ORIGINAL ARTICLE

A decision support system for demand and capacity modelling of an accident and emergency department

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ABSTRACT

Accident and emergency (A&E) departments in England have been struggling against severe capacity constraints. In addition, A&E demands have been increasing year on year. In this study, our aim was to develop a decision support system combining discrete event simulation and comparative forecasting techniques for the better management of the Princess Alexandra Hospital in England. We used the national hospital episodes statistics data-set including period April, 2009 – January, 2013. Two demand conditions are considered: the expected demand condition is based on A&E demands estimated by comparing forecasting methods, and the unexpected demand is based on the closure of a nearby A&E department due to budgeting constraints. We developed a discrete event simulation model to measure a number of key performance metrics. This paper presents a crucial study which will enable service managers and directors of hospitals to foresee their activities in future and form a strategic plan well in advance.

ARTICLE HISTORY

Received 28 September 2018
Accepted 15 December 2018

KEYWORDS

Demand and capacity modelling; discrete event simulation; forecasting; accident and emergency department; health care; decision support system

1. Introduction

Accident and emergency (A&E) units are the busiest departments within hospitals working under immense financial pressures resulting in shortage of clinicians, nurses, beds, and equipment. For the last decades, A&E departments in the United Kingdom (UK) have been struggling with issues related to increasing waiting times and length of stay, as well as lack of resources, which all have a negative impact on day-to-day functioning of A&E services. Increasing waiting times and length of stay have been observed and the 4-h target (the percentage of patients spending 4 h or more in hospital should be less than 5%) determined by the government has not been achieved since the financial year 2014–2015 (National Health Services England, 2014, 2017a).

The population has been increasing and ageing around the world, which causes increasing demands on hospitals (Hong & Ghani, 2006). The considerable increase (i.e., approximately 23.5% from 2006/2007 to 2016/2017 financial year) in the number of admissions has been observed in the UK A&E departments (National Health Services England, 2014, 2017a). In addition, the bed occupancy rates of hospitals in the UK from 2010/2011 to 2016/2017 financial year have presented an upward trend on occupied beds used overnight and day only, 4.87% and 13.51%, respectively (National Health Services England, 2017b).

Proportion of the younger population is decreasing compared to an increasing proportion of the elderly population. According to Cracknell (2010),

the 65 years and over age group in the UK was around 10 million (a one-sixth of the population) in 2010 and expected to reach 19 million by 2050, which is approximately a quarter of the population. Blunt (2014) mentioned in his report that the number of elderly people who visit A&Es in the UK is much higher than other age groups. In addition, he emphasised that most elderly patients spend 4 h or more, and thus hospitals are not able to achieve that 95% of patients are seen, treated and then admitted or discharged within 4 h in A&E, as the target set by the NHS Constitution.

The NHS employs 1.3 million staff in England and Wales, caring for approximately 1 million patients every 36 h, which is equivalent to around 243 million patients per year. This means NHS staff will continue to face challenges in terms of health and well-being due to severe patient demand and financial constraints (Royal College of Physicians, n.d.). Therefore, resources (e.g., staff, beds) may not be sufficient to meet demand for A&E, where doctors and nurses are sometimes forced to work flat out. Reducing the quality of hospital services may lead to loss of motivation in human resources, not to mention the negative effect it might have on service satisfaction for patients. In addition, the NHS has come up against financial constraints and it needs to generate £20 billion (equal to approximately 4% productivity annually) of net savings in the next few years (Hamm, 2010). Taking into account limited capacity (i.e., bed, staff) and financial constraints, as well as increasing patient arrivals, it is clear that A&E departments will continue to struggle (i.e., longer waiting

times) to use their resources efficiently. Due to increasing demand, hospital administrations will need to provide higher productivity rates by enhancing the match of demand and capacity of A&Es. Therefore, key decision-makers would need to model the level of resources needed by patients in A&E as a function of demand factors with a range of supply issues, thus it is crucial to understand patient pathway in order to demonstrate the full impact of change.

In this study, the objective is to develop a demand and capacity model for an A&E department by combining the methods of quantitative forecasting and discrete event simulation techniques. Using the English Hospital Episodes Statistics (HES) data-set, we forecasted daily A&E demand by comparing four forecasting methods and selected the best model according to the forecast accuracy measure. The forecasted demands are then inputted into the simulation model under the expected demand condition. In addition, we have also considered unexpected demand conditions as requested by the directors of the hospital, and examined the impact of the closure of a nearby A&E department at another hospital. We obtained the unexpected demand by increasing the expected demand by various rates. Capacity of the A&E has been investigated through the simulation model for future years. We have taken many inputs into account including demographic features (age groups, gender), staff shifts, number of resources (doctors, nurses, beds, triage rooms, and clinic rooms), salary of human resources, cost of treatment, distributions (investigation for treatment (severity of injuries), waiting time for treatment, and overall waiting time), and laboratory tests. We established distributions based on age groups, so that the related times vary; hence, a more robust model could be built. In addition, we tested several “what-if” scenarios in order to observe how performance metrics are changed. Thus, many outputs have been computed under expected and unexpected demand conditions: capacity (number of patients discharged), utilisation rates of doctors, nurses and beds, demand coverage ratio (DCR), financial implications, and many more.

The first contribution to knowledge is the development of a decision support system (DSS) combining discrete event simulation and comparative forecasting in modelling demand and capacity. To our knowledge, the literature does not contain such an extensive study which has successfully combined these two approaches. Therefore, we generate A&E demand using forecasting techniques, including the seasonal and trend decomposition using loess function (STLF) method, which has not been applied within the health care context. The objective is to enable service managers to better understand future demand and act accordingly to prevent issues related to system performances and capacity. We then take into account the request from the hospital management to

evaluate possible demand increases in the case of the closure of an A&E department at a nearby hospital. Thus, we model unexpected demand conditions by increasing the expected demand by various rates determined in case studies. As a result, service managers will be prepared against possible increasing demand. If they project that demand would increase in future years according to the results of this study, they might need to increase staffing level (i.e., additional staff). Therefore, they will prevent increasing staff utilisation rates and staff will continue to work without severe workloads.

Almost all of the discrete event simulation-oriented research papers do not provide further details in relation to the practical aspects of simulation modelling, for example the validation process, how to determine the warm-up period, calculating the optimal number of replications (i.e., trials), etc. We therefore provide a step by step guide to modelling A&E and thus an opportunity for researchers, practitioners, and analysts to replicate our study within their setting.

Section 2 reviews the literature on forecasting and discrete even simulation; Section 3 presents a flow diagram for the step by step guide. Section 4 shows how A&E demand is forecasted. Section 5 illustrates the conceptualised patient pathway, develops the model, and showcases the validation stage in greater detail. Sections 6 and 7 discuss results and present the conclusion, respectively.

2. Literature review

2.1. Forecasting A&E demand

Many studies have been conducted using time series analysis to forecast patient demand (see Table 1). Batal, Tench, McMillan, Adams, and Mehler (2001), who estimated demand for an urgent care clinic, used stepwise linear regression model in order to optimise staffing levels for patient demand. Champion et al. (2007) compared two forecasting techniques to estimate future admissions. Jones et al. (2008) used regression models including climate variables to compare a number of forecasting methods to estimate A&E demand. Sun, Heng, Seow, and Seow (2009) forecasted daily admissions to A&E by autoregressive integrated moving average (ARIMA) and generalised linear model, including weather variables for planning resources and staff. Kam, Sung, and Park (2010) used a variety of ARIMA techniques (seasonal ARIMA (SARIMA) and multivariate SARIMA) and compared them with moving averages to calculate daily demand. Boutsioli (2010) carried out a study on forecasting A&E demand of 10 hospitals in Greece using a time series method and determined the amount of unforeseen admissions using the residuals generated by the regression model. In another study,

Table 1. A literature review on forecasting hospital demands using time series analysis, ARMA: autoregressive moving average, ARIMA: autoregressive integrated moving average.

