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Patterns of temporal scaling of groundwater level fluctuation

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SUMMARY

We studied the fractal scaling behavior of groundwater level fluctuation for various types of aquifers in Puerto Rico using the methods of (1) detrended fluctuation analysis (DFA) to examine the monofractality and (2) wavelet transform maximum modulus (WTMM) to analyze the multifractality. The DFA results show that fractals exist in groundwater fluctuations of all the aquifers with scaling patterns that are anti-persistent ($1 < \beta < 1.5$; 1.32 ± 0.12 , 18 wells) or persistent ($\beta > 1.5$; 1.62 ± 0.07 , 4 wells). The multi-fractal analysis confirmed the need to characterize these highly complex processes with multifractality, which originated from the stochastic distribution of the irregularly-shaped fluctuations. The singularity spectra of the fluctuation processes in each well were site specific. We found a general elevational effect with smaller fractal scaling coefficients in the shallower wells, except for the Northern Karst Aquifer Upper System. High spatial variability of fractal scaling of groundwater level fluctuations in the karst aquifer is due to the coupled effects of anthropogenic perturbations, precipitation, elevation and particularly the high heterogeneous hydrogeological conditions.

Keywords

Detrended fluctuation analysis (DFA); Wavelet; Mono- and multi-fractality; Karst aquifer; Puerto Rico

1. Introduction

Fractal analysis is a method developed in recent decades to study the irregular features that are not characterized by conventional geometry (Mandelbrot, 1983). The concept has been extended to and applied in many disciplines such as physiology, geophysics, hydrology, and social sciences, as well as the study of nonlinear or stochastic processes. The fractal is

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at <http://dx.doi.org/10.1016/j.jhydrol.2016.03.018>

defined as an object that reproduces itself invariant to scales, whereas it can be either deterministic fractal that has exactly the same geometry, or stochastic fractal that is statistically similar to the object in other scales. Homogeneous variability processes are characterized as monofractal, while complex nonlinear heterogeneous processes are described by multifractal structures (Stanley et al., 1999).

Groundwater levels reflect the potentiometric state of water recharge, storage and discharge of the aquifer (Law, 2001; Conlon et al., 2005). A metastable equilibrium state is easily upset by variations of the hydrogeological conditions such as precipitation, evaporation, transpiration, runoff, infiltration, soil moisture, and anthropogenic activities such as pumping, dams, irrigation, and other water management methods (Li and Zhang, 2007; Zhou et al., 2012). Consequently, groundwater levels fluctuate continuously to achieve potentiometric balance. The temporal fluctuations may exhibit periodic and seasonal cycling due to climatological conditions. The fluctuations are often not stationary and are not suitable to be simulated using ordinary linear and deterministic models. The non-stationary fluctuations of groundwater levels, which may be characterized as fractional Brownian motion (fBm; Hardstone et al., 2012), may not be completely random but, to a large degree, long-range correlated (Bak et al., 1987; Matsoukas et al., 2000; Veneziano and Langousis, 2010). The long-range correlation or self-affinity is also termed as fractals because the variations of groundwater levels, to some extent, follow a logarithmic linear power-law relationship with time windows, i.e. $F(l) \sim l^\gamma$ (fluctuations $F(l)$ vs. time windows l , γ is the scaling coefficient). The fractal scaling coefficients suggest both anti-persistent and persistent correlations, where anti-persistence means that variability in one direction is likely to be followed by variability in the opposite direction, and persistence means there is a trend of the fluctuations (Dingwell and Cusumano, 2010).

Studies on the persistent fractal scaling behavior of hydrology variables date back to the pioneering work of Hurst (1951) studying water levels and storage capacity in the Nile River. Researchers have reported on fractal scaling behavior for groundwater levels, where the power of the fractal correlations was proposed to decrease from persistent scaling in the short scale of about 10 days, to anti-persistent at a few months to a year, and to $1/f$ noise at larger time scale (Zhang and Schilling, 2004; Li and Zhang, 2007; Little and Bloomfield, 2010; Labat et al., 2011). Zhang and Schilling (2004) and Li and Zhang (2007) further suggested that aquifers tend to behave as a fractal filter where the temporal fractal correlation decreases from rainfall to stream flow, to groundwater, and to base flow due to damping effects of land surface, soil deposits and aquifers. Crossovers of the fractals were detected for groundwater level fluctuations, which indicate that the notion of monofractal may not be enough to describe these complex processes. Multifractal analysis is required to detect and quantify irregularities in the time series from point to point. In hydrology, fractal scaling analysis of groundwater level fluctuations is fundamental to more realistic hydrology modeling studies by considering the temporal scaling cascading issues, because every model is based on some spatial and temporal scales whereby the hydrogeology parameters may vary under different scales (Bergstrom and Graham, 1998). Fractal scaling analysis of the mechanistic properties of groundwater level fluctuations may also be useful in revealing intrinsic hydrogeological conditions and in evaluating extreme climatological events and anthropogenic perturbations (Liang and Zhang, 2013).

