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The role of physical digit representation and numerical magnitude representation in children's multiplication fact retrieval

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Abstract

Arithmetic facts, in particular multiplication tables, are thought to be stored in long-term memory and to be interference prone. At least two representations underpinning these arithmetic facts have been suggested: a physical representation of the digits and a numerical magnitude representation. We hypothesized that both representations are possible sources of interference that could explain individual differences in multiplication fact performance and/or in strategy use. We investigated the specificity of these interferences on arithmetic fact retrieval and explored the relation between interference and performance on the different arithmetic operations and on general mathematics achievement. Participants were 79 fourth-grade children ($M_{\text{age}} = 9.6$ years) who completed a products comparison and a multiplication production task with verbal strategy reports. Performances on a speeded calculation test including the four operations and on a general mathematics achievement test were also collected. Only the interference coming from physical representations was a significant predictor of the performance across multiplications. However, both the magnitude and physical representations were unique predictors of individual differences in multiplication. The frequency of the retrieval strategy across multiplication problems and across individuals was determined only by the physical representation, which therefore is suggested as being responsible for memory storage issues. Interestingly, this impact of physical representation was not observed when predicting performance on subtraction or on general mathematical achievement. In contrast, the impact of the numerical magnitude representation was more general in that it was observed across all arithmetic operations and in general mathematics achievement.

Keywords

Multiplication; Interference parameter; Problem size; Numerical magnitude representation; Arithmetical knowledge; Numerical cognition development

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Introduction

Arithmetic facts are suggested to be organized in an interrelated network in long-term memory, which allows individuals to quickly retrieve the answers to simple arithmetic problems such as 3×4 . This retrieval strategy in calculation is particularly useful in adult daily life because numbers and calculations are used to act on and adapt to the surrounding environment. In addition, arithmetic fact retrieval is necessary for more complex calculations, which would be laborious if counting procedures were required for intermediate calculation steps. Retrieval use depends on individual variability; however, factors such as operation play a large role, with retrieval being most common for single-digit multiplications and, to a lesser extent, additions (Bailey, Littlefield, & Geary, 2012; Barrouillet, Mignon, & Thevenot, 2008; Campbell & Xue, 2001; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Grabner et al., 2009; Imbo & Vandierendonck, 2008; Lemaire & Siegler, 1995; Vanbinst, Ceulemans, Ghesquière, & De Smedt, 2015). The extensive training of multiplication tables and simple additions in primary school probably explains these operation differences in retrieval. In spite of this extensive training, substantial individual differences are found in this ability to retrieve arithmetic (multiplication) facts from memory (Dowker, 2005; Lemaire & Siegler, 1995; Mabbott & Bisanz, 2003; Vanbinst, Ceulemans, et al., 2015). Recently, De Visscher and colleagues (De Visscher & Noël, 2013; De Visscher & Noël, 2014a, 2014b; De Visscher, Szmalec, Van Der Linden, & Noël, 2015) suggested that these individual differences in arithmetic fact learning might stem from the variability in an individual's sensitivity to similarity interference in memory. This sensitivity to interference is largely determined by two factors, namely the general sensitivity to interference in memory and the quality of the specific representations that are being manipulated. Indeed, representations that are fuzzy are less distinctive and more susceptible to interference. The network interference theory of Campbell (1995) assumes that there are two types of representations that can be sources of interference in arithmetic problems: the physical code structure, which includes the operands and operation sign, and the magnitude code representation, which corresponds to the magnitude of the answer. The total interference of one problem then corresponds to the sum of the similarity of the physical representation and the magnitude representation of the problem with the other problems. This model of an adult arithmetic fact network thus accounts for the variability in performance and retrieval across problems. However, little is known about the interference taking place during the development of the arithmetic fact network or about the individual differences stemming from these interferences. The current study therefore aimed at investigating the interference related to the physical code (digits) and the interference related to the numerical magnitude representation involved in multiplication facts.

