Reducing radiation dose in grating based x-ray phase contrast CT with twin-peaks in its phase stepping curves

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Purpose: Differential phase contrast CT has been recognized as an x-ray imaging method with the potential to greatly improve the differentiation of soft tissues. Talbot interferometry has been one of the promising solutions allowing implementation with commercially available x-ray tubes with a polychromatic spectrum. Mainly due to imperfections in grating fabrication and the polychromatic spectrum of x-ray beam, a twin-peaks phenomenon may exist in phase stepping curves (PSCs) and degrade the performance of phase retrieval. The authors have previously proposed a Fourier analysis based method for phase retrieval in the scenario wherein the twin-peaks phenomenon occurs in PSCs. In this work, the authors propose a 5-step algebraic method for phase retrieval and investigate the potential of reducing radiation dose while both the Fourier and algebraic methods are being utilized for phase retrieval.

Methods: The algebraic method to deal with the twin-peaks phenomenon, in which a set of linear equations with five unknown variables is needed for phase retrieval, is an extension of the so-called 3-step method that has been used in the scenario wherein only single-peak exists in the PSCs. In addition to a numerical phantom, two sets of experimental data (a phantom made of organic materials and a small animal) acquired by a prototype differential phase contrast CT system are employed to evaluate the performance of the Fourier and algebraic phase retrieval methods and their potential in radiation dose reduction.

Results: The evaluation by both numerical phantom and experimental data shows that the algebraic method works as well as the Fourier method in phase retrieval if the twin-peaks phenomenon in the PSCs is appropriately dealt with. In addition, while the radiation dose associated with data acquisition is being reduced via fewer phase shifting steps, the algebraic method can maintain a better performance compared to the Fourier method.

Conclusions: Along with the Fourier method, the proposed 5-step algebraic method can cope with the twin-peaks phenomenon in phase retrieval. With decreased phase shifting steps and thus radiation dose, the proposed algebraic method performs better than the Fourier method, providing a practical solution for implementation of grating based differential phase contrast CT. © 2016 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4964451>]

Key words: x-ray CT, phase contrast imaging, Talbot interferometry, differential phase contrast imaging, differential phase contrast CT, phase shifting, phase stepping curve, phase retrieval

1. INTRODUCTION

With the capability of simultaneously retrieving the signals corresponding to attenuation, refraction, and scattering (dark field) contrasts in a single scan, the grating based differential phase contrast CT has been recently investigated and

recognized as an emerging x-ray imaging modality with the potential to greatly improve soft tissue contrast.¹⁻³ Initially, this imaging method can only be implemented with highly brilliant synchrotron x-ray sources.^{1,2} Recently, the requirement for an x-ray source with complete spatial coherence has been relaxed via the adoption of Talbot interferometry,⁴

which can be implemented with a micro-focus tube⁵⁻⁸ or a multi-slit source grating in front of an x-ray tube with a finite focal spot.⁹⁻¹¹ By fulfilling the requirement for partial spatial coherence, the grating based differential phase contrast CT is compatible with polychromatic x-ray sources that are readily available in practice. Consequently, the likelihood for x-ray phase contrast CT to become a practical imaging modality for clinical and preclinical applications increases significantly.

In x-ray Talbot interferometry,^{4,10} the phase grating G_1 and absorption grating G_2 work together as a shearing interferometer to detect the x-ray's wave-front alteration caused by objects in the x-ray beam. In phase retrieval, one grating (referred as the shearing grating hereafter) is shifted in the transverse direction over a distance that is equal to the period of the absorption grating G_2 ,¹² while the other one is kept static. At each detector cell, the detected x-ray intensity oscillates as a sinusoidal function that is referred to as the phase stepping curve (PSC). Under ideal conditions with only a single-peak in PSCs, the differential phase signal as a function over the transverse direction can be extracted from the initial phase of the PSCs' first-order Fourier component.¹³

Due to imperfections, such as nonuniform gold layer thickness, in the absorption grating induced in the fabrication process,¹⁴ it has been reported that twin-peaks at significant different heights may exist in the PSCs.^{8,15,16} In general, as noted in Ref. 17, the polychromatic X-ray spectrum is another important factor that may cause the twin-peaks phenomenon in the PSCs. The twin-peaks phenomenon distorts the PSCs and may disturb the phase retrieval in the Fourier analysis based method (namely Fourier phase retrieval). In order to improve the accuracy of Fourier phase retrieval, it has been proposed that the shearing grating should be shifted over twice the period of absorption grating G_2 .^{15,16} A theoretical framework⁸ has been previously developed by us to characterize the Fourier components of the PSCs distorted by the twin-peaks phenomenon. Based on this theoretical framework, we have proposed an approach to extract the differential phase contrast signals from the second-order Fourier component, which improves the accuracy of the Fourier phase retrieval substantially.⁸ It is important to note that the relative amplitudes of the first- and second-order Fourier components depend on the phase shift of grating G_1 . In this work, we focus on the case in which the phase shift generated by grating G_1 is π (180°).