Aquifers are widely distributed throughout Puerto Rico with a concentration in the North Coast featuring karst limestone formations, which provides significant water resources to Puerto Rico. Research on groundwater level fluctuations of Puerto Rico's aquifers has significant scientific and engineering meaning. In this paper, we systematically investigate the temporal fractal scaling behavior of groundwater level fluctuations in wells associated with various types of aquifers in Puerto Rico. We use the well-developed methods of detrended fluctuation analysis (DFA) to study monofractality and wavelet transform maxima modulus (WTMM) to study multifractality. Our goal is to analyze the mono- and multifractality of groundwater level fluctuations, examine the origin of the fractality, and investigate the spatial patterns of the fractal scaling behaviors.

2. Methods

2.1. Site description

Puerto Rico ($17^{\circ}55' - 18^{\circ}33' \text{N}$, $65^{\circ}33' - 67^{\circ}17' \text{W}$) is a relatively large island located in the Caribbean Sea covering a total area of 8710 km^2 . The island is categorized into four areas based on geological formations: the North Coast, South Coast, Alluvial Valley, and Volcaniclastic-, Igneous-, and Sedimentary-Rock. The North Coast features karst limestone formations which can be found in the upper and lower systems as well as the confining units (Fig. 1). The geophysical properties of these aquifers are summarized from previous studies and presented in Table 1 (Lugo et al., 2001; Renken et al., 2002). Note that the area reported in Table 1 refers to the outcrop area, and may not reflect additional area extending beneath shallower formations. The total areas for the Confining Unit and the North Coast Lower Aquifer, which extend beneath the Aguada and Aymamon Limestone Formations, are larger than the reported outcrop area. The northern karst aquifers provide around 50% of the total groundwater production of Puerto Rico, where the total groundwater production accounts for 16% of the total water production in Puerto Rico by 2005 (Molina-Rivera and Gómez-Gómez, 2008). The karst aquifer is evolving stochastically as the combined effects of waterborne matter filling the openings and fractures encroaching through the flushing and dissolution of the limestone, which is likely to impact water storage and groundwater levels. Groundwater flow in karst aquifers is essentially stochastic due to the high heterogeneity in its pathways, unknown fractures and channels, and complex formation matrix (Ghasemizadeh et al., 2012; Yu et al., 2014). Water management activities also have considerable effects on groundwater levels such as pumping of many wells in the north karst aquifers. The closing of the pumping wells due to concerns of groundwater contamination and opening of new wells for water use or pollution remediation further complicate the hydro-potentiometric state of the water levels.

2.2. Data collection and analysis

Groundwater level data were collected from the National Water Information System: Web Interface of U.S. Geological Survey monitoring wells (<http://waterdata.usgs.gov/nwis>; accessed June 2014). We explored the relationship between the monofractal scaling behaviors of precipitation and groundwater level fluctuations. The precipitation data were obtained from the National Oceanic and Atmospheric Administration (NOAA), which monitors 25 stations across the main island of Puerto Rico recording 15-min precipitation

data (<http://www.ncdc.noaa.gov/cdo-web/datasets>; accessed June 2014). Based on data availability, the study period was chosen from January 1, 1982 to December 31, 2012. There are currently 148 monitoring wells in the main island of Puerto Rico recording daily groundwater levels as the depth to land surface. We chose the 22 wells which have at least 2500 water level records (Fig. 1). USGS publishes only verified and approved data, so missing data and data gaps are frequent in the time series records. We used line joining to fill the small number of missing data and concatenated the large data gaps. The identification of the aquifer type for these wells was based on the surface geological formation maps in Fig. 1 and the average groundwater level to the land surface elevation. Note that this study is limited by the low number of wells used, which means that they may not be spatially representative, especially the North Coast Karst Aquifer Lower System and the Confining Units and the wells with high surface elevation are not included.

The seasonal and periodic patterns of the time series were removed by subtracting the daily mean groundwater level $\bar{h}$ of all the sampling years from the original time series $h(t)$ and normalizing by the standard deviation. The deseasonalized time series $\Phi(t)$ is calculated by:

$$\Phi(t) = \frac{h(t) - \bar{h}}{\sqrt{h^2(t) - \bar{h}^2}} \quad (1)$$

Statistical calculations were performed using the software Statistical Analysis System (SAS 9.3, SAS Institute Inc., Cary, NC). Pearson's correlation analysis was performed using SAS PROC CORR to examine the relationship of the fractal scaling results. The tool PROC REG was used to perform multiple linear regressions to filter out the most relevant factors with both entry and leaving levels at 0.05. Group means were compared using SAS PROC GLM and TUKEY's multiple comparisons. The software Geographic Information System (ESRI ArcGIS 9.3) was used to perform the spatial data representation and analysis.