The interference related to the physical code was recently studied by De Visscher and Noël (2014b). Adopting a developmental perspective, they investigated the proactive interference—that is, the interference coming from stored items affecting the current items to be remembered—that stemmed from the *physical similarity* across problems. These authors measured the proactive interference weight of the physical representation that affected each multiplication problem by computing the digit similarity between a problem and the previously learned problems based on the theory of Oberauer and Lange (2008). Concretely,

all possible combinations of two of the three or four digits that compose the association of a problem with its answer were looked for in the previously learned problems (according to the learning order of tables from 2 to 9) to estimate the degree of proactive interference in learning a specific problem. For instance, for the problem $6 \times 8 = 48$, different combinations of the four digits composing the problem (4688) were already present in 11 previously learned problems (e.g., $\underline{6} \times 7 = \underline{42}$, $3 \times \underline{6} = 18$), which gives the problem a proactive interference score of 11. This measure, called the interference parameter, was shown to predict the variability of performance across problems in third graders, fifth graders, and undergraduate students beyond the problem size (De Visscher & Noël, 2014b). That is, the higher the interference parameter of a problem, the lower the performance on that problem. This supports the idea that proactive interference is created by the similarity of physical representation during the storage of arithmetic facts. Moreover, De Visscher and Noël (2014b) showed that the sensitivity to the interference parameter explained a significant part of the individual variability of multiplication performance in third, fourth, and fifth graders as well as in undergraduates, revealing that the more sensitive someone was to the interference parameter, the weaker her or his performance in multiplication was.

Turning to the role of numerical magnitude representations in arithmetic, there is consistent evidence that these representations play a role in individual differences in mathematics achievement (for a review, see De Smedt, Noël, Gilmore, & Ansari, 2013; for a meta-analysis, see Schneider et al., 2016). Recent data also indicate that these magnitude representations correlate with and predict individual differences in simple arithmetic (Vanbinst, Ceulemans, et al., 2015; Vanbinst, Ghesquière, & De Smedt, 2015), although these studies were limited to the retrieval of addition and subtraction facts. Hardly any studies have examined this issue in the context of multiplication, presumably because studies in brain-damaged patients, who had numerical magnitude impairments but spared multiplication facts knowledge, have suggested that numerical magnitude representations were not necessary for solving arithmetic facts (e.g., Lemer, Dehaene, Spelke, & Cohen, 2003). Interestingly, a recent study by Didino, Knops, Vespignani, and Kornpetpanee (2015) provided evidence in favor of the involvement of numerical magnitude representations in multiplication. This study investigated the operand-related lure effect occurring in a multiplication verification task, namely that false answers (lures) belonging to the same multiplication table as the presented problem require more time to be rejected compared with unrelated lures. Didino and colleagues compared two types of operand-related lures, which were either larger [$X \times (N+1)$ lure] or smaller [$X \times (N-1)$ lure] than the correct answer. For instance, the problem 3×7 was presented with the lure 24 [i.e., the $X \times (N+1)$ condition] and with the lure 18 [i.e., the $X \times (N-1)$ condition]. Although the absolute difference between the correct answer and the lures was the same in both conditions, the $X \times (N+1)$ condition resulted in significantly higher latencies than the $X \times (N-1)$ condition, which suggests the involvement of magnitude representations in multiplication. Indeed, magnitude representations have been conceptualized as numerosities on a left-to-right mental number line on which numerosities follow a psychophysical scale, so that large numerosities are more compressed and thus show more overlap with each other compared with small numerosities (Dehaene, 1992). In other words, larger lures have fuzzier magnitude representations than smaller lures, which explains the findings of Didino and

colleagues (2015) and exemplifies that numerical magnitude processing can interfere when solving multiplication facts.

On a related note, numerical magnitude representations might also explain the well-known and robust problem size effect in arithmetic, namely that small problems are solved more easily than large problems (De Brauwer, Verguts, & Fias, 2006; Zbrodoff & Logan, 2005). This effect is already present in children and decreases until sixth grade, where it is similar to that of adults (De Brauwer et al., 2006). Various explanations of this problem size effect have been proposed such as frequency effects (small problems are more frequent than large problems; Ashcraft, 1987; Ashcraft & Christy, 1995; McCloskey & Lindemann, 1992), the number of wrong answers associated with a problem (large problems are more often associated with wrong answers, and therefore their association with the correct answer is less strong/peaked but instead is spread over the different wrong answers; Siegler, 1988), the number of cooperating neighbors (small problems have more cooperating neighbors; Verguts & Fias, 2005), and strategy effects (small problems are more frequently solved by fact retrieval, which is more efficient than procedural strategies, with the latter being more often used in large problems; Zbrodoff & Logan, 2005). The problem size effect, however, might also be explained by numerical magnitude representations. Indeed, Campbell (1995) suggested that each arithmetic problem activates the numerical magnitude representation of its answer. In view of the above-mentioned mental number line (Dehaene, 1992), large numbers have less precise representations than small numbers, and consequently the magnitude representations of large problems' answers will be more overlapping than those of the small ones, which could account for the problem size effect. As a result, individual differences in numerical magnitude representations might be related to individual differences in the problem size effect, with individuals with more precise numerical magnitude representations showing smaller problem size effects; however, this hypothesis has not been tested empirically.

