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Prediction of sound absorption by a circular orifice termination in a turbulent pipe flow using the Lattice-Boltzmann method

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Abstract

The Lattice Boltzmann method was used to perform numerical simulations of the sound and turbulent flow inside a standing wave tube terminated by a circular orifice in presence of a forced mean flow. The computational domain comprised a standard virtual impedance tube apparatus in which sound waves were produced by periodic pressure oscillations imposed at one end. An orifice plate was located between the driver and the tube termination. All waves transmitted through the orifice were effectively dissipated by a passively non-reflecting (i.e. anechoic) boundary at the tube termination. A turbulent jet was formed at the discharge of the orifice by the forced mean flow inside the tube. The acoustic impedance and sound absorption coefficient of the orifice plate were calculated from a wave decomposition of the sound field upstream of the orifice. Simulations were carried out for different excitation frequencies, and orifice Mach numbers. Results and trends were in good quantitative agreement with available analytical solutions and experimental data. The Lattice Boltzmann method was found to be an efficient numerical scheme for prediction of sound absorption by realistic three dimensional orifice configurations.

Keywords

Impedance tube; Sound absorption; Orifice; Numerical methods; Large eddy simulations; Lattice Boltzmann method

1. Introduction

The absorption of sound waves by single or multiple orifice plates has broad applications for the design of liners and other devices for the suppression of tonal and broadband aerodynamic noise. It is well known that sound waves in ducts with flow are absorbed during transmission and reflection by orifices or nozzles. The primary sound absorption mechanism involves the conversion of sound energy into turbulence kinetic energy and also the formation and shedding of vortices at the orifice discharge [1,2]. This phenomenon is of great importance in the design of quiet exhaust systems, absorbing sound barriers, aircraft nacelle liners, and many other applications. Shear flow instabilities, the shape and profile of the orifice, and possible interactions with the tube wall are factors that may affect the acoustic characteristics of the orifice plate in presence of a superimposed mean flow. Most of the energy transferred (i.e. absorbed) through the orifice is supplied by the kinetic energy

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of the mean flow inside the channel. Bechert [1,2] has argued that flow disturbances at the orifice inlet and the Kutta condition at the edge cause toroidal vortical structures to be formed and shed from the orifice plate. Bechert's predictions and semi-empirical model were corroborated by measurements, and comparisons with previous experimental data [1]. A theoretical framework for the problem was proposed by Howe [3,4]. He has proposed linearized models, assuming that the Mach number of the orifice jet flow is relatively low. Howe [4] assumed that the incident acoustic energy is dissipated by two distinct mechanisms. First the acoustic characteristics of medium at orifice downstream, in which the directivity of transmitted sound waves is more universal and equivalent to magnitude, produced by monopole and dipole sources. Second is the formation of vortex waves excited by the shedding of vortices from the nozzle edge, which are triggered by the large-scale instabilities of the jet. Most of Howe's derivations are based on the extended vortex sound theory [5], which originated from Powell's vortex sound hypothesis [6]. Wendoloski [7] extended Howe's theory to deal with orifice plates in ducts with mean flow using a Green's function expansion, using a novel renormalization technique.

Three dimensionless parameters define the orifice flow regime: (1) the Mach number at the orifice plate, M_o , based on the orifice mean flow velocity; (2) the nominal Strouhal number, St , based on the excitation frequency and velocity amplitude as well as the orifice diameter (D_o); and (3) the open area ratio, σ , defined as the ratio of the orifice area and the tube area (D_o^2/D_T^2). For the case of multiple orifices in parallel, the opening area ratio is replaced by the porosity of the plate [7].

The driving pressure amplitude has a great impact on the orifice absorption phenomenon. Ingard and Ising [8] performed parametric studies of the effects of excitation amplitude on the absorption coefficient. The relation between the absorption coefficient and the acoustic pressure and velocity amplitudes was found to be strongly nonlinear for relatively high excitation amplitudes [8].

The acoustic characteristics of orifice plates have been investigated in more recent experimental studies. Ahuja et al. [9] have conducted comprehensive experiments to measure the impedance of a single orifice plate in presence of a bias flow. Parametric studies were performed on the effects of various driver set points as well as mean flow strengths in terms of orifice Mach number. The absorption coefficient and orifice impedance values were reported [9]. Hughes and Dowling [10] and Jin and Sun [11] measured the impedance of perforated orifice plates. The results were found to be in good agreement with Howe's Rayleigh conductivity model [12].

