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Determining Roe and Metz model parameters for simulating multireader multicase confidence-of-disease rating data based on real-data or conjectured Obuchowski-Rockette parameter estimates

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Abstract

The most frequently used model for simulating MRMC data that emulate confidence-of-disease ratings from diagnostic imaging studies has been the Roe and Metz model, proposed in 1997. The RM model generates continuous confidence-of-diseases ratings based on an underlying equal-variance binormal model for each reader, with the separation between the normal and abnormal rating distributions varying across readers. A problem with the RM model is that the parameters are expressed in terms of the rating distributions, as opposed to the reader performance outcomes. Because MRMC analysis results are almost always expressed in terms of the reader performance outcomes, and not in terms of the rating data distributions, it has been difficult to assess how similar the simulated data are to MRMC data encountered in practice. To remedy this situation, recently Hillis (in 2018) derived formulas expressing parameters that describe the distribution of empirical AUC outcomes computed from RM simulated data as functions of the RM parameters. An examination of these values revealed several problems with the realism of the simulated data. This paper continues that work by providing the inverse mapping, i.e., by deriving an algorithm that expresses the RM parameters as functions of the AUC empirical distribution parameters. This result will enable the creation of a recalibrated RM model that more closely emulates real-data studies.

Keywords

Roe and Metz; Obuchowski and Rockette; AUC; ROC curve; diagnostic radiology

1. INTRODUCTION

For the typical diagnostic radiology study, several readers (typically radiologists) assign confidence-of-disease ratings to each case (i.e., subject) based on one or more corresponding radiologic images. The resulting data are called multireader multicase (MRMC) data. These studies are often used to compare different imaging modalities with respect to reader performance. Often measures of reader performance are functions of the estimated receiver-operating-characteristic (ROC) curve, such as the area under the ROC curve (AUC). A

commonly used method for analyzing reader performance outcomes that allows conclusions to generalize to both the reader and case populations is the Obuchowski and Rockette method (OR).¹

The most frequently used model for simulating MRMC data that emulate confidence-of-disease ratings from such studies has been the Roe and Metz model (RM).² Numerous studies have used this model for evaluating MRMC analysis and sample size methods. As discussed by Hillis,³ the RM model generates continuous confidence-of-diseases ratings based on an underlying equal-variance binormal model for each reader, with the separation between the normal and abnormal rating distributions varying across readers.

An outline of this paper is as follows. In Section 2 I discuss the original Roe and Metz model and a more general extension of it. In Section 3 I discuss the OR model. In Section 4 I describe the algorithm that provides the one-to-one mapping from the OR model parameters to the RM model parameter, followed by a discussion in Section 5.

2. ROE AND METZ MODELS: ORIGINAL AND CONSTRAINED UNEQUAL-VARIANCE

2.1 Original RM model

Let X denote a confidence-of-disease rating (also called a decision variable value) assigned by a reader to a case. The simulation model proposed by Roe and Metz² is a mixed four-factor (test, reader, case, and truth) ANOVA model for X with case nested within truth; test, reader, and truth crossed; test and truth treated as fixed factors; and reader and case treated as random factors. Using their notation, the model is given by

$$X_{ijkl} = \mu_t + \tau_{it} + R_{jt} + C_{kt} + (\tau R)_{ijt} + (\tau C)_{ikt} + (RC)_{jkt} + (\tau RC)_{ijkt} + E_{ijkl} \quad (1)$$

$i = 1, 2; j = 1, \dots, r; k = 1, \dots, n_t$; and $t = -, +$, with “ $-$ ” indicating a nondiseased case and “ $+$ ” indicating a diseased case. Here μ_t is the effect of truth state t , τ_{it} is the interaction effect of test i and truth state t , R_{jt} is the interaction effect of reader j and truth state t , C_{kt} is the effect of case k nested within truth state t , the multiple symbols in parentheses denote interactions, and E_{ijkl} is the error term. All effects are random except for μ_t and τ_{it} . The random effects are mutually independent and normally distributed with zero means. Roe and Metz denote the corresponding variance components by σ_R^2 , σ_C^2 , $\sigma_{\tau R}^2$, $\sigma_{\tau C}^2$, σ_{RC}^2 , $\sigma_{\tau RC}^2$ and σ_E^2 . They note that $\sigma_{\tau RC}^2$ and σ_E^2 cannot be estimated separately for this model with no replications, and hence define

$$\sigma_E^2 \equiv \sigma_{\tau RC}^2 + \sigma_E^2$$

Although not mentioned by Roe and Metz, the omission of test, reader, and test-by-reader effects that do not depend on truth is justified by the invariance of the ROC curve to location shifts; thus inclusion of these terms would not change the ROC curve for a given reader. Note that interactions with truth are denoted only by a t subscript in (1). I refer to this as the *original RM model*.

