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Autocorrelation and Cepstral Methods for Measurement of Tibial Cortical Thickness

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Abstract

Cortical thickness of the tibia is related to stress fracture risk and overall skeletal status. Two methods are proposed for estimating tibial cortical thickness based on power spectra of ultrasonic echoes containing reflections from front and back surfaces of the cortex. The locations of the peaks in the autocorrelation function and the cepstrum are related to cortical thickness. Data were acquired on plastic plates in order to validate the methodology. These data indicate high correlations between estimated and true thickness with correlation coefficients $r = 0.99$, (95% confidence interval: 0.993–1.00) for the autocorrelation method and $r = 0.99$ (95% CI: 0.996 – 1.00) for the cepstral method. Data on six tibia samples *in vitro* indicate correlation coefficients of $r = 0.92$ (95% CI: 0.72 – 1.00) for the autocorrelation method and $r = 0.85$ (95% CI: 0.62 – 0.94) for the cepstral method. Estimates of precisions of the two methods were 0.3 ± 0.1 mm (autocorrelation method) and 0.5 ± 0.2 mm (cepstral method). One measurement in a human volunteer *in vivo* demonstrated clinical feasibility of the measurement and good agreement with cortical thickness assessed using peripheral quantitative computed tomography (QCT). This technology offers the promise of an inexpensive, fast, portable, simple, nonionizing technique for assessing skeletal status.

Introduction

Human long bones such as the tibia, radius, and femur possess a thick outer layer of compact (cortical) bone [1]. Cortical thickness is related to fracture risk and has been shown to predict fracture load in radius, femur, and lumbar vertebrae [2]. For example, correlation coefficients between cortical thickness and fracture load have been found to be $r = 0.88$ (radius) and $r = 0.71$ (femur) [2]. The speed of sound along long bones, which depends on cortical thickness, has also been shown to have diagnostic value [3–9].

Prediction of osteoporotic fracture risk is an important diagnostic task. A less prevalent but still important condition, that would seem to be ideally addressed by a diagnostic tibial cortical thickness measurement, is stress fracture in female military recruits during basic training. These young women suffer unacceptably high rates of tibial stress fractures during basic training. [10]. Within the U.S. military, reported rates for stress fractures range from 2% to 12% [11] contributing to significant training losses in all branches of the military [12–

13]. It would be beneficial to have a diagnostic technique that could identify those individuals at greatest risk for fractures prior to training so that preventative measures could be taken.

X-ray based methods for this measurement are relatively expensive, require a trained technologist, and produce ionizing radiation. A new ultrasound technique for measuring tibial cortical thickness is described in this paper. The method offers advantages with respect to cost, speed, and portability. It is a more direct measure of tibial cortical thickness than the longitudinal sound speed measurement. The method presented here entails estimation of tibial cortical thickness by measurement of autocorrelation functions and cepstra of ultrasonic echoes from the tibial cortex. This approach is related to previously published methods for measurements of thicknesses of layers or vessels [13–16]. This method reported in this paper is related to a technique that has been shown to be useful for estimating scatterer spacing in phantoms containing regularly spaced scatterers embedded in an inhomogeneous medium containing randomly distributed scatterers [17].

Theory

Consider a process, $x(t)$ (e.g., the received radio frequency signal from the ultrasound transducer as a function of time), which may be represented as the convolution of an impulse (planar reflector) response, $p(t)$ (which incorporates the electromechanical characteristics of the transducer and diffraction), $a(t)$ (which describes the effects of attenuation as the signal propagates through the cortical layer) and a function that describes scattering and reflection from tissue, $r(t)$.

$$x(t) = p(t) * a(t) * r(t) \quad (1)$$

where the asterisk denotes the convolution operator. This formula ignores attenuation between the insonifying transducer and the front surface of the tibia. The transducer may be positioned in order to make this attenuation negligible. For example, for insonification from the medial orientation at a point 33% (of the tibial length) distal to the tibia-femur cleft level, the front surface of the tibia is within a millimeter or so of the skin surface. (See Methods Section).