Author/s (Year)	Study type	Method/s used, best method (*)	Independent variables
Current study	Daily	ARIMA Exponential smoothing Stepwise linear regression (*) STLF	Days of week, month of year, a day before a holiday, holiday, a day after a holiday
Aboagye-Sarfo et al. (2015)	Monthly	ARMA Vector-ARMA (*) Exponential smoothing	Time <i>Dependent variables:</i> age group, place of treatment, triage category, disposition
Bergs et al. (2013)	Monthly	Exponential smoothing	-
Boutsioli (2013)	Daily	ARMA Multiple linear regression	Weekends, summer holidays, official holidays, duty
Marcilio et al. (2013)	Daily	Generalised estimating equation (*) Generalised linear model (*) Seasonal ARIMA	Days, months, public holidays, after and before days of a holiday, temperature
Kam et al. (2010)	Daily	Moving average Seasonal ARIMA Multivariate seasonal ARIMA (*)	Days, months, quarters of years, seasons, weather factors, daily temperature, holidays, near-holidays
Boutsioli (2010)	Daily	Multivariate regression model	Weekends, summer holidays, official holidays, duty
Sun et al. (2009)	Daily	ARIMA (*) General linear model	Days, months, public holidays, weather factors
Jones et al. (2008)	Daily	Artificial neural network Exponential smoothing Seasonal ARIMA Time series regression (TSR) (*) Time series regression with climate variables (TSRCV)	Days, months, holiday, near-holiday, interaction terms (for TSR), in addition to these daily min-max temperature, daily precipitation (for TSRCV)
Champion et al. (2007)	Monthly	ARIMA Single exponential smoothing (*)	-
Batal et al. (2001)	Daily	Stepwise linear regression	Days, months, seasons, holidays, after and before days of a holiday

Boutsioli (2013) investigated the unpredictable hospital demand variations by using two types of forecast errors (firstly, only positive errors and secondly, both positive and negative forecast errors). Marcilio, Hajat, and Gouveia (2013) found generalised estimating equation and generalised linear model as successful methods against SARIMA. On the other hand, Aboagye-Sarfo et al. (2015) used a new technique (Vector-ARMA) to compare with others on estimating A&E demand.

Table 1 gives detailed information of the literature related to the forecasting hospital demand. We have drawn on the literature to select forecasting methods to be used in the study. We have used three forecasting methods (ARIMA, exponential smoothing (ES), and multiple linear regression) since they have been widely used and recommended as the best methods in the literature. One of the contributions of this study is the use of the STLF method; we tried this method because the hospital data contain both trend and seasonal components. In the study, we include a section comparing the performance of forecasting methods. Most importantly, as shown in Table 3, the STLF method has better forecast accuracy than ARIMA and ES methods which have been widely used in the literature. The STLF is a different forecasting approach which has not previously been applied to forecast demand for A&E. According to Hyndman and Athanasopoulos (2014, p. 163), the STL method is a reliable technique to separate time series data-sets into seasons and trends. This method is explained in Section 4.

2.2. Discrete event simulation modelling

Simulation is an approach which allows characteristic features of any system to be built into a computer environment and for experiments to be conducted (Pidd, 2004, pp. 3–4). Simulation gives useful results to users. Some of its advantages, according to Banks et al. (2005, p. 6), are as follows: firstly, operations of the system can be better understood. Secondly, what-if analyses can be tested without interrupting the system. Finally, blockages can be determined by analysing the system. In addition, Pidd (2004, pp. 9–10) states that simulation is cheaper than real experiments and simulation methods can simulate systems for long periods such as months, or years in a short time and simulation is replicable, therefore an average value can be obtained by rerunning simulation models many times.

As can be seen from the literature review, health care services are systems where simulation techniques have been carried out extensively. This situation is confirmed by Pidd (2004, p. 5) who stresses that simulation in an appropriate implementation allows the restricted resources of hospitals to be effectively used in health care services.

One of the most widely used application areas of simulation methods is the A&E department as seen in the literature review study conducted by Gul and Guneri (2015). System analysis and development is crucial for this kind of department where limited resources are used and emergency medical interventions are necessary. In addition, most studies have examined current

performances of A&Es by means of triage systems which classify patients according to their urgency. Existing versus redesigned triage systems have been compared by a number of researchers (Connelly & Bair, 2004; Gunal & Pidd, 2006; Medeiros, Swenson, & DeFlicht, 2008; Ruohonen & Teittinen, 2006). On the other hand, some studies focus on classifying and prioritising patients, for instance Ozdagoglu, Yalcinkaya, and Ozdagoglu (2009) and Virtue, Kelly, and Chaussalet (2011). A number of studies in the literature have developed systems of A&Es by means of scenarios. Alternative scenarios are generated and compared by measuring the performances of A&Es, for example Komashie and Mousavi (2005), Duguay and Chetouane (2007), Meng and Spedding (2008), Gul, Celik, Guneri, and Taskin Gumus (2012), Wang, Li, Tussey, and Ross (2012), Ahmad, Ghani, Kamil, Tahar, and Teo (2012), Gul and Guneri (2012), Al-Refaie, Fouad, Li, and Shurrab (2014), and Oh et al. (2016).

A&Es have been exhaustively investigated by many researchers around the world, with the aim of assisting key decision-makers to find the most effective and efficient way of running their service. For instance, redesigned triage systems, tackled by means of what-if scenarios and prioritised patients according to their health status. These studies have a number of limitations, firstly the number of staff in each shift are generally assumed to be fixed, and secondly, the lack of availability of real data to capture reality within A&E. In some cases, data are obtained through observations, while others are able to access limited data-sets, and thus without real data no simulation model can be deemed to be accurate, robust, or reliable. Table 2 compares the current study with previous studies related to A&E departments.

Simulation modelling has been developed as an alternative solution method in different departments of hospitals. Within this framework, inpatient and outpatient departments have been considered as study areas. VanBerkel and Blake (2007) examined a general surgery's practice in order to reduce waiting times and operation room times, and according to their findings long waiting times were associated with the number of beds. In this study, it is suggested that alternative scenarios must be combined to decrease patient waiting times. Rohleder, Lewkonja, Bischak, Duffy, and Hendijani (2011) measured performance of an outpatient pathway at an orthopaedic department. A combination of optimum number of staff, patient schedules, and staff punctuality was tested. As a result, significant reductions in waiting times and total patient times were found. Zhu, Hen, and Teow (2012) analysed how two growth rates in demand changed the optimum bed numbers in an intensive care unit. Demir, Gunal, and Southern (2017) developed a decision support tool to better understand future key performance metrics of 10 specialities of

Table 2. Comparison of studies related to accident and emergency (A&E) department, NG: not given.

Author/s and years	Arrival process	Data	Examination of different demand conditions	Waiting time for treatment based on age group	Treatment time based on age group	Overall waiting time based on age group	Warm-up period	Replication number	Shift	Software
Current study	Stochastic	46 months	✓ - by forecasting	✓	✓	✓	2 months	10	✓	Simul8
Oh et al. (2016)	Deterministic	5 months	X	X	X	X	2 days	5	✓	Arena
Al-Refaie et al. (2014)	Stochastic	NG	X	X	X	X	NG	10	X	NG
Wang et al. (2012)	Deterministic	1 month	✓ - by presumptive	X	X	X	NG	NG	✓	Simul8
Gul et al. (2012)	Stochastic	NG	X	X	X	X	NG	NG	✓	ServiceModel
Virtue et al. (2011)	Deterministic	12 months	X	X	X	X	24 hours	50	X	Simul8
Ozdogoglu et al. (2009)	Stochastic	33 days	X	X	✓	✓	3 days	10	X	Arena
Medeiros et al. (2008)	NG	1 month	X	X	X	X	NG	30	X	Arena
Meng and Spedding (2008)	Stochastic	1 month	X	X	X	X	NG	NG	X	MedModel
Duguay and Chetouane (2007)	Stochastic	90 days	X	X	X	X	NG	10	✓	Arena
Gunal and Pidd (2006)	Stochastic	2 months	X	X	X	X	X	50	X	Micro Saint Sharp
Ruohonen and Teittinen (2006)	Stochastic	2 weeks	X	X	X	X	NG	NG	✓	MedModel
Komashie and Mousavi (2005)	Stochastic	NG	X	X	X	X	NG	NG	X	Arena

Table 3. Forecast accuracy (MASE) values of this study. ARIMA: autoregressive integrated moving average, ES: exponential smoothing, STL: the function of the seasonal and trend decomposition using loess.

Forecasting methods	Forecasting models	Forecast accuracy (MASE)	
		Training set	Validation set
ARIMA	(2, 0, 4)	0.7357	0.9984
ES	ETS (M, N, N)	0.7671	0.9977
Multiple linear regression	Stepwise linear regression	0.7998	0.8651
STLF	STL + ETS (A, N, N)	0.6945	0.9781

a hospital. Hospital demand was estimated for the next 6 years by assuming population growth rates of the catchment area which the hospital serves.

Bed capacity issues of health care services are directly proportional to patient demands, making it difficult for health care planners to manage services. Therefore, service managers are forced to take precautions, such as the reallocation of beds, building new departments with an increased capacity. Vasilakis and El-Darzi (2001) analysed the crises coming in sight during winter seasons and revealed the available bed capacity “before crisis” and “during crisis”. Cochran and Bharti (2006) reallocated beds at an obstetrics hospital and increased the bed capacity by a small rate to enable more patients to be admitted. Levin et al. (2008) found that determining the optimal capacity of cardiology enables a reduction in admission times of A&E.

