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Directivity and Frequency-Dependent Effective Sensitive Element Size of Membrane Hydrophones: Theory vs. Experiment

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Abstract

It is important to know hydrophone frequency-dependent *effective* sensitive element size in order to account for spatial averaging artifacts in acoustic output measurements. Frequency-dependent effective sensitive element size may be obtained from hydrophone directivity measurements. Directivity was measured at 1, 2, 3, 4, 6, 8, and 10 MHz from $\pm 60^\circ$ in two orthogonal planes for 8 membrane hydrophones with nominal *geometrical* sensitive element radii (a_g) ranging from 100 to 500 μm . The mean precision of directivity measurements (obtained from four repeat measurements at each frequency and angle) averaged over all frequencies, angles, and hydrophones was 5.8%. Frequency-dependent effective hydrophone sensitive element radii $a_{\text{eff}}(f)$ were estimated by fitting the theoretical directional response for a disk receiver to directivity measurements using the sensitive element radius (a) as an adjustable parameter. For the 8 hydrophones in aggregate, the relative difference between effective and geometrical sensitive element radii, $(a_{\text{eff}} - a_g) / a_g$, was fit to $C / (ka_g)^n$ where $k = 2\pi/\lambda$ and λ = wavelength. The functional fit yielded $C = 1.89$ and $n = 1.36$. The root mean square difference between data and model was 34%. It was shown that, for a given value for a_g , $a_{\text{eff}}(f)$ for membrane hydrophones far exceeds that for needle hydrophones at low frequencies (*e.g.*, < 4 MHz when $a_g = 100 \mu\text{m}$). This empirical model for $a_{\text{eff}}(f)$ provides information required for compensation of spatial averaging artifacts in acoustic output measurements and is useful for choosing an appropriate sensitive element size for a given experiment.

Keywords

hydrophone; directivity; membrane; acoustic output measurement

I. INTRODUCTION

DIRECTIVITY describes hydrophone response as a function of angle of incidence of quasi-planar pressure waves. Frequency-dependent *effective* sensitive element size is different from *geometrical* sensitive element size and may be derived from directivity measurements. It is essential to know the frequency-dependent effective sensitive element size in order to account for spatial averaging effects of hydrophone measurements. This is particularly important for nonlinear pressure waves because harmonics have beam widths that decrease with frequency, leading to increased susceptibility to spatial averaging effects.

Rigid piston models have been proposed to predict sensitivity [1–4] and directivity [4] of needle and reflectance-based fiber-optic hydrophones. One rigid piston model [4] has been validated to predict directivity and frequency-dependent effective sensitive element size for needle [5] and reflectance-based fiber-optic [6] hydrophones. This model was subsequently shown to be useful for compensating for spatial averaging artifacts in hydrophone measurements of nonlinear pressure waves [7, 8]. Similar analysis is now needed for membrane hydrophones, which (along with needle and fiber-optic hydrophones) are among the most common hydrophones used in ultrasonics [9, 10].

Membrane hydrophones are often used for acoustic output measurements because, compared with other hydrophone types, they have relatively broad bandwidth and uniform frequency response. Although most common hydrophones cannot withstand pressure levels from high intensity therapeutic ultrasound (HITU) sources, one robust membrane hydrophone design has been validated for HITU applications [11, 12].

Besides spatial averaging artifacts, the other major source of distortion of signals measured with hydrophones is frequency-dependent sensitivity. The effects of filtering by frequency-dependent sensitivity (such as errors in estimates of peak compressional pressure, peak rarefactional pressure, and pulse intensity integral) may be suppressed using inverse filtering (deconvolution) [13–22]. A spatiotemporal transfer function model for hydrophone distortion based on the product of frequency-dependent sensitivity and a spatial averaging filter has previously been validated for needle and fiber-optic hydrophones [7, 8].

Bacon reported pioneering modeling and measurements of membrane hydrophone directivity [23]. Bacon's model accounted for reduction of hydrophone response with increasing angle due to phase cancellation of plane waves across the sensitive element. In addition, Bacon's model accounted for Lamb waves in the membrane hydrophone film that can become important at large angles of incidence. Bacon validated his model using three unbacked, coplanar, single-membrane hydrophones that were characteristic of 1982 technology and had sensitive element diameters of 1 mm – 4 mm, which would be considered relatively large today.