Roe and Metz constrain the sum of the error variance and variance components involving case to be equal to one:

$$\sigma_C^2 + \sigma_{\tau C}^2 + \sigma_{RC}^2 + \sigma_\epsilon^2 = 1. \quad (2)$$

It follows from this constraint³ that the fixed-reader nondiseased and diseased decision variable (DV) distributions have unit variances (and hence their ROC curves are symmetric), with the fixed-reader AUCs varying across the reader population.

Without loss of generality, Roe and Metz impose the constraints

$$\mu_- = \tau_{1-} = \tau_{2-} = \tau_{1+} = \tau_{2+} = 0 \quad (3)$$

which result in the same DV distributions for both tests 1 and 2. Under this constraint, it can be shown³ that the mean and median separation of the nondiseased and diseased DV distributions across the reader population is given by μ_+ and the median reader-specific AUC is given by $A_z = \Phi(\mu_+/\sqrt{2})$.

Although Roe and Metz only consider simulations for equal test DV distributions, the model can be easily modified to allow for test DV distributions that differ in their median AUC values by not setting τ_{2+} to zero, that is, only the constraints

$$\mu_- = \tau_{1-} = \tau_{2-} = \tau_{1+} = 0 \quad (4)$$

are imposed. It follows that the median AUCs for test 1 and 2 are equal to $A_z^{(i)} = \Phi(\xi_i/\sqrt{2})$, $i = 1, 2$, respectively, where

$$\xi_i = \mu_+ + \tau_{i+}, i = 1, 2. \quad (5)$$

From constraints (4) it follows that $\xi_1 = \mu_+$ for test 1 and $\xi_2 = \mu_+ + \tau_{2+}$ for test 2.

Note that the RM model that allows for different modality AUCs is completely defined by 7 parameters:

$$\xi_1, \xi_2, \sigma_R^2, \sigma_{\tau R}^2, \sigma_C^2, \sigma_{\tau C}^2, \text{ and } \sigma_{RC}^2. \quad (6)$$

2.2 Constrained unequal-variance RM model

In practice, estimated binormal nondiseased and diseased distribution variances for a fixed reader are often different, with diseased subjects typically having more variable test results. Thus to better emulate real data, Hillis⁴ modified the original RM model by allowing variance components involving case to depend on truth, with variance components involving diseased cases set equal to those involving normal cases multiplied by the factor $1/b^2$, $b > 0$. Specifically, the model is given by (1) with variance components (using an obvious notation) denoted by $\sigma_R^2, \sigma_{\tau R}^2, \sigma_C^2(-), \sigma_{\tau C}^2(-), \sigma_{RC}^2(-), \sigma_\epsilon^2(-), \sigma_C^2(+), \sigma_{\tau C}^2(+), \sigma_{RC}^2(+)$ and $\sigma_\epsilon^2(+)$ with,

$\sigma_{C(+)}^2 = \frac{1}{b^2}\sigma_{C(-)}^2$, $\sigma_{\tau C(+)}^2 = \frac{1}{b^2}\sigma_{\tau C(-)}^2$, $\sigma_{RC(+)}^2 = \frac{1}{b^2}\sigma_{RC(-)}^2$, $\sigma_{\epsilon(+)}^2 = \frac{1}{b^2}\sigma_{\epsilon(-)}^2$. Similar to Eq. (2), the constraints

$$\sigma_{C(-)}^2 + \sigma_{\tau C(-)}^2 + \sigma_{RC(-)}^2 + \sigma_{\epsilon(-)}^2 = 1$$

are imposed. It follows that

$$\sigma_{C(+)}^2 + \sigma_{\tau C(+)}^2 + \sigma_{RC(+)}^2 + \sigma_{\epsilon(+)}^2 = \frac{1}{b^2}.$$

Following Hillis,³ I refer to this as the *constrained unequal-variance RM model*. The adjective “constrained” refers to the ratio of each variance component involving diseased cases with respect to the corresponding variance component for nondiseased cases constrained to be $1/b^2$.