The contribution of this study to the field of simulation modelling in health care systems is as follows: (1) we develop a DSS which combines discrete event simulation technique and comparative forecasting method to specify demand and capacity of a health care department (A&E). To determine scientifically the A&E demand for expected demand conditions, we compare four forecasting methods and select the best model instead of relying on a single forecasting method. (2) In comparison with existing studies, this study provides a step by step guide presented in Section 3 to simulating an A&E department, explaining all steps in greater detail, including the model validation stage, warm-up period, and the optimum replication number. In the majority of instances, researchers, practitioners, and analysts find it difficult to replicate a study, hence our objective was to provide all the details to ensure our model can be replicated in other settings.

3. The decision support system

In this study, we develop a DSS combining comparative forecasting techniques and discrete event simulation for demand and capacity planning in an A&E department. For this, the projected demand is obtained from forecasting techniques instead of using presumptive demand to embed it as input in the simulation model. A step by step guide is

presented as a flow diagram illustrating how two techniques are combined in Figure 1. We extracted all required A&E data from the “big data” corresponding to the hospital of interest, i.e., 248,910 A&E arrivals (with 86 variables) over the period of the study. The required data were used in both demand forecasting and parameter estimation of the statistical distributions for the simulation model. These inputs along with model parameters, financial inputs, and local data provided by the hospital were embedded into our A&E simulation model. The model then generated future levels of key output metrics (i.e., capacity, DCR, bed occupancy rate, utilisation rates of doctors and nurses, total revenue, and surplus). All steps mentioned in the flow diagram are explained in Sections 4 and 5 in greater detail.

4. Forecasting A&E demand

Daily demand of the A&E department is predicted by using quantitative forecasting methods since patient admissions are used as an input to the simulation model. This study has been carried out in the A&E department of the Princess Alexandra Hospital (PAH) working 24/7 in England. In this study, 46 months of data were used for the period April, 2009–January, 2013 and the data were extracted from the national HES. The data were divided into two: the training set (April, 2009–January, 2012) and the validation set (February, 2012–January, 2013).

Many forecasting methods have been compared in A&E demand forecasting in the literature. As seen in Table 1, the ARIMA, ES, and multiple linear regression have been widely used. On the other hand, Hyndman and Athanasopoulos (2014, p. 163) mention that the STL method is a reliable decomposition technique to separate the time series data-sets into seasons and trends. Therefore, the STL method may be effective at forecasting. Thus, we have compared the method with three other methods.

The ARIMA method is a forecasting technique which has been widely used and generates forecasts by means of autocorrelations in the time series (Hyndman & Athanasopoulos, 2014, p. 213). The ARIMA method has three parameters (p , d , and q) where p denotes the order of autoregression, d is the order of differencing, and q is the order of the

Figure 1. The structure of the decision support system.

moving average (DeLurgio, 1998, p. 270). ES is one of the most widely used forecasting methods. A feature is that “the ES implies exponentially decreasing weights as the observations get older” (Makridakis, Wheelwright and Hyndman, 1998, p. 140). Multiple linear regression seeks a relationship between independent (explanatory) variables and a dependent variable. In other words, one variable is forecasted using two or more independent variables in the multiple linear regression (Makridakis et al., 1998, p. 241). Stepwise linear regression, which is one of the multiple linear regression methods, selects the explanatory variables relevant to the dependent variable from the initial model including all explanatory variables. In this study, the stepwise linear regression involves the use of dummy variables. For example, the stepwise linear regression model for the daily estimation includes days-of-week, months of year, and variables related to UK public holidays (a holiday, a day before a holiday and a day after a holiday). The STLF method converts data to seasonal data using the STL decomposition. A non-seasonal

forecasting technique is used to get the estimated values. The estimated values are then re-seasonalised by using the “the last year of the seasonal component” (Hyndman, O’Hara-Wild, Bergmeir, Razbash, & Wang, 2016). In this study, the following functions in R are applied in order to select the best ARIMA, ES, the STLF methods, and stepwise linear regression, respectively: the `auto.arima()`, the `ets()`, the `stlf()` functions (Hyndman & Khandakar, 2008), and the `stepAIC()` functions (Ripley et al., 2016).

4.1. Choosing the best forecasting method

In this study, an A&E demand for projection is obtained from forecasting techniques instead of using presumptive demand to embed it as input in simulation model. Therefore, forecasting and simulation is combined for the development of the DSS in demand and capacity modelling. Thus, four forecasting methods are used: ARIMA, ES, stepwise linear regression, and STLF. Using these methods, the daily A&E demand is estimated. At this point, the

important issue is to select the best forecasting method. A number of metrics are available for this purpose. Gneiting (2011) reviewed the surveys on this matter and found that the measure most widely used in organisations is the mean absolute percentage error (MAPE). Unfortunately, it is not widely known that MAPE is a biased measure: it does not treat positive and negative errors symmetrically and consequently selects methods whose forecasts tend to be too low. The mechanism by which this occurs is explained in Tofallis (2015). We have chosen to use the mean absolute scaled error (MASE) method which also has the advantage that if zero occurs in the observations, MASE avoids the infinities which occur with MAPE (Hyndman & Koehler, 2006). MASE is based on a simple quantity that managers can comprehend, namely the average prediction error (irrespective of sign). MASE is a ratio which compares this with the corresponding value from using the naïve forecasting method as a benchmark. In the MASE, the numerator is the mean absolute error of the forecasting method and the denominator is the mean absolute error of the naïve method, i.e., when the forecast is the previous observation. The denominator is therefore the same for all methods studied. Hence, the MASE compares the errors with those from the naïve method:

$$q_t = \frac{e_t}{\frac{1}{n-1} \sum_{i=2}^n |Y_i - Y_{i-1}|} \quad (1)$$

$$\text{MASE} = \text{mean}(|q_t|) \quad (2)$$

where q_t represents a scaled error, e_t is an error term, and y_i denotes the observation at time i (Hyndman & Koehler, 2006).

According to Table 3, the stepwise linear regression is the best method judging by the lowest MASE value with 0.8651. As a result, this means that the daily A&E demand will be forecasted using the stepwise linear regression method.

One of the important issues in forecasting is to validate the forecasts. We use a paired t -test (see Equation (4) for the formula) for validation of forecasts and compare the actual data and forecasted demand from the regression model for the validation set period (February, 2012–January, 2013) in forecasting process. Table 4 shows that the forecasted demand is validated at 99% confidence interval.

In order to estimate the distribution of interarrival times to be used as input in the simulation model, daily A&E demand is forecasted by using the developed stepwise linear regression model for the period February,

2013–January, 2014. The distributions related to patient arrivals are explained in Section 5.3.

5. Discrete event simulation modelling

In our study, patient arrivals, investigation for treatment (severity of injuries) waiting time for treatment, treatment time, and overall waiting time are probabilistic and thus, statistical distributions are considered. In addition, patient arrivals and processes of the hospital are discrete and have discrete time intervals. Moreover, Gunal (2012) states that DES is a successful technique in modelling systems which have queuing processes. Furthermore, Agent-Based Simulation (ABS) is a newer simulation approach, whereas DES has appeared extensively in the literature and is widely accepted and utilised for decision-making purposes by health care organisations in the UK, including the NHS and “The National Institute for Health and Care Excellence”, which has recognised DES as a valid way of simulating complex patient pathways (Davis, Stevenson, Tappenden, & Wailoo, 2014). In the light of these reasons, DES method is applied and Simul8 software is used in our study.

5.1. Data

The data used in the simulation model are obtained in two ways: firstly, the following are derived using the national HES data-set covering period April, 2009–January, 2013: patient arrival date and time, demographic features, treatment time, conclusion time, laboratory tests, and discharge destination. The local data were provided by the hospital, that is, the number doctors, nurses, beds, triage room, etc. In addition, all input parameters and their references are given in Appendix 1.

5.2. Conceptualisation of the A&E department

To develop a discrete event simulation model, it is required that elements of the system are specified and their relationships among each other are mapped out (Pidd, 2004, pp. 35–36). This means that firstly, a hospital should be conceptualised and after that, a simulation model should be developed.

The conceptualisation stage is required to understand the system better and build a simulation model correctly. In this study, the A&E is conceptualised in high level and presented in Figure 2. The conceptualised A&E model is validated in collaboration with directors of the hospital (i.e., clinical directors, director of finance,

Table 4. Validation of the forecasted demand.

Parameter	t Test value	t Critical value	Average number of patients (monthly)	99% Confidence interval
Forecasted demand	2.25	3.11	6781	(6358, 7204)

turnaround director) and consultants in the hospital. In this pathway, four different patient arrivals are shown: patients can be referred from General Practitioners (GP)s, self-admission, by ambulance, or referral from educational establishments and general dental practitioner. Patients are registered and pre-assessment process (triage process) is carried out by a nurse. Patients then wait to be seen by a doctor. Doctors may request further investigations, such as X-ray, urinalysis, biochemistry, and so on. Depending on patient's condition, they can either be admitted to inpatient care, discharged back to primary care; discharged to an outpatient department, discharged by death, or discharged home with no further action.

