots, z_t^{n_s})$, where i -th element is the measurement at time t by i th electrode. The EEG data collected by n_s electrodes is defined as $z_t = \sum_{i=1}^N L_i(x_{t,i}) s_{t,i} + v_t$.

Here v_t is a measurement noise from sensors, $L_i(\cdot)$ is a lead field matrix and $x_{t,i}$ is a source parameters vector which consists of dipole location, orientation of the i th dipole. $L_i(\cdot)$ depends on i th dipole location $r_{t,i}$ and i th dipole orientation $o_{t,i}$ at time t . Leadfield matrix can be calculated using Mosher et al. [30].

The lead field matrix (the source signal) for dipole i is denoted by $L_i(x_{t,i}) = \mathbb{R}^{n_s \times 3}$. A combination of Head geometry, electrode positions and dipole localization gives the non-linear function $L_i(x_{t,i})$. Sensors measure the electrical activity due to dipole source. And these dipole sources contain unity moments along x , y and z directions. These have zero moment in other directions. To compute forward model ‘ z_t ’, a priori knowledge of head geometry, electrode position is necessary.

Now Eq. (1) can be rewritten in a matrix form as

$$Z = L(X)S + V \quad (2)$$

This equation can be taken as measurement equation, and for the state equation, since it is unknown as how the states evolve, we can take this as a random walk model in the cerebrum source localization problem,

$$x_t = x_{t-1} + u_t \quad (3)$$

Equations (2) and (3) can be considered as a measurement or observation equation and state equation. From this, the EEG can be modeled as state space model. In this model, to evaluate source parameters X , consider the posterior probability density of the X based on the EEG measurements can be written in terms of Bayes theorem as $p(X|Z) = \frac{p(Z|X)p(X)}{p(Z)}$. Where $p(X)$ is the probability distribution of source parameters X , encoding of all unknown information available prior to measurement. $p(Z|X)$ is the probability function, including the forward model information and the noise statistical characteristics. From $p(X|Z)$ the estimation of source parameters such as location and amplitude of the dipoles can be computed. To evaluate such dynamic parameters we use particle filter.

2.2 Particle filter

Particle filter is a sequential Monte Carlo method and is used primarily for non-linear and non-Gaussian models. By taking EEG measurements into account, particle filter [5] is used at each time point to find the variables of

the dynamic state space model. This system can be represented in terms of x_t and z_t as:

$$x_t = f(x_{t-1}) + u_t \quad (4)$$

$$z_t = h(x_t) + v_t \quad (5)$$

where x_t consists of N number of unidentified parameters (locations of the sources in EEG model) at time t , z_t contains N_s number of observations (EEG measurements) at time t , $f(\cdot)$ and $h(\cdot)$ are considered to be nonlinear functions where $f(\cdot)$ is related to state-transition and $h(\cdot)$ relates to the state vector along with the observation vector, u_t is the state model error, and v_t is the observation noise.

The main job of the particle filter is to estimate the posterior probability density function (PDF). In this filter it is assumed that the PDF comprised of particles where each particle has the state $x_t^{(i)}$ and weight $w_t^{(i)}$. The weight of the particle becomes more means that the state of the particle is correct. The particle filter's accuracy depends on the number of particles. More particle count gives the more accurate PDF.

Particle filter is used to determine the joint posterior probability density function of x_t at time t using N number of random particles $x_t^{(i)}$ together with weights $w_t^{(i)}$, $i = 1, 2, \dots, N$ as:

$$p(x_t|z_t) \approx \sum_{i=1}^N w_t^{(i)} \delta(x_t - x_t^{(i)}) \quad (6)$$

where $\delta(\cdot)$ is Dirac delta function. Using Eq. (6), the estimated state vector can be written as

$$\hat{x}_t \approx \sum_{i=1}^N w_t^{(i)} x_t^{(i)} \quad (7)$$

Importance density plays a vital role in assigning weights for particles; the sequential importance resampling (SIR) PF [15] can be used to estimate the states based on the importance density. The particle filter's final form was given in algorithm 1.

