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Measurement of Viscoelastic Material Model Parameters using Fractional Derivative Group Shear Wave Speeds in Simulation and Phantom Data

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Abstract

While ultrasound shear wave elastography originally focused on tissue stiffness under the assumption of elasticity, recent work has investigated the higher order, viscoelastic properties of tissue. This paper presents a method to use group shear wave speeds at a series of derivative orders to characterize viscoelastic materials. This method uses a least squares fitting algorithm to match experimental data to calculated group shear wave speed data, using an assumed material model and excitation geometry matched to the experimental imaging configuration. Building on a previous study that used particle displacement, velocity, and acceleration signals, this study extends the analysis to a continuous range of fractional derivative orders between 0 and 2. The method can be applied to any material model. Herein, material characterization was performed for three different two-parameter models and three different three-parameter models. This group speed based method was applied to both shear wave simulations with ultrasonic tracking and experimental acquisitions in viscoelastic phantoms (similar to the Phase II Quantitative Imaging Biomarkers Alliance (QIBA) phantoms). In both cases, the group speed method produced more repeatable characterization overall than fitting the phase velocity results from the peak of the 2D Fourier transform. Results suggest the linear attenuation model is a better fit than the Voigt model for the viscoelastic QIBA phantoms.

I. Introduction

Shear wave elasticity imaging (SWEI) can be used to provide quantitative measurements of tissue stiffness [1]. Through the use of an acoustic radiation force impulse (ARFI), a small displacement can be generated in tissue, which can then be tracked through traditional ultrasonic methods [2]. In most commercial scanners, the imaged material is assumed to be homogenous, linear, elastic, and isotropic, and thus the speed (c) and stiffness (μ) can be related by $\mu = \rho c^2$, where ρ is density [3, 4]. However, soft tissue exhibits viscoelastic effects, with both phase velocity $\alpha(\omega)$ and shear modulus $\mu(\omega)$ dependent on frequency, $f = \omega/2\pi$ [5]. The best method to quantify viscosity is still under debate [6-8]. It is possible to quantify the complex shear modulus by measuring the phase velocity and attenuation at each frequency [9]. For simplification of analysis, these shear moduli can be parameterized to a

material model with a small number of model constants. Many have chosen to use the Voigt model [8, 10, 11], which has a complex shear modulus given by

$$\mu(\omega) = \mu_0 + i\omega\eta \quad (1)$$

where μ_0 is the stiffness and η is the viscosity of the material.

An alternative group speed based method to determine the viscoelastic material parameters has been proposed by Rouze *et al.* [12]. The work herein is an extension of that method. We provide a brief summary of the approach and conclusions of the previous work, as well as the limitations that motivated the extended method presented herein. Rather than calculating a phase velocity, Rouze *et al.* observed that the measured group shear wave speed (gSWS) is related to both the material properties and the frequency content of the shear wave. That previous work considered the propagating shear wave signal in terms of particle displacement (u), velocity (v), and acceleration (a). These signals are related through

$$v(r, t) = \frac{du(r, t)}{dt}, a(r, t) = \frac{dv(r, t)}{dt} = \frac{d^2u(r, t)}{dt^2} \quad (2)$$

where r is the lateral position and t is the time. In the Fourier domain

$$\tilde{v}(r, \omega) = (i\omega)\tilde{u}(r, \omega), \quad \tilde{a}(r, \omega) = (i\omega)\tilde{v} = (i\omega)^2\tilde{u}(r, \omega) \quad (3)$$

where $\omega = 2\pi f$ is the angular frequency. As can be seen in (3), the particle displacement, velocity and acceleration signals represent different weightings of the shear wave frequency content, influencing the relationship between the gSWS of these three signals and the shear modulus $\mu(\omega)$. Assuming a homogeneous material, Rouze *et al.* demonstrated that differences in the group speeds V measured from these signals ($v_d = V_{vel} - V_{disp}$, $a_v = V_{acc} - V_{vel}$, $a_d = V_{acc} - V_{disp}$) are first-order measures of viscosity.