5.3. Inputs–outputs

In this study, inputs and outputs are shown in Figure 3. We used five types of inputs: patient input (patient demand by forecasting), physical inputs (beds, triage, and clinic rooms), staff inputs (doctors, nurses), financial inputs (Healthcare Research Group (HRG) tariff, payments to doctors and nurses indicated in NHS Staff Earnings Publications), and other inputs (distributions, all laboratory tests, shifts, demographic features, such as age groups and gender). HRG is an indicator which classifies similar clinic “conditions” or “treatments” in

terms of level of resources used in health care systems (NHS England, 2017). In this study, reference costs based on HRG (NHS Digital, n.d.) are used to estimate average revenue of the A&E. Appendix 1 shows all input parameters, estimates, distributions, and references.

All laboratory tests (X-ray, electrocardiogram, haematology, biochemistry, urinalysis, and others) in the A&E department are taken into account. Number of resources provided by the hospital are used as inputs in the simulation model (see Appendix 1).

Two age groups (i.e., 20–40, 80+) are associated with waiting times for treatment in A&E departments compared against age group 40–60. A 10% demand increase by younger group means a 0.49% increase on A&E waiting times. In addition to this, same increase on demand by elderly group causes a 1% decrease on the performances (Monitor, 2015). A 1% increase may seem like a small effect but when 100's of patients is considered (per day) this can make a huge difference in terms of efficiency and effective running of A&E department. In addition, we took into account age groups for waiting time for discharge. According to our econometric analysis, the average waiting time based on age groups is clearly different, for example, 62 min for age group 1; 71 min for age group 2; 86 min for age group 3; 101 min for age group 4; and 131 min for age group 5. We

Figure 2. High level conceptualisation of the accident and emergency department at the Princess Alexandra Hospital in England.

Figure 3. Inputs and outputs, HRG is Healthcare Resource Group.

therefore established the distributions based on age groups because the relevant times vary according to age group. Distributions for “waiting time for treatment” and “waiting time for discharge” are computed. Appendix 2 illustrates values of goodness of fit (i.e., Kolmogorov–Smirnov and Anderson–Darling) for 18 different distributions of “waiting time for treatment” for each age group. The best fitting distributions for each age group are selected judging by the lowest goodness of fit value, which are highlighted in bold and their parameter values are stated in Appendix 2. Probability density function graphs for the best fitting distributions of “waiting time for treatment” for each age group are given in Appendix 3.

Appendix 4 illustrates values of goodness of fit (i.e., Kolmogorov–Smirnov and Anderson–Darling) for 18 different distributions of “waiting time for discharge” (by each age group). The best fitting distributions for each age group are selected judging by the lowest goodness of fit value, which are highlighted in bold and their parameter values are stated in Appendix 4. Probability density function graphs for the best fitting distributions of “waiting time for discharge” (by each age group) are given in Appendix 5.

We established the observed frequency distributions for various group patients depending on the severity of their injuries (investigation for treatment), such as waiting time to be seen by a doctor, waiting time for discharge, treatment time, and cost of treatment. According to the HES data-set, there are eight HRG codes for the PAH (i.e., from “VB01Z” to “VB08Z”). These are used for classifying the investigation for treatment. These observed frequency distributions are established to assign individual patients according to the severity of their injuries (investigation for treatment). The reference costs are therefore based on severity of injuries for A&E patients. To give an example, different tariffs are applied based on HRG code (i.e., £237 for VB01Z whereas £110

for VB08Z) as shown in Appendix 1. In conclusion, HRG code is independent from age groups, where all patients are assigned the same HRG code depending on the severity of their condition. These risk adjustments enable us to better capture detailed treatment processes within A&E, financial implications, impact on resources, etc.

We calculate daily average interarrival times of the A&E by dividing total time of a day by daily demand estimated by the stepwise linear regression model. The rational is that there are significant variations on daily A&E interarrival times (as seen in raw data), thus using daily averages of the interarrival times is more realistic than using monthly averages. Therefore, we generate all monthly distributions of the interarrival times based on days-of-weeks pattern by using EasyFit software for each case study. The EasyFit software selects the best distribution according to goodness of fit (i.e., Kolmogorov–Smirnov and Anderson–Darling) (Mathwave Technologies, n.d.). Table 5 gives the monthly distributions of patient interarrival times used in this study. For example, as seen from Table 5, patients arrive to the A&E in accordance with the Poisson distribution ($\lambda = 6.2667$) for the period (April, 2013) whereas they arrive to the A&E according to the geometric distribution ($p = 0.13478$) for the period (March, 2013) in Case 1.

As seen from Figure 3, we obtain four kinds of outputs from this study: patient outputs (capacity), physical outputs (bed utilisation rates, DCR), staff outputs (staff utilisation rates), and financial outputs (average revenue, cost, and surplus). Outputs are obtained quarterly and annually. We developed an output metric: DCR. Therefore, we can measure the percentage of patients admitted to an A&E and discharged with available resources. Its formula is shown in Equation (3). This output shows the A&E’s ability to meet demand. For example, 100% DCR means that all patient demands are met with the available

resources, whereas DCR would be less than 100% depending on the number of patients who are not discharged from A&E.

$$\text{DCR} = \frac{\text{Number of patients who are discharged}}{\text{Number of patients who are admitted to the A\&E}} \quad (3)$$

Our financial outputs are associated with NHS Staff Earnings Publications by applying payments to doctors and nurses determined in NHS Staff Earnings Publications (NHS Digital, 2013; 2014) when calculating average cost of treatment. On the other hand, NHS reference costs (Department of Health, 2013; 2014) are considered as revenue to estimate average revenue of the A&E department.

5.4. Simulation model

The conceptualisation stage enables us to better understand the system prior to developing the simulation model. As presented in Figure 4, the A&E simulation model is modelled using Simul8 simulation software. The “AandE Arrival” entry point is made up of four arrival modes (i.e., GP referral, self-referral, emergency, and other) as shown in Figure 2. Patients arrive at A&E according to the distribution of the interarrival times specified in Table 5. Patients are labelled in terms of age group and gender according to their statistical distributions. Patients wait for pre-assessment which is normally carried out by a nurse and a label related to severity of injuries is assigned to patients for triage process. Patients are then asked to further wait to be seen by an A&E doctor according to a waiting time distribution as indicated in Appendix 2. In the “AandE Treatment” work centre, if a doctor wants a further investigation, patients are referred to the laboratory area, such as X-Ray, electrocardiogram, and so on. An investigation bundle is assigned to each patient according to the distribution obtained from data. For example, if a patient has investigation 1 (X-Ray) and investigation 2 (electrocardiogram), the patient visits firstly X-Ray area and then takes an electrocardiogram test. Patients are then further assessed by the A&E doctor and relevant treatment is decided. After that, patients are prepared to be discharged by “AandE Discharge Preparation”. Then, patients are discharged based on health care provider’s decision by “AandE Discharged” using five discharge modes as shown in Figure 2 (i.e., they can either be admitted to inpatient care, discharged back to primary care; discharged to an outpatient department, discharged by death, or discharged home with no further action). In this model, there are four distinct types in relation to waiting times: (1) waiting for pre-assessment (triage), (2) waiting time for treatment (by clinician), (3) waiting time for discharge (post-treatment), and (4)

overall waiting time, i.e., from arrival to discharge. Relevant distributions have been established for (1), (2), and (3) whereas (4) is an output. In the data collection period of the model, overall waiting time (4) is obtained by adding (1), (2), and (3).

5.5. Verification and validation

The simulation model is verified by a number of directors in the hospital. The model is run for the period February, 2012–January, 2013 and the simulation results (number of admission, waiting time for treatment, and overall waiting time) are obtained for validating the model. We have compared these simulation results and actual values by using a paired *t*-test which is determined as a formula in Equation (4):

$$t_0 = \frac{\bar{d} - \mu_d}{S_d / \sqrt{K}} \quad (4)$$

where $\bar{d}$ denotes average observed differences between actual values and simulation result, μ_d is mean difference, S_d denotes the standard deviation, and K is the number of input data-set (Banks et al., 2005, p. 377). As a result, the model is validated since *t*-test values ($|t_0|$) are less than or equal to *t* critical values ($t_{\alpha/2, K-1}$) at 95% significance level. Table 6 presents the results of the validation test.