One goal of this paper is to provide additional experimental validation for Bacon's model ([23], Eq. 6) using a larger number (8) of membrane hydrophones characteristic of current technology, including backed and unbacked designs, single-layer and bilaminar designs, differential designs, and sensitive element diameters as low as 200 μm . Another goal is to obtain a quantitative, empirical formula for the relationship between geometrical and

frequency-dependent effective sensitive element sizes, which is needed to account for spatial averaging effects for linear and nonlinear pressure waves [7].

II. THEORY

Spatial averaging effects may be modeled by integrating the free field (*i.e.*, the field in the absence of a hydrophone) over the surface of an imaginary hydrophone sensitive element with an appropriate “effective” sensitive element size [24, 25]. The effective sensitive element radius a_{eff} can differ from the geometrical sensitive element radius a_g and depends on frequency.

Real hydrophone sensitive elements may not be perfectly circular and in principle might be more accurately described by ellipses. Some hydrophones even use square-electrode-overlap designs. However, a circular model is often employed and has been shown to be useful for modeling hydrophone directional response [24–27]. Therefore, for practicality and parsimony, a circular model is adopted here.

The method for estimation of $a_{\text{eff}}(f)$ recommended by IEC 62127-3 Section 5.6 is roughly equivalent to fitting directivity measurements to the theoretical directional response for a circular disk receiver from diffraction considerations [7, 22, 24–26, 28–30]

$$D(a, k, \theta) = \frac{2J_1(k a \sin \theta)}{k a \sin \theta} \quad (1)$$

where $k = 2\pi / \lambda$, λ is wavelength, a is sensitive element radius (frequency-dependent adjustable fitting parameter), θ is the angle of an incident plane wave, and $J_1()$ is a Bessel function of the first kind [25]. Prediction of spatial averaging effects based on the disk directivity model as a function of the empirically-obtained $a_{\text{eff}}(f)$ is often a convenient alternative to using the true directivity as a function of a_g [7].

The disk model accounts for reduction of hydrophone response with increasing angle due to phase cancellation of oblique plane waves across the sensitive element. Phase cancellation affects all hydrophones, including membrane, needle, and fiber-optic designs. Membrane hydrophone directivity may also be affected by Lamb waves that propagate in the film [23].

Equation (1) has been referred to as the “rigid baffle” model because it corresponds to the diffraction pattern of a rigid-planar-baffled, disk receiver [29, 31, 32] with geometrical radius a . Usage of (1) with $a = a_{\text{eff}}(f)$ ($D[a_{\text{eff}}(f), k, \theta]$ instead of $D[a_g, k, \theta]$) to model directivity of membrane hydrophones does not imply that the physical mechanisms underlying membrane hydrophones and rigid baffles are similar. Rather, it is a useful modeling method that allows measured directivity data (produced by whatever physical processes) to be summarized by a functional fit (1) that yields $a_{\text{eff}}(f)$, which has a practical physical interpretation: the frequency-dependent radius of an imaginary disk over which the free field would be integrated in order to predict hydrophone output. The function $a_{\text{eff}}(f)$ is helpful for predicting spatial averaging in hydrophone measurements. For example, this method has been shown to be useful for predicting spatial averaging with needle

hydrophones [7, 8] even though needle hydrophones are more accurately represented by the rigid piston model than by the rigid baffle model [5].

III. METHODS

A. Experimental Methods

Table I lists the hydrophones investigated. The hydrophones span a wide range of sensitive element diameters, from 200 to 1000 μm .

Directivity measurements were performed using the National Physical Laboratory (NPL) Scanning Tank with the hydrophone immersed in deionized water. The method employed nonlinear sawtooth waveforms generated by nonlinear propagation so that measurements could be performed at multiple harmonics of the source transducer fundamental frequency [33].