To sum up, arithmetic fact retrieval is known to be interference prone, but little is known about which representations, physical and/or magnitude, cause a detrimental interference effect in storing arithmetic facts. Even less is known regarding the impact of these types of interference on the performance across problems and across individuals. In the current study, we aimed at measuring the unique influence of the physical representation and the numerical magnitude representation on multiplication fact performance in children. We studied 79 fourth-graders who had already stored a lot of multiplication facts but who were still in the process of consolidating these facts in long-term memory. All children solved a multiplication production task with verbal strategy reports, which allowed us to register speed and accuracy as well as the strategy used for each multiplication problem. We further measured children's magnitude representation of the products by administering a products comparison task. We also evaluated children's general intellectual abilities and their number typing speed, which were included as covariates in the analyses to account for potential confounds of these factors. Finally, we investigated children's performance in all arithmetic operations and their general mathematics achievement in order to test for the specificity of the interference effects. We first analyzed the interference effect of the physical representation (interference parameter score) and of the problem size on multiplication performance across problems. Next, we investigated whether the sensitivity to the physical

representation (to the interference parameter) and to the numerical magnitude representation (using both the sensitivity to the problem size and the products comparison) explained individual differences in multiplication performance and strategy use. Finally, we examined the specificity of the two types of interference across individuals by testing their link with performance on other arithmetic operations and on general mathematics achievement.

Method

Participants

Participants were 86 fourth-grade children from six Flemish primary schools. All parents of the participating children gave informed consent. This research was approved by the social and societal ethics committee of KU Leuven. Among the original sample, 3 participants were excluded from the analyses: 2 who did not pay attention during task administration and 1 who did not understand the instructions. In the products comparison task, 1 child scored nearly at the chance level (55%) and therefore was excluded. When exploring the distribution of all variables used, 3 children were considered as outliers in the number typing task (see online Supplementary material) and therefore were excluded from all analyses. The final sample consisted of 79 children (50 boys and 29 girls) with a mean age of 116 months ($SD = 4.3$). All participants had normal intellectual abilities as determined by performance on Raven's matrices ($M = 38.97$, $SD = 7.3$) (Raven, Court, & Raven, 1992); none of the children obtained a score lower than $-1 SD$.

Materials

Multiplication production—The 36 possible combinations of the digits 2 to 9 (no commutative pairs) were used in this task. The position of the larger operand (first vs. second) was counterbalanced so that the mean size of the products was similar in both conditions (larger operand first vs. larger operand second). Each trial started with a 1000-ms fixation cross, followed by the multiplication problem that was displayed in white on a blue background in the center of the screen until participants responded. Participants needed to type the answer on a numerical keypad, which appeared at the right side of the problem, and needed to validate their answer with the Enter key (marked with a green sticker). After responding, children were asked whether they had retrieved the answer, used a procedural strategy, or did not know how they had solved the item. Three pictures illustrated the three possibilities, and examples were given before the start of the task. Children's answers were recorded by the experimenter except if behavioral observations were in conflict with children's answers (e.g., counting on fingers while saying the answer was retrieved), in which case the strategy was classified as "other". The problems were presented in a pseudo-randomized order so that two successive trials never shared similar operands or answers.

Accuracy and reaction time (RT) were recorded. The RT corresponded to the time elapsed from the presentation of the problem until the Enter key was pressed by children, validating the answer. We corrected RT for answering to multiplications by subtracting the time for retyping the numbers displayed on the screen (see number typing speed task below). This way of collecting RTs in multiplication production has several advantages compared with the oral production of answers. First, when using oral production, several trials typically

need to be dismissed because of technical issues with the voice key (false or non-detection of the answer), whereas very few problems occur with the numerical keypad. Second, the time to produce numbers (cognitive and motor processing) is taken into account, which is usually not the case in oral response, where the time of oral production (processing before the articulation) is not considered. This typing method also avoids the bias found in oral response when participants continue calculating after they started to pronounce the decade of the answer (e.g., “twentyyyy ... four”), whereas the voice key detects the onset of the answer. Third, the typing method reduces participants’ anxiety, whereas oral responses seem to increase stress (because of the presence of the experimenter). Finally, children find the typing method fun, which fosters their motivation and attention. This mode of response is a valid method and was used in several other studies (e.g., De Visscher & Noël, 2013, 2014b; De Visscher et al., 2015; Fendrich, Healy, & Bourne, 1993).