Algorithm 1.

1. Initialization of the particles: draw the particles from initial distribution $x_t^{(i)} \sim p(x_t^{(0)})$
2. Particle propagation: Particles are taken from $q(x_t|x_{t-1}, z_{1:t})$ which is an importance density function, where $z_{1:t} = \{z_1, \dots, z_t\}$.
3. Weight update: Particle weight can be calculated as

$$w_t^{(i)} \propto w_{t-1}^{(i)} \frac{p(z_t|x_t^{(i)})p(x_t^{(i)}|x_{t-1}^{(i)})}{q(x_t^{(i)}|x_{t-1}^{(i)})} \quad (8)$$

4. Weight normalization: the sum of all weights should be equal to 1 i.e. $\sum_{i=1}^N w_t^{(i)} = 1$.

The importance density should happen in such a way that $q(x_t|x_{t-1}, z_{1:t}) = p(x_t|x_{t-1})$.

Now Eq. (8) becomes $w_t^{(i)} \propto w_{t-1}^{(i)} p(z_t|x_t^{(i)})$.

Normalize the weights. $\tilde{w}_t^{(i)} = w_t^{(i)} / \sum_{j=1}^N w_t^{(j)}$

5. Resampling: It is used to avoid rapid degeneracy of particles, that is, most of the particles with high weights will dominate smaller weight particles [11]. The dominance of these particles degeneracy leads to poor posterior likelihood density function. Resample and replace the particles $x_{0:t}^i, i = 1, 2, 3, \dots, N$ from $\tilde{x}_{0:t}^i, i = 1, 2, 3, \dots, N$. In accordance with normalized weights, $\tilde{w}_t^i$ perform resampling till next observation arrives.

2.3 Resampling

Depending on duplication or removal of high or low-weight particles, updated particle distribution is resampled separately. Resampling steps are constantly followed and weights tend to be focused on fewer particles after several cycles in particle propagation, and the larger part of the particles has insignificant weights which cause degeneration. Ideally, the propagation of the particles should be extended to manage degeneracy, involving the use of different methods of resampling [18].

(1) Multinomial resampling

Multinomial resampling is also called as random resampling [16, 23]. The main idea behind resampling is to produce $w_t^{(n)}$ which is N number of random measures from the distribution $(0, 1]$ and these numbers are used to pick the particles from x_t . For the particle $x_t^{(m)}$, the n th random number can be taken that satisfy the following condition,

$$Q_t^{(m-1)} < w_t^{(n)} \leq Q_t^{(m)} \quad (9)$$

where $Q_t^{(m-1)} = \sum_{i=1}^m w_t^{(i)}$. In the “wheel” analogy shown in Fig. 2, this method of resampling estimates by picking N

Fig. 2 The Multinomial resampling

independent random directions from the center of the wheel and taking the pointing particle.

(2) Stratified resampling

In stratified resampling, where the total weights are added and divided into equal N pieces, i.e. pre-partitioning the $(0, 1]$ period into N sub-intervals $U\left(0, \frac{1}{N}\right] \dots U\left(1 - \frac{1}{N}, 1\right]$, these sub parts are called as strata as shown in Fig. 3. Then $w_t^{(n)}$ is draws in each strata independently, which means that the particles of small weight are sampled at most once and at least once the particles of large weight are sampled [12, 21].

$$w_t^{(n)} \sim U\left(\frac{n-1}{N}, \frac{n}{N}\right), n = 1, 2, \dots, N \quad (10)$$