The mean water temperature, measured with a calibrated mercury-in-glass thermometer (GH Zeal Ltd, London UK), was $21.0 \pm 1.0^\circ\text{C}$. The signal was acquired with a calibrated Tektronics (Beaverton, OR) DPO-7254 digital phosphor oscilloscope. The active element of the hydrophone under test was aligned to the beam alignment axis of each of two plane-piston Olympus (Tokyo, Japan) source transducers with nominal center frequencies of 1 MHz and 2 MHz, in turn. The nominal diameters of the transducers were 25.4 mm and 12.7 mm. The source transducers were operated at their nominal center frequency in a short burst mode (< 20 cycles) via a Keysight (Santa Rosa, CA) 33250A arbitrary waveform generator. The signal was amplified using an Amplifier Research (Souderton, PA) 150A100B RF amplifier. The hydrophone was positioned in the transducer far field where -6 dB beam widths at all frequencies of interest were larger than 20 times the largest hydrophone diameter. Hydrophone - transducer separations were approximately 600 mm for the 1 MHz transducer and 400 mm for the 2 MHz transducer. This positioning minimized the likelihood of misalignment affecting results and ensured quasi-planar waves across the sensitive element. The hydrophone sensitive element was aligned to the axis of rotation by comparing the time-of-flight of the ultrasound pulse at two angles of incidence, and adjusting the hydrophone position to minimize the difference between them. The drive voltage was set high enough to generate nonlinear fields containing multiple harmonics. Directivity was calculated with the two source transducers at 1, 2, 3, 4, 6, 8 and 10 MHz.

The hydrophone was first rotated through its horizontal plane about its reference center in steps of 10° to generate angles of incidence ranging from -60° to 60° , with 0° indicating the angle of maximum signal. At each position, between 64 and 512 waveforms were acquired from the hydrophone and averaged. The mean waveforms were then windowed using a 5- μs Blackman-Harris window function to remove reflections from the hydrophone mounting, which were particularly noticeable at large angles of incidence. The windowed waveforms were processed with a Fast Fourier Transform, and the magnitudes of the relevant harmonics were extracted and normalized relative to 0° .

Alternative methods for directivity measurement include a pulsed near-field method [34], a time-delay-spectrometry (TDS) based method [24], a method based on sequential

measurement of tone bursts [12, 29], and a method based on using a photoacoustic source consisting of a blackened planar surface illuminated by a laser [35, 36]. The nonlinear approach used for the present study was chosen because of its established consistency with multiple other methods for hydrophone calibration [33].

Type-A (random) uncertainty, also known as precision, for each frequency, angle, and hydrophone was assessed from four normalized repeat measurements.

B. Estimation of Effective Sensitive Element Radii

Frequency-dependent effective hydrophone sensitive element radius $a_{\text{eff}}(f)$ was estimated using a method similar IEC 62127-3 Section 5.6 and entailed fitting the disk model (1) to directivity measurements using the sensitive element radius (a) as an adjustable parameter [7, 29]. The value for $a_{\text{eff}}(f)$ chosen for each frequency and each hydrophone was the value that minimized the root mean square difference (RMSD) between the experimental directivity and model directivity (1) over angles from -30° to 30° . This angular range optimized the directivity model for beams with angular spectra mostly confined to $|\theta| < 30^\circ$, which can accommodate transducers with f-numbers > 1 [7, 8, 37]. The relative difference between effective and nominal geometrical sensitive element radii

$$\frac{\Delta a}{a_g} = \frac{a_{\text{eff}} - a_g}{a_g}$$

was fit to a power-law function of ka_g ,

$$\frac{\Delta a}{a_g} = \frac{C}{(ka_g)^n}$$

The power law fit parameters were obtained from linear fits to log-transformed data. Exponential functions have similar shape and have also been used successfully to model $a_{\text{eff}}(f)$ [29].

IV. RESULTS

Figs. 1 and 2 show directivity measurements (dashed lines) for hydrophones with nominal geometrical sensitive element diameters equal to 200 μm and 1 mm. The central lobes of the directivity patterns become narrower as frequency increases. This is expected because the amount of phase cancellation for oblique plane waves increases with frequency. The hydrophone with the larger sensitive element (Fig. 2) exhibits narrower directivity patterns than the hydrophone with the smaller sensitive element (Fig. 1). This is expected because the amount of phase cancellation for oblique plane waves increases with sensitive element size.

Figs. 1 and 2 show disk model functions for directivity $D(a_g, k, \theta)$, calculated based on nominal geometrical sensitive element size a_g (dotted lines). $D[a_g, k, \theta]$ performs well for the hydrophone with the larger geometrical sensitive element diameter (1 mm) for frequencies > 3 MHz but shows noticeable discrepancies with measurements at 1 MHz and 2

MHz (Fig. 2). $D[a_g, k, \theta]$ (dotted lines) does not perform well for the hydrophone with the smaller geometrical sensitive element diameter (200 μm) for frequencies ranging from 1 MHz to 10 MHz (Fig. 1). The shortcomings of the disk model based on a_g underscore the need to model directivity by $D[a_{\text{eff}}(f), k, \theta]$ instead of $D[a_g, k, \theta]$.