Attempts were made to find the acoustic properties of orifice plates using computational fluid dynamics techniques. Tam et al. [13] performed direct numerical simulations (DNS) of flow through a slit resonator. They investigated the orifice impedance over the frequency range of 0.5–3 kHz for two slit geometries (i.e., 90° straight and 45° beveled slits). The Navier–Stokes equations were discretized using the Dispersion Relation Preserving (DRP) scheme. A wave decomposition method based on the virtual two-microphone model was used to determine the reflection factor. The absorption coefficients were found to agree with experimental data obtained at the NASA Glen Research Center [13]. In a previous study,

Tam et al. [14] used the same DNS method for numerical simulations of acoustic flows through straight slit orifices. The energy dissipation rate was calculated and used to determine the absorption coefficient. Results were compared to experimental data obtained by Ahuja et al. [9,14] and found to be in good agreement. Despite its accuracy, the DNS method is not always practical for realistic engineering applications because of its prohibitive costs. For the low frequency excitation as well as the low Mach number at orifice discharge, two-dimensional simulations have yielded relatively accurate results at reasonable cost, as argued by Ji and Zhao [15]. The 2D planar flow assumption was also found to be useful in predictions of tonal whistling phenomena by Kierkegaard et al. [16]. In this study, the Reynolds Averaged Navier-Stokes (RANS) method was first used to determine the mean flow. The linearized Navier-Stokes method was then used for the calculation of transient flow characteristics.

Development of a generic and reliable computational method for the accurate prediction of orifice at normal flow incidence can also help to tackle the important problem of sound absorption in presence of grazing flows [17]. This configuration is used in the design of acoustic liners for ventilation ducts, turbofan engine nacelles and the exhaust systems of internal combustion engines [18].

For the case of high speed flows through the orifice, the accurate prediction of sound absorption will highly depend on the quality of turbulent jet simulation downstream of the orifice. For higher Mach numbers, a three-dimensional numerical scheme as well as sufficient grid resolution is required to simulate the turbulent shear layer right at the edge of the orifice. The selected numerical scheme should have reasonable computational cost and should be capable of capturing details of orifice geometry.

In the present study, the Lattice-Boltzmann Method (LBM) coupled with the large eddy simulation methodology was used for numerical simulations to predict sound absorption by an orifice plate in the presence of mean flow in a full-scale, 3D virtual impedance tube apparatus. A broad frequency range of 380 Hz to 6 kHz and Mach numbers, 0.05–0.2, were considered. The three-dimensional 19-stage LBM could resolve the turbulent jet formed through the orifice. Three-dimensional assumption will help to accurately model the vortex stretching phenomena which contributes to the formation of turbulent jet downstream of the orifice as well as the energy absorption. This method was combined to the Very Large Eddy Simulation (VLES) method in which the large scale turbulent structures are simulated directly and sub-grid scales are modeled. This methodology has considerable less computational cost comparing to the DNS schemes [14]. The setup can be used for even higher Mach numbers and higher frequency ranges. Also unlike RANS simulation [16], the LBM-VLES is highly accurate in simulation of flow separation at the edge of the orifice plates and since the method is intrinsically transient, there is no need to combine it with a transient solver to get acoustic characteristics and could save several computational steps comparing to the RANS simulations. All dimensions, scales and flow characteristics used in this study were based on the experimental setup of Ahuja et al. [9]. The complex reflection factor was determined using a wave decomposition technique which involved simultaneously recording the pressure and velocity history at multiple locations upstream of

the orifice plate. The use of the VLES method was shown to be accurate and computationally affordable for such case studies.

2. Lattice Boltzmann method

The Lattice-Boltzmann Method is a computational fluid mechanic tool for the numerical simulation of complex hydrodynamic problems from first principles. It has known advantages for weakly compressible aeroacoustic modeling over other commonly used numerical schemes [19]. Numerical simulations of fluid flows based on the Navier–Stokes equations involve the calculation of macroscopic properties. The LBM, on the other hand, is based on kinetic gas theory. In this method, a particle density distribution function is evolved over a discrete lattice domain. Transient macroscopic fluid properties are obtained by imposing streaming and collision laws governed by the Lattice-Boltzmann Equation (LBE). Through the Chapman–Enskog expansion [20], the LBE recovers the compressible Navier–Stokes equation at the hydrodynamic limit [21]. The conserved variables such as density, momentum and internal energy are obtained by performing a local integration of the particle distribution function.

