Efficient directionality-driven dictionary learning for compressive sensing magnetic resonance imaging reconstruction

Anupama Arun
Thomas James Thomas
J. Sheeba Rani
R. K. Sai Subrahmanyam Gorthi

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Anupama Arun,^a Thomas James Thomas,^a J. Sheeba Rani,^{a,*}
and R. K. Sai Subrahmanyam Gorthi^b

^aIIST Trivandrum, Department of Avionics, Kerala, India

^bIIT Tirupati, Department of Electrical Engineering, Andhra Pradesh, India

Abstract. Compressed sensing is an acquisition strategy that possesses great potential to accelerate magnetic resonance imaging (MRI) within the ambit of existing hardware, by enforcing sparsity on MR image slices. Compared to traditional reconstruction methods, dictionary learning-based reconstruction algorithms, which locally sparsify image patches, have been found to boost the reconstruction quality. However, due to the learning complexity, they have to be independently employed on successive MR undersampled slices one at a time. This causes them to forfeit prior knowledge of the anatomical structure of the region of interest. An MR reconstruction algorithm is proposed that employs the double sparsity model coupled with online sparse dictionary learning to learn directional features of the region under observation from existing prior knowledge. This is found to enhance the capability of sparsely representing directional features in an MR image and results in better reconstructions. The proposed framework is shown to have superior performance compared to state-of-art MRI reconstruction algorithms under noiseless and noisy conditions for various undersampling percentages and distinct scanning strategies. © 2020 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: [10.1117/1.JMI.7.1.014002](https://doi.org/10.1117/1.JMI.7.1.014002)]

Keywords: compressive sensing; dictionary learning; magnetic resonance imaging.

Paper 19167R received Jul. 15, 2019; accepted for publication Jan. 7, 2020; published online Jan. 24, 2020.

1 Introduction

Magnetic resonance imaging (MRI) is a noninvasive medical diagnostic imaging modality that enables visualization of the anatomical and physiological functions of the human body. Conventional MRI acquisition in the Fourier domain, commonly referred to as k -space acquisition, is inherently slow. Compressed sensing (CS)¹ advances the prospect of reduced scan time by undersampling the k -space, while faithfully reconstructing the MR slice. Unlike methods, such as those in Refs. 2 and 3, which employ redundant hardware, CS offloads complexity from the acquisition side to the recovery side. The theory of CS facilitates reconstruction of MR images from undersampled k -spaces by enforcing their sparse representation in some transform domain, provided the undersampling operator is incoherent with that transform domain.⁴ While random undersampling of the k -space using suitable trajectories ensures incoherence, suitable sparsifying transforms need to be adopted for better sparse representations. Thereby, CS theory permits MR image reconstruction using sparsity exploiting iterative nonlinear recovery algorithms.⁴ Let x be the P -pixel MR image to be sampled and $F_u \in \mathbb{C}^{m \times P}$ be the undersampling Fourier operator consisting of m rows of the Fourier matrix. CSMRI acquisition can be formulated as $S = F_u x$, where S is the sampled Fourier coefficients. The starting point for most algorithms is an initial reconstruction of the image obtained by taking the inverse Fourier transform of the undersampled k -space, assuming the unsampled locations to be zero. This is referred to as the zero-filled (ZF) image,⁴ and this image has incoherent artifacts, provided the undersampling is random. Practical CSMRI recovery algorithms alternate between a sparsity promoting and

*Address all correspondence to J. Sheeba Rani, E-mail: sheeba@iist.ac.in

a data fidelity stage starting with this ZF image.⁵ The drawback of conventionally used transforms, such as wavelets,⁵ contourlets,⁶ etc., in the sparsity promoting denoising stage is that they can sparsely represent only certain types of image features. Therefore, the achievable acceleration factors are often minimal when using fixed sparsifying bases.