Given the dose required at each step, an immediate consequence of shifting the shearing grating over twice the period of absorption grating G_2 , i.e., twice what is supposed to be in the case wherein no twin-peaks phenomenon exists, is a proportional increase in radiation dose, if the shifting step size remains unchanged. For example, in the Fourier phase retrieval method proposed in Ref. 8, instead of shifting six steps over its period, the absorption grating G_2 shifts 12 steps (twice its period). Apparently, a straightforward way to reduce the radiation dose is to enlarge the step size of phase shifting and thus the number of x-ray exposures is reduced proportionally. However, because of such an enlargement in the shifting step, the accuracy of phase retrieval in the Fourier phase retrieval may degrade and thus needs to be investigated.

It is well known that only three variables, i.e., magnitude, DC offset, and phase, are needed to characterize a sinusoidal signal at a known frequency. Hence, in the grating-based x-ray phase contrast CT with only a single-peak in its PSCs,^{12,18} one of the solutions to reduce x-ray dose in data acquisition is to complete the phase retrieval by solving a set of three linear equations that are obtained at three phase shifting steps (namely, algebraic phase retrieval henceforth). In addition, this algebraic phase retrieval method has been proved to be more flexible and faster than the Fourier phase retrieval method.¹⁹ We speculate that the algebraic phase retrieval method can also be utilized to reduce radiation dose in the grating based x-ray phase contrast CT with twin-peaks in its PSCs, in which the shearing grating may have to be shifted more than three steps. Hence, in this paper, we carry out an investigation to see how the algebraic phase retrieval can be employed in the grating based differential phase contrast CT to deal with the twin-peaks phenomenon in its PSCs, using the data simulated by computer and acquired by a prototype grating based differential phase contrast CT system. Furthermore, we quantitatively investigate the potential of saving radiation dose in both the Fourier and algebraic methods for phase retrieval in the grating based differential phase contrast CT with the twin-peaks phenomenon in PSCs.

2. METHODS AND MATERIALS

Based on the Talbot interferometry theory,^{4,20} the intensity of x-ray irradiance at a detector cell can be written as^{21,22}

$$I_{mn}(x) = a_{0,mn} + \sum_{l=1}^{\infty} a_{l,mn} \cos \left(2\pi l \frac{x}{g_2} + \phi_{l,mn} \right), \quad (1)$$

where g_2 is the period of absorption grating G_2 ; x stands for the phase shifting locations along the transverse direction; $a_{l,mn}$ and $\phi_{l,mn}$ represent the amplitude and phase of the l th order Fourier component; m and n index the detector cells. Equation (1) is a Fourier series and the parameters $a_{l,mn}$ and $\phi_{l,mn}$ can be extracted from $I_{mn}(x)$ via Fourier analysis as briefly introduced below.

2.A. Fourier phase retrieval with single-peak and twin-peaks in PSCs

In an ideal Talbot interferometry,^{4,20} it has been well established that the PSCs can be described by the first-order Fourier component in Eq. (1) as

$$I_{mn}(x) = a_{0,mn} + a_{1,mn} \cos \left(2\pi \frac{x}{g_2} + \phi_{1,mn} \right). \quad (2)$$

The PSCs thus can be obtained by shifting the shearing grating over a distance that is equal to G_2 's period. A Fourier analysis of the PSC at each detector cell yields the initial phase $\phi_{1,mn}^r$ of the reference image (without object in x-ray beam) and the initial phase $\phi_{1,mn}^s$ of the object image (with the object in x-ray beam). Then, the differential phase contrast signal can be acquired via the equation^{20,21}

$$\frac{\partial \Phi_{mn}}{\partial x} = \frac{g_2}{d_{12}\lambda} (\phi_{1,mn}^s - \phi_{1,mn}^r), \quad (3)$$

where d_{12} denotes the distance between the phase and absorption gratings and λ refers to the x-ray's wavelength.