Rouze *et al.* connect a given first order measurement to material model properties through lookup tables created with a model describing shear wave propagation in a viscoelastic material. They considered two material models: the Voigt model, described in equation (1), and a linear attenuation model. This second model is characterized by a linear attenuation $\alpha(\omega) = \alpha_0\omega$, where α_0 is a constant. Using the Kramers-Kronig relations with one subtraction, the following equation for phase velocity $c(\omega)$ can be obtained [13]:

$$c(\omega) = \frac{1}{\frac{1}{c(\omega_0)} - \frac{2}{\pi}\alpha_0 \ln\left(\left|\frac{\omega}{\omega_0}\right|\right)}. \quad (4)$$

The reference frequency (f_0 , $\omega_0 = 2\pi f_0$) was chosen to be 200 Hz. Note that this reference frequency is part of the two-parameter linear attenuation model definition, and changing the reference frequency becomes a different model with different expected parameterization, though many of the overall characteristics and trends will be the same. For convenience, this equation is rewritten to replace α_0 with $c'(\omega_0)$, the instantaneous slope of the phase velocity at the reference frequency, using eq. (36) from [12]. The phase velocity then becomes:

$$c(\omega) = \frac{c(\omega_0)}{1 - \frac{\omega_0 c'(\omega_0)}{c(\omega)} \ln\left(\left|\frac{\omega}{\omega_0}\right|\right)}. \quad (5)$$

This leaves the variables in this model to be $c(\omega_0)$ and $c'(\omega_0)$. Because only two-parameter models were considered, 2D lookup tables were developed for any given pair of V_{disp} , V_{veh} , V_{acc} . Thus, from any two group speed measurements, two parameters could be determined. A major result from this analysis was that the Voigt model was not a good fit for the viscoelastic phantoms used in the study; specifically, for the same underlying signal, the Voigt parameters changed depending on which pairing of V_{disp} , V_{veh} , V_{acc} was used for the analysis. This was not true in the linear attenuation model, which found consistent parameters regardless of the pairing of V_{disp} , V_{veh} , V_{acc} used [12].

The technique described by Rouze *et al.* has two shortcomings that motivated this work. First, because only two group speeds are used to determine the two parameters, there is no additional information for quality-of-fit analysis or higher order material models. Second, it can sometimes be difficult to obtain group speed measurements of particle acceleration (V_{acc}) due to increasing noise with higher derivatives.

In this paper, we extend the gSWS method described by Rouze *et al.* to include multiple fractional derivative orders and apply the extended method in simulations and viscoelastic phantoms using several different two and three parameter viscoelastic material models. The fractional derivative of order ν can be applied to a particle displacement signal u , resulting in a shear wave signal S_ν , such that

$$S_\nu(r, t) = \frac{d^\nu u(r, t)}{dt^\nu} \quad (6)$$

or, in the Fourier domain [14],

$$\tilde{S}_\nu(r, \omega) = (i\omega)^\nu \tilde{u}(r, \omega). \quad (7)$$

While similar to an integer derivative, the fractional derivative allows for a more gradual variation of the sensitivity of the group speed measurement to the frequency content of the wave. By using multiple orders within a range of $0 < \nu < 2$, additional information about the frequency response of the material is obtained, allowing us to overcome the limitations of the previous work [12]. While this study uses derivative order through acceleration ($\nu = 2$), the information at fractional orders can be used even when noise may prevent the use of the acceleration signal. However, because the present work only evaluates phantoms with high quality signal, the effects of maximum derivative order are not fully explored herein. This work instead focuses on the benefits of fractional derivatives for overcoming the first limitation of Rouze *et al.*

By including gSWS measurements at fractional derivative orders, both the consistency of a model's fit to experimental results can be evaluated and the use of higher order models can

be investigated. Six different material models are considered in this work, including the two-parameter Voigt and linear attenuation models mentioned previously, as well as the two-parameter spring-pot model [13, 15]. Additionally, three-parameter generalizations of these models are considered.

II. Methods

The fractional derivative gSWS method is evaluated using both tracked simulation and phantom data. For clarity, the details of the generation of both datasets are presented first, followed by the details of the fractional derivative group speed method.