To overcome the disadvantage of conventional transforms, patch-based techniques that provide local sparse representation have been used. This includes techniques, such as BM3D⁷ and PANO,⁸ which use nonlocal block matching techniques to filter image blocks in the sparsity promoting denoising stage. More recently, adaptive transforms using dictionary learning techniques, such as DLMRI,⁹ which can provide local sparse representation, have been employed on image patches. Using these dictionary learning techniques, the image can be recovered by solving the optimization problem shown below:⁹

$$\arg \min_{x, \Gamma} \sum \|R_{ij}x - D\alpha_{ij}\|_2^2 + \nu \|F_u x - S\|_2^2 \quad \text{s.t. } \|\alpha_{ij}\|_0 \leq T_0 \quad \forall i, j, \quad (1)$$

where R_{ij} is an $n \times P$ operator that extracts a $\sqrt{n} \times \sqrt{n}$ dimensional square patch from the image ($x_{ij} = R_{ij}x \in \mathbb{R}^{n \times 1}$) indexed by the left-corner pixel (i, j) , D is the learned dictionary, α_{ij} is the sparse representation for the patch x_{ij} , and ν and T_0 represent the regularization parameter and the maximum sparsity of each patch representation, respectively. Equation (1) finds a reconstruction x such that each image patch has a sparse representation under the dictionary D (referred to as sparse coding), while ensuring that the reconstruction is consistent with the measurements S . Dictionary learning techniques thus enable a patch to be represented as a sparse combination of elementary patches, which are columns (called atoms) of the matrix D .

A distinguishing trait in MR images (e.g., T2-weighted scans) is that they have a large diversity in directional features that need to be accurately represented. DLMRI,⁹ however, does not account for directionality of the features and also induces a smoothing effect on the reconstructed image, thus blurring out essential sharp features in the image. Patch-based directional wavelets (PBDW)¹⁰ employ a directional wavelet approach to enable the sparser wavelet domain representation by performing a pixel rearrangement on the zero-filled image patches. FDL-CP¹¹ classifies patches of an initial shift invariant discrete wavelet transform (SIDWT)-based reconstruction according to the geometrical directions determined by PBDW and trains dictionaries for each class. This makes it computation and memory intensive as it involves training numerous dictionaries. Alternate strategies have also been employed as in Ref. 12, where a joint dictionary learning and thresholding technique is used to improve reconstruction quality. Dictionaries can also be learned simultaneously while reconstructing the images in a method called blind compressed sensing,¹³ and these methods have been further refined by training various parameters, such as the thresholds of transform-based algorithms for image reconstruction.¹⁴ Nevertheless, the above-discussed techniques are restricted to learning dictionaries from the ZF reconstruction of the current undersampled slice and forego vast existing knowledge of persistent anatomical and contextual features pertaining to each scan type. Deep dealiasing generative adversarial networks (GANs)^{15,16} are deep learning-based initiatives that pursue accelerated CSMRI reconstructions by employing networks of neurons to learn the anatomical and contextual features, albeit with high-computational complexity in a batch setting. The inference stage simply applies the learned model to new undersampled data to swiftly yield unaliased reconstructions, but this is not amenable to adaptively update the weights based on the incoming images on-line. The work in Ref. 15 achieves only comparable performance with state-of-art methods, such as BM3D⁷ and PANO.⁸ Further, GANs have also been exploited to tackle the problem of yielding high-resolution images from their low-resolution counterparts for lesion detection.^{17,18}

In this work, a dictionary learning method imitating the human learning process is proposed, which learns directional features of a specific scan type from existing prior knowledge at low-computational costs. This is a significant improvement from conventional dictionary learning methods that utilize information only from a single MR slice. The double-sparsity model coupled with the stochastic gradient descent method ensures fast learning and swift application of this learned dictionary.¹⁹ The proposed scheme called directionality-driven dictionary learning for CSMRI (DDL-CSMRI) ensures that the dictionary learned from patches rearranged in their respective optimal directions provides superior sparse representations and thereby better

reconstructions from any undersampled k -space of similar scan type. Furthermore, it is suitable to adapt the learned dictionary to incoming undersampled MR images using the online sparse dictionary learning (OSDL) technique in realistic time. The proposed framework is experimentally shown to enable better recovery for CSMRI in terms of peak signal-to-noise ratio (PSNR) and structural similarity index (SSIM) under realistic scenarios.

In the following section, the proposed DDL-CSMRI method will be described in detail. Further, Sec. 3 discusses the numerical simulations and comparisons with other state-of-art CSMRI recovery algorithms, while Sec. 4 details the conclusion.

2 Proposed Directionality-Driven Dictionary Learning for CS-Based MRI Reconstruction

The fundamental attributes of adaptive dictionaries are the computational efficiency of their learning process, enforcement of patch sparsity on the target image, and ease of application.

