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A Stochastic SPICE Model for Superconducting Nanowire Single Photon Detectors and Other Nanowire Devices

Adam Nykoruk McCaughan,

National Institute of Standards and Technology, Boulder, CO 80305 USA

Dylan M. Oh,

University of Colorado Boulder, Boulder, CO 80305 USA

Sae Woo Nam

National Institute of Standards and Technology, Boulder, CO 80305 USA

Abstract

Superconducting nanowire devices, such as the superconducting nanowire single photon detector (SNSPD) or nanocryotron, have a time-dependent stochasticity that depends on the current flowing through them. When modeling complex circuits made of several such devices (for instance, an array of SNSPDs), the ability to include this randomness can be important for predicting unwanted effects and interactions within the circuit. We present a modification of the model described by Berggren *et al.* that allows for the inclusion of this stochasticity into the nanowire device model. We then verify the model against experiment using a tungsten silicide SNSPD, and show that the modified model replicates the stochasticity of the physical device.

Keywords

Superconducting integrated circuits; circuit simulation; thin film circuits; SPICE

I. INTRODUCTION

SUPERCONDUCTING nanowire single photon detectors (SNSPD) have found increasing use in fields such as quantum information [1] and classical communications [2]. In addition, a recent class of superconducting circuit elements has emerged that uses the superconducting nanowire as their basic construction element—these electronic elements have been used as superconducting amplifiers, memories, and comparators, among other applications [3]–[6]. To date, these applications have used a few elements at a time integrated into small circuits, but new applications such as nanowire-based memories and SNSPD readout arrays may drastically increase their scale of integration. As these circuits become increasingly complex, they will require robust modeling techniques such as electrical simulation and Monte Carlo analysis. Recently, in Berggren *et al.* [7], the authors constructed a compact SPICE model for a nanowire device and showed that it reproduced several salient features of superconducting nanowires such as critical current, latching, and hotspot stabilization.

(Corresponding author: Adam Nykoruk McCaughan. adam.mccaughan@nist.gov).

However, one feature that was not captured in the model was the inherent stochasticity of these nanowires. Here, we modify that SPICE model to introduce and allow for the simulation of this stochasticity.

Modeling stochasticity is important as it allows the circuit designer to simulate potentially unwanted interactions between elements—for instance, random counts produced in an SNSPD array may cause crosstalk or readout issues. Using a proper model that captures stochasticity allows simulations to match implemented systems; for example by representing quantum fluctuations, electrical noise, or light leakage. Dark counts (sometimes called false counts), which have long been present in the SNSPD literature, are random triggers of a detector that are not caused by legitimate photon arrival events (or correspond to photons that are not part of the signal). Any model that introduces stochasticity must account for these. But other, non-detector devices such as the nTron or yTron may be more concerned with electrical noise and quantum or thermal fluctuations. So any model must also be able to account for these features. Fortunately, in nanowires both of these types of stochasticity can be modeled as statistical processes with an exponential dependence on the nanowire bias current, and so can be modeled in a unified manner (albeit with different parameters for each independent process). Additionally, these features can be added to SPICE without too much additional computation complexity. We note that because the model we are discussing applies to any nanowire device and not just SNSPDs, these random device triggers will be referred to as “switching events” for the remainder of this work.

II. BACKGROUND

Superconducting nanowires are stochastic due to four primary factors: thermal fluctuations, quantum fluctuations, electrical noise, and radiation-based (e.g., black body) photon noise. Of these, only the first two are intrinsic to the devices; the others are properties of the system in which they reside. Thermal and quantum fluctuations have been thoroughly treated in the literature; in present-day applications of superconducting nanowires, which involve temperatures between 1K and 4K, the thermal fluctuations usually dominate over the quantum fluctuations [8]. One particularly useful treatment of these fluctuations was presented in Clem *et al.* [9], where the authors showed that the thermal fluctuations in a superconductor nanowire due to temperature caused the critical current of the nanowire to fluctuate, forming a distribution.