Similar to the original RM model, imposing constraint (3) results in the null model with $A_z = \Phi(\mu_+/\sqrt{1+b^{-2}})$ and imposing constraint (4) results in the nonnull model with

$$\begin{aligned} A_z^{(i)} &= \Phi\left((\mu_+ + \tau_{i+})/\sqrt{1+b^{-2}}\right) \\ &= \Phi\left(\xi_i/\sqrt{1+b^{-2}}\right), i = 1, 2, \end{aligned}$$

where again $A_z^{(i)}$ denotes the median AUC across the reader population for test i and ξ_i is defined by Eq. (5).

The algorithm discussed in this paper will be for this constrained unequal-variance RM model, which includes the original RM model, discussed in the previous section, as a special case with $b = 1$. Note that the constrained unequal-variance RM model that allows for different modality AUCs is completely defined by 8 parameters:

$$b, \xi_1, \xi_2, \sigma_R^2, \sigma_{\tau R}^2, \sigma_{C(-)}^2, \sigma_{\tau C(-)}^2, \text{ and } \sigma_{RC(-)}^2.$$

3. OBUCHOWSKI-ROCKETTE (OR) MODEL

Obuchowski and Rockette¹ proposed a test $\times$ reader factorial ANOVA model for the AUC estimates, but unlike a conventional ANOVA model, the errors are assumed to be correlated to account for correlation due to each reader evaluating the same cases. Their model, which I refer to as the *OR model*, is given by

$$\hat{\theta}_{ij} = \mu_{OR} + \tau_{i:OR} + R_{j:OR} + (\tau R)_{ij:OR} + \epsilon_{ij:OR} \quad (7)$$

where μ_{OR} is the intercept term, $\tau_{i:OR}$ denotes the fixed effect of test i , $R_{j:OR}$ denotes the random effect of reader j , $(\tau R)_{ij:OR}$ denotes the random test $\times$ reader interaction, and $\epsilon_{ij:OR}$

is the error term. The $R_{j:OR}$ and $(\tau R)_{ij:OR}$ are assumed to be mutually independent and normally distributed with zero means and respective variances $\sigma_{R:OR}^2$ and $\sigma_{\tau R:OR}^2$. (I include “OR” in effect and variance component subscripts to distinguish OR effects and variance components from similarly notated RM-model quantities.) The $\epsilon_{ij:OR}$ are assumed to be normally distributed with mean zero and variance $\sigma_{\epsilon:OR}^2$ and are assumed independent of the $R_{j:OR}$ and $(\tau R)_{ij:OR}$. Three possible error covariances are assumed:

$$\text{Cov}(\epsilon_{ij:OR}, \epsilon_{i'j':OR}) = \begin{cases} \text{Cov}_1 & i \neq i', j = j' \text{ (different test, same reader)} \\ \text{Cov}_2 & i = i', j \neq j' \text{ (same test, different reader)} \\ \text{Cov}_3 & i \neq i', j \neq j' \text{ (different test, different reader)} \end{cases}$$

It follows from (7) that $\sigma_{\epsilon:OR}^2$ is equal to the variance of AUC for a single fixed reader and test and Cov_1 , Cov_2 , and Cov_3 are the AUC covariances for a single fixed reader and two tests, two different fixed readers and one test, and two different fixed readers using two different tests, respectively. These error variance-covariance parameters are typically estimated by averaging corresponding estimates computed using a fixed-reader method, such as the jackknife,⁵⁻⁷ bootstrap,^{7,8} or the method proposed by DeLong et al⁹ (for empirical AUC estimates). The $\epsilon_{ij:OR}$ can be interpreted as AUC measurement error attributable to the random selection of cases and within-reader variability that describes how a fixed reader interprets the same image in different ways on different occasions. The OR model can alternatively be described with population correlations $r_i = \text{Cov}_i / \sigma_{\epsilon:OR}^2$ replacing corresponding Cov_i .