Number typing speed—Because RTs in the multiplication task also involve the speed of number typing, we administered a number typing task in which participants needed to retype (and validate with the Enter key) numbers displayed in the center of the screen. The format, procedure, and data logging were exactly the same as those of the multiplication production. The numbers were the 31 different products that formed the answers to the multiplication production task. This provided us with a measure of the number typing speed (including motor speed processing and number processing), which was subtracted from the multiplication production speed.

Products comparison—To measure the numerical magnitude representation of the products, a symbolic number comparison task was administered. After a 1000-ms fixation cross, two Arabic numbers were displayed on each side of a vertical line situated in the center of the screen. Children needed to press the key corresponding to the side of the largest number (i.e., the “s” key for the left answer and the “l” key for the right answer). The numbers remained on the screen until children answered. To directly test whether the numerical magnitude representation of the products was influencing multiplication performance, the stimuli were only composed of the products of all tables from 2 to 9. Each product was compared with its closest products, providing us with 30 pairs that were repeated twice (e.g., 24 was compared with 21 or 25; see Supplementary material for the list of stimuli). The side of the largest number was counterbalanced. The order of the 60 trials was pseudo-randomized so that no more than three successive answers were similar. Black numbers were displayed on a gray background. Both accuracy and RT were recorded. Because several studies cast doubt on the use of distance or ratio effects and reported higher explanatory power for the overall RT or accuracy (Lyons, Nuerk, & Ansari, 2015; Schneider et al., 2016), the measure used in the analyses was the global median RT in the task. Accuracy was not used due to ceiling effects.

Calculation and general mathematics achievement—We collected two standardized math ability measures: a timed calculation test and an untimed general mathematics achievement. The timed calculation task was the Tempo Test Rekenen (TTR [Tempo Test Arithmetic]; de Vos, 1992), which comprises five columns of arithmetic problems that increase in difficulty. The test includes one column of each operation

(addition, subtraction, multiplication, and division) and one column of mixed problems. A time limit of 1 min was allocated to solve each column. Children's general mathematics achievement was assessed using the (untimed) curriculum-based standardized mathematics achievement test from the Flemish Student Monitoring System (Dudal, 2000), the content of which corresponded to the mathematics curriculum until the middle of the fourth grade. The test included various aspects of mathematics such as number knowledge, understanding of operations, simple arithmetic, multi-digit calculation, word problem solving, measurement, and geometry. The school's teachers provided us with the scores of this general mathematics achievement test.

Intellectual ability—Raven's Standard Progressive Matrices (Raven et al., 1992) was administered to measure children's intellectual ability. In this test, children needed to indicate which piece of a puzzle fit into a pattern. Children were given a maximum of 45 min to complete the task, which was a sufficient amount of time for all children to finish the 60 questions.

Procedure

All participants took part in two sessions (i.e., one group-administered session and one individual session) comprising a collection of tasks in the context of a larger project. In the current article, only the experimental tasks of interest and the control tasks are reported. The group-administered session included Raven's matrices and the TTR. The individual session included the multiplication production, the number typing speed, and the products comparison. The collection of individual data took place in a quiet room at children's school.

Results

The results are reported in two major subsections. The first subsection investigates the effect of physical and numerical magnitude interference across the multiplication problems (averaged across individuals). We compare the explanatory power of the physical interference parameter and problem size on multiplication RT and on the probability of retrieval. The second subsection investigates the variability between individuals in multiplication and examines how this variability between individuals is explained by individual differences in the sensitivity to both interference from physical and numerical magnitude representations. We begin the second subsection by describing how individual sensitivity to these two types of interference was computed and then investigate the extent to which the degree of sensitivity to each type of interference, as well as the magnitude comparison speed, predicted the speed of solving multiplication problems, the retrieval percentage in multiplication, and the performance on the different operations in the calculation test and the general mathematics achievement test.

The RTs consisted of the RTs of correct trials only. We always used median RTs in order to account for potential outliers in RT.