The LBM offers potential advantages over methods based on the Navier–Stokes equations for subsonic compressible flows. Because of the use of simple particle-based models for solid–fluid boundaries, alleviating the need for Jacobians to compute grid metrics, complex nozzle geometries such as chevrons and corrugated shaped-orifices may be included in the computational domain without a considerable increase in computational cost. The method is intrinsically parallelizable with minimal floating point operations (FLOP). The convection operator in the LBE is linear; however, nonlinear features are recovered by particle advection-collision relations together with multi-scale expansions. The main disadvantage of the standard implementation of the method is the compressibility criterion associated with the commonly used Bhatnagar–Gross–Krook (BGK) approximation [21]. The use of BGK approximation simplifies the implementation, reduces costs, but limits the highest achieved velocity to a Mach number of ~ 0.5 . This limitation may be circumvented through the use of higher order Lattice Boltzmann schemes [22].

Many complex fluid phenomena have been accurately modeled using the LBM [21], as well as benchmark fluid-flow problems, such as flow over airfoils [23] and flow inside rectangular cavities [19]. The LBM has frequently been used for low speed aeroacoustic applications. Crouse et al. [19] have studied a series of canonical acoustic sound propagation problems and those involving strong interactions between fluid flow and acoustic response. Li et al. [24] studied the accuracy of a modified LB scheme for canonical wave propagation problems. Lew et al. [25] have shown the accuracy of the LBM for simulations of jet flows and associated radiated sound. They found that the LBM results were comparable in accuracy with those from a Navier–Stokes Large Eddy Simulation code, while they were obtained at a significantly lower computational cost.

By considering Δx as lattice unit length and Δt as lattice unit time step, the Lattice-Boltzmann equation may be written by the following form [26].

$$f_i(\hat{x} + c_i \Delta x, t + \Delta t) - f_i(\hat{x}, t) = -\frac{\Delta t}{\tau} (f_i - f_i^{eq}), \quad (1)$$

where the distribution function $f_i(\hat{x}, t)$ can conveniently be considered as a typical histogram representing a frequency of occurrence at position $\hat{x}$ with particle velocity e_i in the i direction at time t .

The frequencies may be considered to be direction-specific fluid densities. The left hand side of Eq. (1) represents the propagation of particles in a discretized lattice domain, whereas the right hand side is the collision operator adopted from the BGK approximation [27]. A relaxation time parameter, τ , used in this model is related to the kinematic viscosity, ν , such that the equilibrium distribution function, f_i^{eq} , relates the Lattice-Boltzmann model to hydrodynamic properties and is essential for local conservation criteria to be satisfied. In this study, f_i^{eq} was obtained from the *D3Q19* model [26]. The *D3Q19* scheme allows discrete velocities along eighteen directions towards neighboring cells as shown in Fig. 1. The 19th direction belongs to the particle at rest. This model is second-order in both time and space [21]. The conservative fluid properties such as density and momentum are obtained by summing the moments along velocity directions i.e.

$$\rho(x, t) = \sum_{i=1} f_i(\hat{x}, t), \quad (2)$$

and

$$\rho \hat{u}(x, t) = \sum_{i=1} c_i f_i(\hat{x}, t). \quad (3)$$

A time explicit CFD solver, *PowerFLOW 4.3b*, based on the D3Q19 Lattice-Boltzmann kernel was the primary tool used for simulations in the present study. The time steps used in *Power-FLOW* are typically very small and allowing flow dynamics at frequencies up to 10 kHz. The simple structured-grid generation capability makes the incorporation of solids with an arbitrary and complex geometry in the computational domain relatively easy. The D3Q19 model can predict the macroscopic flow characteristics for weakly compressible flow conditions restricted to relatively low Mach numbers (i.e. less than ~ 0.5).

3. Problem description and main parameters

One common experimental method to determine the absorption coefficient and the acoustic impedance of a material is to use the impedance tube (IT). For permeable samples, the apparatus consists of an acoustic driver at one end of a rigid tube, and an anechoic termination is provided at the other end. The permeable or porous sample is located somewhere in the tube. An IT device that allows measurements of acoustic properties in presence of mean flow was described by Ahuja et al. [9]. In the experimental setup, a vacuum pump was connected to the downstream section of tube in order to maintain a mean flow inside the channel. The acoustically insulated terminals were located upstream to

balance the pressure inside the channel. The IT measurements are usually performed in accordance with international standards such as *ASTM-E1050* [28]. The bandwidth of the experiments is limited by the tube size. The upper frequency limit is to ensure plane wave propagation through the pipe while the lower frequency limit depends on the microphones spacing and the accuracy of the phase measurements for the finite difference approximations of the wave speed.