To evaluate the origins and types of the fractality in a time series, shuffling was performed by randomly sorting the deseasonalized data by a newly created random variable using SAS RANUNI function. Then we investigated the fractal scaling behaviors for the shuffled data and compared them with the results from the unshuffled data. There are two types of origins of the multifractality: one due to broad probability density functions of the values in the time series, and one due to different long-range correlations of the small and large fluctuations (Kantelhardt et al., 2002). If the fractal scaling coefficients after shuffling are the same as those unshuffled, then it will indicate the first type of multifractality because the probability density function is not related to the order of the time series. If the fractal scaling coefficients turn out to be 0.5 (white noise), it indicates the second type of multifractality because all the fractal correlations were destroyed by the shuffling processes. If the shuffled results are weaker than the unshuffled ones, it indicates the existence of both types of multifractality.

2.3. Groundwater levels

The average land surface elevation of the wells is 19 ± 24 m (mean $\pm$ STD, standard deviation) with a range of 1–84 m. The average groundwater level to the land surface

elevations for all the records was 7 ± 6 m, with a range of 0.5–21 m ($n = 125,700$). The group means of the groundwater levels in different types of aquifers were: 3 ± 2 (Alluvial Aquifer, $n = 71,700$), 9 ± 6 (South Coast Aquifer, $n = 23,900$), and 14 ± 5 (North Coast Aquifer Upper System, $n = 30,100$), respectively, and were significantly different ($p < 0.0001$). There were more deviations of groundwater levels in the North Coast Karst Aquifer Upper Systems.

2.4. DFA method

Many methods have been developed to determine the fractal scaling behaviors, such as power spectrum, rescaled range analysis, probability distribution function, variation method, wavelet analysis and detrended fluctuation analysis (Dubuc et al., 1989; Sivakumar, 2000; Zhang and Schilling, 2004; Shang and Kamae, 2005; Hardstone et al., 2012). DFA is considered more powerful than conventional methods due to its ability to detect the evolving non-stationarities and also its ability to effectively remove the seemingly long-range correlations due to external effects (such as periodic cycling of the climatological conditions) and leave only the fluctuations from internal aquifer properties (Peng et al., 1994; Kantelhardt et al., 2002; Bryce and Sprague, 2012). The DFA method has been described in much detail (Peng et al., 1994, 1995; Alvarez-Ramirez et al., 2005). Hence, only a brief description of the calculation algorithm is summarized in the following steps:

1. Convert the seasonally adjusted time series of groundwater levels $\Phi(t)$ ($t = 1, 2, \dots, N$, and N is the final day of the records) into unbound random walk processes $\tilde{\Phi}(i)$. Note that this process may not be necessary because $\tilde{\Phi}(i)$ has already been integrated.

$$\tilde{\Phi}(i) = \sum_{t=1}^i [\Phi(t) - \overline{\Phi(t)}] \quad (2)$$

2. Divide $\tilde{\Phi}(i)$ into m non-overlapping windows of the same size l ($l = N/m$).
3. Within each window, a local polynomial trend of order p is fitted where p may be first (DFA1) or higher orders (DFA2, DFA3, etc.).
4. Calculate the root-mean-square deviation from the trend over every window at every time scale:

$$F(l) = \sqrt{\frac{1}{N} \sum_{i=1}^N [\tilde{\Phi}(i) - \tilde{\Phi}_l(i)]^2} \quad (3)$$

5. Perform linear regressions for the series of the logarithm transformed fluctuation function $F(l)$ and window size l , where the slope (β) denotes the DFA coefficients:

$$\log_{10} F(l) = \beta \log_{10} l + a \quad (4)$$

The tool PROC ROBUSTREG was used to perform the linear regression analysis for the logarithm based fluctuation functions $F(l)$ and window sizes l , where the Huber penalty function was used to iterate the weighted least squares (Little and Bloomfield, 2010). As only the most collinear points were used by the robust regression method, the detection of crossovers may be effectively circumvented when breakpoints were found in the power-law plots. Hence, we analyzed those power-law functions with potential crossovers separately. The DFA coefficient β of the linear power-law relationship is roughly equal to the Hurst coefficients (H ; Peng et al., 1994) with $0 < H < 0.5$ as anti-persistent correlation, $H \approx 0.5$ as white noise, $0.5 < H < 1$ as long-range positive correlation, and $H \approx 1$ as 1/f or pink noise. As groundwater level fluctuations are nonstationary and should be characterized by Brownian motion in this study, the actual Hurst coefficients were determined as $\beta - 1$, and with $\beta = 1.5$ as Brownian noise or fBm. Pink noise randomly distributes in low frequency band with the spectrum density inversely proportional to the frequency (Halley and Kunin, 1999). Larger β values imply lower fluctuations, weaker long-range correlation, and flatter time series.