Consider two approximately planar surfaces (e.g. proximal and distal surfaces of tibial cortical layer) with reflection coefficients, R_1 and R_2 , separated by a distance d .

$$r(t) = R_1 \delta(t - t_0) + R_2 \delta(t - t_0 - 2d/c) \quad (2)$$

where $\delta(t)$ denotes the Dirac delta function, and c is the speed of sound. In this case, the expected value of $x(t)x^*(t - \tau)$ (the autocorrelation function) will exhibit a peak at $\tau = 2d/c$. The application of the expectation operator here allows for the possibility that the signal may possess some degree of random character such as uncertainty in d or additive noise.

The power spectrum of the received signal is

$$\begin{aligned} |X(f)|^2 &= |P(f)|^2 |R_1 + R_2 e^{-2\beta f d} e^{-i2\pi f(2d/c)}|^2 \\ &= |P(f)|^2 \{ [R_1^2 + R_2^2 e^{-4\beta f d} + 2R_1 R_2 e^{-2\beta f d} \cos[2\pi f(2d/c)]] \} \end{aligned} \quad (3)$$

where β is the attenuation slope of the cortical layer and f is frequency. The logarithm of equation (3) is

$$2\log |X(f)| = 2\log |P(f)| + 2\log |R_1 + R_2 e^{-2\beta f d} e^{-i2\pi f(2d/c)}| \quad (4)$$

The last term has a factor of the form $e^{-i2\pi f a}$, which has an inverse Fourier transform of $\delta(s - a)$ where s is the transform variable and $a = 2d/c$. The cepstrum [18, 19] corresponding to $x(t)$, which is the inverse Fourier transform of Equation (4), will have a peak at $s = t = 2d/c$ (where the transform variable has been called t in order to facilitate comparison with the result derived from the autocorrelation function). One early application of the cepstrum [18, 20] was for removal of signals corresponding to reverberations, a problem somewhat analogous to the present topic.

Methods

Water Tank Measurements

A Panametrics (Waltham, MA) 5800 pulser/receiver and a Panametrics 2.25 MHz center frequency, circular, 1.5" focal length, broadband transducer were used. Received ultrasound signals were digitized (8 bit, 10 MHz) using a LeCroy (Chestnut Ridge, NY) 9310C Dual 400 MHz oscilloscope and stored on computer (via GPIB) for off-line analysis. The usable bandwidth of the transducer was approximately 1.0 – 2.8 MHz. In this frequency range, the spectrum remained at least 10 dB above the noise floor.

Five acrylic plates (thickness: 1.4, 3.3, 7.2, 9.1, 12.2 mm) and four polycarbonate plates (thicknesses: 1.4, 5.5, 9.2, 12.1 mm) were scanned in order to validate the method.

Six human tibia samples (genders, ages unknown) were obtained. Two examples are shown in Figure 1. They were defatted using a trichloro-ethylene solution prior to ultrasonic measurements. Sample thicknesses were measured using calipers. Ultrasound measurements were only performed at locations where thickness was relatively well-defined. Excessively rough regions were avoided.

Ultrasonic measurements were performed in distilled water at room temperature. The surface of the specimen was oriented perpendicular to the ultrasound propagation direction as shown in Figure 2. Cortical thicknesses of tibia specimens and plastic plate thicknesses were measured using calipers. Three sets of 15–20 ultrasonic echoes were obtained for each plastic sample or tibia specimen *in vitro*. Each set of 15–20 corresponding power spectra were averaged.

In Vivo Measurements

A human volunteer was scanned using the same pulser/receiver and digitizer as above with a Panametrics 1.0 MHz center frequency, circular, 2" focal length transducer as shown in Figure 3. The usable bandwidth was 0.7 – 1.6 MHz. In this frequency range, the spectrum remained at least 10 dB above the noise floor. A standoff pad was used in order to ensure that the tibial cortical layer coincided with the focal zone of the transducer. Aquasonic gel (Parker laboratories, Fairfield, NJ) was used to improve the acoustic coupling both between the skin and the standoff pad as well as between the standoff pad and the transducer. Measurements were performed at a point 33% (of the tibial length) distal to the tibia-femur cleft level. Measurements were performed on the medial side of the tibia, where a relatively smooth outer surface could be identified by palpation. At this site, the thickness of intervening skin is minimal, especially when the transducer and standoff pad are compressed against the skin. In addition, the condition of parallelness of the outer and inner surfaces of the cortex is approximately achieved at this site.