A. Simulations with Ultrasonic Tracking

Simulations were performed to assess the accuracy and bias of the fractional derivative gSWS method. First, the theoretical displacement from an ARFI push was calculated using the 3D Green's function solution for shear wave propagation in an infinite homogenous material as described by Rouze *et al.* ([12], (40)). For an accurate model of the push function, the experimental 3D ARFI push was modeled in Field II based on the transducer geometry used for the experimental acquisitions, and all points within 1% of the maximum intensity value were included in the forcing function for the Green's function calculations [16]. Material parameters used in the Green's function calculations were chosen to model the estimated parameters of the B phantom described below and are given in Table I. A volume of data was generated, with dimensions (assuming the origin at the lateral and elevational centerline and axial focal depth) of: lateral 0 to 20 mm, elevational -5 to 5 mm, and axial -2.5 to 2.5 mm. These volumes were used to displace random scatterers (normalized to a maximal displacement of 10 microns) and generate RF data in Field II according to the procedure described by [17]. The Field II imaging parameters were chosen to match the experimental transducer configuration. The number of scatterers per estimated resolution cell was chosen as 21.8, which was determined to approach the theoretical signal-to-noise ratio limit of fully developed speckle ($\text{SNR} = 1.91$). One hundred independent speckle realizations were generated. Displacement estimation on the RF data was performed using normalized cross correlation with a kernel of 2.5λ and a search region of 0.25λ . These simulations have known viscoelastic material parameters, and thus allow us to evaluate both the bias and variance in our methods.

B. Phantom Acquisitions

Three viscoelastic phantoms were imaged using an Philips C5-2 curvilinear array on a Verasonics Vantage scanner, as described by Deng *et al.* [18]. The phantoms used are copies of the phantoms used in the Quantitative Imaging Biomarkers Alliance (QIBA) Phase II study, specifically, E2297-A3, E2297-B2, and E2297-C3, referred to as A, B, and C respectively [7]. For all data presented, the ARFI push frequency was 2.36 MHz, focal depth was 50 mm, and the push geometry F-number was 2.0. The tracking frequency was 3.1 MHz, with diverging wave tracking, and beamformed to give images with 0.2 mm lateral spacing. Each phantom was imaged 8 times, with rotation of the phantom between acquisitions to interrogate different speckle realizations. The two lateral directions of shear wave propagation were considered independently for a total of 16 shear wave acquisitions.

Material displacements were estimated from the IQ data using a Loupas algorithm with a 3λ kernel [19], then averaged axially over a 2 mm depth-of-field centered at the focal depth. The resulting displacement data were processed using standard methods, including reverberation removal [18], zero-padding, a temporal Tukey window, and a 1 kHz low pass filter.

C. Group SWS from Fractional Derivative Signals

To obtain the gSWS for each fractional derivative order, the following steps were taken. First, the particle displacement at each lateral position was converted into the temporal Fourier domain, and differentiated according to (7) for each desired derivative order ν , then transformed back into the time domain to give the shear wave signal $S_\nu(r, t)$ from (6). This signal was then upsampled to a pulse repetition frequency of 40 kHz. Finally, the group speed for each derivative order was determined using the Radon sum method for a lateral range from 5 to 15 mm [20]. The Radon sum method is more robust than the time-to-peak method used by Rouze *et al.* in [12], and thus was used for this work in anticipation of future *ex vivo* and *in vivo* applications requiring robustness. The results from Rouze *et al.* are expected to hold for any gSWS calculation method, as long as a consistent method is used throughout the analysis [21].

The minimum distance in the lateral range was chosen to ensure that all analysis was performed outside the near-field of the source and that the shear wave signal at all derivative orders was fully captured without any artifacts from reverberation. The maximum distance was chosen so that the shear wave signal was still visible (above the noise) in all experimental acquisitions. Similar lateral ranges (10mm, with a minimum distance based on avoiding near-field effects and the ability to capture the waveform after the push reverberation) have been used by [22] and [23].

Sample shear wave signals experimentally measured in a viscoelastic phantom are shown in Fig. 1. Each column represents a derivative order as labeled. Note that derivative orders 0, 1 and 2 correspond to particle displacement, velocity and acceleration, respectively, and derivative orders 0.5 and 1.5 appear as intermediate steps between the integer derivative orders in all rows. The first row shows selected wave profiles through time at given positions. Normalized signals are displayed, but typically particle displacement is measured in microns, and other derivative orders have units of $\mu\text{m/s}^\nu$. The second row shows the entire normalized shear wave signal over space and time as well as the Radon sum trajectory, and is labeled with the gSWS as determined by the Radon sum method. The third row shows the two dimensional Fourier transforms (2DFT) of the signals in the second row. We notice that with increasing derivative order in the second row, the wave profiles become narrower and appear more susceptible to noise. While these characteristics are visible in the space-time display, the Radon sum method is able to determine the wave trajectory in an accurate manner. Additionally, the shape of the Fourier transform remains similar throughout the derivative orders, but higher temporal frequencies progressively account for more of the energy content of the wave.