Despite thermal and quantum fluctuations being the only fixed properties of the nanowires, it is nonetheless useful to be able to characterize environmental contributions as well in a given setup; this experiment-based characterization will be our focus in this work. In a real system operated at Kelvin-scale temperatures, thermal, electronic, and photon-based noise will all have contributions—this behavior was expressly described in Ref. [10]. As shown in Ref. [9], the thermal-based fluctuations take the form Ae^{jB} , where j is the bias current flowing through the nanowire. One example of the switching-rate statistics from these fluctuations can be seen in Fig. 1(a), that displays the switching statistics for one of the nanowire arms of a yTron. Additionally, the dark counts seen in superconducting nanowire detectors have been shown experimentally [11]–[14] to take the form Ae^{jB} , with one example shown in Fig. 1(b). Thus, by creating a stochastic model that can produce these

exponential switching-rate statistics, we can capture the relevant stochasticity with a single modeling approach. We note, however, that these statistics do not model the underlying dynamics of the fluctuations—they only reproduce the phenomenological output of the nanowires which arise when those fluctuations induce switching events.

III. THE MODIFIED SPICE MODEL

Shown in Fig. 2 is the circuit model modified from Ref. [7]. Note that despite the difference in layout, it is functionally equivalent to the dynamic model—it was drawn this way to simplify understanding of how additional stochastic elements (highlighted with a red dashed box in Fig. 2) can be added independently. Aside from the stochastic element, the circuit only differs from Ref. [7] in that the switch S is open when $v_{se} > 0.5$ V, which is functionally equivalent to the original model.

The goal of the stochastic elements is to create a random process in SPICE that can produce switching events with the same bias-dependent statistics as actual devices. Specifically, the elements must produce an inhomogeneous Poisson-process with a rate parameter that has an exponential dependence on the current in the nanowire. Creating this distribution is possible despite the constraints of SPICE (common SPICE implementations do not have the ability to produce arbitrary random distributions). In our implementation, the stochastic distribution is generated by modulating the critical current of the nanowire in a way that reproduces the desired Poisson-process. We can reproduce the desired switching rate versus bias current $\Gamma(i)$, using a formula of the form

$$\Gamma(i) = \frac{1}{dt} \exp((iI_{c0})/\Delta I_c) \quad (1)$$

where i is the current through the nanowire device, I_{c0} and I_c are fitted parameters, and $1/dt$ is set to the maximum desired dark count rate in the simulation. By choosing the right parameters to match our experimental setup, we can get the desired log-linear rate of switching events using only SPICEs uniformly-distributed random variable rand and mapping it onto $I_c(t)$. This is accomplished by assigning each independent stochastic process (i.e., each log-linear fit line in Fig. 3) to one of the I_{stN} current sources in Fig. 2. Each stochastic current source takes the form:

$$I_{st} = i > I_c(t) \quad (2)$$

$$I_c(t) = I_{c0} - \Delta I_c(-\ln(\text{rand}(t/dt))) \quad (3)$$

where t is the current simulation time and the other parameters are those described in 1. Since the rand function generates a random uniformly-distributed value every time its integer value changes, this form essentially means that every dt time interval, the critical current of the nanowire is altered. When the test $i > I_c(t)$ becomes true, the stochastic current source I_{stN} is turned on and set to 1 A, causing v_{se} to be >0.5 V, opening the switch $S(v_{se})$, and starting the hotspot growth process. The resulting distribution of I_{stN} is

exponentially distributed, and so creates a Poisson process with an exponential rate dependence on i , just as originally desired.

One potential hazard can occur if dt is set too large due to the nature of the rand variable in SPICE; since this variable changes value exactly every dt time interval, switching from multiple nanowire devices may occur simultaneously. This can be ameliorated by decreasing dt (at the cost of longer computation time due to more time steps), or alternatively by implementing a second rand stage in SPICE that randomizes the delay between the $i > I_c(t)$ statement becoming true and the triggering of the switching event. In our simulations, we found that a maximum timestep dt of 1 ns was sufficiently small to reproduce the stochastic behavior we desired, but large enough to allow speedy completion of the simulation (over several hundred microseconds).