However, in the scenario with twin-peaks in PSCs, the frequency of the first Fourier component becomes half that of the original PSCs. Therefore, the first- and second-order Fourier components should be included to characterize the PSCs with twin-peaks⁸

$$I_{mn}(x) = a_{0,mn} + a_{1,mn} \cos \left(2\pi \frac{x}{2g_2} + \phi_{1,mn} \right) + a_{2,mn} \cos \left(4\pi \frac{x}{2g_2} + \phi_{2,mn} \right). \quad (4)$$

It is proposed to shift the shearing grating over a distance that spans twice G_2 's period to fully obtain the entire PSCs.^{15,16} Then the differential phase contrast signal can be acquired as the second order Fourier component of the PSCs via⁸

$$\frac{\partial \Phi_{mn}}{\partial x} = \frac{g_2}{d_{12}\lambda} (\phi_{2,mn}^s - \phi_{2,mn}^r). \quad (5)$$

Equation (5) is the foundation for grating-based x-ray phase contrast imaging to deal with the twin-peaks phenomenon in PSCs. It should be noted that the radiation dose and acquisition time required by Eqs. (4) and (5) double in comparison to those by Eqs. (2) and (3), as the number of phase shifting steps in the former doubles, if the step size is kept unchanged.

2.B. Algebraic phase retrieval with single-peak PSCs

Alternatively, Eq. (2) can be rewritten as

$$I = a + b \cos \theta_x \cos \phi - b \sin \theta_x \sin \phi, \quad (6)$$

where $\theta_x = 2\pi x/g_2$, $a = a_0$, and $b = a_1$, and the detector cell index m and n are omitted for simplicity. By recording the x-ray intensity at three different phase shifting locations, we can have

$$\begin{aligned} I_1 &= a + b \cos \theta_1 \cos \phi - b \sin \theta_1 \sin \phi, \\ I_2 &= a + b \cos \theta_2 \cos \phi - b \sin \theta_2 \sin \phi, \\ I_3 &= a + b \cos \theta_3 \cos \phi - b \sin \theta_3 \sin \phi, \end{aligned} \quad (7)$$

where $\theta_i = 2\pi x_i/g_2$ ($i = 1, 2, 3$). With $b \cos \phi$ and $b \sin \phi$ being noted as two independent unknown variables, and $C = b \cos \phi$ and $S = b \sin \phi$, Eq. (7) can be rewritten as¹²

$$\hat{I}_3 = \mathbf{A}_3 \hat{d}_3, \quad (8)$$

where

$$\begin{aligned} \hat{I}_3 &= (I_1, I_2, I_3)^T, \\ \hat{d}_3 &= (a, C, S)^T, \\ \mathbf{A}_3 &= \begin{bmatrix} 1 & \cos \theta_1 & \sin \theta_1 \\ 1 & \cos \theta_2 & \sin \theta_2 \\ 1 & \cos \theta_3 & \sin \theta_3 \end{bmatrix}. \end{aligned} \quad (9)$$

Equation (8) can be readily solved as $\hat{d}_3 = \mathbf{A}_3^{-1} \hat{I}_3$ (or, via least squares using the Moore–Penrose pseudo inversion²³ if more

than three data points are recorded in the PSC). Then, ϕ can be obtained by

$$\phi = \arctan \frac{S}{C}. \quad (10)$$

After the reference and sample phase signals are acquired using Eq. (10), the differential phase contrast can be obtained according to Eq. (3). In principle, this algebraic method can accomplish the phase retrieval in only three phase shifting steps, thereby lowering the radiation dose in the phase retrieval [Eqs. (6)–(10)] compared to the Fourier phase retrieval method [Eqs. (2) and (3)].

2.C. Algebraic phase retrieval with twin-peaks in PSCs

However, in the scenario with twin-peaks in PSCs, Eq. (6) is no longer valid and needs to be extended by adding a half frequency component as

$$\begin{aligned} I &= a + b_2 \cos \theta_{x,2} \cos \phi_2 - b_2 \sin \theta_{x,2} \sin \phi_2 \\ &\quad + b_1 \cos \theta_{x,1} \cos \phi_1 - b_1 \sin \theta_{x,1} \sin \phi_1, \end{aligned} \quad (11)$$

where $\theta_{x,i} = i\pi x/g_2$ ($i = 1, 2$). Noting that $C_i = b_i \cos \phi_i$ and $S_i = b_i \sin \phi_i$, we can rewrite Eq. (11) as

$$I = a + C_2 \cos \theta_{x,2} - S_2 \sin \theta_{x,2} + C_1 \cos \theta_{x,1} - S_1 \sin \theta_{x,1}. \quad (12)$$