Fig. 3 The Stratified resampling

(3) Systematic resampling

Systematic resampling is also called as universal sampling; In this common technology, we draw only one random number, that is, one direction in the ‘wheel’ for first particle and other particle with $N - 1$ directions, fixed at increments of $1/N$ of that randomly picked a direction, as shown on Fig. 4. Presently, $w_t^{(1)}$ is drained from the regular distribution on $\left(0, \frac{1}{N}\right]$, and whatever is left over the ‘ u_t ’ information is acquired conclusively [4, 8, 19], i.e. $w_t^{(n)} \sim U\left(0, \frac{1}{N}\right)$,

$$w_t^{(n)} = w_t^{(1)} + \frac{n-1}{N}, n = 2, 3, \dots, N \quad (11)$$

(4) Partial stratified resampling

To reduce the computational complexity, resampling has to be done on only few particles. The main idea behind partial stratified resampling is to complete the resampling only on few particles by joining the particles with higher weights and lower weights based on the high threshold and lower threshold [24]. The lower weight particles which are below the lower threshold (T_l) and the higher weight particles above the higher threshold (T_h) will be combined and resampled where low weight particles will be replaced with high weight particles. The particles which are in between low and high thresholds will not be resampled and will be considered for next time step. The advantage of using PSR is, it will perform resampling in less execution time as only some of the particles will undergo resampling.

PSR algorithm can be executed in two steps; first step is to classify (negligible, moderate and dominant) the particles based on their weight, second step is to perform stratified resampling on the particles which are the combination

Fig. 4 The Systematic resampling

Table 1 Threshold values for PSR

Threshold	Low threshold value $< T_l$	High threshold value $> T_h$	Moderate ($T_h - T_l$)
T_1	0.1	0.9	0.8
T_2	0.15	0.85	0.7
T_3	0.2	0.8	0.6
T_4	0.25	0.75	0.5
T_5	0.3	0.7	0.4
T_6	0.35	0.65	0.3
T_7	0.4	0.6	0.2
T_8	0.45	0.55	0.1
T_9	0.5	0.5	0

of negligible and dominant weights. The threshold values have taken in between (0, 1], both low threshold, high threshold values started at 0.5. T_l Starts decreasing by 0.05 and T_h starts increasing by 0.05. We have defined total nine threshold values (Table 1).

3 Simulation results

To model the neural activity of the cerebrum, the amount of dipole sources are unknown and are time-variant. In source localization, we first compute the leadfield matrix (this can be done with the help of fieldtrip toolbox [31]). By applying the leadfield matrix and EEG data on the particle filter, one can estimate the positions of the dipoles and their amplitudes with respect to time. Therefore, the problem under this condition is to find N number of dipole sources at each time point t . Here, there is no prior information about the number of dipoles. Particle filter is used to deal with the unknown and time-varying dipoles.

The neural sources have different dipoles where they appear at different time points. Despite this, they represent the same neural source. So it is preferable to cluster them by concerning their 3-dimension location. The clustering gives diverse dipole source collection into different neural source. To find out the 3-dimensional location of the same source, it is ideal to take average location of dipoles belonging to that group.

The mean square error and relative accuracy are utilized as measurements for the performance assessment to look at the outcomes from utilizing the resampling methods of the particle filter for localization of neural sources [20]. As the neural source localization is nonlinear and is an inverse problem, for real data, there is no accurate position of the source dipoles, and thus cannot say which resampling method is better. For this reason, synthetic data is used in which the positions of dipoles are known so that one can go

for the analysis of the resampling methods. Then it becomes easy to analyze the algorithms using real EEG data.

3.1 Synthetic data

The algorithm is applied to the synthetic data [6] in which four dipoles are located to validate the location of the neural sources through the use of particle filter. The volume of the brain is 10 cm in radius. The dipole positions are shown in Table 2.

We have taken 10,000, 20,000, 50,000, 100,000 particle for each resampling method. The higher the number of particles, the better the approximation of good localization. The comparison of resampling methods has been done with respect to mean square error between true and estimated positions, relative accuracy and computational time. The synthetic data has a total of 400 ms in that analysis has been done from 100 to 150 ms. The 0–100 ms interval is the pre stimulus interval. The sampling frequency of the data is 1000 Hz.