Physical representation and numerical magnitude representation: Across problems

In the multiplication production task, the mean accuracy was 97.4% ($SD = 2.8$) and the average median RT across problems was 3938 ms ($SD = 967$). The average median RT in the number typing speed across problems was 2212 ms ($SD = 365$). By subtracting the median RT of the number typing speed from the median RT in multiplication solving, we obtained an average RT across problems corrected for motor speed ($M = 1727$ ms, $SD = 778$).

First, we tested whether the interference parameter and the problem size influenced the RT across the different multiplication problems. We did not analyze accuracy due to ceiling effects. Both the interference parameter, which measures the similarity interference of the physical code in learning (De Visscher & Noël, 2014b), and the problem size (products) were entered as independent variables in a linear regression on the corrected average median RT across multiplication problems (i.e., 36 problems). The results showed a large and significant effect for only the interference parameter (Table 1). The problem size zero-order correlation of .448 dropped to .144 when the interference parameter was taken into account due to some covariance (correlation between interference parameter and problem size: $r = .544$). The model explained 44% of variance in RT across multiplication problems.

Second, we tested the hypothesis according to which the similarity interference of the physical code (measured with the interference parameter) or the size of the problem influenced the storage of multiplication during the learning stage and consequently the probability of retrieval. To that end, we determined for each problem the percentage of correct trials for which individuals indicated to have used a retrieval strategy and predicted this percentage of retrieval by the problem size and the interference parameter.

The mean percentage of correct retrieval was 82.4% ($SD = 11.8$). The physical interference parameter was a strong and significant factor that determined retrieval across multiplications, whereas the problem size was not significant (see Table 2). However, the zero-order correlation of the problem size was $-.530$ and dropped to $-.224$ after adding the interference parameter to the model due to some covariance. The model explained 57% of the variance in percentage of retrieval across multiplication problems.

Physical representation and numerical magnitude representation: Individual differences

Accuracy was at ceiling levels in the multiplication production task ($M = 97.4\%$, $SD = 4.0$). The average of the median RT was 3920 ms ($SD = 827$). Across individuals, the average of the median RT in the number typing task was 2181 ms ($SD = 367$). The multiplication speed, corrected for the encoding and typing speed, was computed by subtracting the median RT in number typing from the median RT in multiplication. The average corrected multiplication speed was 1739 ms ($SD = 626$).

Following De Visscher and Noël (2014b), we computed the individual sensitivity to the physical similarity interference (interference parameter) and to the problem size (products) by entering the two factors in a multiple regression on the RT for each individual and took the respective slopes for the subsequent models. The average slope for the interference parameter was 123 ($SD = 111$) and for problem size was 23 ($SD = 32$); both were different

from zero [interference parameter slope: $t(78) = 9.886$, $p < .001$; problem size slope: $t(78) = 6.422$, $p < .001$]. Regarding the strategy use, children reported a retrieval strategy in 30 of 36 problems ($SD = 5.6$) on average.

In the products comparison task, the average accuracy was 93.6 ($SD = 4.4$) and the average median RT was 1101 ms ($SD = 169$).

Regarding the sensitivity to interference to numerical magnitude representation, two measures were considered: the problem size slope and the RT in products comparison. The problem size slope directly measured how sensitive one is to the size of the problem (slower as the product increases) in multiplication solving. The RT in products comparison measured how fast one is to compare the magnitude of close products, assuming that the faster someone is, the more precise her or his magnitude representation of products is. Our data indicated no relationship between these two variables, $r(77) = .109$, $p = .338$, suggesting that the quality of the magnitude representation of products is not related to the sensitivity to the problem size in multiplication solving.

To test the impact of the physical representation and the magnitude representation on the individual differences in multiplication, we ran a multiple regression on the corrected multiplication speed. The Raven score was entered in order to control for intellectual ability. In addition, the interference parameter slope, the problem size slope, and the RT in products comparison were entered as independent factors. This model, $F(4, 74) = 9.573$, $p < .001$, explained 34.1% of the variance. The interference parameter slope, which tapped the physical representation, and the two different measures related to the products' magnitude (i.e., problem size and products comparison speed) were significant (see Table 3). That is, the more children are sensitive to the physical representation (larger interference parameter) or to the numerical magnitude representation (larger problem size and longer RTs in comparing products), the slower they are in multiplication.