Figs. 1 and 2 show side lobes that depart from the disk model and become more prominent as frequency decreases. Similar side lobes were observed by Bacon and attributed to Lamb waves in the film [23]. Figs. 1 and 2 also show minima near 40° , particularly noticeable at low frequencies, which is similar to Bacon's theory and experiments ([23], Fig. 10).

In order to check for field dependence and to validate the harmonic-based methodology, comparisons were made between directivity patterns obtained using the second harmonic measured with the 1 MHz source transducer and the fundamental measured with the 2 MHz source transducer. The absolute value of the difference between the two directivities, averaged over all angles from -80° to $+80^\circ$, was 0.04 ± 0.05 (mean $\pm$ standard deviation), suggesting that the two methods were essentially equivalent.

In addition to the main hydrophone time-domain signal, there was an unexpected wave at high angles of incidence. The time of arrival shortened as the angle increased, so the signal started interfering with the main hydrophone signal for angles larger than 40° (dependent on hydrophone geometry). For large angles of incidence, the signal arrived before the main wave, suggesting that it traveled in a medium with speed of sound faster than water, probably from the mounting ring through the film. The effect of this signal was to increase uncertainty in the measurements at large angles of incidence. However, the amplitude of this signal was not sufficient to completely explain the side lobes, which were still likely due to Lamb waves in the film.

Fig. 3 shows the average (over all 8 hydrophones and all angles between $\pm 60^\circ$) precision of measurements as a function of frequency. Average precision increased approximately linearly with frequency at a rate of about one percent per MHz. This may be due to signal-to-noise ratio diminishing with frequency.

Fig. 4 shows the average (over all 8 hydrophones and all 7 frequencies) precision of measurements as a function of angle. Average precision was in the range of 4 to 8 percent and did not show a strong trend with angle. The mean precision averaged over all frequencies, angles, and hydrophones was 5.8%.

Fig. 5 shows effective radius a_{eff} as a function of ka_g for the GEC Marconi hydrophone with nominal $a_g = 250 \mu\text{m}$. For low values of ka_g , $a_{\text{eff}} \gg a_g$. As ka_g increases, a_{eff} asymptotically approaches a value close to a_g .

The RMSD between measured directivities and functional fits, $D[a_{\text{eff}}(f), k, \theta]$, averaged over all 8 hydrophones, all frequencies (1, 2, 3, 4, 6, 8, and 10 MHz), and all angles from -30° to 30° , was 2.6%, supporting the disk model (1) (using $a_{\text{eff}}(f)$ instead of a_g) for modelling directivity.

Fig. 6 shows the relative difference between a_{eff} and a_g as a function of ka_g for all 8 hydrophones. A curve fit of the form $C/(ka_g)^n$ with $C = 1.89$ and $n = 1.36$ is also shown. The RMSD between the curve fit and the measurements is 34%. Relatively large differences were exhibited by two of the hydrophones with sensitive element radius of 100 μm (D1202 and HMB-0200, but not the MH06).

Fig. 7 shows directivity full-width at half-maximum (FWHM) θ_{FWHM} for all 8 hydrophones plotted vs. the product of frequency f (MHz) and nominal geometrical sensitive element diameter d_g (mm), where $d_g = 2a_g$. The variable on the horizontal axis, fd_g , was chosen to facilitate comparison with Fig. 11 in Bacon [23]. (If f is measured in MHz and d_g is measured in mm, then $ka_g = 2.12fd_g$). Following Bacon, the solid blue curve shows the circular disk model (1) based on a_g : $D[a_g, k, \theta]$ (Note that $D[a_{eff}(f), k, \theta]$ fits the data better than $D[a_g, k, \theta]$. Fig. 7 is similar to Fig. 11 in Bacon [23]. In both figures, measurements of θ_{FWHM} are consistent with $D[a_g, k, \theta]$ for $fd_g > 4$ MHz·mm and lower than $D[a_g, k, \theta]$ for $fd_g < 4$ MHz·mm. At $fd_g = 1$ MHz·mm (the lowest value measured by Bacon), Bacon reported experimental and theoretical values for θ_{FWHM} in the range of $60^\circ - 70^\circ$. In the present investigation the range was similar: $62^\circ - 73^\circ$. For $fd_g < 1$ MHz·mm in the present investigation, θ_{FWHM} continues the trend established for $fd_g > 1$ MHz·mm, reaching a value of near $80 - 100^\circ$ at $fd_g = 0.2$ MHz·mm. The present investigation provides extended validation for Bacon's model for a larger number of hydrophones (8 vs. 3), a broader variety of membrane hydrophone designs, and smaller minimum geometrical sensitive element diameters (200 μm vs. 1000 μm) than Bacon's original investigation [23].