(1) Mean Square Error (MSE)

Mean square error between the position of the true dipoles and estimated dipoles can be calculated. It could be written as,

Table 2 The positions of dipoles located in synthetic data

Dipole	Position (3-dimension) in cm		
	X axis	Y axis	Z axis
1	−1.37	−5.43	7.34
2	3.74	4.54	5.66
3	−2.04	3.73	9.56
4	2.96	2.11	9.42

Fig. 5 Comparison of average MSE for source localization with 20,000 number of particles using PSR threshold values

Fig. 6 Comparison of average RA for source localization with 20,000 number of particles using PSR threshold values

$$MSE = \frac{(x_{true} - x_{estimated})^2 + (y_{true} - y_{estimated})^2 + (z_{true} - z_{estimated})^2}{3}$$

In the above equation $(x_{true}, y_{true}, z_{true})$, $(x_{estimated}, y_{estimated}, z_{estimated})$ represent the 3-dimensional positions of the true and estimated dipoles.

(2) Relative Accuracy (RA)

The calculation of normalized error values between true positions and estimated dipole positions is relative accuracy. It is mentioned as,

$$RA = 1 - \frac{|TruePosition - Estimatedposition|}{TruePosition}$$

(3) Computational time:

Measuring the computational time to implement the resampling methods for the particle filter will also explore the quality of the resampling methods. Figure 5 shows the MSE values between the different threshold values. Figure 6 shows the RA values of different threshold values.

3.2 Real EEG data

The estimation accuracy for the multinomial, stratified, systematic and PSR resampling algorithms with real EEG data is shown in this section. The data was taken from the EEG/Epilepsy dataset of Brainstorm [38]. This dataset is from a person with focal epilepsy, there are no typical epilepsy risk factors. The data set was collected at the Epilepsy Center in Freiburg, Germany during a one night non-invasive telemetry experiment. The data were

Fig. 7 Channel locations

recorded using 29 channels at a frequency of 256 Hz. EEG signals from 29 terminals were recorded (FP1, FP2, F3, F4, C3, C4, P3, P4, O1, O2, F7, F8, T7, T8, P7, P8, Fz, Cz, Pz, T1, T2, FC1, FC2, FC5, FC6, CP1, CP2, CP5, CP6) as per the 10/20 International framework. The channel locations are shown in Fig. 7. The reference coordinate system source is located at the focal point of the spheres with the x-axis indicating from right to left, the y-axis pointing back to front, and the z-axis guiding base to the top [14]. The EEG data includes 921,600 samples. Between 137,000 and 200,000 samples, we have used 10,000 ms of data. The real EEG data is shown in Fig. 8. Consider that we do not have the exact position of the source dipole for real EEG data because it is not linear. Focal seizures are characterized by focal epilepsy in a given part of the brain. In patients with epilepsy the value of localization of EEG data sources has been proven. The technique can locate brain resources, from extra-temporal to low-temporal locations, as demonstrated in previous studies [22, 32, 37]. For both internal and surgical therapies, the exact location of focal epilepsy is important. The suggested technique can be considered in addition to the diagnostic protocol. The location of EEG sources indicated by focal brain epilepsy around the temporal lobe. Results of source location can

Fig. 8 Real EEG measurements

Fig. 9 The dipole positions and source amplitudes across time of the real data with multinomial resampling using 20,000 number of particles

Fig. 10 The dipole positions and source amplitudes across time of the real data with stratified resampling using 20,000 number of particles

Fig. 11 The dipole positions and source amplitudes across time of the real data with systematic resampling using 20,000 number of particles

Fig. 12 The dipole positions and source amplitudes across time of the real data with PSR resampling using 20,000 number of particles

Table 3 Average MSE and RA for the Multinomial, Stratified, Systematic and PSR resampling methods for different number of particles