In this study, the publicly available C program “DFA.c” from the PhysioToolkit website was used to calculate the series of logarithm-transformed $F(l)$ and l for each of the wells (Goldberger et al., 2000). Then linear regression lines were fitted using a robust regression method to each set of logarithm-transformed $F(l)$ and l to compute the β values. The β values calculated from the second order polynomial detrending (DFA2) were reported, and the values from other orders of polynomial trends (DFA1 and DFA3) were also evaluated to examine the effect of detrending methods on the β values.

2.5. Multifractal analysis

The principles behind computing the wavelet transform modulus maxima (WTMM) were first introduced by Mallat and Zhong (1992) for signal processing and later adopted and explored by Muzy et al. (1993) to account for the multifractal nature of the fully developed turbulent signals. It is based on the basic functions known as wavelets, which are endowed with varying moments used to investigate the recurrence of groundwater level fluctuations. The wavelet transform of a series χ_k of length N is computed with Eq. (5), as follows:

$$W(n, s) = \frac{1}{s} \sum_{k=1}^N x_k \psi \left[\frac{k-n}{s} \right] \quad (5)$$

The function $\psi(x)$ is the analyzing wavelet that can be represented by a polynomial, for

example the m th derivative of a Gaussian, $\psi^{(m)}(x) = d^m (e^{-\frac{x^2}{2}}) / dx^m$. The parameter s is the scale parameter. The WTMM method only uses the local maxima of $|W(n, s)|$ as a function of n , which gives the partition function:

$$Z(q, s) = \sum_{i=1}^{i_{\max}} |W(n_{i, s})|^q \quad (6)$$

Therefore, the partition function $Z(q,s)$ can further be described by the scaling exponent function $\tau(q)$ and the scale s ,

$$Z(q,s) \sim s^{\tau(q)} \quad (7)$$

Multifractal spectrum function $D(h)$ is calculated from scaling the exponential function via a Legendre transformation, where h is the Hölder exponent of a fractal subset and $D(h)$ is the corresponding fractal dimension or the singularity spectrum (Makowiec et al., 2009):

$$h = h(q) = \frac{\partial \tau(q)}{\partial q} \quad (8)$$

$$D(h) = \inf_q [q \cdot h - \tau(q)] \quad (9)$$

The Hölder exponent scaling function $h(q)$ is related to the Hurst exponent, or the DFA coefficient, when $q = 2$, by the following relationship:

$$\tilde{h}(2) = h(q) + 1 \quad (10)$$

In order to quantify the differences of the singularities due to different underlying processes, we fitted a nonlinear quadratic function around the maxima position of the spectrum $D(h)$, where the model is (Yu et al., 2002):

$$D(h) = A[h(q) - h_{\max}]^2 + B[h(q) - h_{\max}] + C \quad (11)$$

The parameter $h_{\max}$ is the corresponding Hölder coefficients at maxima $D(h)$. The parameter B denotes the measurement of the asymmetry for the curve, whereas positive values denote left-skewed and negative for right-skewed spectra, respectively. Left-skewed spectrum suggests relatively low-weighted high-fractal exponents (with more regular or smoother structure), and right-skewed spectrum indicates relatively strong-weighted high-fractal exponents (with fine-structure and more rougher geometry; Yu et al., 2002). We also used the width of the base of the spectrum W to quantify the strength of multifractality, which is defined as the difference between the $h(q)$ values when the fitted quadratic function $D(h)$ equals to 0. A larger value of W suggests a true multifractal statistical structure in time and a smaller width suggests an approximate monofractal structure (Holdsworth et al., 2012).

The publicly available code for WTMM “*multifractal.c*” from the Physionet website was used to perform the multifractal analysis (Goldberger et al., 2000). The moments were chosen as $-5 \leq q \leq 5$ and the interval was set to 0.2.

3. Results and discussions

3.1. Monofractal patterns

The linear power-law relationships between the logarithm based fluctuation functions $F(l)$ and the window sizes l for all the wells are presented in Fig. 2 ($n = 22$). The DFA coefficients (β values) varied from 1.05 to 1.69, with an average of 1.37 ± 0.16 (Fig. 3). The intercepts (a) of the logarithm power-law functions may indicate how groundwater levels are responding to the variations of the hydrogeological conditions of the associated aquifer. The intercepts of the power-law relationships were strongly negatively related to the β values ($r = -0.98$, $p < 0.0001$, $n = 22$). The DFA coefficients were slightly correlated with the mean groundwater levels ($r = 0.40$, $p = 0.06$, $n = 22$), and were notably correlated with the STDs of the water levels ($r = 0.63$, $p = 0.002$, $n = 22$). The shapes of each set of $F(l)$ and l lines were quite similar (Fig. 2). The deviations of the fluctuation function $F(l)$ were greater at larger window sizes l than those at smaller l . The calculated DFA coefficients confirm that fractal scaling exists in groundwater level fluctuations, and that the fractal scaling behaves as anti-persistent ($1 < \beta < 1.5$; 1.32 ± 0.12 , $n = 18$) and persistent ($\beta > 1.5$; 1.62 ± 0.07 , $n = 4$) correlations. The group means of the monofractal scaling coefficients determined by DFA for the Alluvial Aquifer, Northern Karst Aquifer Upper System, and South Coast Aquifer were 1.34 ± 0.16 ($n = 14$), 1.38 ± 0.08 ($n = 4$), and 1.46 ± 0.24 ($n = 4$), respectively, and were not significantly different from each other (Fig. 4; $p = 0.5$).