4. ALGORITHM FOR MAPPING OR MODEL PARAMETERS TO RM MODEL PARAMETERS

From hereon, the term “RM model” will refer to the unconstrained unequal-variance RM model discussed in Sect. 2.2 unless otherwise stated. The mapping from the RM model parameters to the OR model was derived by Hillis³ and is given in Table 1.

In order to facilitate the derivation of the reverse mapping, I define an alternative parameterization for the RM model, which is presented in Table 2. Table 2a expresses the alternative RM parameters in terms of the RM parameters, Table 2b presents the constraints on the alternative parameters, and Table 2c expresses the RM model parameters in terms of the alternative RM parameters. Table 3 expresses the OR parameters in terms of the alternative RM parameters and Table 4 expresses the alternative RM parameters in terms of the OR parameters.

The proposed algorithm is presented in Table 5. It results from replacing the OR parameters in Table 3 by specified values, solving for the corresponding alternative RM parameter values, and then solving for the RM parameter values using the formulas in Table 2c.

5. DISCUSSION

A problem with the RM model is that its parameters are expressed in terms of the rating distributions, as opposed to the reader performance measures. Because MRMC analysis results are almost always expressed in terms of the reader performance measures, and not in terms of the rating data distributions, it has been difficult to assess how similar the simulated data are to MRMC data encountered in practice. Although recently Hillis³ remedied this problem by deriving formulas for computing the OR parameter values that describe the distribution of empirical AUC outcomes computed from RM simulated data, he did not provide the reverse OR-to-RM mapping, which would make it easy to calibrate the RM model to produce simulated data similar to specific real data sets. This paper overcomes that limitation by presenting an algorithm that computes RM parameter values from a specified set of OR parameter values.

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Table 1.

Obuchowski-Rockette parameters expressed in terms of constrained unequal-variance RM model parameters for the empirical AUC. Notes: $F_{\text{BVN}}(\cdot, \cdot; \rho)$ is the standardized bivariate normal distribution function with correlation ρ ; $\delta_i = \mu_+ + \tau_i$; $V = 1 + b^{-2} + 2(\sigma_R^2 + \sigma_{TR}^2)$; $c_1 = \frac{1}{n_0 n_1}$; $c_2 = \frac{n_0 - 1}{n_0 n_1}$; $c_3 = \frac{n_1 - 1}{n_0 n_1}$; $c_4 = \frac{1 - n_0 - n_1}{n_0 n_1}$. This table reprinted, adapted and revised with permission from Hillis [3, Table 3]. Notation is the same except that $(\sigma_{\text{fixed}}^2(-) + \sigma_{\text{fixed}}^2(+))$ has been replaced by $1 + b^{-2}$.

$$\mu_{\text{OR}} + \tau_{i:\text{OR}} = \Phi\left(\frac{\delta_i}{\sqrt{V}}\right)$$

$$\sigma_{R:\text{OR}}^2 = F_{\text{BVN}}\left(\frac{\delta_1}{\sqrt{V}}, \frac{\delta_2}{\sqrt{V}}; \frac{2\sigma_R^2}{V}\right) - \left[\Phi\left(\frac{\delta_1}{\sqrt{V}}\right)\Phi\left(\frac{\delta_2}{\sqrt{V}}\right)\right]$$

$$\sigma_{TR:\text{OR}}^2 = .5 \sum_{i=1}^2 \left\{ F_{\text{BVN}}\left(\frac{\delta_i}{\sqrt{V}}, \frac{\delta_i}{\sqrt{V}}; \frac{2(\sigma_R^2 + \sigma_{TR}^2)}{V}\right) - \left[\Phi\left(\frac{\delta_i}{\sqrt{V}}\right)\right]^2 \right\} - \sigma_{R:\text{OR}}^2$$

$$\text{Cov}_1 = \sum_{m=1}^4 c_m F_{\text{BVN}}\left(\frac{\delta_1}{\sqrt{V}}, \frac{\delta_2}{\sqrt{V}}; \frac{\rho_m(1 + b^{-2}) + 2\sigma_R^2}{V}\right)$$