The schematic configuration of an IT device for measuring the performance of orifices with flow is shown in Fig. 2. Different parameters affect the measurements and the acoustic properties of the orifice. The list of important variables is shown in Table 1. The orifice Mach number and the nominal Strouhal number govern the acoustic behavior of the orifice plates [7]; however, the excitation amplitude and thickness are also reported to affect the physics of sound absorption [8]. In this study, the IT was modeled based on the actual full-scale dimensions provided by Ahuja et al. [9]. The following geometrical variables were used and remained fixed in all simulations: $D_T/D_o = 5.73$, $d_o/D_o = 0.16$, $L_1/D_o = 110.67$, $L_2/D_o = 88.92$, $L_d/D_o = 10.00$.

4. Computational setup

The computational domain was subdivided into structured lattice arrays with variable resolutions (VR). This procedure is analogous to grid refinement and stretching used in finite-difference and finite-volume numerical schemes. The lattice length from one VR to another always varies by a factor of two to ensure appropriate particle convection along pre-assigned discretized velocity directions as shown in Fig. 1. Fig. 3 shows the grid setup and VR regions used in this study. In the numerical setup, four levels of VR regions were used upstream of the orifice. The large size of the computational domain in the streamwise direction downstream of the orifice and the use of coarser grid at outermost layers ensured reflected sound wave attenuation and provided a uniform region for probing the flow history at the upstream boundary for acoustic calculations.

A total of 15.8 million grid cells (voxels) were used. The smallest voxel size of $0.050 D_o$ was located near the orifice entrance, and in the jet plume shear layer. The specified resolution was set to resolve the shortest wave length, at highest frequency limit by over sixty voxels ($x_{max} \sim 60 \lambda_{min}$) at the coarsest level upstream of the orifice. This is sufficiently fine to achieve very low numerical dissipation. As shown in a previous study performed by Brés et al. [29], a minimum of fifty points per wavelength is required to achieve numerical absorption level of $0.01 \text{ dB}/\lambda$ or less for a simulation that includes turbulence modeling.

Based on the orifice diameter (D_o) and mean orifice jet velocity, the Reynolds number was matched to that of the flow in the experimental setup for all set points. Turbulent subgrid scale structures were modeled using the Very Large Eddy Simulation (VLES) method [30]. In the VLES scheme, the large scales are directly simulated. In order to account for subgrid scale turbulent fluctuations, the Lattice Boltzmann equation is extended by replacing its molecular relaxation time with an effective turbulent relaxation time scale. The effective time,

$$\tau_{eff} = \tau + C_\mu \frac{k^2/\varepsilon}{(1 + \tilde{\eta}^2)^{1/2}}, \quad (4)$$

is derived from a systematic renormalization group (RNG) procedure. In Eq. (4), C_μ is constant and $\tilde{\eta}$ is a combination of a local strain parameter ($k|S_{ij}|/\varepsilon$), local vorticity parameter ($k|\Omega_{ij}|/\varepsilon$) and local helicity parameters. A modified, two-equation “ k - ε ” model based on the original RNG formulation describes the sub-grid scale turbulence contributions [30,31]. The turbulence model energy production and dissipation equations are

$$\rho \frac{Dk}{Dt} = \frac{\partial}{\partial x_j} \left[\left(\frac{\rho v_0}{\sigma_{k0}} + \frac{\rho v_T}{\sigma_{kT}} \right) \frac{\partial K}{\partial x_j} \right] + \tau_{ij} S_{ij} - \rho \varepsilon, \quad (5)$$

and

$$\rho \frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_j} \left[\left(\frac{\rho v_0}{\sigma_{\varepsilon 0}} + \frac{\rho \varepsilon_T}{\sigma_{\varepsilon T}} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{\varepsilon 1} \tau_{ij} S_{ij} - \left[C_{\varepsilon 2} + C_\mu \frac{\eta_s^3 (1 - \eta_s/\eta_0)}{1 + \beta \eta_s^3} \right] \rho \frac{\varepsilon^2}{k}, \quad (6)$$

respectively. The parameter, $v_T = C_\mu k^2/\varepsilon$, is the eddy-viscosity in the RNG formulation and σ_{k0} , σ_{kT} , $\sigma_{\varepsilon 0}$, $\sigma_{\varepsilon T}$, $\sigma_{\varepsilon 1}$, $\sigma_{\varepsilon 2}$, and β are constants, either derived from the RNG procedure or tuned for internal and external flow configurations. The above equations are solved using a modified Lax-Wendroff explicit 2nd order finite difference scheme [32].