D. Fractional Derivative Material Model Fitting

Material parameter values are determined from gSWS measurements through a lookup table specific to each material model investigated. These lookup tables are generated using a similar procedure as described in Rouze *et al.* [12]; that is, using a model for shear wave propagation under the assumption of an axially-infinite excitation. This equation ([12],(24)) requires as inputs a known shear modulus and excitation source geometry. The form of each material model's shear modulus from its respective parameters is given in Section II-E. The excitation source geometry is determined using a Fourier decomposition of a Field II model of the ARFI push, using the lateral-elevational force profile at the axial focal depth for the desired experimental push geometry and focal configuration [16]. This source geometry term is numerically estimated, and then used in the otherwise-analytic equation for cylindrical shear wave propagation. Note this procedure differs from the simulations used above: here, the push is assumed to be axially infinite ([12], (28)), while in the Green's function simulations, the full 3D force profile is modeled. The Green's function simulation is prohibitively time consuming for the plethora of materials modeled in the creation of the look up tables, leading to the use of the simplified axially-infinite propagation model.

To avoid introduction of artifacts from the post-processing steps, both the shear wave signals calculated from the axially-infinite propagation model and the experimental shear wave signals were analyzed with the same post-processing steps to: remove reverberation, filter, and measure the group speed. These steps were performed for a range of material parameters for each of the six models tested, creating six separate lookup tables of gSWS as a function of fractional derivative order. The range of parameters was chosen to cover the expected material properties seen in healthy and diseased livers, which the QIBA phantoms were designed to model. Note that at extreme material properties, shear wave propagation is difficult to measure with current experimental set-ups regardless of technique. These lookup tables were then used to fit experimental data. Because of the many fractional derivative orders used, the inversion of the lookup tables used by Rouze *et al.* [12] does not apply. Instead, the most accurate material parameters for a given model were considered those with the least summed squared difference between the propagation model and experimental gSWS measurements across all derivative orders tested (i.e. best fit to curve in Fig. 3). Twenty-one derivative orders were used, from zero to two in steps of 0.1.

E. Material Models

The six material models investigated are the Voigt model, the standard linear model, the springpot model, the fractional Voigt model, the linear attenuation model, and the power law attenuation model. The first four of these are composed of theoretical springs, dashpots, and springpots, and thus, have shear moduli equations easily expressed in the frequency domain. They are as follows [15]:

$$\begin{aligned}\mu_{\text{Voigt}}(\omega) &= \mu_0 + i\omega\eta \\ \mu_{\text{standard linear}}(\omega) &= \frac{\mu_1\mu_2 + i\omega\eta(\mu_1 + \mu_2)}{\mu_2 + i\omega\eta} \\ \mu_{\text{springpot}}(\omega) &= \mu_p(i\omega)^p, \quad 0 < p \leq 1 \\ \mu_{\text{fractional Voigt}}(\omega) &= \mu_0 + \mu_p(i\omega)^p, \quad 0 < p \leq 1\end{aligned}$$

Note the model described as fractional Voigt here is equivalent to the Kelvin-Voigt fractional derivative model as considered in [24].

For the remaining two models (linear attenuation model and power law attenuation model), we assume the form of the shear attenuation, and then find the phase velocity using the Kramers-Kronig relations with one or two subtractions, depending on the power [13]. For the power law case, the attenuation is given by $\alpha(\omega) = \alpha_0 \omega^p$ ($0 < p < 2$), and the phase velocity is given by:

$$c(\omega) = \frac{1}{\frac{1}{c(\omega_0)} + \alpha_0 \tan\left(\frac{\pi p}{2}\right)(\omega^p - \omega_0^p)} \quad (8)$$

In practice, this equation is rewritten to replace the α_0 with $c'(\omega_0)$, using:

$$\alpha_0 = \frac{-c'(\omega_0)}{c^2(\omega_0) \tan\left(\frac{\pi p}{2}\right)(p-1) \omega_0^{(p-2)}} \quad (9)$$

This substitution results in a equation analogous to (5), and leaves the parameters to be fit as $\alpha(\omega_0)$, $c'(\omega_0)$, and the attenuation power p . The reference frequency (f_0 , $\omega_0 = 2\pi f_0$) was chosen as 200 Hz, although any frequency within the bandwidth could be used.

In the linear attenuation case, the power p is set to a fixed value of 1, leaving only variables $\alpha(\omega_0)$ and $c'(\omega_0)$. Solving this special case using one subtraction leads to equations (4) and (5).