2.1 Proposed DDL-CSMRI Formulation

In DDL-CSMRI, the $M \times N$ -order image is split into overlapping patches of size $p \times p$ with distinct directional features. In order to ensure that the learned dictionary captures these distinct features, each patch is arranged corresponding to certain candidate directions.¹⁰ The arrangement that yields the lowest approximation error out of all candidate directions is chosen to be the geometric direction of that patch, and each patch is arranged according to their respective geometric directions:

$$w_{ij} = \arg \min_{\theta_{ij,d} \in \theta} \|\tilde{c}_{ij,d}(\theta_{ij,d}, S) - \Psi^T P(\theta_{ij,d}) x_{ij}\|_2^2, \quad (2)$$

where $P(\theta_{ij,d})$ rearranges the pixels of patch at (i, j) along the candidate direction θ_d , the set of Q candidate directions is given by $\theta = \{\theta_1, \theta_2, \dots, \theta_Q\}$, Ψ^T is the forward one-dimensional Haar wavelet, and $\tilde{c}_{ij,d}(\theta_{ij,d}, S)$ represents the S largest wavelet coefficients (with S set to be 25%) of $\Psi^T P(\theta_{ij,d}) x_{ij}$. The optimal direction is thus that direction θ_d for which the representation error from the S largest wavelet coefficients is minimum (as shown in Fig. 1).

Let the set of rearranged patches in the optimal direction be given as

$$z = \{z_{11}, \dots, z_{ij}, \dots, z_{JJ}\} = \{P(\theta_{11})x_{11}, \dots, P(\theta_{ij})x_{ij}, \dots, P(\theta_{JJ})x_{JJ}\}, \quad (3)$$

where $P(\theta_{11})x_{11}, \dots, P(\theta_{ij})x_{ij}, \dots, P(\theta_{JJ})x_{JJ}$ represents the patches at coordinates $(1,1), \dots, (i,j), \dots, (J,J)$ rearranged in the optimal direction. DDL-CSMRI learns a dictionary on these reoriented patches, which to the best of our knowledge has not been attempted before. DL-MRI⁹ does not incorporate any directionality in its learning process, while FDL-CP¹¹ merely uses the optimal direction to classify patches.

Fig. 1 Obtaining the candidate directions.¹⁰ The line in red is the optimal direction, which has a sparser representation compared to any random direction, as shown in black for the given patch.

2.1.1 Dictionary learning stage

The dictionary learning process consists of finding a suitable dictionary D so that these patches reoriented in the optimal direction have a sparse representation. This learning problem can be formulated as

$$\min_{A, \Gamma} \frac{1}{2} \|z - \phi A \Gamma\|_F^2 \quad \text{s.t. } \|a_j\|_0 = k \quad \forall j, \|\gamma_i\|_0 \leq p \quad \forall i, \quad (4)$$

where the dictionary $D = \phi A$ is the product of a fixed base dictionary ϕ , a sparse adaptable matrix A . $\Gamma = [\gamma_{11}, \dots, \gamma_{ij}, \dots, \gamma_{JJ}]$ is the set of sparse representations of the reoriented patches $z = [z_{11}, \dots, z_{ij}, \dots, z_{JJ}]$, and k and p represent the maximum sparsity of the columns of the matrix A and the sparsity of the representation, respectively. The OSDL algorithm²⁰ that employs the stochastic normalized iterative hard thresholding is adopted to effectively learn the sparse matrix A , while employing a cropped wavelet-based fixed basis. Once the sparse dictionary A has been learned, this dictionary can be used to determine the sparse representation of any new image of the same class using standard CS recovery algorithms, such as the greedy algorithm OMP.²¹ With the learned dictionary, the sparse representation for the (i, j) 'th patch from the ZF image can be determined by enforcing a constraint on the representation error thus obtaining denoised image patches.

2.1.2 Recovery stage

The CSMRI reconstruction of an image x from the undersampled k -space can now be formulated as

$$\min_{x, \Gamma} \sum_{i,j} \|z_{ij} - \phi A \Gamma_{ij}\|_2^2 + \nu \|F_u x - S\|_2^2 \quad \text{s.t. } \|\gamma_{ij}\|_0 \leq T_0 \quad \forall i, j. \quad (5)$$