A specific volumetric boundary scheme [33] was used to implement a particle bounce-back algorithm and simulate the no-slip wall boundary conditions. This algorithm is well customized for complex geometries. Enhanced turbulent wall boundary conditions were applied using a generalized LBM slip algorithm and a modified wall-shear stress model. This scheme significantly reduces the near wall grid resolution required for capturing turbulent structures near the solid boundaries. A non-reflective boundary condition was imposed at the end of the tube to simulate the anechoic termination, using viscous-damping layers available in the PowerFLOW code. The outermost VR regions near the outlet boundary were relatively coarse and caused further dissipation of outgoing waves, thus acting as ‘sponge’ zones. The inclusion of the orifice plate within the computational domain ensures sufficient initial perturbations at the orifice exit for the jet to breakup close to the orifice trailing edge, even at low Reynolds numbers. This approach eliminates the need to apply artificial forcing terms which are used in most Large eddy simulations (LES) techniques, and usually causes spurious sound waves reflected back upstream. Such spurious waves cause unwanted noise at the virtual microphone locations. A positive sinusoidal fluctuating pressure was imposed at the inflow boundary i.e.

$$P/P_{amb} = 1 + (A^*/2)(1 + \cos(\omega t)), \quad (7)$$

where $A^* = A/P_{amb}$, A is the acoustic pressure amplitude and P_{amb} is the ambient pressure. A steady mass flow rate was imposed at the tube outlet. Its magnitude was selected to match available experimental data [9]. The mass flow rate value provided the desired Mach number at the orifice outlet.

5. Acoustic calculations

Six virtual probes were located at random intervals upstream from the orifice plate, as indicated schematically in Fig. 4. The average value of the pressure and velocity over four lattice lengths in the vicinity of each probe was calculated. The spatial averaging process is necessary to reduce noise and account for the effects of finite microphone size. A random spacing was selected for probe distribution. Jones and Parrot [34] argue that a uniform spacing might be inaccurate at certain frequencies. They might be located near acoustic nodes and hence sensing very small amplitudes of the same order of magnitude as the experimental or numerical errors. The acoustic calculations included a wave decomposition procedure of the standing sound wave field upstream of the orifice plate. This process was initiated by extracting the pressure and velocity history at the probe locations. These data were used to calculate the reflection factor, from which the absorption coefficient and the acoustic impedance of the orifice plate were obtained. The calculations were done in the frequency domain. Each probe sampled the pressure and velocity signals at the same location. Using a linearized wave equation, taking into account the presence of a mean flow, the relationship between the acoustic velocity and the pressure was obtained. The complex pressure and velocity at each probe location were obtained using a Fourier transform. The sampling frequency and signal length are of great importance to achieve high accuracy. Around 2×10^5 time steps were required to achieve a steady state periodic wave field inside the tube, as shown in Fig. 5. Sampling started after the steady periodic state was achieved, and extended over a time period of about 0.08 s, sufficient to get a maximum spectral resolution of $f \sim 12$ Hz. To minimize the bias due to the spectral leakage errors, the amplitude of the tonal signals was obtained by summing up the energy of the signals in neighboring frequency components using a five-point stencil. This energy summation method was also compared to the case in which Hann windowing was applied and resulted in almost the same magnitude with at most a 3% difference.

The acoustic field upstream of the orifice was decomposed into incident and reflected waves. The complex acoustic pressure at the j th probe may be generally decomposed as

$$\hat{P}_j = \hat{P}_j^I e^{i\Gamma I x_j} + \hat{P}_j^R e^{i\Gamma R x_j}, \quad (8)$$

where superscripts I and R denote the incident and reflection values respectively, subscript J refers to the location of the j th probe. The reference point (i.e. $x = 0$) is the orifice location. The complex wave number, Γ , in Eq. (8), is defined as

$$\hat{\Gamma}_{I,R} = \pm (k_{I,R} + i\alpha_{I,R}), \quad (9)$$

The wave number, k , was modified to account for the mean flow [34], which is given as

$$k_{I,R} = (\omega/c)/(1 \pm M_T), \quad (10)$$

where M_T is the mean Mach number of the tube flow upstream from the orifice. Turbulence and thermo-viscous losses can be modeled using the correlations by Ingard and Singhal [35].