The algorithm is expected to perform best when the inner and outer surfaces of the cortical layer are oriented approximately perpendicular to the ultrasound propagation direction. In the perpendicular orientation, the strengths of the signals emanating from the interfaces are maximized, enhancing their conspicuity relative to backscatter from the cortical layer and electronic noise. The angle and the location of the transducer were adjusted in order to maximize intensities of front and back specular reflections from the cortical layer (visible in real time on the oscilloscope display), in order to achieve as closely as possible the perpendicularity condition. Another sign of perpendicularity was minimization of the time delay between the front and back specular reflections, which could be monitored on the oscilloscope display. One set of six ultrasonic echoes was acquired from a human tibia *in vivo*. Again, a single average power spectrum was computed.

For purposes of comparison, cortical thickness was also measured on the human volunteer using a Stratec QCT2000 (Stratec Medizintechnik, Pforzheim, Germany) peripheral quantitative computed tomography system.

Data Analysis

It is important to understand the limits of the range of target layer thicknesses for which this technique can be successful. Equation (3) allows prediction of the rate of spectral oscillations if sound speed and layer thickness are known. Excessively thick targets will produce relatively gradual oscillations in the power spectrum. However, the system transfer function, $|P(f)|^2$, will also produce relatively gradual variations in the power spectrum, which cannot be separated from the oscillations of interest, causing a method based on Equation (3) to fail.

In the case of acrylic ($c = 2650$ m/s) plates ranging from 1.4 mm to 12.2 mm, spectral oscillations with periods $c/2d$ (see Equation 3) in the range of 109 kHz – 946 kHz could be expected. In the case of polycarbonate ($c = 2190$ m/s) plates ranging from in the same range of thicknesses, spectral oscillations with periods in the range of 90 kHz – 782 kHz could be expected. Cortical thicknesses in the range of 2.5 – 12 mm yield spectral oscillations with

periods in the range of 158 – 760 kHz (assuming a cortical sound speed of 3800 m/s [21]). These oscillations are generally more rapid than those due to the transfer function, $P(f)$, which in the case of the 2.25 MHz center frequency transducer was an approximately Gaussian-shaped function with a center frequency of 2.25 MHz and a full width half maximum (FWHM) of about 1 MHz.

Prior to inverse Fourier transformation (for computation of either autocorrelation function or cepstrum), power spectral values outside of the usable bandwidths (see above) were zeroed out. What remained was an oscillating function riding on a slowly varying background trend (corresponding roughly to $|P(f)|^2$). This background was subtracted from the spectrum in order to isolate the function of interest (the right hand factor in Equation 3) and also to suppress the abrupt discontinuities in the bandpass filtered power spectrum at the endpoints of the bandpass filter, which could result in artifacts in the inverse Fourier transform.

The background trend was computed by convolving the original power spectrum with a Gaussian function, $\exp(-f^2/2\sigma^2)$, with a width of $\sigma = 0.2$ MHz. The effective total width of the Gaussian filter (4σ) was then 800 kHz. The smoothing process removed the more rapid (in frequency domain) oscillations. By taking the difference between the unfiltered and smoothed power spectra, the more rapid oscillations were restored and the uninteresting more gradual oscillations (corresponding to $|P(f)|^2$ and perhaps hypothetical thicknesses much larger than the range of interest) were suppressed. After background removal, a Hamming window was applied to the data. Then, the dc level was subtracted.

The thickness was measured from the location of the highest peak in the autocorrelation function (the inverse Fourier transform of the processed average power spectrum) or cepstrum in the range between 2 and 9 mm (tibiae) and 2 and 14 mm (plastic samples). The independent variable, t (see Theory section), was scaled so that the autocorrelation function and cepstrum could be represented as functions of hypothetical layer thickness ($d = c \cdot t/2$).

Results

The ultrasonic echo from an acrylic plate (thickness: 7.2 mm) is shown in Figure 4. Reflections from the front and back surfaces may be seen. The corresponding power spectrum in Figure 5 exhibits oscillations as predicted from Equation (3). The period of spectral oscillation ($c/2d$) is 184 kHz. These oscillations ride upon a slowly varying background.