The above equation requires every rearranged patch z_{ij} to have a sparse representation under the dictionary $D = \phi A$, while ensuring that the reconstruction x , constructed by reorienting patches to their original direction followed by weighted averaging, is consistent with the measurements S . This problem is solved via an alternating minimization procedure.⁹ In the first stage, x is updated so that each reoriented patch has a sparse representation, while in the second stage, the reconstruction is updated to ensure data consistency. The second requirement of enforcing data consistency reduces to

$$Fx(k_x, k_y) = \begin{cases} S(k_x, k_y), & (k_x, k_y) \notin \Omega \\ \frac{S(k_x, k_y) + \nu S_0(k_x, k_y)}{1 + \nu}, & (k_x, k_y) \in \Omega \end{cases} \quad (6)$$

In the noiseless case, this reduces to simply replacing the sampled locations in the k -space of the reconstructed image with that which was originally sampled. Whereas in the noisy case, this amounts to weighted averaging of the k -space values. The significant steps in DDL-CSMRI are represented mathematically in Algorithm 1, where $F_x(k_x, k_y)$ denoted the Fourier coefficient of the image x at position (k_x, k_y) .

2.2 CSMRI Framework Using DDL-CSMRI

Figure 2 shows the proposed framework for DDL-CSMRI in CSMRI. Fully sampled datasets available from previous scans are used to train a patch-based DDL-CSMRI dictionary using the framework explained above. In order to speed up the convergence, many dictionary learning techniques start with a reconstruction from conventional CSMRI methods such as the SIDWT.¹¹ It was also experimentally observed that geometrical directions to rearrange the pixels can be better obtained from the SIDWT image. The overlapping patches of this image are rearranged in the optimal direction to obtain the rearranged vector z (steps 3 and 4). Denoising the vector z (in step 4 of Fig. 2) consists of finding the sparse representation for the patches under the learned dictionary using recovery algorithms in CS such as OMP.²¹ The denoised patches are stitched

Algorithm 1 Recovery for DDL-CSMRI-based CSMRI.

1 CS-MRI recovery D, S
Input: S , undersampled k -space measurements, base dictionary ϕ , trained sparse dictionary A , ITER

Output: $\hat{x}$, reconstructed MR image

2 Initialize $x = F_u^{-1}S$ (ZF image), $S_0 = FF_u^{-1}y$, $\Omega =$ sampled positions

3 **for** $i = 1: \text{ITER}$ **do**

4 $z = H_W x$

5 Sparse coding stage: find sparse α_{ij} s.t. $\|z_{ij} - D\alpha_{ij}\|_2^2 \leq \epsilon \quad \forall i, j$ using OMP

6 $x = H_W^{-1}z$

7 $F_x(k_x, k_y) = \begin{cases} S(k_x, k_y), & (k_x, k_y) \notin \Omega \\ \frac{S(k_x, k_y) + \nu S_0(k_x, k_y)}{1 + \nu}, & (k_x, k_y) \in \Omega \end{cases}$

8 Update x using $x \leftarrow \text{IFFT}(F_x)$

9 **end**

Fig. 2 CSMRI framework using DDL-CSMRI.

together after the inverse rearrangement (step 6) to obtained the denoised reconstruction (step 7) following which the reconstruction consistent with the measured k -space is determined in step 7. In the noiseless case, this consists of replacing the sampled positions in the original k -space in the reconstruction.⁹ Taking the inverse Fourier transform of this updated k -space completes one iteration of the recovery algorithm. This process is now repeated for the updated x for ITER iterations.

3 Results and Discussion

In order to validate the techniques, experiments were carried out to determine the capability of the proposed DDL-CSMRI algorithm to recover the MRI images from the undersampled k -space. Retrospective sampling is done on the full k -space for Gaussian, radial, and Cartesian sampling schemes. The experiments are conducted for a dataset of 100 T2-weighted axial brain MR images of dimension 256×256 , which is constructed from the MRI slices of 20 distinct patient databases available in Ref. 22. The recovery performance of the proposed algorithms was

compared with the state-of-art methods, such as DLMRI,⁹ FDLCP,¹¹ PANO,⁸ BM3D,⁷ and PBDW,¹⁰ in MATLAB 2018a running on an Intel Xeon core CPU, 3.7 GHz, 128 GB memory, 64-bit Windows 10 operating system. A patch size of 8×8 , overlap stride of $r = 2$, and a linear error threshold varying from $\varepsilon = 0.1$ to $\varepsilon = 0.01$ in 100 iterations was used in the OMP-based sparse coding stage of DDL-CSMRI. The patch size for DLMRI was set to 8×8 , with the number of dictionary atoms set to 256 and the overlap stride factor $r = 1$. The atom sparsity was set to 10 and threshold to 0.023 as recommended in Ref. 9. For FDL-CP, the patch size was set to 8 and the regularization parameter $\lambda = 10^4$. For PANO, BM3D, and PBDW, the parameters were set as recommended in the respective papers. The reconstruction quality is quantified by the PSNR and SSIM.²³ PSNR is defined as