The model on the percentage of correct retrieval was the same except that it included the number typing speed measure as a measurement of global processing speed. Only the interference parameter slope was a significant predictor (see Table 4), which indicates that the more sensitive one is to the interference parameter (physical representation) in multiplication solving, the lower the percentage of correct retrieval is.

We further investigated performance on the standardized achievement tests. On the measure of calculation fluency (TTR), children on average correctly solved 23.9 additions ($SD = 3.7$), 21.8 subtractions ($SD = 4.4$), 23.0 multiplications ($SD = 3.9$), 20.3 divisions ($SD = 4.7$), and 21.5 mixed problems ($SD = 4.3$), which corresponds to normal scores in fourth grade. For each of the arithmetic operations, a multiple regression was run with five independent variables: Raven score, number typing speed (to have a measure of global speed), interference parameter slope, problem size slope, and RT products comparison (see Table 5). Results indicated that the interference parameter slope explained a significant part of the performance in addition, multiplication, and division. The problem size slope explained part of the variance in addition and multiplication, whereas performance in products comparison was a significant predictor in all five conditions.

The mean score of the general mathematics achievement test was 43.11 ($SD = 8.1$). Table 6 shows a multiple regression in which (untimed) general mathematics performance is predicted. Only Raven's matrices and the RT on products comparison emerged as unique and significant predictors that explain 41% of the variance, $F(5, 73) = 10.126$, $p < .001$.

Discussion

For a long time, interference effects have been revealed in arithmetic fact retrieval (e.g., Campbell, 1987; McCloskey & Cohen, 1989). More precisely in the network interference theory (Campbell, 1995; Graham & Campbell, 1992), similarity interference in arithmetic fact retrieval is predicted to stem from two types of representations: a physical code representation (digits) and a magnitude code representation. Studies of general memory processes have shown that the similarity of the stimuli creates interference at the learning stage (Lewandowsky, Geiger, Morrell, & Oberauer, 2010; Oberauer, 2009; Oberauer & Lange, 2008). These interference-based forgetting theories of memory might explain the development of arithmetic facts in children. The findings of these studies showed that when an item needs to be memorized, its similarity with the previously learned and stored items determines the memory trace of the current item. That is, the more similar the item to remember is to the stored items, the weaker its memory trace will be. Hence, the similarity interference can affect both the storage and retrieval phases.

Corroborating these theories, the detrimental effect of interference has recently been shown to affect the storage phase of arithmetic facts (and not only the retrieval phase), and the sensitivity to the physical similarity interference was found to be related to individual differences in arithmetic fact performance in children and adults (De Visscher & Noël, 2013, 2014a, 2014b; De Visscher et al., 2015). Much less is known about the impact of the numerical magnitude representation similarity on the storage and retrieval of arithmetic facts across problems and across individuals. The current study therefore investigated the unique impact of the physical representation and the numerical magnitude representation on solving multiplication facts. This was done in a sample of fourth-grade children, considering both performance and strategies.

First, the physical representation interference assessed via the interference parameter (De Visscher & Noël, 2014b) was found to explain variability of performance across multiplication problems. Moreover, the sensitivity to this physical similarity interference explained individual differences in multiplication. Only the interference parameter slope remained a significant predictor in the multiple regression, including the problem size slope, in multiplication speed across problems, indicating that the more interfering problems had longer RTs. These findings corroborate previous studies showing the detrimental interference effect of the physical similarity across problems, and also across individuals, in children as well as in adults (De Visscher & Noël, 2014b). In addition, our study showed that the physical similarity interference also determined the use of retrieval strategies; more interfering problems (across problems model) and individuals with higher sensitivity to physical interference had lower frequencies of retrieval use. Our findings constitute new evidence in favor of the hypothesis that similarity interference hampers individuals from

storing multiplication facts in long-term memory and consequently from retrieving them from memory (De Visscher & Noël, 2014b).

Second, our study aimed at assessing the role of the numerical magnitude representation in solving single-digit multiplication by using two measures: the sensitivity to the problem size and the performance in a products comparison task. Our findings show that the numerical magnitude representation plays a role in accounting for individual differences in multiplication speed but not in the use of retrieval strategy. More precisely, both the sensitivity to the problem size in multiplication and the speed in the numerical magnitude comparison tasks uniquely accounted for individual differences in multiplication speed.