The number of groundwater level records varied considerably from well to well. The power-law relationship intrinsically suggests that the scaling coefficient is invariant to the scales. However, our data showed that there were some variations of the DFA coefficients with the length of the time series, i.e. the finite number of records (Fig. 5). Fig. 5a shows the gradual variations of the DFA coefficients with increasing time series size. Generally, the DFA coefficients decreased slightly with increased record size. The range of the DFA values was generally small over the change of time series size (with all STDs < 0.1), which suggests that monofractality exist in the fluctuation process. The small variations of the DFA coefficients especially those changes from persistent ($\beta > 1.5$) to anti-persistent ($1 < \beta < 1.5$) may possibly be due to the irregular geometric shape of the time series in these wells, which indicate that these processes are not homogeneous but show heterogeneity from location to location in the time scales. There are correlations between the STDs of DFA coefficients with the change of record size and the average groundwater levels for South Coast Aquifer ($r = 0.74$, $p = 0.26$, $n = 4$) and Northern Karst Aquifer Upper System ($r = 0.78$, $p = 0.22$, $n = 4$), but no correlation for the Alluvial Valley. This result shows that the variations of DFA coefficients are greater in the shallow wells in the South Coast Aquifer and the Northern Karst Aquifer Upper System.

Analyzing the power-law function of the logarithm based $F(l)$ and l is useful to understand the progress of fractals in the time series (Podobnik et al., 2007). We found distinct fractal crossovers in two wells associated with the Alluvial Valley Aquifer (Fig. 6). The patterns of crossovers in these two wells were quite similar. The power-law correlation was persistent ($\beta > 1.5$) within the short time scale (24 or 26 days), and anti-persistent in the long time scale. The short time scale crossovers principally reflect the persistence of the immediate response

of aquifers to hydro-potentiometric perturbations such as rainfall events and boundary conditions (Li and Zhang, 2007). The large time scale crossovers were proposed to be due to the intervals between rainfall events (Matsoukas et al., 2000; Li and Zhang, 2007). This is explained in our case as water levels in the Alluvial Valley Aquifer were generally shallower and more influenced by rainfall.

The effect of using different order of polynomial detrending methods was examined. The scaling coefficients were generally greatest from DFA3, then DFA2 and DFA1 (Fig. 7). However, there were no significant differences among the group means ($p = 0.07$). The relationships in Fig. 7 are not completely linear, which means that the DFA coefficients are not increasing systematically with the increase of detrending orders. This is further evidence of the effect of high heterogeneity of the fluctuation processes.

The values of the DFA fractal scaling coefficients from the shuffled deseasonalized data were all close to 0.5, which is white noise without any correlation of the fractals. This indicates that shuffling destroyed the embedded fractal correlations of the fluctuation processes.

The DFA fractal scaling coefficients of precipitation intensities for the 25 NOAA stations ranged from 0.53 to 0.64, with a mean of 0.59 and STD of 0.03. These results of the long-range correlations of the precipitation intensities are similar to the results for the daily precipitation records of several stations from representative climate zones of the world (Kantelhardt et al., 2006), the daily temperature records over Australia (Kiraly and Jánosi, 2005), and monthly temperature data in U.S. (Kurnaz, 2004).

3.2. Multifractal analysis

The multifractal results include the scaling exponent spectrum $\tau(q)$, the Hölder exponent spectrum $h(q)$, and singularity spectrum $D(h)$ corresponding to a series of moments ($-5 < q < 5$). The multifractal analysis results were presented based on the aquifer type and the asymmetry of the singularity spectrum in Fig. 8 (Alluvial Valley Aquifer), Fig. 9 (South Coast Aquifer), and Fig. 10 (Northern Karst Aquifer Upper System). Our analysis also shows that there is a strict requirement for the sampling size of WTMM method, whereas small record sizes give irregularly-shaped singularity spectra which are difficult to explain.