V. DISCUSSION

A. Summary

Bacon's model for membrane hydrophone directivity ([23], Eq. 6) originally validated for three 1982-vintage membrane hydrophones with $2a_g \approx 1$ mm, has been validated for 8 more modern membrane hydrophones with a variety of designs and manufacturers and $2a_g \approx 200$ μm .

In the present investigation, membrane hydrophones were characterized by their nominal a_g rather than their true a_g . This approach is useful for many membrane hydrophone users since the nominal a_g is far easier to ascertain and is often specified by the manufacturer. However, it is important to understand the limits of this simplification. Nominal a_g often corresponds to the radius of the circular electrode or the overlapping area of the two electrodes on top and bottom of the PVDF foil. This visible geometrical size can be measured directly. The nominal a_g can be approximately equal to the high-frequency a_{eff} [38]. However, the actual sensing element can be larger than this electrode size due to the spot-poling process when activating the piezoelectricity in the PVDF foil. For example, Wilkens and Molkenstruck applied two different voltages to similar membrane hydrophones during the spot-poling process. The hydrophone that received a 20% higher spot-poling voltage exhibited a 3 dB increase in sensitivity at the cost of a 20% increase in high-frequency effective radius [29].

Spatial averaging artifacts (frequency-dependent signal reduction due to phase cancellation) are common for hydrophone measurements. They are particularly important when

measuring nonlinear signals, which have beams with many harmonic components. The cross-sectional beam widths of these harmonic components decrease with harmonic frequency. Therefore, as harmonic frequency increases, the potential for spatial averaging artifacts also increases. In order to compute an inverse spatial averaging filter to compensate for this effect, it is necessary to know the frequency-dependent effective sensitive element radius $a_{\text{eff}}(f)$ [7, 8]. Ideally, $a_{\text{eff}}(f)$ could be measured for every hydrophone individually (e.g., IEC 62127-3 Sec. 5.6). However, this is not always practical, and it is not always possible in the planning stages of an experiment, when investigators determine the appropriate sensitive element size of a hydrophone yet to be acquired. Therefore, it is useful to have a generic expression for $a_{\text{eff}}(f)$ as obtained in this investigation.

B. Comparison with Previously Published Data

The results presented in this paper may be analyzed in the context of prior reports of measurements of frequency-dependent directivity, which may be summarized by $\theta_{\text{FWHM}}(f)$ or $a_{\text{eff}}(f)$ for membrane hydrophones [12, 22, 24, 29, 30, 32, 33]. The two parameters convey similar information and are related by $a_{\text{eff}}(f) = 2.22 \text{ c} / [2 \pi f \sin \theta_{\text{FWHM}}(f)]$ [25, 32, 39], where $\theta_{\text{FWHM}} = 2 \theta_{\text{HWHM}}$. Previous investigators reported their own multiple-frequency measurements on one or two membrane hydrophones but did not quantitatively relate them to measurements by other investigators or to Bacon's theory ([23], Eq. 6). Therefore, a quantitative synthesis of these reports and the present results would be valuable. While some investigators reported $\theta_{\text{FWHM}}(f)$ [23, 32, 33], others reported $a_{\text{eff}}(f)$ [22, 24, 29, 30]. To facilitate a comprehensive comparison, reported values of $\theta_{\text{FWHM}}(f)$ will be converted here to $a_{\text{eff}}(f)$ using $a_{\text{eff}}(f) = 2.22 \text{ c} / [2 \pi f \sin \theta_{\text{HWHM}}(f)]$.