where $\rho_1 = \frac{\sigma_{RC}^2(-) + \sigma_C^2(-) + \sigma_{RC}^2(+) + \sigma_C^2(+)}{1 + b^{-2}}$, $\rho_2 = \frac{\sigma_{RC}^2(-) + \sigma_C^2(-)}{1 + b^{-2}}$, $\rho_3 = \frac{\sigma_{RC}^2(+) + \sigma_C^2(+)}{1 + b^{-2}}$, $\rho_4 = 0$

$$\text{Cov}_2 = \frac{1}{2} \sum_{i=1}^2 \sum_{m=1}^4 c_m F_{\text{BVN}}\left(\frac{\delta_i}{\sqrt{V}}, \frac{\delta_i}{\sqrt{V}}; \frac{\rho_m(1 + b^{-2})}{V}\right)$$

$\rho_1 = \frac{\sigma_C^2(-) + \sigma_{TC}^2(-) + \sigma_C^2(+) + \sigma_{TC}^2(+)}{1 + b^{-2}}$, $\rho_2 = \frac{\sigma_C^2(-) + \sigma_{TC}^2(-)}{1 + b^{-2}}$, $\rho_3 = \frac{\sigma_C^2(+) + \sigma_{TC}^2(+)}{1 + b^{-2}}$, $\rho_4 = 0$

$$\text{Cov}_3 = \sum_{m=1}^4 c_m F_{\text{BVN}}\left(\frac{\delta_1}{\sqrt{V}}, \frac{\delta_2}{\sqrt{V}}; \frac{\rho_m(1 + b^{-2})}{V}\right)$$

where $\rho_1 = \frac{\sigma_C^2(-) + \sigma_C^2(+)}{1 + b^{-2}}$, $\rho_2 = \frac{\sigma_C^2(-)}{1 + b^{-2}}$, $\rho_3 = \frac{\sigma_C^2(+)}{1 + b^{-2}}$, $\rho_4 = 0$

$$\sigma_{\epsilon:\text{OR}}^2 = \frac{1}{2} \sum_{i=1}^2 \sum_{m=1}^4 c_m F_{\text{BVN}}\left(\frac{\delta_i}{\sqrt{V}}, \frac{\delta_i}{\sqrt{V}}; \frac{\rho_m(1 + b^{-2}) + 2(\sigma_R^2 + \sigma_{TR}^2)}{V}\right)$$

where $\rho_1 = 1$, $\rho_2 = \frac{\sigma_{TC}^2(-) + \sigma_{RC}^2(-) + \sigma_C^2(-) + \sigma_{\epsilon}^2(-)}{1 + b^{-2}}$, $\rho_3 = \frac{\sigma_{TC}^2(+) + \sigma_{RC}^2(+) + \sigma_C^2(+) + \sigma_{\epsilon}^2(+)}{1 + b^{-2}}$, $\rho_4 = 0$

Table 2.

Alternative parameterization for constrained unequal-variance RM model parameterization.

(a) Alternative parameters expressed in terms of original parameters. Here, in terms of the original parameterization, we have defined $V = 1 + b^{-2} + 2(\sigma_R^2 + \sigma_{TR}^2)$ and $\xi_i = .\mu_+ + \tau_i + .$ Note that $x_6 = x_5 + \sigma_{C(-)}^2$, $x_7 = x_6 + \sigma_{RC(-)}^2$ and $1 - x_4 = [1 + b^{-2}]/V$

Alternative RM model parameters		Original RM model parameters
b	=	b
x_1	=	$(\xi_1)/\sqrt{V}$
x_2	=	$(\xi_2)/\sqrt{V}$
x_3	=	$2\sigma_R^2/V$
x_4	=	$[2(\sigma_R^2 + \sigma_{TR}^2)]/V$
x_5	=	$\sigma_{C(-)}^2$
x_6	=	$\sigma_{TC(-)}^2 + \sigma_{C(-)}^2$
x_7	=	$\sigma_{RC(-)}^2 + \sigma_{TC(-)}^2 + \sigma_{C(-)}^2$