The inverse Fourier transform of the power spectrum (or log power spectrum) gives the autocorrelation function (or cepstrum). An example is shown in Figure 6. When the time axis is converted to distance ($d = c \cdot t/2$), the location of the peak corresponds to the layer thickness. Estimated (based on the autocorrelation function) and true (measured using calipers) thicknesses for nine plastic plates are shown in Figure 7. The correlations, $r = 0.99$, 95% CI: 0.993–1.00 for the autocorrelation method and $r = 0.99$, 95% CI: 0.996–1.00 for the cepstrum are excellent.

The autocorrelation function and cepstrum measured from a tibia sample *in vitro* is shown in Figure 8. Since the surfaces of the tibia are not as planar and parallel as in the plastic plate

examples, the peak is wider and the data are somewhat noisier. Nevertheless, a peak may be discerned. In Figure 9 measured and true cortical thicknesses for six tibia samples are shown. The correlation coefficient for the autocorrelation method was 0.92 (95% CI: 0.79, 0.97). The precision (average value of the standard deviations of measurements) was 0.3 mm. The correlation coefficient for the cepstral method was $r = 0.85$ (95% CI: 0.62 – 0.94). The corresponding precision was 0.5 ± 0.2 mm.

One measurement in a human *in vivo* is shown in Figure 10. The locations of the peak (Autocorrelation: 6.1 cm; Cepstrum: 5.7 cm) agree moderately well with each other and correspond fairly closely to the cortical thickness measured using QCT (5.9 mm). In converting from times to distances, a sound speed of 3800 m/s was assumed for tibia data (both *in vitro* and *in vivo*) [21]. Unlike the idealized low-noise experiment with the plastic plates, in the more noisy experiment, surfaces are not as smooth or as parallel to each other, perpendicularity to the ultrasound beam is achieved to a lesser degree, and noise corresponding to backscatter from the tissue between the surfaces degrades the signal. Therefore it is perhaps not surprising to see a small discrepancy between the two methods emerge *in vivo*.

Discussion

A new ultrasound-based approach for cortical thickness measurement has been validated in plastic samples and in tibia specimens *in vitro*. Clinical feasibility has been demonstrated with an *in vivo* application.

The two methods (autocorrelation and cepstrum) performed comparably on plastic plates. The autocorrelation method seemed to perform somewhat better on tibia samples *in vitro* although there was considerable overlap in the 95% confidence intervals for correlation between measured and true values of thickness.

A constant value (3800 m/s) for the radial ultrasonic velocity in tibial cortex was assumed in this study [21]. It is likely that in actuality this velocity varies somewhat among individual tibiae. Nevertheless, a good correlation between autocorrelation peak and cortical thickness was observed here. Evidently, the degree of variation of radial velocities in the samples investigated *in vitro* was not great enough to overshadow the correlation with thickness. This is true even though a rather wide range of cortical thicknesses (2.4 to 5.4 mm) was encountered. Whether the range of radial velocities *in vivo* is small enough to permit reliable estimates of cortical thickness by this method remains to be verified.

A fast, portable, simple, nonionizing technique for assessing skeletal status has been presented. It could be useful as a screening tool and could potentially be applied to other skeletal sites.

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1.
Tibia samples.

2.

Experimental setup for measurements on plastic plates and tibia specimens in vitro.

3.
In vivo data acquisition.

4.

Radio frequency echo from plastic plate showing reflections from front and back surfaces.

5.
Power spectrum of radio frequency echo from Figure 4.

6.
Autocorrelation function (solid line) and cepstrum (dashed line) measured from a 9.2 mm thick polycarbonate plate.

7.

Measurements of autocorrelation peaks in plastic samples. Similar results were obtained using the cepstral approach.

8.
Autocorrelation function (solid line) and cepstrum (dashed line) from a tibia sample with a cortical thickness of 3.0 mm.

9.
Measurements of autocorrelation peaks in six tibia samples. Error bars denote standard errors.

10.

Autocorrelation function (solid line) and cepstrum (dashed line) measured in a human volunteer *in vivo* at 1 MHz. The two methods achieve slightly different results.