This model estimates the effect of the tube wall absorption coefficient in the presence of mean flow using the relations

$$\alpha_{I,R} = (\beta_T + \beta_v)/(1 \pm M_T), \quad (11)$$

where β_T indicates the dissipation of acoustic energy due to turbulence inside the channel, β_v quantifies thermo-viscous losses at the channel walls, and M_T is the Mach number in the upstream direction. The losses caused by turbulence in the channel may be accounted for as suggested by Ingard and Singhal [35] as

$$\beta_T = (2\psi M_T / D_T)[1/(1 + 0.869\sqrt{\psi})] \quad (12)$$

where D_T is the tube diameter, ν is the kinematic viscosity, and c is the speed of sound. The friction coefficient, ψ , can be calculated using Prandtl's universal resistance law [34], stated as

$$1/\psi = 2\text{Log}_{10}(\text{Re}\sqrt{\psi} - 0.8). \quad (13)$$

Neglecting heat transfer through walls, and assuming a fully turbulent flow, viscous losses can be estimated by Kirchoff's relation [34] as

$$\beta_v = (1/D_T c)(2\omega\nu)^{1/2}. \quad (14)$$

An alternate method to include the visco-thermal attenuation in which the turbulent losses are neglected has been suggested by Dokumaci [36]. As the Mach number in the tube far from the orifice is small ($M_T \sim 0.017$), based on Eq. (12), β_T would have a small magnitude and the effect on the absorption coefficient would be less than one percent and can indeed be neglected. The thermal losses, on the other hand, cannot be distinguished from viscous losses in LBM simulation due to the isothermal assumption and the presence of the numerical dissipation. The turbulence dissipation, however, is resolved by the LBM. To capture the viscous losses, an auxiliary energy equation that contains viscous dissipation terms must be coupled with the momentum equations. This approach was not used in the current study. Instead, the viscous attenuation factor, β_v , was replaced with the numerical dissipation factor, β_n . To evaluate the dissipation rate, a numerical experiment was performed inside a three dimensional tube of the same dimension scales as the main orifice setup using the finest grid resolution and viscosity. The fundamental frequencies and amplitudes were also selected to be similar to those for the main orifice case. The length of the channel was $L_T = 18D_T$. Assuming to be P_0 , the pressure distribution along tube was assumed to obey the relation:

$$P = \Re\{P_0 e^{-\alpha x} e^{ik\chi}\}. \quad (15)$$

By tracking the location of the pressure magnitude maxima, the pressure decay rate corresponding to the numerical dissipation for the LBM simulation was calculated. This

procedure is illustrated in Fig. 6 for a frequency of 6 kHz. This study was repeated for other frequencies (i.e. 1–6 kHz), and the results are listed in Table 2. The data in Table 2 suggest that numerical dissipation is a weaker function of frequency and is mostly related to grid resolution and bulk viscosity [19]. In the numerical study, the thermo-viscous factor, β_v , was replaced with the numerical dissipation factor, β_n , which was extracted from Table 2 based on the frequency and the grid resolution. The acoustic velocity was calculated based on the linearized Euler equation, taking into account non-zero mean flow, was as

$$\hat{V}_j = (1/Y) \left(\hat{P}_j^I e^{i\Gamma I x_j} + \hat{P}_j^R e^{i\Gamma R x_j} \right), \quad (16)$$

where Y is the modified characteristic impedance taking into account the losses [37],

$$Y = \rho c \left[1 - (\beta_T + \beta_n)/k + i(\beta_T - \beta_n)/k \right]. \quad (17)$$

Eqs. (8) and (16) constitute one set of two complex equations with two unknowns; $\hat{P}_j^I$ and $\hat{P}_j^R$. The solution yields the absorption coefficient of the orifice plate, α_o , defined as the ratio of the acoustic energy absorbed by the orifice plate and the incident energy. The absorption coefficient and the impedance values at the orifice location (i.e. $x = 0$) were obtained using the relations

$$\alpha_0 = 1 - |P_j^R / P_j^I|^2, \quad (18)$$

and

$$\hat{Z}_0 = R + Xi = Y \left(1 + P_j^R / P_j^I \right) / \left(1 - P_j^R / P_j^I \right). \quad (19)$$