From Eq. (12), it can be concluded that five unknown variables are needed to describe the PSCs with twin-peaks. If the x-ray intensity is recorded at five different phase shifting locations, Eq. (12) can be expanded in a way similar to that arriving at Eq. (8),

$$\hat{I}_5 = \mathbf{A}_5 \hat{d}_5, \quad (13)$$

where

$$\begin{aligned} \hat{I}_5 &= (I_1, I_2, I_3, I_4, I_5)^T, \\ \hat{d}_5 &= (a, C_2, S_2, C_1, S_1)^T, \\ \mathbf{A}_5 &= \begin{bmatrix} 1 & \cos \theta_{1,2} & \sin \theta_{1,2} & \cos \theta_{1,1} & \sin \theta_{1,1} \\ 1 & \cos \theta_{2,2} & \sin \theta_{2,2} & \cos \theta_{2,1} & \sin \theta_{2,1} \\ 1 & \cos \theta_{3,2} & \sin \theta_{3,2} & \cos \theta_{3,1} & \sin \theta_{3,1} \\ 1 & \cos \theta_{4,2} & \sin \theta_{4,2} & \cos \theta_{4,1} & \sin \theta_{4,1} \\ 1 & \cos \theta_{5,2} & \sin \theta_{5,2} & \cos \theta_{5,1} & \sin \theta_{5,1} \end{bmatrix}. \end{aligned} \quad (14)$$

Then Eq. (13) can be solved as $\hat{d}_5 = \mathbf{A}_5^{-1} \hat{I}_5$ (or, via least squares using the Moore–Penrose pseudoinversion²³ if more than five data points are recorded in the PSC), and thus ϕ_2 can be obtained as

$$\phi_2 = \arctan \frac{S_2}{C_2}. \quad (15)$$

2.D. Selection of shearing grating's step size

In principle, with the algebraic phase retrieval method, the x-ray intensity only needs to be recorded at three (or five) phase shifting locations to retrieve the phase signals in the scenario wherein a single-peak (or twin-peaks) occurs in the PSCs. To make the phase retrieval via Eq. (8) or Eq. (13)

Fig. 1. The determinants of $\mathbf{A}_3$ (a) and $\mathbf{A}_5$ (b) as functions of the angular step size $\Delta\theta$ of phase shifting.

robust, the step size of phase shifting needs to be optimized as described below.

In the 3-step algebraic method, it is recommended to perform the phase shifting at invariant steps.^{24,25} Since the PSC is a periodic function, it is straightforward to assume in Eq. (9) that

$$\theta_i = (i-1)\Delta\theta \quad (i = 1, 2, 3) \quad (16)$$

with the initial phase being included in ϕ as a constant. The determinant of $\mathbf{A}_3$ is plotted in Fig. 1(a) as a function of the angular step size $\Delta\theta$ (equivalent to the step size of phase shifting). According to the theory of linear algebra, the 3-step algebraic method is of the highest stability while the phase shifting step size $\Delta\theta$ is selected as 120° ($2\pi/3$).

Following the practice in the 3-step method, the phase shifting is also carried out at invariant steps in the 5-step algebraic method. Since the frequency of the first-order Fourier component is half of the second-order Fourier component, the phase shifting step size of the first-order Fourier component is half the step size of the second-order Fourier component. With the initial phases of the two components being included in ϕ_1 and ϕ_2 , respectively, the shifting phases in Eq. (14) can be expressed as

$$\begin{aligned} \theta_{i,2} &= (i-1)\Delta\theta \quad (i = 1, 2, 3, 4, 5), \\ \theta_{i,1} &= 0.5 \cdot \theta_{i,2}. \end{aligned} \quad (17)$$

The determinant of $\mathbf{A}_5$ is plotted in Fig. 1(b) as a function of the phase shifting step size $\Delta\theta$. Again, according to the theory of linear algebra, the 5-step algebraic method is of the highest stability while the step size is selected as 144° ($4\pi/5$).

2.E. Evaluation

A grating based x-ray differential phase contrast CT prototype system, as illustrated in Fig. 2, has been built in our laboratory.⁸ It consists of a microfocus x-ray tube (TFX-8100 SW, Trufocus, Co.), a rotary stage (B5990TS, Velmex), a flat panel detector (RadEye4, Rad-icon Imaging), and the lab-fabricated phase grating G_1 and absorption grating G_2 . The x-ray tube operates in continuous mode at 40 kVp and 0.22 mA with an $8 \mu\text{m}$ focal spot. The detector cells are of $48 \times 48 \mu\text{m}^2$ pitch and 1024×2048 matrix. The periods of grating G_1 and G_2 are 8 and $4.6 \mu\text{m}$, respectively. The distance between the x-ray tube and the system's rotation

axis is 120 cm and that between the two gratings is 18.4 cm. The phase grating G_1 is located right behind the sample table and the absorption grating G_2 is right in front of the x-ray detector.