$$\text{PSNR} = 20 \log \frac{255^2}{\|x - \hat{x}\|_F}, \quad (7)$$

where x is the original fully sampled image, $\hat{x}$ is the reconstructed image, and $\|\cdot\|_F$ corresponds to the Frobenius norm of a matrix. A higher PSNR indicates the reconstructed image more closely resembles the fully sampled image. The SSIM is defined as

$$\text{SSIM}(x, \hat{x}) = \frac{(2\mu_x\mu_{\hat{x}} + c_1)(2\sigma_{x\hat{x}})}{(\mu_x^2 + \mu_{\hat{x}}^2 + c_1)(\sigma_x^2 + \sigma_{\hat{x}}^2 + c_2)}, \quad (8)$$

where μ_x , $\mu_{\hat{x}}$, σ_x , $\sigma_{\hat{x}}$, and $\sigma_{x\hat{x}}$ correspond to the means, standard deviations, and covariance of x and $\hat{x}$ with constants c_1 and c_2 . A high SSIM indicates a greater consistency with the original fully sampled image.

3.1 Performance in the Noiseless Scenario

The evaluation of the algorithms' performances in the noiseless scenario provides an insight into the best or ideal reconstructions that can be achieved. Figure 3 portrays the performance of DDL-CSMRI on the T2 slice in Fig. 3(a), employing two-dimensional (2-D) random 20% undersampling as shown in Fig. 3(f).

The reconstruction errors in Fig. 3 visually ratify the superior performance of the proposed scheme over state-of-art techniques. DDL-CSMRI considerably outperforms DLMRI⁹ by overcoming the errors near edges and smooth regions observed in case of DLMRI [Fig. 3(b)] and achieves a 6-dB gain in PSNR and 26% gain in SSIM over it. Though visual inspection of

Fig. 3 Reconstructed brain images and errors using random Gaussian sampling pattern with sampling percentage 20%: (a) fully sampled brain image; (b)–(e) reconstructed images using DLMRI, FDLCP, BM3D, and the proposed DDL-CSMRI; (f) 20% Gaussian undersampling mask; (g)–(j) error magnitude corresponding to the above reconstructions.

Fig. 3 is unable to judge the best-performing algorithm, it is observed that DDL-CSMRI attains a gain in PSNR of 1.2 and 3.8 dB over BM3D and FDL-CP, respectively, for 20% sampling as shown in Fig. 4(b). A comprehensive evaluation of the algorithms is provided in Fig. 4 by plotting the PSNR and SSIM versus the sampling percentage. It is observed that the gain in PSNR of DDL-CSMRI over the state-of-art is more pronounced for higher sampling percentages. Moreover, DDL-CSMRI attains the highest SSIM in each case, which demonstrates that features are more accurately preserved by it.

The performance of DDL-CSMRI with the radial and Cartesian sampling patterns is also tested, and the results are tabulated in Tables 1 and 2. For the radial sampling strategy, FDL-CP achieves marginally higher PSNR than DDL-CSMRI for 10% and 20% undersampling, whereas DDL-CSMRI attains the highest PSNR for all other cases, as shown in Table 1. DDL-CSMRI is also observed to attain superior SSIM in all cases. For the case of Cartesian undersampling, DDL-CSMRI achieves the optimum PSNR and SSIM for all undersampling percentages as shown in Table 2. However, compared to the corresponding 2-D random and radial sampling masks, the Cartesian mask results in lower PSNR and SSIM for all algorithms. This is due to the fact that undersampled Cartesian masks are not as incoherent as their Gaussian and radial counterparts and introduce considerable aliasing artifacts. Thus, the extensive analysis of DDL-CSMRI with multiple sampling trajectories and sampling percentages validates its effectiveness in handling multiple sampling strategies and sampling ratios.

Fig. 4 Reconstruction quality of Fig. 3(a) versus the sampling %-age. (a) and (b) PSNR and SSIM versus various sampling %-ages, respectively.