(b) Constraints on alternative parameters (these follow from (a) and the RM model parameter constraints)

$$0 \leq x_i \leq 1, i = 3, \dots, 7$$

(c) Original parameters expressed in terms of alternative parameters. For the alternative parameterization,

$$V = (1 + 1/b^2)/(1 - x_4)$$

Original parameter		Alternative parameter
b	=	b
ξ_1	=	$x_1\sqrt{V}$
ξ_2	=	$x_2\sqrt{V}$
σ_R^2	=	$\sigma_R^2 = .5x_3V$
σ_{TR}^2	=	$.5x_4V - .5x_3V$
$\sigma_{C(-)}^2$	=	x_6
$\sigma_{TC(-)}^2$	=	$x_6 - x_5$
$\sigma_{RC(-)}^2$	=	$x_7 - x_5$

Table 3.

Obuchowski-Rockette parameters for the empirical AUC expressed in terms of the alternative parameterization for the constrained unequal-variance RM model given in Table 2. This table results from replacing the original RM model parameters in Table 1 by the alternative RM model parameters, as defined in Table 2a. Notes: F_{BVN} ($\cdot, \cdot; \rho$) is the standardized bivariate normal distribution function with correlation $\rho; \delta_i = \mu_+ + \tau_{i+}$;

$$V = 1 + b^{-2} + 2(\sigma_R^2 + \sigma_T^2); c_1 = \frac{1}{n_0 n_1}; c_2 = \frac{n_0 - 1}{n_0 n_1}; c_3 = \frac{n_1 - 1}{n_0 n_1}; c_4 = \frac{1 - n_0 - n_1}{n_0 n_1}.$$

$$\mu_{\text{OR}} + \tau_{i:\text{OR}} = \Phi(x_i), i = 1, 2$$

$$\sigma_{R:\text{OR}}^2 = F_{\text{BVN}}(x_1, x_2; x_3) - [\Phi(x_1)\Phi(x_2)]$$

$$\sigma_{T:\text{OR}}^2 = .5 \sum_{i=1}^2 \left\{ F_{\text{BVN}}(x_i, x_i; x_4) - [\Phi(x_i)]^2 \right\} - \sigma_{R:\text{OR}}^2$$

$$\sigma_{\varepsilon:\text{OR}}^2 = \frac{1}{2} \sum_{i=1}^2 \sum_{m=1}^4 c_m F_{\text{BVN}}(x_1, x_2; \rho_m(1 - x_4) + x_4)$$

$$\text{where } \rho_1 = 1, \rho_2 = \frac{1}{1 + b^{-2}}, \rho_3 = \frac{1}{1 + b^2}, \rho_4 = 0$$

$$\text{Cov}_3 = \sum_{m=1}^4 c_m F_{\text{BVN}}(x_1, x_2; \rho_m(1 - x_4))$$

$$\text{where } \rho_1 = x_5, \rho_2 = \frac{x_5}{1 + b^{-2}}, \rho_3 = \frac{x_5}{1 + b^2}, \rho_4 = 0$$

$$\text{Cov}_2 = . \frac{1}{2} \sum_{i=1}^2 \sum_{m=1}^4 c_m F_{\text{BVN}}(x_1, x_2; \rho_m(1 - x_4))$$

$$\rho_1 = x_6, \rho_2 = \frac{x_6}{1 + b^{-2}}, \rho_3 = \frac{x_6}{1 + b^2}, \rho_4 = 0$$

$$\text{Cov}_1 = \sum_{m=1}^4 c_m F_{\text{BVN}}(x_1, x_2; \rho_m(1 - x_4) + x_3)$$

$$\text{where } \rho_1 = x_7, \rho_2 = \frac{x_7}{1 + b^{-2}}, \rho_3 = \frac{x_7}{(1 + b^2)}, \rho_4 = 0$$

Table 4.