Two sets of experimental data were utilized to evaluate the performance of the Fourier and algebraic phase retrieval methods and their potential in radiation dose reduction. One set was acquired by scanning an organic material phantom (namely MA phantom),⁸ which is a cylinder water phantom consisting of four small tubes filled with glycerol, isopropanol, ethanol, and air, respectively. The other data set was acquired by scanning a sacrificed lab mouse. In the scans, a total of 180 projections were acquired with a 2° angular interval. At each angular position, the absorption grating G_2 was shifted 12 steps at $0.8 \mu\text{m}$ interval and the x-ray exposure time at each step was 6.5 s. This data set is defined as the base data set and note that $0.8 \times 12 = 9.6 \mu\text{m}$ is as large as twice the period of the absorption grating G_2 . Corresponding to each set of the scanned data, six experiments, as depicted in Table I, were designed to evaluate the performance of the phase retrieval methods and their potential to reduce radiation dose.

A concise explanation of Table I may be informative to the readers. In DOE-a, the data set is formed by trimming the first six data points from the PSCs, and the Fourier phase retrieval method is applied on the trimmed PSCs ($0.8 \times 6 = 4.8 \mu\text{m}$ is equal to the absorption grating G_2 's period). In DOE-b, the Fourier method is applied on the base data set for phase retrieval. The purpose of running DOE-a and DOE-b is to demonstrate the necessity of appropriately dealing with the twin-peaks phenomenon in PSCs while the Fourier method is employed for phase retrieval. In DOE-c and DOE-d, the

Fig. 2. Photograph of the prototype grating based differential phase contrast imaging system: (a) microfocus x-ray tube; (b) the rest of the system.

TABLE I. Data sets for evaluation of phase retrieval performance and the potential of radiation dose reduction.

DOE	Data points	Shift step (μm)	Multiple of g_2	Method
a	6	0.8	1	Fourier
b	12	0.8	2	Fourier
c	6	0.8	1	Algebraic
d	12	0.8	2	Algebraic
e	5	1.8 (140.9°)	2	Fourier
f	5	1.8 (140.9°)	2	Algebraic

data sets are correspondingly identical to that in DOE-a and DOE-b, but the algebraic methods of 3- and 5-variables are utilized for phase retrieval, respectively. As such, the necessity of taking the twin-peaks phenomenon into account in phase retrieval by the algebraic method can be demonstrated. It should be noted here that, to match the cases reported in the literature,^{12,18} only the 3-variable algebraic method is used in DOE-c. In addition, if the number of data points in each PSC is larger than that of variables in the algebraic method, the phase retrieval is carried out by least squares, using the Moore–Penrose pseudoinversion.²³ In DOE-e and DOE-f, five data points at the sampling distance of 1.8 μm are formed in each PSC, via data sorting and interpolation from the 12 data points in each PSC of the base data. The Fourier and 5-variable algebraic methods are used for phase retrieval, respectively, as such the potential of reducing radiation dose in the grating based differential phase contrast CT can be studied.

It is worthwhile to note that, to obtain the data sets in DOE-e and DOE-f, it would be better to reacquire the data at 1.8 μm step if the experiments could be repeatedly carried out under the identical condition. However, due to a low output power of the microfocus X-ray tube and the small detector cell size, it takes about 10 h, which is very long from the perspective of data acquisition for imaging, to acquire one single set of data for phase contrast CT imaging. During such a long duration of time, the performance of the detector and X-ray tube changes substantially, as evidenced by significant shrinkage in dynamic range and increase in background noise, due to, probably, a continuous x-ray irradiation on the detector and the increase in x-ray tube's temperature. Therefore, we believe that data sorting and interpolation are better strategies in terms of feasibility and objectivity for us to evaluate and compare the performances of the different phase retrieval methods under the same condition.

Prior to the experimental evaluation using scanned data, a one-dimensional (1-D) numerical phase contrast phantom with no attenuation was simulated to verify the accuracy of the phase retrieval methods. In the simulation, the PSCs were formed using protocols identical to those in Table I. The relative amplitudes of the zeroth, first, and second order Fourier components were set as 1.0, 0.3, and 0.09, respectively, and a Gaussian noise with zero mean and standard deviation of 0.018 (1/5 of the amplitude of the second-order Fourier component) was added as the background.

In evaluation of the Fourier phase retrieval method, the reference and sample signals were acquired using Eqs. (2) and (4), respectively, while the counterparts in the algebraic

phase retrieval method were acquired using Eqs. (10) and (15). The differential phase contrast signals were dewrapped with a 3-D dewrapping algorithm.²⁶ To generate the phase contrast CT images, the dewrapped differential phase contrast signals were reconstructed with an FDK-like algorithm,²⁷ in which the ramp filtering was replaced by the Hilbert filtering.¹³