The relationships between $\tau(q)$ and q (Figs. 8–10) are all not linear, which suggests that multifractal behavior exists for groundwater level fluctuations in all wells. The shapes of the curves $h(q)$ vs q were distinctive from well to well. Fig. 8b compares the curves $h(q)$ vs q between different shaped spectra, where the left-skewed spectra were generally higher than the right-skewed spectra. This phenomenon might be due to the relatively rougher shaped time series for the right-skewed spectra in Fig. 8. The values of $\tilde{h}(2)$ were quite similar to the monofractal scaling coefficients calculated by DFA2 ($r = 0.94$, $p < 0.0001$, $n = 22$). The curves of $D(h)$ vs q and $D(h)$ vs $h(q)$ reveal the singularities of the processes. Our results suggest that the singularity spectrum of groundwater level fluctuations were rather site specific, even for wells associated with the same types of aquifers. Therefore, for rigorous modeling, the simulated time series should produce similar singularity spectrum.

Table 2 presents the mean maxima values of $D(h)$, the corresponding $q(h)$, and the average quadratic modeled parameters and the spectra base width for different types of aquifers. The generalized quadratic models for $D(h)$ vs $h(q)$ are presented in Fig. 11. The modeled spectra of $D(h)$ vs $h(q)$ were all right-skewed. The maxima $h(q)$ value and the width W values were the smallest in the Alluvial Aquifer, followed with Northern Karst Aquifer Upper System and South Coast Aquifer. This indicates that the fluctuation processes were smoother and were less multifractal in the Alluvial Aquifer than other types of aquifers. The fluctuation processes were more likely to be multifractal and heterogeneous in the South Coast Aquifers, which was also coincident with the relatively shallower water depth than the Karst Aquifer. The multifractal behavior of the fluctuations in the Karst Aquifer is in the middle of those from the other two types of aquifers, which may be due to more turbulent groundwater flow as well as the presence of multi-porosity (fractures, conduits, and matrix) in the karst aquifer (Shevenell and Goldstrand, 1997).

The Hölder exponent spectrum $h(q)$ turned out to be values close to zero when shuffling groundwater level data in all the wells. This result suggests that shuffling destroyed the intrinsic fractal correlations of the time series completely and hence the fractal scaling behaviors were highly temporal dependent in describing the various magnitudes of fluctuations.

3.3. Further discussion

We provide one of the few studies on the fractal scaling analysis of the subsurface system for various types of aquifers including the karst limestone aquifer in a moderately large tropical coastal island (e.g. Larocque et al., 1998; Majone et al., 2004). This study can be extended to other coastal islands with similar tropical climates and limestone geology, such as Barbados and Guam (Jones and Banner, 2003), and also to karst aquifers widely distributed around the world (Back et al., 1992). The monofractality scaling results in this study are in accordance to fractal studies in other regions (e.g. Walnut Creek, Iowa: Li and Zhang, 2007; karstic watersheds, France: Labat et al., 2011 and also the references herein; local sandstone aquifer, Sauk County and Dane County, Wisconsin: Rakhshandehroo and Amiri, 2012; unconfined Chalk aquifer in the Pang-lambourn catchment, UK: Little and Bloomfield, 2010). Previous studies suggest that soil storage processes and the intermittent spatial behavior of precipitation are more important than the fractal scaling of precipitation for river runoff fractal behavior (Kantelhardt et al., 2006). This hypothesis is particularly relevant to our study due to: (1) large variations of the fractal scaling coefficients for wells in close proximity, and (2) similar fractal scaling behaviors of the temporal precipitation intensities among different stations in Puerto Rico (all β values close to 0.5).

We have found that both the monofractal and multifractal behaviors of the groundwater level fluctuation processes are highly site specific. Although we did not find statistical differences among the group means of the DFA coefficients for different type of aquifers, it should be noted that this simple comparison may not be representative as there is wide spatial distribution of the wells and marked differences of the hydrogeological conditions even for the same type of aquifer. Fig. 12 presents the spatial distribution of the DFA coefficients. A multifractal scaling analysis on the sea water changes suggests longer temporal fractal

scaling of about 16 year in addition to the seasonal cycling of the sea water (Zhang and Ge, 2013). Sea water is more likely to interact with the subsurface system in the South Coast Aquifer and the Alluvial Valley Aquifer due to their more permeable geologic formations with deposits of sand, silt and clay than the Northern Karst Aquifer with rock wedges near the coast. This is coincidence with our analysis that the DFA coefficients in wells closer to the ocean in the South Coast Aquifer and Alluvial Valley Aquifer tend to be smaller than those near the mountainous areas, while there is no such trend for the karst aquifers.