No of particles	Average MSE				Average RA			
	MN*	Str*	SYST*	PSR	MN	Str	SYST	PSR
10,000	0.022	0.020	0.018	0.015	0.66	0.67	0.67	0.70
20,000	0.024	0.018	0.018	0.015	0.57	0.67	0.70	0.74
50,000	0.027	0.024	0.025	0.020	0.56	0.60	0.58	0.61
100,000	0.018	0.017	0.017	0.013	0.65	0.71	0.70	0.76

MN* Multinomial, SYST* Systematic, Str* Stratified

Table 4 Computational time for synthetic data between Multinomial and Systematic resampling methods for different number of particles

No of particles	Multinomial	Stratified	Systematic	PSR
10,000	60.58 s	42.24 s	39.12 s	34.32 s
20,000	185.13 s	109.98 s	93.53 s	70.17 s
50,000	871.62 s	354.32 s	221.79 s	195.78 s
100,000	8019.25 s	576.85 s	359.86 s	283.57 s

confirm the EEG measurements that enhance the correlation between epileptic position and brain anatomy.

The dipole positions and source amplitudes of the real EEG data with multinomial, stratified, systematic and PSR resampling methods in particle filter were shown in Figs. 9, 10, 11 and 12 respectively. For real EEG data we have shown the figures for 20,000 particles only. As can be observed from the figure, the dipoles are positioned across the brain. The source amplitudes are represented from the time 10,000–20,000 ms where the activations of various dipoles are also given.

4 Discussion

We calculated MSE and RA for four different particle numbers for each dipole to determine the accuracy for synthetic data. A high MSE value means the estimated data is too far from the true data, whereas a low MSE indicates good accuracy in the estimate. A high RA value close to 1 corresponds to a low error between the estimated and true positions, making the estimate very accurate. The comparison analysis between resampling algorithms was evaluated in

this article by taking into account both low MSE and high RA values. From Table 3, it can be observed that the PSR resampling MSE and RA are impressive for different number of particles than other conventional methods of resampling. With regard to computational time, multinomial resampling takes more time to estimate the dipoles compared to other methods of resampling, the PSR method shows that it estimates the source positions with very less time of execution given in Table 4.

The number of dipoles is present at each sample and the amplitudes of those dipoles are shown in Figs. 9, 10, 11 and 12. The average MSE is low with different number of particles for PSR resampling and also the RA is high compared to other methods of resampling. The computational time for resampling PSR is very low. In Fig. 5, the PSR method compares the average MSE by changing the threshold values. We can observe that MSE is very low in the threshold values T_7 and T_8 . The comparison of average RA for the PSR threshold values were given in Fig. 6, it is shown that high RA is observed for T_7 and T_8 threshold values. For T_1 , only the particles are resampled with very high weight and very low weight and most particles are considered to be moderate

Table 6 Computational time between Multinomial, Stratified, Systematic and PSR resampling methods for different number of particles for real EEG data

No of particles	Multinomial	Stratified	Systematic	PSR
10,000	2025 s	1865 s	1598 s	1313 s
20,000	9398 s	2596 s	3239 s	2477 s
50,000	58,840 s	6571 s	7054 s	5662 s
100,000	97,787 s	16,487 s	18,133 s	11,761 s

Table 5 Comparison of RMSE of source position for synthetic data with other PF methods

Approach	Number of particles	Number of dipoles	Knowledge of dipole number	RMSE of position
PF [36]	100,000	4	Known	5.4 mm
RBPF [5]	50,000	2	Known	6.3 mm
DPF [35]	100,000	3	Unknown	6.2 mm
PF-PHDF [26]	16,000	3	Unknown	6.9 mm
Proposed PSR-PF	20,000	4	Unknown	4.4 mm

Fig. 13 Dipole sources from real EEG dataset using DIPFIT

weight particles and are not sampled again. The majority of the useful data contained in moderate particles are therefore lost, leading to very bad predictability.