3. EVALUATIONS AND RESULTS

3.A. Evaluation by 1-D numerical differential phase phantom

Presented in Fig. 3(a) is the ground truth of the 1-D numerical differential phase phantom and the rest are sequentially the differences between the ground truth and the phase retrieved in DOE-a and DOE-c [Fig. 3(b)], DOE-b and DOE-d [Fig. 3(c)], and DOE-e and DOE-f [Fig. 3(d)]. It is shown that, in the scenarios wherein the twin-peaks phenomenon occurs in the PSCs but is not taken into account in phase retrieval, both the Fourier and algebraic methods [Fig. 3(b)] may generate an error as large as $\pi/6$ in phase retrieval. However, by taking the twin-peaks into account, the phase retrieval error can be almost eliminated with both the Fourier and algebraic phase retrieval methods [Fig. 3(c)]. In DOE-e and DOE-f [Fig. 3(d)], though each PSC consists of only five data points, both the

FIG. 3. The 1-D numerical differential phase phantom (a) and the differences between the ground truth (a) and the phases retrieved in DOE-a and DOE-c (b), DOE-b and DOE-d (c), and DOE-e and DOE-f (d).

Fourier and algebraic methods still perform well in phase retrieval, compared to their counterpart in DOE-a and DOE-b.

3.B. Quantitative evaluation by imaging an organic material phantom

The MA phantom was utilized to evaluate the performance of the phase retrieval methods and the potential of radiation dose reduction. As displayed in Fig. 4(a) (indicated by arrow heads), without taking the twin-peak phenomenon into account (DOE-a), artifacts appear in the projection image obtained by the Fourier method, due to the error in phase retrieval. By taking the twin-peaks phenomenon into account, the Fourier method (DOE-b) generates differential phase contrast projection [Fig. 4(b)] that is free of artifact and good in terms of noise and edge definition. This demonstrates that it is necessary for the Fourier method to take the twin-peaks phenomenon into account in phase retrieval. It should be noted that the noise reduction in Fig. 4(b) compared to Fig. 4(a) is attributed to the fact that a doubled number of phase shifting steps (and thus doubled radiation dose) are utilized in data acquisition. We take the performance of the Fourier method corresponding to DOE-b as the reference from now on. Presented in Figs. 4(c) and 4(d) are results of the 3-variable and 5-variable algebraic methods (DOE-c and DOE-d), respectively, using the data identical to those in DOE-a and DOE-b. It is observed that the quality of the image in Fig. 4(d) is significantly better than that in Fig. 4(c), showing that it is also necessary for the algebraic

phase retrieval method to take the twin-peaks phenomenon into account. The results corresponding to DOE-e and DOE-f, in which each PSC consists of five data points spreading over a distance that is equal to twice the period of grating G_2 , are displayed in Figs. 4(e) and 4(f), respectively. It is observed that both the Fourier and algebraic methods work well in the phase retrieval while the radiation dose is reduced by more than half. Meanwhile, it is noted in Figs. 4(e) and 4(f) that the noise is obviously higher compared to that in Figs. 4(b) and 4(d) because of the reduction in radiation dose.

To be quantitative in the performance evaluation of phase retrieval, we measured the mean and standard deviation (SD) within an ROI [see Fig. 4(f)] in each of the projections. Taking the phase retrieval accuracy of DOE-b as the reference, the relative performance of each DOE is quantified in Table II. A quantitative inspection of Table II (left column) reinforces our visual inspection, as we scrutinize images (a) vs (b), (c) vs (d), (a) vs (e), and (c) vs (f) in Fig. 4, that the accuracy of phase retrieval by both the Fourier and algebraic methods is indeed improved substantially by considering the twin-peaks phenomenon. In addition, under identical data conditions, the algebraic method outperforms the Fourier method in noise (right column of Table II), as we scrutinize images (a) vs (c), (b) vs (d), and (e) vs (f) in Fig. 4.

The CT images of the MA phantom corresponding to the projections in Fig. 4 are presented in Fig. 5, where the Hounsfield unit is defined and calibrated using the refractive index decrement of water of the reference [ROI 2 of Fig. 5(f)]. As displayed in Fig. 5(a) (indicated by arrows), without taking the twin-peaks phenomenon into account (DOE-a), the uniformity of the image obtained by the Fourier method is poor, due to the error in phase retrieval. By taking the twin-peaks phenomenon into account, the Fourier method (DOE-b) generates the image [Fig. 5(b)] with significantly improved uniformity. The noise reduction in Fig. 5(b) compared to Fig. 5(a) is attributed to the fact that a doubled number of phase shifting steps (and thus doubled radiation dose) is utilized in data acquisition. Presented in Figs. 5(c) and 5(d) are the results of the 3-variable and 5-variable algebraic methods (DOE-c and DOE-d), respectively, using the data identical to those in DOE-a and DOE-b, which again shows the necessity for the algebraic phase retrieval method to take the twin-peaks phenomenon into account. The results corresponding to DOE-e and DOE-f, in which each PSC consists of five data points spreading over a distance equal to twice the period of grating G_2 , are displayed in Figs. 5(e) and 5(f), respectively. It is observed that both the Fourier and algebraic methods work

FIG. 4. Projections of the MA phantom obtained by the Fourier method corresponding to DOE-a (a), DOE-b (b), DOE-e (e); and by the algebraic method corresponding to DOE-c (c), DOE-d (d), and DOE-f (f) in Table I (display window: $[-\pi/4 \pi/4]$).