IV. PARAMETER EXTRACTION

To get the fitting parameters for the switching rate vs bias current $\Gamma(i)$, one can directly measure switching rates as a function of bias current. Alternatively, one can measure statistics of the critical current distribution. Particularly for nanowire devices that may latch, the critical-current method is simpler and preferable—if the device cannot self-reset and latches after a single switching event, it becomes difficult to gather switching rate statistics. This current-distribution measurement is performed by applying a current ramp into the device at a rate of dI/dt and measuring the point at which it switches. This process is repeated multiple times, creating a distribution of switching current as is shown in Fig. 3(a). After binning the critical current measurements into a histogram, the statistics can be readily converted into switching rate vs bias statistics using the Fulton-Dunkelberger transform [8], [15]:

$$\Gamma(K) = \frac{dI}{dt} \frac{1}{\Delta I} \ln \left(\sum_{j=1}^K P(j) / \sum_{i=1}^{K-1} P(i) \right) \quad (4)$$

where K corresponds to the k th bin of the critical current histogram (starting at the lowest current-value with an occupied bin), ΔI is the width of each bin of the histogram, and $P(k)$ is the number of counts in the k th bin. An example is shown in Fig. 3, where a critical current distribution (Fig. 3(a)) has been transformed into switching rate data (Fig. 3(b)). Given that the form of the critical current distribution is approximately normal, the uncertainty of the resulting distribution scales approximately as $1/\sqrt{n}$, where n is the number of samples taken. Practically, this meant that to achieve an uncertainty of less than 1% in the critical current distribution, n of the order of 10,000 samples were required.

Once the switching rate versus bias data is available, the desired dt should be chosen and then I_c and I_c can be used to fit to the data. care should be taken to ensure higher-rate switching data is more heavily weighted; the lower-rate switching data can be significantly scattered by Poisson noise. After all the parameters are extracted, the model can be run. It is important to ensure that the maximum allowable timestep in SPICE is set to dt , so that the $i > I_c(t)$ inequality is evaluated for each new value of rand (which changes its value every dt seconds). Shown in Fig. 4 are fits to the combined stochastic contributions to the WSi

SNSPD measured here: the dark-count switching rate data from Fig. 1, and the critical-current-sweep switching rate data from Fig. 3. The values used for the fits are listed in Table I.

The fitted parameters are compared to the realized simulation in Fig. 4. To validate that the stochastic modification to the model does not affect the dynamics of the SNSPD, we compared the output trace to one recorded from the experimental device and graphed them in Fig. 5.

When performing Monte-Carlo analyses with this model, care should be taken to provide the rand function with an appropriate random-seed offset to guarantee each run is independent of the last. We also note that to test the model without stochasticity, the I_c parameter may be set to zero, which returns the model to a deterministic mode of operation where switching events only occur when $i > I_{c0}$.

V. CONCLUSION

We have presented a stochastic addition for the superconducting nanowire model described in Berggren *et al.*, and shown how it can replicate the inhomogeneous Poisson-processes that characterize the most common noise processes in these devices. This modeling procedure is also applicable to other, future SPICE models of superconducting nanowires; since it relies only on a time-dependent manipulation of the device critical current, the technique could be applied to any model with an otherwise-fixed critical current. Additionally, this type of analysis may be useful in performing jitter analyses of purely-electronic nanowire devices such as the nTron or yTron. Since those devices rely on manipulation of a nanowire critical current to produce output, the model presented here may shed light on how the input slew rate affects output timing uncertainty.

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Fig. 1.

Exponential dependence of the switching rate on the bias current for two different nanowire devices. (a) Measured statistics for a WSi SNSPD. (b) Measured statistics for a WSi yTron.

Fig. 2. SPICE circuit model based on Berggren *et al.* [7]. The stochastic elements added to the model are outlined with the red dashed box.

Fig. 3. (left) Measured I_c distribution for a WSi SNSPD using a ramp rate dI/dt of $203 \mu\text{A/s}$. (right) Transformation of the I_c distribution in switchingrate data using the Fulton-Dunkelberger transform.

Fig.4.

(top) Fits to the dark-count switching rate data from Fig. 1 and critical-current-sweep switching rate data from Fig. 3. (bottom) Simulation of combined switching rate using two stochastic current sources, showing agreement with fits.

Fig. 5. SPICE simulation (black) and experimental oscilloscope trace (cyan) of the WSi SNSPD showing the same agreement between simulation and experiment as the original model; the stochastic modification leaves the timing dynamics of the original model intact.

TABLE I**STOCHASTIC ELEMENT FIT PARAMETERS**

Element #	I_c	I_{c0}	dt
1	27.3×10^{-8}	10.3×10^{-6}	10^{-9}
2	3.27×10^{-8}	6.53×10^{-6}	10^{-9}

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