We investigated the relationship between the fractal scaling strength with the mean groundwater level depth in each well. We found that the DFA coefficients generally increased with the increase of water level depth (Alluvial Aquifer: $r = 0.63$, $p = 0.02$, $n = 14$; South Coast Aquifer: $r = 0.78$, $p = 0.22$, $n = 4$), or there tends to be fewer fluctuations for deeper wells. Note that the above large p -values were mainly due to the limited sampling sizes for those types of aquifers in this study. Generally, the depth of the ground-water levels to the land surface of the aquifers is an important factor relating to the fractal scaling coefficients of groundwater level fluctuations. The power of the fractal scaling correlations in the relatively shallower wells is mostly greater than those in the deeper wells. The shallow aquifers are more dynamic to precipitation, transpiration, discharge and the soil geophysical effects such as capillarity. As a result, groundwater levels in the shallow wells are more responsive and have weaker ability in memorizing the past effects. In contrast, there was no correlation between the fractal scaling coefficients and the water levels in the Northern Karst Limestone Aquifer Upper System ($r = -0.18$, $p = 0.77$, $n = 5$). The elevation effects for the karst aquifer may not be as distinct as for the other types of aquifers. The dampening effects of soils maybe somewhat mitigated by the more turbulent groundwater flow in the karst aquifers. As a consequence, fractal scaling may not be very sensitive to elevation effects in the more permeable karst aquifer, especially the Upper System. The filtering abilities of the fractals of karst aquifers may be governed by the local hydrogeological conditions to a larger degree than the soil dampening effects.

Approximated from transmissivity values and the aquifer formation thickness (Table 1), the general hydraulic conductivity for North Coast Upper Karst Aquifer is estimated as 186 m d^{-1} ($T: 2600 \text{ m}^2 \text{ d}^{-1}$; $Depth: 14 \text{ m}$), South Coast Aquifer 75 m d^{-1} ($T: 750 \text{ m}^2 \text{ d}^{-1}$; $Depth: 10 \text{ m}$), and Alluvial Valley Aquifer 33 m d^{-1} ($T: 100 \text{ m}^2 \text{ d}^{-1}$; $Depth: 3 \text{ m}$). The higher monofractal and less multifractal behavior of the time series fluctuations in the Alluvial Valley Aquifer than those in the North Coast Upper Karst Aquifer are most probably due to the higher storage capacity as reflected by the lower conductivity in the Alluvial Aquifer. The fluctuations of groundwater levels in the North Coast Upper Aquifer exhibit more monofractal and less multifractal behaviors than those in the South Coast Aquifer. This is largely caused by the combined effect of the lower storage capacity (negative to the power of both monofractal and multifractal scaling) and the abundance of pumping wells in the karst aquifer (positive to multifractal scaling), as demonstrated by Zhou et al. (2012) that anthropogenic perturbation such as construction of reservoirs increases the complexity degree of the hydrological processes. Our study is limited by the lack of detailed systematical data on hydrogeological parameters, such as conductivity and transmissivity, which will better facilitate characterization and interpretation of the hydrogeology and groundwater dynamics of the fractal analysis results. Extra effort to correlate the results with

local hydrogeology such as conductivity and storage coefficient is needed to complement our understanding of the function of different types of aquifers on the scaling behaviors of groundwater level fluctuations.

4. Conclusions

We used DFA and a robust regression method to study the monofractal and WTMM method to study the multifractal behaviors of groundwater level fluctuations in wells associated with different types of aquifers in Puerto Rico. The analyses show that fractal scaling exists in all types of aquifers and the scaling properties were anti-persistent or persistent correlations closing to fBm ($\beta = 1.5$). The multifractal analysis shows that the fluctuation processes should be described by multifractality, and the origin of the multifractality is most likely from the complex fractal correlations for the heterogeneous fluctuations. The multifractal spectrum or the singularity spectrum shows that the fluctuations are smoother in the Alluvial Aquifer and rougher in the South Coast Limestone Aquifer and the Northern Karst Aquifer Upper System. Each of the singularity spectra was distinct, which suggests that the fluctuation patterns of groundwater levels are site specific. In general, the fractal scaling coefficients were smaller in shallow aquifers, except for the karst aquifer, whereas the high spatial variability of fractal scaling behavior of groundwater level fluctuations in the karst aquifer is the coupled effect of anthropogenic perturbations, precipitation, elevation and particularly the high heterogenic hydrogeological conditions.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Fig. 1.

Location of USGS groundwater monitoring sites (different black and white points denote wells in different types of aquifers; $n = 22$) in Puerto Rico. The geophysical properties of the study wells and associated aquifers are in Table 1 and Supplementary Material.

Fig. 2. Power-law relationship of the logarithm fluctuation function $F(l)$ and window size l for the long-range correlations of groundwater level fluctuations in all wells ($n = 22$).

Fig. 3.

Histogram of well frequencies of the DFA coefficients. The dotted line is the fitted Gaussian model with the equation of $y = 7.6 * \exp(-0.5 * ((x - 1.31)/0.14)^2)$, $r = 0.87$, $p = 0.06$, $n = 22$. The dashed line represents $\beta = 1.5$, which means the monofractal following a fractional Brownian motion (fBm).