TABLE II. Phase retrieval accuracy in each design of experiment relative to the reference (DOE-b).

$[(\text{mean}_{\text{ROI}} - \text{mean}_{\text{ROI(b)}}) / \text{mean}_{\text{ROI(b)}}] \times 100\%$		$[(\text{SD}_{\text{ROI}} - \text{SD}_{\text{ROI(b)}}) / \text{SD}_{\text{ROI(b)}}] \times 100\%$	
(a) +111.0	(b) +0.0	(a) +12.8	(b) +0.0
(c) +104.2	(d) +4.7	(c) +12.0	(d) -4.9
(e) +16.9	(f) +10.9	(e) +60.4	(f) +47.3

FIG. 5. Tomographic images of the MA phantom obtained by the Fourier method corresponding to DOE-a (a), DOE-b (b), DOE-e (e), and by the algebraic method corresponding to DOE-c (c), DOE-d (d), and DOE-f (f) in Table I (display window: [−400 200]).

well in the phase retrieval while the radiation dose is reduced by more than half.

To be quantitative in the imaging performance evaluation, we measure the mean and standard deviation (SD) of the CT number within the ROIs [Fig. 5(f)] in each of the DOEs and itemize them in Table III. It is noted that, while the radiation dose is reduced by more than half, the CT number gauged at the target (ROI 1) obtained by the algebraic phase retrieval method (DOE-f) is more accurate than that by the Fourier

TABLE III. Mean and SD measured within the ROI [see Fig. 5(f)] in each DOE.

	Mean value (HU)		Standard deviation (HU)	
	ROI 1	ROI 2	ROI 1	ROI 2
a	−150.1	−59.3	13.3	22.0
b	−218.9	0.0	14.6	15.3
c	−156.4	−66.3	13.1	26.4
d	−223.5	−12.1	14.3	14.1
e	−164.2	−24.9	19.2	30.4
f	−211.1	−37.8	19.4	32.8

method (DOE-e). However, this does not hold for the CT number gauged at the water background (ROI 2) under the same conditions.

3.C. Qualitative evaluation by imaging small animals

A sacrificed laboratory mouse was scanned using the prototype differential phase contrast CT system for further performance evaluation, in which the scan protocols and phase retrieval process were the same as those used in the experiment with MA phantom. The projections of the mouse corresponding to the DOEs specified in Table I are displayed in Fig. 6.

It is observed that, similar to the case of the MA phantom, by taking the twin-peaks phenomenon into account and using a sufficient number of phase shifting steps, the Fourier phase retrieval method improves its performance substantially [Figs. 6(a) and 6(b)], and so does the algebraic method [Figs. 6(c) and 6(d)]. A close inspection of Figs. 6(b) and 6(d) tells us that the algebraic phase retrieval method outperforms the Fourier method in noise while the data are sufficient. This observation holds in the scenario wherein each PSC consists of five data points spreading over a distance that is equal to twice the period of grating G_2 [Figs. 6(e) and 6(f)], while the radiation dose is reduced by more than half.

The tomographic images of the mouse corresponding to the projection data in Fig. 6 are presented in Fig. 7. It is observed in Figs. 7(a) and 7(c) (DOE-a and DOE-c) that, if the twin-peaks phenomenon is not appropriately dealt with, both the Fourier and algebraic phase retrieval methods cannot work well, as evidenced by the poor image quality (accuracy and

FIG. 6. Projections of the lab mouse obtained by the Fourier series method corresponding to DOE-a (a), DOE-b (b), DOE-e (e); and by the algebraic method corresponding to DOE-c (c), DOE-d (d), and DOE-f (f) in Table I (display window: [− $\pi/4$ $\pi/4$]).