F. Phase Velocity Material Model Fitting

As a point of comparison, the phase velocity of the shear wave data was also considered and parameters fit using the appropriate $\alpha(\omega)$ equation for each material model. The phase velocity was determined using the spatial frequency corresponding to the peak of the 2DFT as described by [9] and [25]. The same axial depth and lateral range was used for constructing the 2DFT signal as was used for the gSWS measurements. For each frequency, phase velocities outside the range of three times the standard deviation were discarded as outliers. Then, for each material model, the remaining frequencies in each acquisition were fit using MATLAB's (The MathWorks, Natick, MA, USA) fit function with nonlinear least squares regression. The phase velocity fits were performed over the frequency range f_{min} to 600 Hz, with the lower cutoff chosen such that the asymptotic wave propagation assumption $kr \gg 1$ is satisfied [26]. A threshold of $kr > 1.5$ was chosen empirically and applied based on the average wavenumber k across acquisitions, leading to an f_{min} of 90, 120 and 150 Hz for the A, B, and C phantoms respectively. For the simulation data, this $kr > 1.5$ threshold leads to f_{min} of 120 Hz for all models except the Standard Linear and Voigt models, where the f_{min} is 110 Hz. The upper cutoff was chosen to exclude frequencies at which low SNR lead to high variability across acquisitions and phase velocity speeds inconsistent with measurements at lower frequencies. The mean and standard deviation of the fitted

parameters across the total number of acquisitions (100 in simulation, 16 in phantoms) were considered.

III. Results

A. Simulations with Ultrasonic Tracking

Each material model was fit to the simulation data using the gSWS and phase velocity methods. Note that the assumed material model for the fitting process matches the material model used to generate the underlying simulated data, i.e. different data used to evaluate the different material models. As expected, each fit model approximated the corresponding data well for both methods, and no major artifacts or fitting errors were observed. To quantify the goodness of each fit, the mean square error (MSE) was calculated for each fit, with MSE defined as $\frac{1}{df} \sum (x - x_{fit})^2$, where df is the number of degrees of freedom in the fit, and x is the data used for the fit (gSWS or phase velocity, with the data from all 100 speckle realizations fit simultaneously). The MSE was <0.01 for all models and both methods of fitting. However, because of the different underlying data used for fitting each model, it is inappropriate to compare the MSE of the different material models.

Fig. 2 shows the fit parameters from the simulations for both the fractional derivative and phase velocity methods. The errorbars show the mean and standard deviation of the parameter results from the 100 speckle realizations (fit independently). The dashed black line represents the true known value (0% error). Results are shown as a percentage error for each parameter to allow for comparison among the different material models on consistent axes (otherwise, parameter values from different models cannot be directly compared). Overall, minimal bias is observed, except for the fractional Voigt model. The standard deviations of the fractional derivative gSWS method are an average of 9% smaller than those of the phase velocity method.

B. Phantom Experiments

Fig. 3 shows the results of model fits to gSWS as a function of fractional derivative orders in phantom experiments. The discrete points represent the phantom group speed at a particular derivative order, with the error bars representing the mean and standard deviation across all 16 acquisitions. The experimental data is the same in each plot, and only the fits are different. We note that the Voigt model approximates the gSWS over the derivative orders analyzed fairly well, but it does not share the downward curvature of the experimental data. In contrast, the linear attenuation model and springpot model capture the curvature of the data more accurately. Likewise, each of the three-parameter models approximate the shape of the gSWS curves accurately. The fits pictured for each phantom were fit to all data points simultaneously. To quantify the goodness of each fit, the mean square error (MSE) listed on each plot shows the average gSWS MSE for all three phantoms, with MSE defined as $\frac{1}{df} \sum (x - x_{fit})^2$, where df is the number of degrees of freedom in the fit, and x is the data used for the fit (for this case, the gSWS). The MSE is lower for the Linear Attenuation model than the other two parameter models, and lower for the Power Law Attenuation model than the other three parameter models.

Fig. 4 shows the results of model fits to phase velocity curves. The errorbars represent the mean and standard deviation of phase velocity over all 16 acquisitions. Note that all the experimental data are the same, and only the fits are different. Again, the Voigt fit does not share the shape of the experimental data, while all other models appear to give better qualitative fits to the data. The average MSE is listed for each model (here, x in the MSE equation is the phase velocity), and is lower for the Linear Attenuation than the other two parameter models. In the phase velocity data, the three parameter model with the lowest MSE is the Standard Linear model.