Table 1 PSNR and SSIM comparison for noiseless radial acquisition of Fig. 3(a).

Sampling % age	ZF	FDL-CP ¹¹	BM3D ⁷	DLMRI ⁹	PANO ⁸	PBDWS ¹⁰	Proposed DDL-CSMRI
10%	23.62/ 0.3021	36.08/ 0.9548	34.53/ 0.9506	29.48/ 0.7441	32.84/ 0.9090	33.87/ 0.9451	35.78/ 0.9583
20%	25.44/ 0.3698	40.83/ 0.9729	40.34/ 0.9792	33.26/ 0.7999	38.70/ 0.9662	38.70/ 0.9738	40.78/ 0.9820
30%	27.14/ 0.4241	44.18/ 0.9799	45.20/ 0.9902	35.93/ 0.8258	43.18/ 0.9832	43.03/ 0.9865	45.27/ 0.9919
40%	28.75/ 0.4689	47.11/ 0.9839	49.28/ 0.9950	37.97/ 0.8435	47.20/ 0.9917	46.99/ 0.9930	50.02/ 0.9970
50%	30.13/ 0.5031	49.38/ 0.9859	52.93/ 0.9973	39.74/ 0.8570	51.82/ 0.9970	50.42/ 0.9964	54.48/ 0.9987

Note: The bold values represent the best result in terms of PSNR/SSIM.

Table 2 PSNR and SSIM comparison for noiseless Cartesian acquisition of Fig. 3(a).

Sampling % age	ZF	FDL-CP ¹¹	BM3D ⁷	DLMRI ⁹	PANO ⁸	PBDWS ¹⁰	Proposed DDL-CSMRI
10%	22.7/ 0.5983	27.51/ 0.8630	26.24/ 0.8328	24.82/ 0.7199	25.56/ 0.7650	27.17/ 0.8603	27.99/ 0.8686
20%	24.53/ 0.6552	32.85/ 0.9367	30.31/ 0.9108	27.46/ 0.7975	29.62/ 0.8664	33.03/ 0.9396	33.18/ 0.9386
30%	26.78/ 0.7235	40.64/ 0.9750	39.95/ 0.9794	32.79/ 0.8778	36.80/ 0.9485	40.04/ 0.9775	40.92/ 0.9829
40%	27.72/ 0.7405	43.09/ 0.9800	43.01/ 0.9873	35.73/ 0.9005	40.25/ 0.9671	41.96/ 0.9833	43.95/ 0.9902
50%	28.67/ 0.7632	45.82/ 0.9838	47.37/ 0.9937	37.70/ 0.9127	42.72/ 0.9762	45.71/ 0.9892	48.42/ 0.9956

Note: The bold values represent the best result in terms of PSNR/SSIM.

Fig. 5 (a)–(c) Three distinct T2-weighted brain images; (d) 2-D random sampling pattern; (e) radial sampling pattern; and (f) Cartesian sampling pattern.

In order to probe the superiority of the proposed scheme for multiple brain image slices, three distinct T2-weighted brain images shown in Figs. 5(a)–5(c) are subjected to the sampling patterns shown in Figs. 5(d)–5(f) with 10% sampling and the PSNR and SSIM are comparatively studied.

From the results tabulated in Table 3, it is observed that the proposed DDL-CSMRI performs best in all scenarios with the highest PSNR and SSIM and marginally lower PSNR than FDL-CP in the radial sampling case of Fig. 5(b).

3.2 Performance in the Noisy Scenario

In addition to the aliasing artifacts due to undersampling, noise due to the measuring modality also needs to be addressed. This noise is modeled as white Gaussian noise with zero mean and standard deviation of 18.8, which is a reasonable approximation to the Rician model of MRI images²⁴ and is adopted in many state-of-art methods such as DLMRI.⁹ This noise is then added to the actual k -space to obtain the measured noisy k -space, which is passed to all the algorithms.

Table 3 PSNR (dB)/SSIM results for brain images in Fig. 5.