The absorption coefficient values were calculated for all probes (i.e. $j_{max} = 6$) locations. The final absorption coefficient, $\alpha_{o,t}$ for each set point was obtained by averaging the six values.

$$\alpha_{o,t} = \frac{1}{j_{max}} \sum_{j=1}^{j_{max}} \alpha_{o,j}, \quad (20)$$

6. Results and discussion

The methods were validated for the acoustic excitation amplitude fixed at 145 dB and a low frequency set point of 380 Hz (i.e. $kD_T = 0.2$) and five high frequency cases of 1, 2, 3, 4 and 6 kHz. The high frequency cases were used to compare LBM results with experimental data [9]. The analytical solutions proposed by Wendoloski [7] covers both high and low frequency ranges. The mean flow strength was characterized by the orifice Mach number. Both the experiments and the numerical simulations were done for Mach number values of 0.05, 0.1, 0.15, and 0.2. These Mach number values are less than 0.5, and hence they fall well inside the validity range of the D3Q19 LBM. In the absence of acoustic waves, velocity

profiles along the tube cross section in the vicinity of the orifice plate at $x = 10D_o$ were compared with hot wire measurement data [9]. Non-dimensional velocity profiles for different orifice Mach numbers are shown in Fig. 7 with respect to the distance from the lower wall (i.e. $0 \leq y \leq D_T$). This comparison allowed the verification of the predicted tube outlet boundary condition used in the simulations, by comparing to the mass flow rate values given by the experimental data [9]. The velocity profile for $M_o = 0.05$ is parabolic and laminar as shown in Fig. 7. For higher Mach numbers (i.e. $Re_T > 1200$) the pattern is more logarithmic and similar to that for fully turbulent flows inside the confined tubes, as expected.

The periodic acoustic flow through the orifice causes the formation of ring vortices at the orifice trailing edge. This phenomenon was studied by observing the pressure contours inside the IT. Fig. 8 shows the shedding of vortices from the orifice edge. Fig. 8b depicts a velocity iso-surface of a turbulent jet formed downstream of the orifice, the strength of which characterizes sound absorption of the orifice plate. The absorption coefficient obtained from the LBM simulation for the case of $kD_T = 0.2$ was compared with the analytical solution [7] and plotted in Fig. 9 as a function of orifice Mach number. The optimum Mach number corresponding to the maximum absorption is predicted to be 0.09 for this condition. As explained in [7], the area ratio or the porosity in cases where there are several orifices is the key factor affecting the absorption efficiency should be taken into account for more detailed parametric studies. The frequency dependence of the orifice absorption for a fixed Mach number of 0.005 from LBM results, experimental data [9] and the analytical solution [7] is shown in Fig. 10. It shows that an increase in excitation frequency reduces the sound absorption of the orifice over the frequency range above 1 kHz. According to Fig. 10, the absorption coefficient for the low frequency range (i.e. $kD_T < 1.5$) was better predicted by LBM comparing to the analytical solution. This is due to the fact that large-scale turbulent structures in the vicinity of the orifice edge together with energy transmission from upstream channel are accurately simulated via LBM. Effects of sub-grid scale structures are also included in the turbulence model implemented in the LBM code. Numerical method has several advantages over the analytical formulation proposed in reference [7] as well as similar analytical solution using Howe's Rayleigh conductivity concept [12]. First, the large eddy simulation method used in this study can better capture the physics of the turbulent jet which becomes more important as Mach numbers increases. Second, the details of the orifice and duct geometries can be included into computational domain without any simplifying assumptions. Fig. 11 shows the predicted sound absorption comparing to measured values [9] for high frequencies. The trends and magnitudes of the absorption coefficient are in good agreement with the measured data. One interesting observation, which is discernible in Fig. 11, is that the Mach number dependence of the sound absorption coefficient varies with the frequency. From a physical point of view, this implies that the contribution of high frequency acoustic waves to supply energy for vortex shedding increases when the Mach number is increased; hence, more energy is absorbed by increasing the Mach number (i.e. larger α). For the low frequency waves, the mean flow seems to have more contribution on supplying required kinetic energy for vortex formation process and hence, less acoustic energy required from the upstream channel (i.e. smaller α). This observation is consistent with the acoustic model developed by Howe [12] and

Wendoloski [7]. Figs. 12 and 13 show the real part (R) and imaginary part (X) of the orifice impedance (Z) respectively, again averaged over all probe locations. The calculated impedance value at each frequency and orifice Mach number was normalized by the characteristics impedance (ρc). The results show consistent variations of impedance values comparing to experimental data [9].