5.6. Determination of replication number and warm-up period

Using fixed-sample-size procedure, we calculate the optimum replication number for the simulation model. Equation (5) presents formula for fixed-sample-size procedure:

$$n_y^*(\gamma) = \min \left\{ i \geq n; \frac{t_{i-1, 1-\alpha/2} \sqrt{S^2(n)/i}}{|\bar{X}(n)|} \right\} \leq \gamma' \quad (5)$$

where n is initial replication number, i is required replication number, S is standard deviation, γ' is “adjusted” relative error, and $\bar{X}$ is average estimates of key parameter (Law & Kelton, 2000, p. 513). It is recommended that $\gamma \leq 0.15$ and at $n_0 \leq 10$ (Law & Kelton, 2000, p. 515). Minimum value of replication number is chosen as optimum replication number if $n_y^*(\gamma)$ is less than or equal to γ' (Law & Kelton, 2000, p. 513). In this study, initial replication number is determined as 10 and we calculate $n_y^*(\gamma)$ is less than or equal to γ' for the key performance metrics (i.e., average waiting time and average length of stay). As a result, we use the optimum replication number as 10 replications in our simulation model.

Welch’s method is a widely used technique for determining the length of the warm-up period. This

Table 5. Monthly distributions of interarrival times based on days-of-weeks pattern.

Simulation's period	Distributions and parameters					
	Case 1 (base model)	Case 2 (5% increase)	Case 3 (10% increase)	Case 4 (15% increase)	Case 5 (20% increase)	Case 6 (25% increase)
Warm-up period			Poisson ($\lambda = 5.9355$)			
Data collection period	December 2012– January 2013					
	February 2013	Poisson ($\lambda = 6.2857$)	Binomial ($n = 6, p = 0.96753$)	Poisson ($\lambda = 5.8571$)	Geometric ($p = 0.15556$)	Poisson ($\lambda = 5.2857$)
	March 2013	Geometric ($p = 0.13478$)	Poisson ($\lambda = 5.9677$)	Geometric ($p = 0.15271$)	Poisson ($\lambda = 5.3871$)	Poisson ($\lambda = 5.3871$)
	April 2013	Poisson ($\lambda = 6.2667$)	Poisson ($\lambda = 5.7000$)	Geometric ($p = 0.15957$)	Poisson ($\lambda = 5.2667$)	Binomial ($n = 5, p = 0.97749$)
	May 2013	Poisson ($\lambda = 6.3548$)	Poisson ($\lambda = 5.8065$)	Geometric ($p = 0.15736$)	Poisson ($\lambda = 5.3548$)	Poisson ($\lambda = 5.1935$)
	June 2013	Poisson ($\lambda = 6.3000$)	Poisson ($\lambda = 5.7000$)	Geometric ($p = 0.15873$)	Poisson ($\lambda = 5.3000$)	Binomial ($n = 5, p = 0.96038$)
	July 2013	Geometric ($p = 0.13778$)	Poisson ($\lambda = 5.8387$)	Geometric ($p = 0.15578$)	Geometric ($p = 0.15979$)	Poisson ($\lambda = 5.2581$)
	August 2013	Poisson ($\lambda = 6.8065$)	Poisson ($\lambda = 6.3548$)	Poisson ($\lambda = 5.9677$)	Geometric ($p = 0.15423$)	Poisson ($\lambda = 5.3548$)
	September 2013	Poisson ($\lambda = 6.2667$)	Poisson ($\lambda = 5.6667$)	Geometric ($p = 0.15957$)	Poisson ($\lambda = 5.2667$)	Binomial ($n = 5, p = 0.96287$)
	October 2013	Geometric ($p = 0.13778$)	Poisson ($\lambda = 5.8710$)	Geometric ($p = 0.15578$)	Geometric ($p = 0.15979$)	Poisson ($\lambda = 5.2581$)
	November 2013	Geometric ($p = 0.13636$)	Poisson ($\lambda = 5.9000$)	Geometric ($p = 0.15464$)	Poisson ($\lambda = 5.3333$)	Poisson ($\lambda = 5.3333$)
	December 2013	Geometric ($p = 0.13537$)	Poisson ($\lambda = 5.9032$)	Geometric ($p = 0.15271$)	Poisson ($\lambda = 5.3548$)	Poisson ($\lambda = 5.3548$)
	January 2014	Poisson ($\lambda = 7.0323$)	Poisson ($\lambda = 6.2903$)	Poisson ($\lambda = 6.1613$)	Poisson ($\lambda = 5.8710$)	Geometric ($p = 0.15897$)

Figure 4. The structure of the A&E simulation model.

Table 6. The results of validation tests.

Parameters	<i>t</i> Test value	<i>t</i> Critical value	Average value (monthly)	95% Confidence intervals
Number of admissions	1.49	2.20	7052	(6959, 7144)
Waiting time for treatment	2.02		64.21	(63.76, 64.67)
Overall waiting time	1.15		153.61	(152.93, 154.29)

method determines warm-up period through four steps: (1) simulation is run n replication times. (2) For each observation, all replication values ($\bar{Y}_i$) of a key performance metric (i.e., waiting time) are averaged. (3) Moving averages of $\bar{Y}_i(w)$ by using formula in Equation (6):

$$\bar{Y}_i(w) = \begin{cases} \frac{\sum_{s=-w}^w \bar{Y}_{i+s}}{2w+1} & \text{if } i = w+1, \dots, m-w \\ \frac{\sum_{s=-(i-1)}^w \bar{Y}_{i+s}}{2i-1} & \text{if } i = 1, \dots, w \end{cases} \quad (6)$$

Equation (4) graphs of moving averages of $\bar{Y}_i(w)$ are obtained for each key performance metric. Then, the point where moving averages are smoothed is selected (Law & Kelton, 2000, pp. 520–521).

In this study, the warm-up period is investigated for key performance metrics (i.e., waiting time for treatment and overall waiting time). In the simulation model, the warm-up period consists of two months: December, 2011 (31 days) and January, 2012 (31 days) and totally the warm-up period is 62.

5.7. Case study

We have compared four forecasting methods and selected the one giving the best forecast accuracy measure. By using the forecasting method selected, we estimate daily A&E demand and compute monthly patient interarrival times to embed in the simulation model as input. In this study, six case studies are developed as given in Table 7. Case 1 (base model) consists of only A&E demand obtained from the stepwise linear regression model. Capacity for Case 1 is modelled and the

simulation model including warm-up period is run 10 times (replication is 10 according to the fixed-sample-size procedure). Therefore, Case 1 is investigated under expected demand conditions since forecasting provides the foreseen demand of the A&E department. Following the request of the management of hospital, we also examine how the balance of demand and capacity is affected in case the nearby hospital is closed. In this situation, more patients than expected will visit the A&E department. Thus, we examine these possible increases under unexpected demand conditions. Case studies covering Case 2–Case 6 are developed based on the base model (Case 1). For example, the A&E demand in Case 2 is 5% higher than in Case 1. Five different increases in demand levels are taken into account to observe possible effects on the A&E's performance.

In addition, we generate “what-if” scenarios by considering the bottlenecks in the A&E department. In this regard, we develop six scenarios (see Table 8) related to how demand is met with additional resources. Each scenario contains previous scenarios cumulatively. For example, Scenario 3 includes Scenarios 1 and 2. Scenario 1 is the base model (demand is provided by forecasting method). Scenario 2 includes increase in the following waiting times by 20% since possible increases in demand could provide longer length of stay: we therefore increase, (1) waiting for pre-assessment (triage) by 20%, (2) waiting time for treatment (clinician) by 20%, and (3) waiting time for discharge (post-treatment) by 20%. In Scenario 3, an additional X-Ray is added to the A&E system in addition to Scenario 2. In Scenario 4, a total of three nurses are employed, i.e., one nurse for each shift. Thus, we investigate how capacity is affected by this

Table 7. Case studies.

Demand conditions	Case studies	Explanations
Expected demand	Case 1	Base model
Unexpected demand	Case 2	5% Increase
	Case 3	10% Increase
	Case 4	15% Increase
	Case 5	20% Increase
	Case 6	25% Increase

Table 8. Scenarios in this study.

Scenarios	Explanations
Scenario 1	Base model
Scenario 2	Scenario 1 + increase on overall waiting time by 20%
Scenario 3	Scenario 2 + one more X-Ray
Scenario 4	Scenario 3 + one more nurse per shift
Scenario 5	Scenario 4 + one more bed
Scenario 6	Scenario 5 + one more doctor per shift

additional resource and whether performance metrics (i.e., utilisation rates of nurses and beds) are increased or not. Scenario 5 has one additional bed in comparison with Scenario 4. Finally, Scenario 6 involves an additional doctor per shift compared with Scenario 5. Each scenario is analysed under expected and unexpected demand conditions and therefore, simulation outputs determined in [Figure 2](#) are calculated.