The efficiency of the proposed algorithm was compared with that of [5, 26, 35, 36] as illustrated in Table 5. The proposed PSR-based system has a 4.4 mm tracking error compared to [36], which has the best tracking efficiency. The PSR only utilizes 20,000 particles and also has no knowledge of dipole numbers. In summary, the proposed system achieves comparable results for estimation of the unknown number of dipoles with considerably less computational complexity.

Since EEG source localization is an inverse problem, the MSE and RA of the real EEG data cannot be found. That's why we consider the computational time of the comparison metric. Table 6 shows the time taken to execute both methods of resampling. It is observed that the particle filter will be executed in a short time for the lower number of particles. The algorithm will take longer if there is a higher number of particles with large data. From Table 5, we can say that it will take very short time for PSR resampling algorithm compared to other conventional methods of resampling.

We tried to compare results of real data with [7], the normal dipole fitting technique. First, to discover the independent channel in the dipole fitting phase, we had to use independent component analysis. The dipole fitting was then used for the independent components. However, with this method, the number of dipoles equals the number of channels (29 channels for our actual data), which means that 29 dipoles exist. We rejected components using maps because we do not know the correct positions and the amount of dipole and we took five dipole in the EEG. We assumed, that is, that the number of dipoles was fixed. Figure 13 shows the results of the dipole fitting. It was very difficult to pick which sections to accept or reject while viewing the component map. The problem with dipole fitting is that the user normally takes a long time to make decisions. The PF technique produces location estimates for the entire time window, with time tracking. This is the expected behavior to describe the neural activity in the brain region.

5 Conclusion

In this paper, the partial stratified resampling method of the particle filter for neural source localization has been discovered and we have presented comprehensive performance analysis for the multinomial, stratified, systemic and partial stratified resampling in the particle filters. Performance metrics such as mean square error, relative accuracy and computation time are evaluated for resampling techniques. Among synthetic and real EEG data computational results are demonstrated. Comparing with other traditional resampling approaches, the performance metrics shows more consistent and accurate for PSR resampling. The results clearly show that the computational time of PSR is lower than traditional resampling techniques, as only a few particles are resampled based the defined threshold values. PSR classifies weights as Th (high threshold) and Tl (low threshold) respectively. The interval between both threshold values is increased from 0.1 to 0.9 to estimate the neural source location for each case, and nine different threshold values are defined. Our results clearly demonstrate that PSR is very reliable; PSR takes 65% less time than stratified resampling, as there are only fewer particles. The PSR algorithm in particular is time-efficient and can be used to approximate the position of neural sources, as the computational complexity is low. In summary, using PSR, it is possible to achieve high speed for the location of the neural source than conventional methods of resampling.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

References

1. Antelis J, Minguez J. Dynamic solution to the EEG source localization problem using Kalman filters and particle filters. In: International conference of the IEEE engineering in medicine and biology society. 2009;77–80.
2. Arulampalam MS, Maskell S, Gordon N, et al. A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking. *IEEE Trans Signal Process*. 2002;50(2):174–88. <https://doi.org/10.1109/78.978374>.
3. Baillet S, Mosher JC, Leahy RM. Electromagnetic brain mapping. *IEEE Signal Process Mag*. 2001;18(6):14–30.
4. Bolic M, Djuric PM, Hong S. Resampling algorithms for particle filters: a computational complexity perspective. *EURASIP J Appl Signal Process*. 2004. <https://doi.org/10.1155/s1110865704405149>.