Alternative RM parameters expressed in terms of OR parameters. These results follow from the Table 3 relationships. Notes: $F_{\text{BVN}}(., .; \rho)$ is the standardized bivariate normal distribution function with correlation ρ ;

$$c_1 = \frac{1}{n_0 n_1}; c_2 = \frac{n_0 - 1}{n_0 n_1}; c_3 = \frac{n_1 - 1}{n_0 n_1}; c_4 = \frac{1 - n_0 - n_1}{n_0 n_1}.$$

$$x_1 = \Phi^{-1}(\mu_{\text{OR}} + \tau_1: \text{OR})$$

$$x_2 = \Phi^{-1}(\mu_{\text{OR}} + \tau_2: \text{OR})$$

$$x_3 = \left\{ x: \sigma_{R: \text{OR}}^2 - F_{\text{BVN}}(x_1, x_2; x_3) + \Phi(x_1)\Phi(x_2) = 0 \right\}$$

$$x_4 = \left\{ x_4: \sigma_{TR: \text{OR}}^2 + \sigma_{R: \text{OR}}^2 - .5 \sum_{i=1}^2 \left\{ F_{\text{BVN}}(x_i, x_i; x_4) - [\Phi(x_i)]^2 \right\} = 0 \right\}$$

$$b = \left\{ b: \sigma_{\epsilon: \text{OR}}^2 - \frac{1}{2} \sum_{i=1}^2 \sum_{m=1}^4 c_m F_{\text{BVN}}(x_1, x_2; \rho_m(1 - x_4) + x_4) = 0 \right\}$$

$$\text{where } \rho_1 = 1, \rho_2 = \frac{1}{1 + b^{-2}}, \rho_3 = \frac{1}{1 + b^2}, \rho_4 = 0$$

$$x_5 = \left\{ x_5: \text{Cov}_3 - \sum_{m=1}^4 c_m F_{\text{BVN}}(x_1, x_2; \rho_m(1 - x_4)) = 0 \right\}$$

$$\text{where } \rho_1 = (1 + b^{-2})x_5, \rho_2 = \frac{x_5}{1 + b^{-2}}, \rho_3 = \frac{x_5}{1 + b^2}, \rho_4 = 0$$

$$x_6 = \left\{ x_6: \text{Cov}_2 - \frac{1}{2} \sum_{i=1}^2 \sum_{m=1}^4 c_m F_{\text{BVN}}(x_1, x_2; \rho_m(1 - x_4)) = 0 \right\}$$

$$\text{where } \rho_1 = x_6, \rho_2 = \frac{x_6}{1 + b^{-2}}, \rho_3 = \frac{x_6}{1 + b^2}, \rho_4 = 0$$

$$x_7 = \left\{ x_7: \text{Cov}_1 - \sum_{m=1}^4 c_m F_{\text{BVN}}(x_1, x_2; \rho_m(1 - x_4) + x_3) = 0 \right\}$$

$$\text{where } \rho_1 = x_7, \rho_2 = \frac{x_7}{1 + b^{-2}}, \rho_3 = \frac{x_7}{(1 + b^2)}, \rho_4 = 0$$

Table 5.

Algorithm steps for computing parameter values for the constrained unequal-variance Roe and Metz model⁴ that correspond to specified Obuchowski and Rockette parameter values Notes: $\hat{\theta}_1$ and $\hat{\theta}_2$ denote specified values of the reader-averaged performance empirical AUCs for tests 1 and 2, respectively. $\hat{\sigma}_{R:OR}^2$, $\hat{\sigma}_{T:OR}^2$, $\hat{\sigma}_{\epsilon:OR}^2$, $\widehat{Cov}_1$, $\widehat{Cov}_2$, and denote specified values of the corresponding OR parameters, and let $\hat{r}_1$, $\hat{r}_2$ and $\hat{r}_3$ denote the specified values for the correlations defined by $r_1 = Cov_{i:OR}/\sigma_{\epsilon:OR}^2$. These specified values can be computed from real data conjectured.

- 1 Solve for x_1 and x_2 :

$$x_1 = \Phi^{-1}(\hat{\theta}_1) \text{ and } x_2 = \Phi^{-1}(\hat{\theta}_2)$$

- 2 Solve for x_3 , using the values for x_1 and x_2 obtained in Step 1:

$$x_3 = \left\{ x_3 : \hat{\sigma}_{R:OR}^2 - F_{BVN}(x_1, x_2; x_3) + \Phi(x_1)\Phi(x_2) = 0 \right\}$$

From the constraint $0 \leq x_3 \leq 1$ (from Table 2b) and the relationship $F_{BVN}(x, y; \rho_1) < F_{BVN}(x, y; \rho_2)$ if $\rho_1 < \rho_2$,¹⁰ where $F(\cdot, \cdot; \rho)$ is the standardized bivariate normal distribution function with correlation ρ , it follows that

$\hat{\sigma}_{R:OR}^2 - F_{BVN}(x_1, x_2; x_3) + \Phi(x_1)\Phi(x_2)$ is an increasing function of x_3 and hence x_3 can be easily determined numerically. Similarly, numerical solutions for x_4 , x_5 , x_6 and x_7 can be obtained.