6. Results and discussion

Simulation is a technique which has been widely used in different research areas and provides better management performance and DSSs to the related companies or organisations by means of operational research. However, simulation on its own uses sampling from historical data distributions but does not deal with upward trends in some inputs, such as demand. Such disadvantages must be avoided, particularly when simulation is used in strategic planning. The simulation technique therefore needs to be combined with forecasting methods in order to estimate the values of parameters for projection. It should be looked at what constitutes a good criterion for comparing forecasting methods, if one undertook a similar study. This is in fact an outstanding issue in the field of forecasting – there is no universally accepted measure of forecast accuracy. In fact, it seems to depend on the research area and the characteristics of the data used. The existence of particular features in the data, such as trend and seasonality, may lead to the use of certain types of forecasting techniques. Therefore, in this study, ARIMA, ES, and multiple linear regression methods are selected since these methods have been widely used and recommended as the best methods in the literature as mentioned in

[Section 2.1](#). In addition, the STL method was also compared with the others because the hospital data contains both trend and seasonal components.

This study presents a DSS to modelling demand and capacity compared to other studies in the literature. It combines discrete event simulation technique and quantitative forecasting in order to investigate demand and capacity of the A&E department by using 46 months of “big” data. In this study, we use demand obtained by quantitative forecasting instead of using presumptive rates in the simulation model. We took all the laboratory processes with more than 18 tests into account in the simulation model. To develop the model that captures variation (uncertainty), statistical distributions are based on age groups so that the related times vary according to age groups. In addition, the warm-up period is determined by using Welch’s method and it is added to the run length of the model. Therefore, we ensure that the system’s queues are embedded in the model to behave as under normal conditions and it is run before collecting statistical results from the model. To prevent any correlations among the results of key performance metrics and reduce variance, we specify optimum replication number as 10 replications.

DCR is a metric that showcases whether the hospital is able to cope with the expected and unexpected demand for A&E. The A&E has the ability in meeting demand if the DCR is around 100%. It means that available resources are sufficient to provide efficient delivery of health care in the A&E department. Otherwise, the management of the department (i.e., service managers and directors of the hospital) will need to take necessary actions against the projected demand.

In [Table 9](#), capacity amounts are given quarterly and annually under expected and unexpected demand conditions. Firstly, the DCR is more than 99% which means future demand is met with available resources in each scenario under the expected demand condition. In Case 2, a 5% increase in demand causes a little reduction in meeting demand. However, this problem is removed by additional resources in Scenarios 3–6. As the unexpected demand rises, the capability of the A&E department in meeting the demand decreases. For example, the capability in coping with demand results in the reduction by around 8%, 16%, 19%, and 23% in Cases 3, 4, 5, and 6, respectively in base scenario. An additional X-Ray is enough to achieve around 100% DCR in Case 3 although it is not adequate for Case 4, increasing DCR from 83.70% to 88.28%. In addition to an additional X-Ray, an additional nurse per shift is required to meet demand in Case 4. Scenarios increase the DCR from 81.08% to 98.92% in Case 5. However, all scenarios are insufficient to

meet all unexpected demand in Case 5. Likewise, more planning for additional reinforcements is required in order to achieve 100% DCR in Case 6. Around 5% of the demand is not met in Case 6 despite all the listed additional resources being applied.

Figures 5–9 present comparative graphs which show the outputs of this study and how performance metrics are changed through scenarios. These graphs use two vertical axes: the axis on the left of the graph represents the DCR as plotted using “bars” whereas the vertical axis on the right is the annual capacity in the A&E represented using “lines”. Note that in Figure 5, out of the six scenarios only three lines are shown. Scenarios 1–2 and Scenarios 4–6 overlap as they produce identical outputs. Figure 5 compares capacity (number of patients discharged) and DCR under expected (Case 1) and unexpected (other cases) demand conditions. The A&E department’s capacity reaches the peak in each case when Scenario 6 is applied. The increase in demand results in decrease in DCR in Scenarios 1 and 2. On the other hand, Scenario 3 is not able to prevent a decrease in DCR in the last three cases even with rises in DCR in the first three cases.

Figure 6 illustrates comparison of capacity (number of patients discharged) and utilisation of beds in A&E. Scenario 2 increases use of beds as additional resources (X-Ray and nurse) are integrated in to the system; the utilisation of beds increase since more patients occupy more beds. In Scenario 5, as expected the addition of a bed has slightly decreased the utilisation of beds. On the other hand, utilisation rates of beds exceed 90% in Cases 4–6. The A&E department’s management should take some precautions to avoid capacity issues before facing severe demands as in Cases 4–6.

Figures 6 and 7 illustrate the results related to utilisation rates of human resources (doctors and nurses). Utilisation rates of doctors are around 84% and rise to over 90% in Cases 4–6. Likewise, utilisation rate of nurses is roughly 100%. In every case, Scenario 6 includes an additional doctor per shift in the system and reduces the utilisation substantially. We should be aware that scenarios such as Scenario 4 increases staffing costs. Although Scenario 4 employs an additional nurse per shift, the utilisation rates of nurses remain higher in Cases 4–6.

In this study, HRG Tariff is used to calculate revenue for the A&E department for the period (February, 2012–January, 2013). The hospitals revenue is proportional to the number of patients treated in A&E depending on patient severity, whereas for costing we have only considered staff costs. Staff cost is calculated by multiplying the number of hours treated by staff with unit cost of staff per hour. Surplus is derived by deducting costs from revenues and calculated on a quarterly and annually basis.

Figure 8 presents comparative results of average revenue and surplus. Scenarios which increase the number of patients admitted provide A&E with the highest revenue. Due to increased capacity, Cases 4–6 dramatically increase revenue under the unexpected demand conditions. However, Scenarios 4 and 5 give higher surplus than Scenario 6 due to doctor’s salary.

7. Conclusion

We developed a DSS which discrete event simulation was combined with comparative forecasting technique to model demand and capacity of the A&E department of the PAH in England in this study. For this, we prepared a step by step guide as presented in the DSS illustrating how the two techniques are combined. We have compared four forecasting methods (ARIMA, ES, stepwise linear regression, and the STL method which has not previously applied to forecast A&E demand) and selected the best according to a forecast accuracy measure. We estimated daily A&E demand using stepwise linear regression and developed two demand conditions, namely the expected demand condition based on predicted activity, and the unexpected demand condition as requested by the hospital management in the case of closure of an A&E department at a nearby hospital. We then modelled capacity of A&E using discrete event simulation under expected and unexpected demand conditions.

The experimental results clearly illustrate that the A&E department will not be able to cope with the demand in most of the unexpected demand conditions although it has the ability of balancing demand and capacity under the expected demand condition. Additional resources tested in the scenarios will not be sufficient to cope with all demands in Case 5 (20% increase in demand) and Case 6 (25% increase in demand) although they do provide efficient delivery of health care in the A&E department under the expected demand conditions.

The existing A&E models were developed based on historical data, where no projections about the future had been made. Given that there is a year on year increase in A&E admissions, this is a crucial piece of information which is missing for modelling purposes. However, our A&E model (combined with forecasting) included demand inputs estimated by forecasting techniques using big data. In addition, it explored the demand–capacity balance and determined key performance metrics for the next period. The A&E model analysed how the unexpected demands are met by testing cumulative scenarios. It therefore provides a crucial decision support for A&E service managers and hospital management. This study suggests that hospitals should take an integrated approach to capturing demand and capacity using forecasting and simulation.

Table 9. Quarterly and annual capacity (number of patients discharged) and demand coverage ratio (DCR) of the A&E department based on case studies and scenarios at 95% confidence interval, DCR is the percentage of patients admitted to an A&E and discharged with available resources.