5. Campi C, Pascarella A, Sorrentino A, et al. A Rao-Blackwellized particle filter for magnetoencephalography. *Inverse Prob.* 2008;24:25015–23. <https://doi.org/10.1088/0266-5611/24/2/025023>.
6. Campi C, Pascarella A, Sorrentino A, et al. Highly automated dipole estimation (HADES). *Comput Intell Neurosci.* 2011. <https://doi.org/10.1155/2011/982185>.
7. Delorme A, Makeig S. EEGLAB: an open source toolbox for analysis of single-trial EEG dynamics. *J Neurosci Methods.* 2004;134(1):9–21.
8. Douc R, Cappe O. Comparison of resampling schemes for particle filtering. In: *Proceedings ISPA 4th international symposium on image and signal processing and analysis.* 2005;64–9. <https://doi.org/10.1109/ispa.2005.195385>.
9. Doucet A, De Freitas N, Gordon NJ. *Sequential Monte-Carlo methods in practice.* New York: Springer; 2001. <https://doi.org/10.1007/978-1-4757-3437-9>.
10. Ebersole JS, Hawes-Ebersole S. Clinical application of dipole models in the localization of epileptiform activity. *J Clin Neurophysiol.* 2007;24(2):120–9.
11. Ebinger B, Bouaynaya N, Georgieva P, et al. EEG dynamic source localization using marginalized particle filtering. In: *2015 IEEE international conference on bioinformatics and biomedicine (BIBM), Washington, DC.* 2015;454–457. <https://doi.org/10.1109/bibm.2015.7359727>.
12. Fearnhead P. *Sequential Monte Carlo methods in filter theory.* PhD thesis, University of Oxford. 1998.
13. Galka A, Yamashita O, Ozaki T, et al. A solution to the dynamical inverse problem of EEG generation using spatiotemporal Kalman filtering. *NeuroImage.* 2004;23:435–53.
14. Georgieva P, Bouaynaya N, Silva F, Mihaylova L, Jain LC. A beamformerparticle filter framework for localization of correlated EEG sources. *IEEE J Biomed Health Inform.* 2016;20(3):880–92.
15. Georgieva P, Mihaylova L, Bouaynaya N, et al. Particle filters and beamforming for EEG source estimation. In: *IEEE world congress on computational intelligence, international joint conference on neural networks, Brisbane.* 2012;10–15. <https://doi.org/10.1109/ijcnn.2012.6252516>.
16. Gordon NJ, Salmon DJ, Smith AFM. Novel approach to nonlinear/non-Gaussian Bayesian state estimation. *IEEE Proc Radar Signal Process.* 1992;140:107–13.
17. Hamalainen M, Hari R, Ilmoniemi RJ, Knuutila J, Lounasmaa O. Magnetoencephalography-theory, instrumentation, and applications to noninvasive studies of the working human brain. *Rev Mod Phys.* 1993;65:413–97.
18. Hol JD. *Resampling in Particle Filters, report,* Linköping. 2004.
19. Hol JD, Schon TB, Gustafsson F. On resampling algorithms for particle filters. *IEEE Nonlinear Stat Signal Process Workshop.* 2006;2006:79–82.
20. Karkulali P, Nagarajan R. A holistic comparison of the different resampling algorithms for particle filter based prognosis using lithium ion batteries as a case study. *Microelectron Reliab.* 2018;91(1):160–9. <https://doi.org/10.1016/j.microrel.2018.08.007>.
21. Kitagawa G. Monte-Carlo filter and smoother for non-Gaussian nonlinear state space models. *J Comput Graph Stat.* 1996;1:1–25.
22. Koessler L, Benar C, Maillard L, Badier JM, Vignal JP, Bartolomei F, Chauvel P, Gavaret M. Source localization of ictal epileptic activity investigated by high resolution EEG and validated by SEEG. *Neuroimage.* 2010;51(2):642–53.
23. Kumar VS, Rao TVKH. Resampling schemes for Rao-Blackwellization Particle Filters. In: *International conference on computing, analytics and security trends (CAST), Pune.* 2016;377–382. <https://doi.org/10.1109/cast.2016.7914998>.