7. Closing remarks and conclusions

A numerical simulation of a noise-induced flow through an orifice plate was performed using the LBM. The solution of the linear wave equation, modified for the presence of mean flow, was used to decompose the wave upstream of the orifice plate and to calculate the absorption coefficient and acoustic impedance. Visco-thermal losses denoted by β_v in semi-empirical correlations were replaced by the numerical dissipation coefficient β_n . The same empirical value β_T was used to account for the turbulent losses. A case study was done for the fixed acoustic amplitude 145 dB. Variations of the sound absorption coefficient with respect to orifice mean velocity and the excitation frequency were studied and compared with the available analytical solution and experimental data. Both flow and acoustic results were in good agreement with previously published data. The LBM was found to be an accurate tool for modeling sound absorption in a full-scale virtual impedance tube. The VLES turbulence scheme was found to be an effective and accurate method to capture realistic behavior of the flow field, correct prediction of velocity profiles as well as the acoustic characteristics of the duct.

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Nomenclature

c	speed of sound (m/s)
d	thickness (m)
D	diameter (m)
e	discrete particle speed (m/LatticeTime)
F	particle distribution function
L	length scale (m)
M	Mach number ($M = U/a$)
P	pressure
R	air constant
Re	Reynolds number ($Re = UD/\nu$)

St	Strouhal number (FD/U)
T	time (LatticeTime)
U	macroscopic flow velocity
V	acoustic velocity
$\vec{x}$	Cartesian coordinate of fluid particle

Greek symbols

γ	specific heat ratio
ρ	fluid density (kg/m^3)
ν	kinematic viscosity (m^2/s)
ψ	friction coefficient

Subscripts

amb	ambient
o	orifice parameter
T	tube parameter
j	probe index
ex	excitation
i	i th discrete velocity direction
1	orifice upstream
2	orifice downstream

Superscripts

eq	equilibrium
^	complex number

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Fig. 1.
Sketch of D3Q19 LBM model.

Fig. 2.
Orifice plate configuration in an impedance tube for acoustic measurements.

Fig. 3.
Structured grid distribution near the orifice plate.

Fig. 4.
Probe locations upstream of the orifice.

Fig. 5.
Acoustic pressure signal recorded by probe at $x = 6D_o$ upstream of the orifice plate.

Fig. 6.
Sound attenuation due to the numerical dissipation vs. analytical thermo-viscous dissipation.

Fig. 7.
Streamwise velocity profile at orifice upstream ($x=10D_o$).

Fig. 8.

(a) Vortex formation and shedding at the edge of orifice and (b) velocity iso-surface (i.e. 0.5 m/s) of turbulent jet originated from the noisy mean flow across the orifice.

Fig. 9.
Sound absorption coefficient with respect to orifice Mach number for low frequency case ($kD_T = 0.2$).

Fig. 10.
Frequency dependence of sound absorption coefficient at fixed orifice Mach number (i.e. $Mo = 0.05$).

Fig. 11.
Sound absorption coefficient variation with mean flow strength and the excitation frequency.

Fig. 12.
Variation of the normalized orifice resistance with the Mach number and the excitation frequency.

Fig. 13.
Variation of the normalized orifice reactance with the Mach number and the excitation frequency.

Table 1

Main parameters.

Variable	Description	Non-dimensional
D_o	Orifice diameter	–
D_t	Tube diameter	D_t/D_o
t_o	Orifice plate thickness	t_o/D_o
L_1	Upstream length	L_1/D_o
L_2	Downstream length	L_2/D_o
L_a	Absorption thickness	L_a/D_o
U_m	Mean flow velocity	Orifice Mach number ($M_o = U_m/c^*$)
f_{ex}	Excitation frequency	Orifice Strouhal number ($f_{ex} D_o/U_{ex}$)
A_{ex}	Excitation amplitude (pressure)	$SPL = 20 \text{ Log } (P/P_0^{**})$

* c denotes speed of sound at the laboratory temperature $c = (\gamma RT)^{0.5}$.

** P_0 denotes the reference pressure: $P_0 = 2 \times 10^{-5}$ Pa.

Table 2Attenuation factor (β) comparison.

Frequency (Hz)	1000	2000	3000	4000	6000
LBM (β_n)	0.0235	0.0242	0.0245	0.0257	0.0273
β_v from Eq. (12)	0.0711	0.100	0.123	0.142	0.1743