Fig. 5 shows the parameter values corresponding to the fitted material models for both the fractional derivative and phase velocity methods. In this figure the error bars represent the standard deviation of the parameter fit results from each of the 16 acquisitions (fit independently). Metrics of stiffness in both methods trend with expectations of phantom stiffness from the group speed measurements. The manufacturer did not provide quantitative data for the phantom properties other than that the stiffness increased from the A to the B to the C phantom, which is consistent with the results from both methods. No consistent trend is seen in the viscosity parameters. While the true values for these phantoms are unknown, for most models, there is reasonable consistency between the parameters reconstructed. The viscosity parameter η in the Voigt model (top row, second column) however, demonstrates a distinct discrepancy in values. Note that overall, the errorbars from the fractional derivative method are similar to or smaller than the errorbars from the phase velocity method. Overall, the standard deviations of the fractional derivative gSWS method are an average of 39% smaller than those of the phase velocity method.

Additionally, Table II reports the average coefficient of variation across parameters for each model and each method, where coefficient of variation is defined as the standard deviation over the mean. Note that the true values for each parameter are not known, so the mean used is that calculated by each method, i.e. different means are used when computing the result for the fractional derivative gSWS method and the phase velocity method. The average coefficient of variation is lower using the fractional derivative gSWS method than the phase velocity method for all models, agreeing with the above comparison of standard deviations. Additionally, the variation increases for all three parameter models relative to their corresponding two parameter models.

IV. Discussion

We have described a new method for characterizing viscoelastic material properties based on group speed measurements of fractional derivatives of the shear wave signal. Traditional methods of viscoelastic characterization rely on the 2DFT or a single Fourier transform [9, 23, 25], however, these techniques can be difficult to interpret and tend to suffer in low SNR scenarios. Reducing viscoelastic analysis to a small number of model parameters can assist with comparison to clinical output as well as intuitive understanding. Thus, our goals for this new method were two-fold. First, by using an increased number of fractional derivative gSWS, we could use this method to compare the quality of fits of multiple different material models in phantoms, including both two-parameter and three-parameter models. Second, we

hypothesized that using a group speed based method, as opposed to a phase velocity method, would lead to more robust characterization of higher order parameters.

It is first helpful to consider the limits on the accuracy of fitting material parameter values using shear wave based methods. Because of the limited frequency bandwidth of an acoustic radiation force excitation, only a certain range of the material frequency response can be analyzed, regardless of whether gSWS or phase velocities are being considered. In general, the two-parameter models (Voigt, springpot, and linear attenuation) are well defined within the range of the fractional derivative gSWS lookup table. However, for certain parameter combinations in the three-parameter material models, the phase velocity curves are similar across the frequency range interrogated. Because the fractional derivative gSWS are effectively weighted sums of the phase velocity frequency response, this leads to similar gSWS curves for different parameters. This can be evaluated in abstract, but is also demonstrated in the simulation data with known material parameters. Specifically, this similarity in gSWS curves leads to the discrepancies seen in the fractional Voigt model (Fig. 2, bottom row), where small differences in the phase velocity curves or gSWS values across speckle realizations can lead to large differences in the parameters determined, and thus high degrees of variability and bias. An attenuation measurement may help differentiate these parameters; however, using speed-based methods alone, certain parameter combinations cannot be uniquely identified. This can influence the choice of material model used for viscoelastic characterization by shear wave imaging. Ideally, all model parameters will be well defined by the speed-based frequency response within the given bandwidth.

Having investigated the limitations of these models, we then used the fractional derivative gSWS method and the phase velocity fitting method to evaluate the applicability of these models to the viscoelastic phantoms tested. Presumably, we would expect reasonable agreement within and between the methods if the model is correct, as is seen in the simulated data with known and matched model fitting (Fig. 2). Additionally, a lower parameter model is preferred if it can be used without significantly sacrificing accuracy, to simplify any potential clinical application and avoid overfitting.

As noted in the results section, the Voigt model has a poor fit to both the fractional derivative gSWS and phase velocity curves. This was also indicated in Rouze et al [12]. Evaluating the resulting parameters from the two methods reinforces the challenges of the Voigt model (Fig. 5, top row). The phase velocity method and fractional derivative gSWS method demonstrate a clear discrepancy in the reconstructed viscosity values. This indicates the Voigt model is not able to capture the phantom's frequency response consistently across analysis methods, and confirms that it is not an appropriate model for these phantoms. In contrast, the other five models demonstrate reasonable agreement between the two methods.