24. Li T, Bolic M, Djuric PM. Resampling methods for particle filtering: classification, implementation, and strategies. *IEEE Signal Process Mag.* 2015;32(3):70–86. <https://doi.org/10.1109/msp.2014.2330626>.
25. Makeig S, Westerfield M, Jung TP, et al. Dynamic brain sources of visual evoked responses. *Science.* 2002;295:690–4. <https://doi.org/10.1126/science.1066168>.
26. Miao L, Zhang JJ, Chakrabarti C, et al. Efficient Bayesian tracking of multiple sources of neural activity: algorithms and real-time FPGA implementation. *IEEE Trans Signal Process.* 2013;61(3):633–47. <https://doi.org/10.1109/tsp.2012.2226172>.
27. Mohseni HR, Wilding EL, Sanei S. Sequential Monte Carlo techniques for EEG dipole placing and tracking. In: *Sensor array and multichannel signal processing workshop.* 2008;95–98.
28. Mosher JC, Leahy RM. Source localization using recursively applied and projected (RAP) MUSIC. *IEEE Trans Signal Process.* 1999;47(2):332–40.
29. Mosher JC, Lewis PS, Leahy RM. Multiple dipole modelling and localization from spatio-temporal MEG data. *IEEE Trans Biomed Eng.* 1992;39(6):541–57. <https://doi.org/10.1109/10.141192>.
30. Mosher JC, Leahy R, Lewis P. EEG and MEG: forward solutions for inverse methods. *IEEE Trans Biomed Eng.* 1999;46(3):245–59. <https://doi.org/10.1109/10.748978>.
31. Oostenveld R, Fries P, Maris E, et al. FieldTrip: open source software for advanced analysis of MEG, EEG, and invasive electrophysiological data, computational intelligence and neuroscience. 2011;Article ID 156869, 9 p. <https://doi.org/10.1155/2011/156869>. <http://fieldtrip.fcdonders.nl>.
32. Ouyang G, Wang Y, Yang Z, Li X. Global synchronization of multichannel EEG in patients with electrical status epilepticus in sleep. *Clin EEG Neurosci.* 2015;46(4):357–63.
33. Pascual-Marqui R. Review of Methods for Solving the EEG Inverse Problem. *Int J Bioelectromagn.* 1999;1(1):75–86.
34. Sarvas J. Basic mathematical and electromagnetic concepts of the biomagnetic inverse problem. *Phys Med Biol.* 1987;32:11–22. <https://doi.org/10.1088/0031-9155/32/1/004>.
35. Sorrentino A, Parkkonen L, Pascarella A, et al. Dynamical MEG source modeling with multi-target Bayesian filtering. *Hum Brain Mapp.* 2009;30(6):1911–21. <https://doi.org/10.1002/hbm.20786>.
36. Sorrentino A, Parkkonen L, Piana M. Particle filters: a new method for reconstructing multiple current dipoles from MEG data. *Int Congr Ser.* 2007;1300:173–6.
37. Stefan H, Schneider S, Abraham-Fuchs K, Bauer J, Feistel H, Pawlik G, Neubauer U, Rohrlein G, Huk WJ. Magnetic source localization in focal epilepsy: multichannel magnetoencephalography correlated with magnetic resonance brain imaging. *Brain.* 1990;113(5):1347–59.
38. Tadel F, Baillet S, Mosher JC, Pantazis D, Leahy RM. Brainstorm: a userfriendly application for MEG/EEG analysis. *Comput Intell Neurosci.* 2011;Article ID 879716, 13 p.
39. Van Veen BD, Buckley K. Beamforming: a versatile approach to spatial filtering. *IEEE ASSP Mag.* 1988;5:4–24.
40. Vivaldi V, Sorrentino A. Bayesian smoothing of dipoles in Magneto-/Electro-encephalography. *Inverse Prob.* 2016;32(045007):1–16. <https://doi.org/10.1088/0266-5611/32/4/045007>.
41. Zhukov L, Weinstein D, Johnson C. Independent Component Analysis for EEG source localization. *IEEE Eng Med Biol Mag.* 2000;19(3):87–96.

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