- 3 Solve for x_4 , using the values for x_1 and x_2 obtained in Step 1:

$$x_4 = \left\{ x_4 : \hat{\sigma}_{T:OR}^2 + \hat{\sigma}_{R:OR}^2 - .5 \sum_{i=1}^2 \left\{ F_{BVN}(x_i, x_i; x_4) - [\Phi(x_i)]^2 \right\} = 0 \right\}$$

- 4 Solve for b using one of the following options.

Option 1: Solve for b , using the values for x_1 , x_2 and x_4 obtained in Steps 1 and 3:

$$b = \left\{ b : \hat{\sigma}_{\epsilon:OR}^2 - \frac{1}{2} \sum_{i=1}^2 \sum_{m=1}^4 c_m F_{BVN}(x_1, x_2; \rho_m(1 - x_4) + x_4) = 0 \right\}$$

$$\text{where } \rho_1 = 1, \rho_2 = \frac{1}{1 + b^{-2}}, \rho_3 = \frac{1}{1 + b^2}, \rho_4 = 0$$

Option 2: Specify a value of b . Then, using the specified value of b and the specified correlations $\hat{r}_1$, $\hat{r}_2$ and $\hat{r}_3$, compute a new specified value for the OR error variance of $\hat{\sigma}_{\epsilon:OR}$, given by

$$\hat{\sigma}_{\epsilon:OR}^2 = 0R = \frac{1}{2} \sum_{i=1}^2 \sum_{m=1}^4 c_m F_{BVN}(x_1, x_2; \rho_m(1 - x_4) + x_4), \text{ where } \rho_1 = 1, \rho_2 = \frac{1}{1 + b^{-2}}, \rho_3 = \frac{1}{1 + b^2},$$

$\rho_4 = 0$. Then compute new specified values of the error covariances:

$$\widehat{Cov}_i = \hat{r}_i \hat{\sigma}_i^2, i = 1, 2, 3.$$

- 5 Solve for x_5 , x_6 , and x_7 , using the following equations and the value for x_1 , x_2 , x_4 and b obtained in Steps 1, 3 and 4. If Option 2 was used in Step 4, then replace $\widehat{Cov}_i$ by $\widehat{Cov}_i$, $i = 1, 2, 3$, in the equations.

$$x_5 = \left\{ x_5 : \widehat{Cov}_3 - \sum_{m=1}^4 c_m F_{BVN}(x_1, x_2; \rho_m(1 - x_4)) = 0 \right\}$$

Where $\rho_1 = (1 + b^{-2})x_5$, $\rho_2 = \frac{x_5}{1 + b^{-2}}$, $\rho_3 = \frac{x_5}{1 + b^2}$, $\rho_4 = 0$

$$x_6 = \left\{ x_6: \widehat{\text{Cov}_2} - \frac{1}{2} \sum_{i=1}^2 \sum_{m=1}^4 c_m F_{\text{BVN}}(x_1, x_2; \rho_m(1 - x_4)) = 0 \right\}$$

where $\rho_1 = x_6$, $\rho_2 = \frac{x_6}{1 + b^{-2}}$, $\rho_3 = \frac{x_6}{1 + b^2}$, $\rho_4 = 0$

$$x_7 = \left\{ x_7: \widehat{\text{Cov}_1} - \sum_{m=1}^4 c_m F_{\text{BVN}}(x_1, x_2; \rho_m(1 - x_4) + x_3) = 0 \right\}$$

where $\rho_1 = x_7$, $\rho_2 = \frac{x_7}{1 + b^{-2}}$, $\rho_3 = \frac{x_7}{(1 + b^2)}$, $\rho_4 = 0$