Images	Mask	DLMRI ⁹	FDL-CP ¹¹	BM3D ⁷	Proposed DDL-CSMRI
Fig. 5(a)	Fig. 5(d)	31.61/0.7895	37.7/0.9670	37.78/0.9715	38.58/0.9741
	Fig. 5(e)	29.5/0.7375	34.93/0.9484	34.08/0.9446	35.44/0.9552
	Fig. 5(f)	24.16/0.6752	25.19/0.7745	24.79/0.7641	26.03/0.7967
Fig. 5(c)	Fig. 5(d)	30.38/0.7936	36.13/0.9617	36.04/0.9660	36.6/0.9673
	Fig. 5(f)	27.71/0.7379	34.02/0.9462	32.19/0.9342	33.96/0.9494
	Fig. 5(f)	22.39/0.6629	23.09/0.7680	22.97/0.7514	23.76/0.7871
Fig. 5(c)	Fig. 5(d)	32.52/0.8176	38.56/0.9672	39.09/0.9734	39.69/0.9750
	Fig. 5(d)	30.79/0.7791	36.32/0.9521	36.05/0.9537	37.02/0.9595
	Fig. 5(e)	23.62/0.6788	25.74/0.8158	24.92/0.7871	26.22/0.8362

Note: The bold values represent the best result in terms of PSNR/SSIM.

The 2-D random acquisition scheme is considered and the PSNR and SSIM achieved with different algorithms are plotted versus the sampling percentage in Fig. 7. The parameter λ in DLMRI and DDL-CSMRI was set to 5.26, and the data consistency stage in the framework shown in Fig. 2 is modified in accordance with the equation in step 7 of Algorithm 1 since ν is finite unlike the simple update when ν is infinite in the noiseless case.⁹ A visual representation of the reconstructed images and the associated errors with different algorithms is shown in Fig. 6, where the original fully sampled image shown in Fig. 6(a) is subjected to a 10% random undersampling of the measured noisy k -space and the magnitude of the noise-only image is shown in Fig. 6(f). It is observed that in the noisy case, the reconstructed image of DDL-CSMRI is more consistent with the original image compared to the other algorithms. This is justified by the PSNR and SSIM versus sampling %-age graphs plotted in Fig. 7 where the proposed scheme is observed to perform much better than all the algorithms for all values of the sampling percentage. For 20% undersampling, DDL-CSMRI achieves roughly a 10% gain in PSNR and 25% gain in SSIM over the second best-performing algorithms, namely DLMRI and BM3D, respectively.

The noisy k -space renders degradation in performance metrics as observed in Fig. 7 as compared to the noiseless case in Fig. 4. Contrary to the trend followed in the previous noiseless

Fig. 6 Reconstructed brain images and errors using random Gaussian sampling pattern with 10% sampling: (a) fully sampled brain image; (b)–(e) reconstructed images using DLMRI, FDLCP, BM3D, and the proposed DDL-CSMRI; (f) noise-only image; and (g)–(j) error magnitude corresponding to the above reconstructions.

Fig. 7 Reconstruction quality of Fig. 6(a) versus the sampling %-age. (a) and (b) PSNR and SSIM versus various sampling %-ages, respectively.

cases, where the PSNR improves with the increase in sampling percentage, the k -space measurements corrupted with noise were found to decrease PSNR and SSIM with increasing sampling in some cases. This could be attributed to the fact that noise affects more measurements when the number of sampling points is increased and thus has a negative impact on the recovery performance.

3.3 Computation Time

The offline dictionary learning from 70 distinct T2-weighted brain slices takes ~ 8 min. As this dictionary is adapted for the specific class of images, it can be used to reliably reconstruct any undersampled k -space belonging to the same scan type. The mean computation time of the algorithms averaged over 10 tests is tabulated in Table 4. The proposed DDL-CSMRI is observed to be faster than PANO,⁸ FDL-CP,¹¹ PBDW,¹⁰ and DLMRI.⁹ However, it is not able to match the speed of BM3D,⁷ which is not a dictionary learning-based strategy, rather a block matching-based filtering technique. Further, the significant achievement of DDL-CSMRI over the BM3D particularly in the noisy scenario outweighs its running time limitation. DDL-CSMRI achieves $\sim 34\%$, 50% , 74% , and 89% improvement in the running time over PANO,⁸ FDL-CP,¹¹ PBDW,¹⁰ and DLMRI,⁹ respectively, thereby taking much lower computational timing for reconstruction.

3.4 Choice of Parameters

In this experiment, the performance of the algorithm is individually evaluated for various values of weighted averaging parameter λ and different patch sizes. The test image of Fig. 3(a) is

Table 4 Comparison of computation time.