FIG. 7. Tomographic images of the lab mouse obtained by the Fourier method corresponding to DOE-a (a), DOE-b (b), DOE-e (e) and by the algebraic method corresponding to DOE-c (c), DOE-d (d) and DOE-f (f) in Table I (display window: $[-400\ 300]$).

uniformity) in the reconstructed images. By taking the twin-peaks phenomenon into account, the accuracy and uniformity are improved substantially, as demonstrated in Figs. 7(b) and 7(d). Presented in Figs. 7(e) and 7(f) are results of the Fourier and algebraic phase retrieval methods under the condition that each PSC consists of five data points spreading over twice the period of grating G_2 . It is observed that the noise in Figs. 7(e) and 7(f) is significantly higher than that of Figs. 7(b) and 7(d), respectively, because the radiation dose in the former is reduced by more than half. Moreover, it is noted that the image reconstructed from the projection data acquired by the Fourier phase retrieval method in Fig. 7(e) (DOE-e) appears significantly brighter than that reconstructed from those obtained by the algebraic method (DOE-f), which suggests that the CT number has been escalated due to the error in phase retrieval.

4. DISCUSSIONS AND CONCLUSIONS

By extending the so-called 3-step method,^{12,18,19} a 5-step algebraic phase retrieval method to deal with the twin-peaks phenomenon in the PSCs is proposed in this paper. Using a numerical phantom and experimental data acquired by the

prototype differential phase contrast CT system in our lab, the performance of the algebraic phase retrieval method in the scenario of twin-peaks is evaluated and compared with that of the Fourier method. Moreover, the potential to reduce radiation dose by both the Fourier and algebraic methods in the scenario of twin-peaks is studied. Based on the preliminary results obtained, a number of observations are in order below.

It has been demonstrated via a phantom and animal study that, by taking the twin-peaks phenomenon into account (Figs. 4–7), the accuracy of phase retrieval in both the Fourier and algebraic methods improves significantly. This necessitates that, in the scenario wherein the twin-peaks phenomenon exists in the PSCs, the phase retrieval should be carried out over a distance equal to twice the period of the absorption grating, if the highest image quality in accuracy and uniformity is to be pursued in the grating based differential phase contrast CT. This is the primary point we want to make in the discussion.

The second point we want to make is that the radiation dose associated with the grating based differential phase contrast CT can be reduced by decreasing the number of phase shifting steps in data acquisition while either the Fourier or algebraic method is used for phase retrieval. The minimum number of phase shifting steps required by the algebraic phase retrieval method is 5, because a total of five variables are involved in the algebraic method. Note that the minimum number of phase shifting steps in the Fourier method may be lowered further. However, as demonstrated in Fig. 7(e), in the case wherein the number of phase shifting steps is equal to 5, the quality of the CT images acquired by the Fourier phase retrieval method becomes poor, which discourages us to go further to decrease the number of phase shifting steps and thus the radiation dose. In addition, it is observed that with fewer phase shifting steps in phase retrieval, the noise increases accordingly in the projection images. It can be anticipated that (i) there always is a trade-off between image quality (noise) and radiation dose and (ii) less radiation dose used in the data acquisition leads to higher noise in the formed image.

It has been noted that there is little difference in the performance of phase retrieval between the Fourier and algebraic methods while the organic phantom is used for performance evaluation, as displayed in Figs. 4 and 5. However, their difference increases when the mouse is utilized for performance evaluation (Figs. 6 and 7). One underlying reason is the fact that the structures of the organic MA phantom are relatively simple and regular, whereas those in the mouse are much more complicated and irregular and thus may suffer more from the twin-peaks phenomenon in the PSCs. Hence, the performance evaluation carried out using the mouse may bear more useful information for us to envision the potential of grating based differential phase contrast CT, regarding its capability and radiation dose for preclinical/clinical applications.

It is worthwhile noting that the relationship of $\phi_1 = \phi_2/2$ holds in Eq. (11), according to the analysis in Ref. 8. Therefore, only four unknown variables may be needed to characterize a PSC with twin-peaks. However, as also indicated in Ref. 8, the amplitude of the first order Fourier

component is far smaller than that of the second order component, and as such the signal corresponding to ϕ_1 may be too noisy. Hence, for reliability, ϕ_1 is considered as an independent variable in the 5-step algebraic method proposed and implemented in this paper. In general, at the expense of increasing radiation dose, the availability of more than five data points in PSCs can improve the performance of phase retrieval, especially in the scenarios wherein data are noisy or inconsistent. If this is the case, the Moore–Penrose pseudoinversion, should be employed for phase retrieval.

Reducing the steps of phase shifting from 12 to 5 at each angular position in the data acquisition can reduce both radiation dose and data acquisition time by more than half. Thus the 5-step algebraic phase retrieval method proposed and evaluated in this work is believed to be practically relevant. However, prior to ending this paper, we want to indicate that five steps of phase shifting still need to be carried out at each angular position in the proposed method. To further reduce radiation dose and data acquisition time, we are investigating more data acquisition schemes and image formation solutions, including the scheme that requires only one phase shifting step at each angular position.²⁸ The outcome of our investigation will be promptly reported in our publications in the future.

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CONFLICT OF INTEREST DISCLOSURE

The authors have no COI to report.

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