Our findings also revealed that the sensitivity to the problem size in multiplication and the speed in products comparison did not correlate with each other, which indicates that they measure different processes. This is not surprising given that the problem size effect is multi-determined. Indeed, it might be due to a frequency effect (e.g., Ashcraft & Christy, 1995), to the distribution of association between the problem and the correct and produced wrong answers (Siegler, 1988), to the different proportion of retrieval versus counting-based strategies to solve the problems (LeFevre et al., 1996), or to the number of cooperating versus competing neighbor problems (Verguts & Fias, 2005). Thus, the sensitivity to the problem size might reflect not just the numerical magnitude processing but, more likely, a combination of the different factors mentioned above. By contrast, the RT in products comparison is probably a purer measure of the ability to process the magnitude of numbers.

Future studies are required to better understand the respective link of the sensitivity to problem size and of the numerical magnitude comparison ability with arithmetic fact performance. Nonetheless, as far as we know, our study is the first showing a clear relationship between numerical magnitude representation and simple multiplication ability in children. Although numerical magnitude representation has been largely investigated in relation to global math performance (for a review, see De Smedt et al., 2013; for a meta-analysis, see Schneider et al., 2016), the link between numerical magnitude representation and single-digit arithmetic has received much less attention. In our study, the importance of numerical magnitude representation is highlighted in the multiplication fact performance of fourth-grade children. This is in line with previous results suggesting that symbolic numerical magnitude processing is related to performance in single-digit problems (mixing additions, subtractions, and multiplications) in children (Bugden & Ansari, 2011; Reigosa-Crespo et al., 2012; Vanbinst, Ceulemans, et al., 2015).

Considering the performance in different arithmetic operations and in general mathematical achievement, the physical and numerical magnitude representations showed different impacts. Indeed, the physical representation was specifically related to those operations that are mainly tapping facts stored in memory (i.e., additions and multiplications) but not to subtractions or mixed operation calculations, which supposedly rely more on procedural strategies. Concomitantly, the percentage of retrieval in multiplication, across problems and across individuals, was uniquely determined by the interference parameter. It needs to be noted that the physical representation was also linked to division, which may reflect the close relationship between multiplication and division facts (Campbell, 1997; De Brauwer &

Fias, 2009). The sensitivity to the physical interference was not associated with the untimed general mathematics achievement that included word problems, geometry, and other mathematical knowledge than arithmetic facts. The physical representation therefore seems to have an important and specific role in storing and retrieving arithmetic facts in long-term memory.

Conversely, numerical magnitude representation showed less impact on pure memory retrieval but affected all different arithmetic operations as well as general mathematical achievement. These findings corroborate previous studies showing a relationship between numerical magnitude representation and math ability in general (De Smedt et al., 2013; Schneider et al., 2016) but also shed light on the importance of numerical magnitude representation in simple arithmetic. Although in braindamaged patients numerical magnitude representation did not seem strictly necessary for retrieving arithmetic facts, our findings suggest that it explains individual differences across all operations. Accordingly, our results show that numerical magnitude representation is contributing to arithmetic fact knowledge and therefore should be taken into account in teaching primary school children and in understanding and treating children with arithmetic fact deficit.

Although the relationship between numerical magnitude representation and single-digit multiplication solving is found in fourth graders, the future development of this association is unknown. It might be possible that with development, this association decreases as multiplication facts are increasingly based on retrieval in adults. The importance of the physical similarity therefore could increase. In De Visscher and Noël (2014b), the interference parameter indeed explained performance across problems and across individuals in undergraduates. Future studies should investigate these associations in longitudinal paradigms and in adults. Besides, these findings concern typically developing children, yet this association should be investigated in the context of atypical development of arithmetic fact network or of general math ability. More specifically, hypersensitivity to interference in memory has been shown in certain profiles of dyscalculia with arithmetic fact deficits (De Visscher & Noël, 2013, 2014a; De Visscher et al., 2015), and a deficit of numerical magnitude representation has also been shown in children with dyscalculia in several studies (see De Smedt et al., 2013, for a review). How interference of both representations could account for different profiles of dyscalculia should be further tested. Finally, models of the development of arithmetic facts should integrate both physical and numerical magnitude representations. Although the interference issue was part of most arithmetic fact models in adults, the interference effect in typical and atypical arithmetic fact learning received less attention and should be considered with regard to both physical and numerical magnitude representations.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Table 1
Linear regression with the interference parameter and the problem size (products) factors on the corrected average median RT across multiplication problems.