Demand conditions	Case studies	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6
Expected demand condition	Case 1 (Base model)	Q1	20708 (20667, 20749)	20709 (20661, 20756)	20475 (20368, 20583)	20459 (20352, 20567)	20459 (20352, 20567)
		Q2	20872 (20825, 20919)	20884 (20828, 20940)	20815 (20769, 20862)	20814 (20767, 20861)	20814 (20767, 20861)
		Q3	20616 (20520, 20712)	20582 (20480, 20684)	20476 (20443, 20510)	20482 (20447, 20515)	20482 (20447, 20517)
		Q4	20144 (20065, 20224)	20161 (20088, 20234)	19971 (19907, 20035)	19967 (19903, 20031)	19967 (19903, 20031)
		Total	82340 (82076, 82604)	82336 (82057, 82614)	81737 (81487, 81989)	81722 (81469, 81975)	81722 (81469, 81975)
	Case 2 (5% Increase)	DCR (%)	99.95 (99.63, 100.00)	99.94 (99.60, 100.00)	99.21 (98.91, 99.52)	99.20 (98.89, 99.50)	99.20 (98.89, 99.50)
		Q1	21058 (21014, 21101)	21056 (21003, 21109)	21196 (21156, 21237)	21184 (21144, 21225)	21184 (21143, 21226)
		Q2	21202 (21157, 21247)	21187 (21136, 21238)	21754 (21717, 21791)	21754 (21717, 21791)	21754 (21717, 21791)
		Q3	21021 (20976, 21065)	21002 (20959, 21044)	21442 (21390, 21494)	21438 (21388, 21489)	21438 (21387, 21488)
		Q4	19959 (19858, 20059)	19969 (19852, 20086)	20528 (20466, 20590)	20536 (20469, 20602)	20536 (20469, 20602)
Unexpected demand condition	Case 3 (10% Increase)	Total	83240 (83006, 83472)	83214 (82949, 83477)	84920 (84728, 85112)	84912 (84718, 85106)	84912 (84716, 85107)
		DCR (%)	97.15 (96.88, 97.42)	97.12 (96.81, 97.43)	99.11 (98.89, 99.33)	99.10 (98.88, 99.33)	99.10 (98.87, 99.33)
		Q1	21376 (21329, 21423)	21379 (21330, 21428)	22320 (22289, 22351)	22560 (22521, 22599)	22560 (22521, 22600)
		Q2	21372 (21331, 21412)	21395 (21348, 21441)	22505 (22475, 22535)	22715 (22682, 22749)	22715 (22683, 22748)
		Q3	21213 (21154, 21272)	21214 (21170, 21258)	22282 (22219, 22345)	22063 (22021, 22104)	22063 (22021, 22105)
	Case 4 (15% Increase)	Q4	18613 (18512, 18714)	18643 (18526, 18759)	22029 (21960, 22098)	21784 (21753, 21814)	21783 (21753, 21813)
		Total	82574 (82327, 82820)	82631 (82373, 82886)	89136 (88943, 89329)	89121 (88976, 89265)	89121 (88977, 89265)
		DCR (%)	91.91 (91.64, 92.18)	91.97 (91.69, 92.26)	99.21 (99.00, 99.43)	99.20 (99.04, 99.36)	99.20 (99.04, 99.36)
		Q1	21433 (21366, 21499)	21430 (21358, 21501)	22045 (21980, 22110)	23448 (23373, 23506)	23448 (23386, 23511)

(Continued)

Table 9. (Continued).

Demand conditions	Case studies	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6
Case 5 (20% Increase)	Q2	21562 (21514, 21610)	21552 (21516, 21589)	21766 (21700, 21833)	24401 (24316, 24486)	24392 (24301, 24483)	24392 (24301, 24483)
	Q3	21422 (21390, 21455)	21397 (21346, 21448)	22092 (22049, 22136)	23961 (23818, 24104)	23961 (23810, 24112)	23961 (23810, 24112)
	Q4	15848 (15738, 15958)	15798 (15665, 15931)	18666 (18475, 18858)	23187 (23003, 23370)	23187 (23003, 23370)	23187 (23003, 23370)
	Total	80265 (80008, 80523)	80177 (79885, 80468)	84569 (84204, 84937)	94989 (94510, 95466)	94988 (94500, 95476)	94988 (94500, 95476)
	DCR (%)	83.79 (83.52, 84.06)	83.70 (83.39, 84.00)	88.28 (87.90, 88.67)	99.16 (98.66, 99.66)	99.16 (98.65, 99.67)	99.16 (98.65, 99.67)
	Q1	21616 (21579, 21653)	21626 (21589, 21662)	21845 (21764, 21926)	24175 (24114, 24235)	24178 (24115, 24242)	24178 (24115, 24242)
	Q2	21653 (21614, 21691)	21668 (21618, 21718)	21749 (21686, 21811)	24349 (24243, 24455)	24367 (24255, 24479)	24367 (24255, 24479)
	Q3	21551 (21508, 21593)	21539 (21492, 21585)	21724 (21662, 21785)	24379 (24265, 24493)	24381 (24274, 24488)	24381 (24274, 24488)
	Q4	15034 (14902, 15165)	15025 (14893, 15157)	16932 (16786, 17078)	24496 (24433, 24559)	24495 (24425, 24564)	24495 (24425, 24564)
	Total	79854 (79603, 80103)	79858 (79591, 80123)	82250 (81898, 82600)	97399 (97054, 97742)	97421 (97068, 97773)	97421 (97068, 97773)
Case 6 (25% Increase)	DCR (%)	81.08 (80.83, 81.34)	81.09 (80.82, 81.36)	83.52 (83.16, 83.87)	98.90 (98.55, 99.25)	98.92 (98.56, 99.28)	98.92 (98.56, 99.28)
	Q1	21574 (21515, 21634)	21578 (21523, 21634)	21712 (21655, 21769)	24194 (24131, 24257)	24189 (24117, 24261)	24189 (24117, 24261)
	Q2	21586 (21533, 21639)	21579 (21522, 21636)	21614 (21550, 21679)	24135 (24020, 24250)	24134 (24024, 24244)	24134 (24024, 24244)
	Q3	21663 (21607, 21720)	21678 (21618, 21739)	21579 (21515, 21643)	24137 (24039, 24236)	24147 (24050, 24244)	24147 (24050, 24244)
	Q4	13768 (13613, 13922)	13748 (13618, 13877)	15470 (15200, 15741)	23999 (23858, 24139)	24027 (23888, 24165)	24027 (23888, 24165)
	Total	78591 (78267, 78914)	78583 (78282, 78885)	80375 (79920, 80832)	96465 (96048, 96882)	96497 (96078, 96915)	96497 (96078, 96915)
	DCR (%)	77.16 (76.84, 77.48)	77.15 (76.86, 77.45)	78.91 (78.47, 79.36)	94.71 (94.30, 95.12)	94.74 (94.33, 95.15)	94.74 (94.33, 95.15)

Figure 5. Comparative graphs of demand coverage ratio (DCR) and capacity.

Figure 6. Comparative graphs of utilisation rates of bed (URB) and capacity.

Figure 7. Comparative graphs of utilisation rates of doctors (URD) and capacity.

Figure 8. Comparative graphs of utilisation rates of nurses (URN) and capacity.

Figure 9. Comparative graphs of average revenue and surplus.

Moreover, hospitals should stress test their systems using such techniques, as it is a useful approach to test complex systems, as illustrated above.

This article will inevitably provide many benefits to management of NHS Trusts. In relation to practical implications, the management is able to foresee patient demands for their hospital in future years and test whether they are able to cope with demand with resources at their disposable. Therefore, this will enable key decision-makers to be alerted well in advance if performance targets and patient needs cannot be achieved.

In addition, decision-makers can observe the impact of possible changes in resources (i.e., staff, beds, rooms) and how it effects the performance of A&E in the safety of a simulation environment. The results will bring a different perspective to the management in terms of strategic planning (both short

and long term) and encourage them to develop a realistic plan. In conclusion, this study provides a crucial and practical decision support tool for hospital managers, which will benefit patients, taxpayers, the NHS, and beyond.

A limitation of the study is that we did not take account of triage system's interactions with other departments (e.g., the medical assessment unit) which may impact activity and utilisation of resources. We will consider this aspect of the A&E system in our future simulation models. Further research will involve the development of similar models for outpatient and inpatient specialities which are in interaction with the A&E department.

Disclosure statement

No potential conflict of interest was reported by the authors.

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Appendix 1. Inputs parameters of the simulation model, DoH: Department of Health, HES: hospital episodes statistics, N/A: not available, NHS: National Health Service

Input parameters	Estimates	Distributions	References
Patient inputs			
- Available demand (2012/2013)	See Table 5	See Table 5	HES data-set
- Forecasted year (2013/2014)	See Table 5	See Table 5	N/A
Physical inputs			
- Number of beds	22	Fixed	Local data
- Number of triage rooms	5	Fixed	Local data
- Number of clinic rooms	4	Fixed	Local data
Staff inputs			
- Number of doctors	12	Fixed	Local data
- Number of nurses	21	Fixed	Local data
Financial inputs			
<i>Revenues in the A&E (HRG codes for severity of injuries):</i>	2012/2013–2013/2014	Fixed	DoH (2013, 2014)
- VB01Z	£235–£237	Fixed	Department of Health (2013, and 2014)
- VB02Z	£235–£210	Fixed	Department of Health (2013 and, 2014)
- VB03Z	£151–£164	Fixed	Department of Health (2013, 2014)
- VB04Z	£151–£139	Fixed	Department of Health (2013, 2014)
- VB05Z	£151–£130	Fixed	Department of Health (2013, 2014)
- VB06Z	£81–£102	Fixed	Department of Health (2013, 2014)
- VB07Z	£112–£119	Fixed	Department of Health (2013, 2014)
- VB08Z	£112–£110	Fixed	Department of Health (2013, 2014)
<i>Costs in the A&E:</i>		Average	NHS Digital (2013, 2014)
- Average monthly payment to a doctor	£6178–£6274	Average	NHS Digital (2013, 2014)
- Average monthly payment to a nurse	£2552–£2563		

(Continued)

(Continued).