Reconstruction methods	Time (s)
DLMRI ⁹	430
BM3D ⁷	9.3
FDL-CP ¹¹	95.1
PANO ⁸	71.8
PBDW ¹⁰	181.3
Proposed DDL-CSMRI	47.6

Note: The bold and bold italic values represent the best and second best performance, respectively, in terms of running time.

Fig. 8 Performance of DDL-CSMRI with various parameter settings (a) PSNR versus λ at $\sigma = 18.8$ (PSNR 1) and $\sigma = 0.01$ (PSNR 2); (b) PSNR and SSIM versus patch size.

subjected to the 10-fold undersampled 2-D random sampling mask. The trend of PSNR with various λ values is plotted in Fig. 8(b) for two distinct Gaussian noise levels of 18.8 and 0.01, spanning almost 65 dB in range. It is observed that λ of around 100 performs best in both scenarios and thus has been adopted in our algorithm. Figure 8(b) plots the PSNR and SSIM for patch sizes of 4×4 , 8×8 , and 16×16 . It is observed that a choice of 8×8 performs optimally with respect to PSNR and SSIM compared to 4×4 and 16×16 . This can be due to the fact that 8×8 achieves higher sparsity and thus better representations compared to 4×4 , whereas, the performance of 16×16 decreases due to the possibility of multiple geometric directions and choice of a single optimal direction, leading to poorer reconstructions.

3.5 Qualitative Analysis

The uncertainty map introduced by Ref. 25 is good measure for qualitative assessment of the reconstruction stage. The approach in Ref. 25 implements a deep cascaded convolutional neural network for CS-based MRI reconstruction in which the network configuration is sampled according to a particular distribution to obtain multiple reconstructed images without using all subnetworks. This strategy is known to improve performance and speed in deep learning-based methods, and the variance of the reconstructed images can be used to obtain the uncertainty map. In this dictionary learning context, the sparsity of the signal can be passed as a constraint in the reconstruction process to select a corresponding number of atoms from the learned dictionary and generate the respective reconstructed images. The uncertainty map relative to the artifact-free reconstructions from 10% radially under-sampled k-space data for sparsity levels 1 to 50 (in steps of 1) is shown in Fig. 9, which shows the challenging areas to reconstruct in the image, e.g., the edges.

Fig. 9 Qualitative evaluation of reconstructions: (a) original image, (b) zero-filled reconstruction, and (c) uncertainty map.

4 Conclusion

CS has tremendous potential to accelerate MRI and can help transform diagnostic imaging with respect to efficiency, cost-effectiveness, and clinical utility.⁴ There has been considerable effort in developing algorithms that yield reliable reconstructions for diagnosis from the poorly sampled k -space. This work proposes a scheme that incorporates directional features into the OSDL framework to efficiently learn a pretrained dictionary on a vast database of T2-weighted slices of the brain and demonstrates competence with the state-of-art CSMRI recovery schemes. The proposed scheme has been shown to achieve the highest reconstruction quality in terms of PSNR and SSIM for a variety of T2-weighted brain images, acquisition trajectories, and sampling ratios. Furthermore, it has been demonstrated to be superior in the noisy acquisition scenario achieving significant gains in the performance metrics over its counterparts, affirming its robustness to noisy acquisitions. DDL-CSMRI also achieves commendable improvement in the running time over state-of-art dictionary learning methods such as FDL-CP,¹¹ PBDW,¹⁰ and DLMRI,⁹ thereby suiting it for fast reconstructions.

Disclosures

The authors declare that there is no conflict of interest, financial or otherwise.

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Anupama Arun received her BTech degree in avionics from Indian Institute of Space Science and Technology, Thiruvananthapuram. Her research interests include compressed sensing, biomedical signal, and image processing.

Thomas James Thomas is currently working toward his PhD at Indian Institute of Space Science and Technology, Thiruvananthapuram. His research interests include compressed sensing, design, and mapping of signal processing algorithms on hardware and biomedical signal processing. He is a student member of IEEE.

J. Sheeba Rani is currently a faculty member in avionics with the Indian Institute of Space Science and Technology, Thiruvananthapuram, India. Her research interests include image analysis, satellite image processing, and design and performance evaluation of hardware solutions for the same. She is a senior member of IEEE.

R. K. Sai Subrahmanyam Gorthi is currently a faculty member in electrical engineering with IIT Tirupati, Tirupati, India. His research interests include visual tracking, recognition, deep learning, and satellite image processing. He is a member of IEEE.