The mean square error statistics indicate that the three-parameter models perform better than any of the two-parameter models even when accounting for the appropriate degrees of freedom. However, these metrics only account for fitting all 16 acquisitions simultaneously. In a clinical setting, a large number of independent speckle realizations may not be attainable, and so reliable measurements from a small number of acquisitions is necessary. The phantom parameter reconstructions from each individual acquisition give a sense of

variability within a model (Fig. 5, Table II). Among the two parameter models tested, the linear attenuation model has the lowest MSE and the lowest coefficient of variation of all models for the fractional derivative gSWS method. The Voigt model has a lower coefficient of variation for the phase velocity method, but this model is not preferred for the reasons discussed above. Additionally, the linear attenuation model has the advantage of having a clear and reliable identification of a stiffness-based parameter, c_0 . For these reasons, we recommend the use of the linear attenuation model over the springpot model or Voigt model when characterizing these viscoelastic phantoms with a two parameter model. Furthermore, we observe that while the three-parameter models have a lower overall MSE, when fitting individual acquisitions, the parameter values they reconstruct are more variable than when only two parameters are fit (Fig. 5). This study suggests that extending to three parameter models is of limited utility under current SWEI SNR and bandwidth limitations, though higher quality data may allow for more reliable three-parameter model analysis. Instead, the results of this comparison indicate that an appropriate two parameter model (for these phantoms, the linear attenuation model) can accurately characterize the material properties with similar MSE and lower variability compared to the three parameter models investigated.

It should be noted that while these phantoms were designed to model the viscoelastic properties of human liver and the stiffness is in the correct range [27], the phantoms are not necessarily an accurate model of human tissue to a higher order. A similar investigation to the one described here could be performed in tissue to validate the type of model most appropriate, and to determine whether the levels of viscosity present in these phantoms are similar to the viscosity expected in tissue. Additionally, it is worth noting that the optimal material model for SWEI characterization may be different from models used in traditional mechanical testing or in MR elastography, due to the differences in frequency ranges considered for the different techniques [24].

In addition to material model comparison, this study demonstrates the robustness of the fractional derivative method. Note that for the phantoms, which are an unknown material, we are also subject to any error in the material model relative to the true material properties of the phantom. However, in the simulations, we performed the fits to a material model that perfectly matches our material model, and so only the effects of tracking are exhibited.

We see in Fig. 2 for the simulated data that the fractional derivative approach produces low error and small variability for all models tested with the exception of the fractional Voigt model. The variability for the phase velocity method is higher than or similar to that of the fractional derivative method in all cases. The same statement can be made about the variability in the phantom data indicated in Fig. 5: while the results differ from model to model, the fractional derivative gSWS method is more repeatable than the phase velocity based characterization. We hypothesize that the basis of this effect is the susceptibility to noise when taking the 2D Fourier transform with limited energy at each frequency. The fractional derivative group speed method weights different frequencies when taking the fractional derivatives, but considers these weighted signals only as a sum. While the higher derivative orders do exhibit some apparent amplification of noise, as would be expected, the main group shear wave signal is still visible and accurately estimated by the Radon sum

group speed algorithm, as is indicated in Fig. 1. Thus, we find this technique tolerates noise well while still providing information about the relative phase velocity information. As such, we conclude that the fractional derivative gSWS technique provides a robust alternative to Fourier transform techniques in phantoms and simulations. Further work will evaluate the robustness of this new technique for *in vivo* viscoelastic characterization.

V. Conclusions

This paper presents a method to characterize viscoelastic materials using group speed measurements at a series of fractional derivative orders, allowing for analysis using two- and three-parameter material models and comparison between material models. Viscoelastic phantom data are analyzed with a nonlinear fitting algorithm incorporating an axially-infinite model of shear wave propagation, and the appropriate parameters for six separate material models were determined. For these viscoelastic QIBA phantoms, we find the linear attenuation model is a more accurate alternative to the Voigt model, a conclusion which agrees with previous work characterizing these phantoms. In addition, we find higher variability of estimated parameters in three-parameter material models, thus limiting the utility of higher order models with current SWEI systems. Finally, we find the fractional derivative method to give more repeatable measurements than fitting phase velocity curves, for both experimental phantom data and in simulations with known material parameters. These same simulations show the fractional derivative method to have minimal bias for most material models. Thus, we conclude that the fractional derivative group speed method is a more robust alternative to fitting phase velocity curves for viscoelastic phantom material characterization.