Input parameters	Estimates	Distributions	References
Other inputs			
<i>Demographic features:</i>			
- Gender			
(1) Male	47%	Multinomial	HES data-set
(2) Female	53%	Multinomial	HES data-set
- Age groups			
(1) Age group 1 (0–15)	23%	Multinomial	HES data-set
(2) Age group 2 (16–35)	28%	Multinomial	HES data-set
(3) Age group 3 (36–50)	16%	Multinomial	HES data-set
(4) Age group 4 (51–65)	12%	Multinomial	HES data-set
(5) Age group 5 (65+)	21%	Multinomial	HES data-set
- Laboratory process:			
- Laboratory service			
(1) What percentage of patients are referred to the laboratory?		Multinomial	HES data-set
(2) What percentage of patients are not referred to the laboratory?	76%	Multinomial	HES data-set
- Percentage of tests	24%		
- First tests – second tests – third tests			
X-Ray		Multinomial	HES data-set
Electrocardiogram	42% – 8% – 12%	Multinomial	HES data-set
Haematology	13% – 22% – 10%	Multinomial	HES data-set
Biochemistry	31% – 26% – 26%	Multinomial	HES data-set
Urinalysis	1% – 32% – 27%	Multinomial	HES data-set
Others	8% – 7% – 16%	Multinomial	HES data-set
Shifts	5% – 5% – 9%	Fixed	Local data
Distributions			
- Severity of injuries	3	Frequency distribution	HES data-set
- Waiting time for pre-assessment	Frequency distribution	Multinomial	Expert opinion
- Pre-assessment process	15 min	Multinomial	Expert opinion
- Waiting time for treatment	10 min	See Appendix 2 and Appendix 3	HES data-set
- Treatment time	See Appendix 2 and Appendix 3	Frequency distribution	HES data-set
- Waiting time for discharge	See Appendix 4 and Appendix 5	See Appendix 4 and Appendix 5	HES data-set

Appendix 2. Comparative test results and parameter values of fitting distributions for waiting time for treatment (by each age group)

Distributions	Age group 1 (0–15)			Age group 2 (16–35)			Age group 3 (36–50)			Age group 4 (51–65)			Age group 5 (65+)		
	Kolmogorov–Smirnov		Anderson–Darling	Kolmogorov–Smirnov		Anderson–Darling	Kolmogorov–Smirnov		Anderson–Darling	Kolmogorov–Smirnov		Anderson–Darling	Kolmogorov–Smirnov		Anderson–Darling
	Statistic		Statistic	Statistic		Statistic	Statistic		Statistic	Statistic		Statistic	Statistic		Statistic
Normal	0.12923		2099.70	0.13879		2422.90	0.14521		1454.30	0.16594		1337.60	0.17456		2315.30
Triangular	0.72610		86722.00	0.69821		94712.00	0.69742		55558.00	0.69360		40774.00	0.70196		83634.00
Rounded uniform	0.17436		16625.00	0.18930		16112.00	0.19679		8996.30	0.22222		5816.10	0.23048		10818.00
Uniform	0.17379		16495.00	0.18657		15988.00	0.19482		8905.70	0.21989		5773.10	0.22971		10754.00
Exponential	0.06663		557.04	0.06177		795.44	0.05583		389.93	0.04596		207.67	0.07516		718.60
Erlang	0.13537		3929.10	0.12344		3171.30	0.10218		1179.80	No fit		No fit	No fit		No fit
Log normal	0.06010		513.37	0.08475		1577.20	0.09208		1081.20	0.10092		971.83	0.13096		2769.40
Weibull	0.04054		173.41	0.13330		3866.90	0.03498		148.87	0.04517		188.78	0.06858		885.00
Gamma	0.02868		99.90	0.03245		222.74	0.03653		170.08	0.05291		247.91	0.06844		664.64
Beta	0.14541		4822.80	0.08022		8362.60	0.05944		1529.80	0.05865		5705.50	0.08795		12296.00
Pearson V	0.13806		3096.40	0.21034		7659.80	0.22249		4900.40	0.24150		3894.60	0.26344		7804.10
Pearson VI	0.03141		109.18	0.03129		192.54	0.06170		486.27	0.05301		240.84	0.06645		678.68
Gauss	0.10574		3296.20	0.09362		6199.20	0.09447		4531.00	0.09259		4234.40	0.13080		21378.00
Poisson	0.47940		2.9599E+5	0.46884		3.4596E+5	0.46908		2.0249E+5	0.48098		1.4773E+5	0.48196		2.6785E+5
Binomial	No fit		No fit	No fit		No fit	No fit		No fit	No fit		No fit	No fit		No fit
Negative binomial	0.02374		94.47	0.02732		155.15	0.03375		148.45	No fit		No fit	No fit		No fit
Bernoulli	No fit		No fit	No fit		No fit	No fit		No fit	No fit		No fit	No fit		No fit
Geometric	0.07811		774.29	0.06938		998.22	0.11182		1486.40	0.03524		161.62	0.06233		477.29
Parameters	(n = 1, p = 0.02035)			(n = 1, p = 0.01572)			(n = 1, p = 0.01491)			(p = 0.01289)			(p = 0.01337)		

Appendix 3. Probability density function graphs for distributions of “waiting time for treatment” for each age group

Appendix 4. Comparative test results and parameter values of fitting distributions of waiting time for discharge (by each age group)

	Age group 1 (0–15)		Age group 2 (16–35)		Age group 3 (36–50)		Age group 4 (51–65)		Age group 5 (65+)	
	Kolmogorov–Smirnov	Anderson–Darling	Kolmogorov–Smirnov	Anderson–Darling	Kolmogorov–Smirnov	Anderson–Darling	Kolmogorov–Smirnov	Anderson–Darling	Kolmogorov–Smirnov	Anderson–Darling
Distributions	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Normal	0.20660	3499.20	0.21681	4758.40	0.18845	2070.30	0.17422	1357.00	0.10933	1495.20
Triangular	0.70813	98696.00	0.68889	1.1465E+5	0.66845	55713.00	0.65161	34414.00	0.62277	44780.00
Rounded uniform	0.26538	14884.00	0.27653	14027.00	0.24699	7684.80	0.23098	5817.80	0.14521	11660.00
Uniform	0.26379	14815.00	0.27396	13862.00	0.24492	7679.80	0.22933	5828.70	0.14491	11648.00
Exponential	0.09458	916.43	0.10103	1934.50	0.08063	644.23	0.07091	419.73	0.13337	1978.80
Erlang	No fit	No fit	No fit	No fit	No fit	No fit	No fit	No fit	0.28874	10603.00
Log normal	0.09328	1614.20	0.08845	1943.00	0.10860	1476.40	0.12836	1393.20	0.17493	4045.80
Weibull	0.06740	305.37	0.06490	454.72	0.06117	375.58	0.08673	452.96	0.14490	2044.90
Gamma	0.05078	278.89	0.05633	410.83	0.05852	311.09	0.06349	366.75	0.08135	1333.40
Beta	0.06579	4573.90	0.06745	19416.00	0.06324	2699.20	0.06462	5746.80	0.15577	9192.60
Pearson V	0.21863	5429.50	0.21196	6395.60	0.22414	4577.20	0.24988	3950.60	0.30445	10262.00
Pearson VI	0.06859	348.64	0.06459	491.18	0.05849	311.08	0.06760	373.05	0.11909	1678.50
Gauss	0.14898	14917.00	0.16125	21451.00	0.15557	14078.00	0.14419	10511.00	0.12057	20250.00
Poisson	0.52731	3.3229E+5	0.53841	4.4603E+5	0.50976	2.2846E+5	0.48435	1.4293E+5	0.41942	2.0181E+5
Binomial	No fit	No fit	No fit	No fit	No fit	No fit	No fit	No fit	No fit	No fit
Negative binomial	No fit	No fit	No fit	No fit	No fit	No fit	No fit	No fit	0.29493	11176.00
Bernoulli	No fit	No fit	No fit	No fit	No fit	No fit	No fit	No fit	No fit	No fit
Geometric	0.07952	518.96	0.08804	1279.00	0.06968	424.11	0.06151	319.86	0.13584	1994.10
Parameters	(α = 0.6916, β = 90.064)		(α = 0.63069, β = 112.98)		(α ₁ = 0.79911, α ₂ = 2.4519E+6, β = 2.6279E+8)		(ρ = 0.00983)		(α = 1.5365, β = 85.049)	

Appendix 5. Probability density function graphs for distributions of waiting time for discharge (by each age group)