Fig. 4.

Box plots of mean DFA coefficients of groundwater level fluctuations in different types of wells, where there are no statistical differences among the group means ($p = 0.06$). The dotted line represents $\beta = 1.5$ (fBm).

Fig. 5.

The effects on the DFA coefficients by gradually increasing the size of the time series (plot a; the x-axis is the days, which were normalized as beginning from 1500 days after January 1, 1982, and in all the wells the normalized days were switched to the first day for easy comparison); and the spatial distribution of STDs of DFA coefficients in each well with the change of the time series size (plot b).

Fig. 6.

Plots of $\log_{10}F(l)$ vs $\log_{10}l$ for the deseasonalized groundwater levels at two confining wells. The crossovers occurred at the vertical dotted lines at $\log_{10}l = 1.42$ or 26 days (Well 15), and $\log_{10}l = 1.38$ or 24 days (Well 18). The dashed lines over the dots are the fitted linear lines.

Fig. 7.

Comparisons of the monofractal scaling coefficients based on different polynomial detrending method, i.e., DFA1, DFA2, and DFA3. There were no significant differences among the group means of DFA1, DFA2, and DFA3 (1.30 ± 0.12 , 1.37 ± 0.16 , and 1.40 ± 0.18 ; $p = 0.08$).

Fig. 8.

Multifractal analysis spectra for groundwater level fluctuations in the Alluvial Aquifer ($n = 14$), where the relationships including the scaling exponent spectrum $\tau(q)$ vs the moments q (plot a), the Hölder exponent spectrum $h(q)$ vs q (plot b), the singularity spectrum $D(h)$ vs q (plot c), and $D(h)$ vs $h(q)$ (plot d). The left-skewed singularity spectrum suggests more smooth-looking structures underlying the time series, while the right-skewed spectrum indicates more common fine structures.

Fig. 9. Multifractal analysis results for groundwater level fluctuations in the South Coast Aquifer ($n = 4$).

Fig. 10. Multifractal analysis results for groundwater level fluctuations in the Northern Karst Aquifer Upper System ($n = 4$).

Fig. 11.

Relationship of the singularity spectrum $D(h)$ as a function of the Hölder exponent spectrum $h(q)$ for different types of aquifers based on the quadratic model $D(h) = A * (h - h_{\max})^2 + B * (h - h_{\max}) + C$, where the parameters were obtained from the average values from the fitted models on the monitored data, and the values were presented in Table 2. All the spectra were right-skewed.

Fig. 12.
Spatial patterns of the fractal scaling Hurst coefficients (DFA2 coefficients).

Table 1

Geophysical properties of aquifers associated with the studied wells in Puerto Rico.

Aquifer	Lithology	T ($\text{m}^2 \text{d}^{-1}$)	Elev (m)	Depth (m)	Area (km^2)	Area %
North Coast Upper	Within the Ayamon Limestone and Aguada limestone and alluvial deposits along the coast	18.6–26,012	18 ± 8	14 ± 5	714	8.2
North Coast Lower	Occurs within various members of the Cibao Formation and the Lares Limestone	0.2–334	ND	ND	557	6.4
Confining Unit	Principally the upper member of the Cibao Formation	Confined	ND	ND	488	5.6
South coast	Alluvial and fan-delta deposits underlie the broad coastal plain	10–7500	49 ± 25	8 ± 6	618	7.1
Alluvial Valley	River alluvium fill valleys incised into Volcaniclastic, igneous intrusive, or limestone bedrock	18.6–186	10 ± 20	3 ± 2	1089	12.5
Rock type	Volcaniclastic-, igneous-, and sedimentary- rock	Poorly permeable	ND	ND	5243	60.2

Note: T : transmissivity, data were obtained from Lugo et al. (2001) and Renken et al. (2002); *elev*: land surface elevation; *depth*: average groundwater level to the land surface elevation; Area: outcrop area; ND: no data.

Table 2

Statistics (mean $\pm$ standard deviation) of the modeled multifractal analysis based on the field data of groundwater level fluctuations in different types of aquifers, including the exponent scaling spectrum $h(q)$, singularity spectrum $D(h)$, asymmetry coefficient B , and spectrum width W .

Aquifer	#	$h(q)$	$D(h)$	A	B	C	W
Alluvial	14	0.51 ± 0.21	1.01 ± 0.08	-4.92 ± 2.97	-0.06 ± 0.29	1.00 ± 0.05	0.99 ± 0.21
South	4	0.74 ± 0.33	1.01 ± 0.06	-3.31 ± 2.08	-0.13 ± 0.07	1.00 ± 0.06	1.20 ± 0.27
Upper	4	0.60 ± 0.18	0.96 ± 0.03	-4.32 ± 2.76	-0.12 ± 0.24	0.94 ± 0.05	1.03 ± 0.28