Factor	R^2	df	β	t	p -Value	Zero-order correlation	Partial correlation
Interference parameter	.442	2, 33	.585	3.775	<.001	.656	.549
Problem size (products)			.129	0.835	.410	.448	.144

Table 2
Linear regression with the interference parameter and the problem size (products) factors
on the percentage of correct retrieval across multiplication problems.

Factor	R^2	df	β	t	p -Value	Zero-order correlation	Partial correlation
Interference parameter	.572	2, 33	-.644	-4.742	<.001	-.741	-.637
Problem size (products)			-.179	-1.322	.195	-.530	-.224

Table 3
Multiple regression on the multiplication speed corrected.

Factor	R^2	df	β	t	p -Value	Zero-order correlation	Partial correlation
Raven	.341	4, 74	-.005	-0.051	.960	.045	-.006
Interference parameter slope			.452	4.437	<.001	.434	.458
Problem size slope			.244	2.457	.016	.153	.275
RT products comparison			.279	2.865	.005	.386	.316

Table 4
Multiple regression on the percentage of correct retrieval.

Factor	R^2	df	β	t	p -value	Zero-order correlation	Partial correlation
Raven	.113	5, 73	.017	0.153	.879	-.050	.018
Number typing speed			.207	1.444	.153	.081	.167
Interference parameter slope			-.362	-2.920	.005	-.268	-.323
Problem size slope			-.122	-1.039	.302	-.001	-.121
RT products comparison			-.010	-0.071	.944	.035	-.008

Table 5
Multiple regressions on additions, subtractions, multiplications, divisions, and mixed problems (TTR).

Addition:	Factor	R^2	df	β	t	p-Value	Zero-order correlation	Partial correlation
	Raven	.340	5, 71	.006	0.064	.950	.015	.008
	Number typing speed			-.116	-0.937	.352	-.425	-.111
	Interference parameter slope			-.222	-2.030	.046	-.268	-.234
	Problem size slope			-.205	-1.978	.052	-.205	-.228
	RT products comparison			-.374	-3.176	.002	-.507	-.353
Subtraction:	Factor	R^2	df	β	t	p-Value	Zero-order correlation	Partial correlation
	Raven	.244	5, 73	.086	0.823	.413	.083	.096
	Number typing speed			-.036	-0.271	.787	-.334	-.032
	Interference parameter slope			-.174	-1.520	.133	-.226	-.175
	Problem size slope			-.054	-0.500	.618	-.057	-.058
	RT products comparison			-.397	-3.135	.002	-.459	-.344
Multiplication:	Factor	R^2	df	β	t	p-Value	Zero-order correlation	Partial correlation
	Raven	.352	5, 72	-.124	-1.277	.206	-.118	-.149
	Number typing speed			-.225	-1.800	.076	-.463	-.208
	Interference parameter slope			-.222	-2.078	.041	-.276	-.238
	Problem size slope			-.258	-2.544	.013	-.245	-.287
	RT products comparison			-.247	-2.069	.042	-.438	-.237
Division:	Factor	R^2	df	β	t	p-Value	Zero-order correlation	Partial correlation
	Raven	.401	5, 73	.137	1.486	.142	.110	.171
	Number typing speed			-.031	-0.261	.795	-.412	-.031
	Interference parameter slope			-.355	-3.485	.001	-.369	-.378
	Problem size slope			-.173	-1.789	.078	-.130	-.205
	RT products comparison			-.419	-3.722	<.001	-.526	-.399
Mixed:	Factor	R^2	df	β	t	p-value	Zero-order correlation	Partial correlation
	Raven	.366	5, 73	.038	0.396	.693	.058	.046
	Number typing speed			-.121	-0.997	.322	-.444	-.116
	Interference parameter slope			-.085	-0.816	.417	-.200	-.095
	Problem size slope			-.005	-0.047	.962	-.052	-.006
	RT products comparison			-.499	-4.300	<.001	-.588	-.450

Table 6
Multiple regression on the general mathematics achievement test.

Factor	R^2	df	β	t	p -Value	Zero-order correlation	Partial correlation
Raven	.410	5, 73	.573	6.236	<.001	.570	.590
Number typing speed			.117	1.006	.318	-.140	.117
Interference parameter slope			-.081	-0.802	.425	.014	-.093
Problem size slope			-.097	-1.007	.317	-.108	-.117
RT products comparison			-.308	-2.754	.007	-.293	-.307