Bacon provided quantitative values for $\theta_{\text{FWHM}}(f)$ for three membrane hydrophones at five frequencies [23]. Shombert *et al.* provided quantitative values for $\theta_{\text{HWHM}}(f)$ for one membrane hydrophone at four frequencies [32]. Wilkens and Molkenstruck provided quantitative values for $a_{\text{eff}}(f)$ for one membrane hydrophone at eight frequencies [29]. Martin and Treeby provided quantitative values for $a_{\text{eff}}(f)$ for two membrane hydrophones at two frequencies [22].

Three other teams provided data in graphical form but with sufficient graphical detail to allow extraction of values with adequate precision for meaningful quantitative comparison with other studies. Smith and Bacon provided directivity plots for a membrane hydrophone at four frequencies (but did not compare them to Bacon's theory [23]) [33]. Their angular scale had divisions of 10° , allowing readers to quantitatively ascertain θ_{FWHM} confidently to within 2.5° [33]. Beard *et al.* provided a plot of $a_{\text{eff}}(f)$ for one membrane hydrophone at eight frequencies. Their scale had divisions of $100 \mu\text{m}$, allowing readers to quantitatively ascertain $a_{\text{eff}}(f)$ confidently to within $25 \mu\text{m}$ or 5% of a_g ($500 \mu\text{m}$) [30]. Radulescu *et al.* provided a plot of $a_{\text{eff}}(f)$ as a quasi-continuous function of frequency. Their scale also had divisions of $100 \mu\text{m}$, allowing readers to quantitatively ascertain $a_{\text{eff}}(f)$ confidently to within $25 \mu\text{m}$ or 12.5% of a_g ($200 \mu\text{m}$) [24].

Preston *et al.* provided directivity plots for 7 membrane hydrophones [40]. Only one hydrophone had a geometrical sensitive element diameter below 1 mm (0.5 mm), and directivity for that hydrophone was measured at one frequency (2.25 MHz). Preston *et al.*

only provided multiple-frequency data for one hydrophone, which appears to be similar or identical to the coplanar shielded 1-mm-sensitive-element hydrophone previously measured by Bacon [23], one of Preston's co-authors. The plots did not have sufficient size and quality to permit confident extraction of quantitative data for estimation of $a_{eff}(f)$. However, the plots did suggest good conformity with Bacon's theory ([23], Eq. 6), especially for angles within $\pm 40^\circ$. For the 0.5 mm hydrophone, a diameter value of 0.7 mm was used to compute the theoretical directivity in order to optimize the fit.

Fig. 8 shows the relative difference between a_{eff} and a_g as a function of ka_g for the aforementioned studies [22–24, 29, 30, 32, 33]. Fig. 8 is similar to Fig. 6. In both figures, relative differences are large for low ka_g and asymptotically approach zero for large ka_g . Similar behavior has been demonstrated for needle [5] and fiber-optic [6] hydrophones. The same curve fit from Fig. 6 of the form $C/(ka_g)^n$ with $C = 1.89$ and $n = 1.36$ is also shown. The RMSD between the curve fit and the previous measurements in Fig. 8 is 24%, somewhat lower than the RMSD for Fig. 6 (34%). The uncertainties associated with extracting numbers from graphs (5% - 12.5% of a_g as discussed above) are not big enough to have a significant effect on the trend shown in Fig. 6.

Figs. 6 and 8 suggest that for $ka_g > 8$, the difference between a_{eff} and a_g is small ($< 12\%$) and therefore there is great potential benefit in reducing the choice for a_g in order to minimize spatial averaging artifacts, as expected. However, for $ka_g < 8$, the difference between a_{eff} and a_g grows rapidly as a_g is reduced and therefore the benefit in reducing the choice for a_g becomes much less than what one might intuitively expect. It should also be noted that the magnitude of signal reduction due to spatial averaging monotonically increases with the ratio of $a_{eff}(f)$ to frequency-dependent beam width [7]. The fact that beam width tends to increase as frequency decreases may mitigate spatial averaging effects at low frequencies.