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Fig. 1.

Examples of experimentally acquired shear wave signals in a viscoelastic phantom at derivative orders $\nu = 0.0, 0.5, 1.0, 1.5$ and 2.0 . The top row shows normalized wave profiles through time at lateral positions of 6, 8, 10, 12 and 14 mm. The middle row shows the normalized spatial temporal shear wave, with the group shear wave trajectory indicated by the white line, and the gSWS given in the title. The bottom row shows the normalized two dimensional Fourier Transform of the shear wave signal, and demonstrates the change in frequency content with derivative order.

Fig. 2.

Parameter value results for the simulations, expressed as percent error. Raw parameter values used to run the simulations can be found in Table I. Each row shows the results from 100 simulated speckle realizations from a different simulated material model, and each plot shows a parameter within the material model. From top to bottom, the models are: Voigt, standard linear, linear attenuation, power law attenuation, springpot, and fractional Voigt. Each material model fit was only performed on the simulation using that same material model, thus, the rows in this figure are independent. The blue points show the results from the fractional derivative fitting, and the red points indicate the results from the phase velocity fitting. The errorbars show the mean and standard deviation over 100 speckle realizations. The dashed black line shows zero percent error. Note that overall, bias is low, and the errorbars from the fractional derivative method are similar to or smaller than the errorbars from the phase velocity method.

Fig. 3.

Fractional derivative gSWS curves and fits from the experimentally acquired viscoelastic phantom data. These graphs show the gSWS as a function of fractional derivative order. The experimental data are the same for all six graphs. The three phantoms (A, B, and C) are shown in different colors, and the errorbars show the mean and standard deviation across 16 acquisitions. The black lines show the respective best fit gSWS curves for a given material model, considering all acquisition points simultaneously. For each model, the mean square error (MSE) values show the average of the MSE values for the 3 phantoms.

Fig. 4.

Phase velocity curves and fits from the experimentally acquired viscoelastic phantom data. This figure shows the phase velocity as a function of frequency, computed with standard methods [9, 25]. As in Fig. 3, the experimental data are the same for all six graphs. The three phantoms (A, B, and C) are shown in different colors, and the errorbars show the mean and standard deviation across 16 acquisitions. The black lines show the respective best fit phase velocity curves for a given material model, considering all acquisition points simultaneously. For each model, the mean square error (MSE) values show the average of the MSE values for the 3 phantoms.

Fig. 5.

Parameter value results for experimentally acquired viscoelastic phantom data. Each row shows the results from a different material model, and each graph a parameter within those models. From top to bottom, the models are: Voigt, standard linear, linear attenuation, power law attenuation, springpot, and fractional Voigt. The blue points show the results from the fractional derivative fitting, and the red points indicate the results from the phase velocity fitting. For both, the errorbars show the mean and standard deviation over 16 experimental acquisitions.

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TABLE I

Green's Function Simulation Parameters

Voigt	$\mu_0 = 5 \text{ kPa}$	$\eta = 1 \text{ Pa} \cdot \text{s}$	$\eta = 1 \text{ Pa} \cdot \text{s}$
Standard Linear	$\mu_1 = 5 \text{ kPa}$	$\mu_2 = 4 \text{ kPa}$	
Linear Attenuation	$c_0 = 2.5 \text{ m/s}$	$c'_0 = 1.25 \text{ m/s/kHz}$	
Power Law Attenuation	$c_0 = 2.5 \text{ m/s}$	$c'_0 = 1 \text{ m/s/(kHz)}^p$	$p = 1.25$
Springpot	$\mu_p = 1.5 \text{ kPa} \cdot \text{s}^p$	$p = 0.2$	
Fractional Voigt	$\mu_0 = 2 \text{ kPa}$	$\mu_p = 0.5 \text{ kPa} \cdot \text{s}^p$	$p = 0.3$

TABLE II

Average coefficient of variation across estimated parameters for all 3 QIBA phantoms for each material model that was explored

Model	Fractional Derivative gSWS (%)	Phase velocity (%)
Voigt	11.0	12.0
Standard Linear	13.8	44.0
Linear Attenuation	7.9	13.0
Power Law Attenuation	10.0	19.7
Springpot	9.4	36.4
Fractional Voigt	33.1	73.7