C. Membrane Hydrophones vs. Needle Hydrophones

Fig. 9 shows relative differences between a_{eff} and a_g as functions of ka_g for membrane and needle hydrophones. The curve for needle hydrophones was obtained from theoretical considerations [7] supported by experimental data [5]. The asterisks correspond to relative differences resulting from a hydrophone with $a_g = 100 \mu\text{m}$ at frequencies of 1, 2, 3, and 4 MHz. (Recall that frequency is directly proportional to k via $2\pi f = kc$, where c is the speed of sound). At low frequencies, the effective sensitive element radius $a_{eff}(f)$ is much closer to a_g for the needle hydrophone than the membrane hydrophone. Therefore, for frequencies of 1 MHz – 4 MHz and $a_g = 100 \mu\text{m}$, a needle hydrophone would appear to be far less susceptible to spatial averaging artifacts than a membrane hydrophone. The difference between the membrane and needle curves is attributable to the different physics of the two types of hydrophones. For membrane hydrophones, $a_{eff}(f)$ is determined by geometry, the spot-poling process, and symmetric Lamb waves in the membrane film that can become important at large angles of incidence [23]. For needle hydrophones, $a_{eff}(f)$ is determined by geometry and diffraction effects [4, 7].

VI. CONCLUSION

The present investigation provides an extension beyond Bacon's validation for his pioneering membrane hydrophone directivity model in 1982 ([23], Eq. 6), including a larger number of hydrophones (8 vs. 3), a broader variety of membrane hydrophone designs (including backed, unbacked, differential, square-electrode-overlap, single-layer and bilaminar designs), and a range of geometrical sensitive element diameters more relevant to current hydrophone technology (200 μm - 1000 μm vs. 1000 μm - 4000 μm) [23]. An empirical formula for frequency-dependent effective sensitive element size for membrane hydrophones, which is necessary for computing inverse spatial averaging filters [7], was obtained. A quantitative comparison of effective sensitive element size for membrane and needle hydrophones was presented. The effective sensitive element radius is much closer to the geometrical sensitive element radius for needle hydrophones than membrane hydrophones at low frequencies (*e.g.*, < 4 MHz when $a_g = 100 \mu\text{m}$). Therefore, needle hydrophones are less susceptible to spatial averaging artifacts at low frequencies. This work provides guidance for choosing an appropriate hydrophone (membrane or needle) and geometrical sensitive element size for a given experiment.

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Fig. 1.

Directivity for the Precision Acoustics D1202 with sensitive element nominal geometrical diameter equal to 200 μm . Measurements are connected with dashed lines. Disk models based on the geometrical sensitive element diameter, $D(a_g, k, \theta)$, are shown in dotted lines.

Fig. 2.

Directivity for GEC Marconi hydrophone with sensitive element nominal geometrical diameter equal to 1 mm. Measurements are connected with dashed lines. Disk models based on the geometrical sensitive element diameter, $D(a_g, k, \theta)$, are shown in dotted lines.

Fig. 3.
Precision of directivity measurements vs. frequency averaged over all 8 hydrophones and all angles between $\pm 60^\circ$. Error bars denote standard deviations.

Fig. 4.
Precision of directivity measurements vs. angle averaged over all 8 hydrophones and all 7 frequencies from 1 MHz to 10 MHz. Error bars denote standard deviations.

Fig. 5. Effective radius a_{eff} as a function of ka_g for the GEC Marconi hydrophone with nominal geometrical sensitive element radius of 250 μm .

Fig. 6.
Relative difference between a_{eff} and a_g as a function of ka_g for all 8 hydrophones.

Fig. 7. Directivity full-width at half-maximum (FWHM) θ_{FWHM} for all 8 hydrophones plotted vs. the product of frequency f (MHz) and nominal geometrical sensitive element diameter d_g (mm). The theoretical directivity $D[a, k, \theta]$ is defined in (1). $D[a_{eff}(f), k, \theta]$ fits the data better than $D[a_g, k, \theta]$.

Fig. 8.

Relative difference between a_{eff} and a_g as a function of ka_g for previous studies.

Fig. 9. Relative difference between a_{eff} and a_g as functions of ka_g for membrane and needle hydrophones.

TABLE I

HYDROPHONES USED FOR MEASUREMENTS

Manufacturer	Model	Notes	Nominal Geometrical Sensitive Element Radius $a_g(\mu\text{m})$
Precision Acoustics	D1202	differential	100
Onda	HMB-0200	backed	100
Gampt	MHB MH06	differential	100
Precision Acoustics	UC1604		200
Marconi	B014	coplanar	250
NTR	HMA-0500		250
Precision Acoustics	UT1606		300
Marconi	IP039	bilaminar	500