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Aerothermoelastic-Acoustics Simulation of Flight Vehicles

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Abstract

This paper describes a novel computational-fluid-dynamics-based numerical solution procedure for effective simulation of aerothermoacoustics problems with application to aerospace vehicles. A finite element idealization is employed for both fluid and structure domains, which fully accounts for thermal effects. The accuracies of both the fluid and structure capabilities are verified with flight- and ground-test data. A time integration of the structural equations of motion, with the governing flow equations, is conducted for the computation of the unsteady aerodynamic forces, which uses a transpiration boundary condition at the surface nodal points in lieu of the updating of the fluid mesh. Two example problems are presented herein to that effect. The first one relates to a cantilever wing with a NACA 0012 airfoil. The solution results demonstrate the effect of temperature loading that causes a significant increase in acoustic response. A second example, the hypersonic X-43 vehicle, is also analyzed; and relevant results are presented. The common finite element-based aerothermoelastic-acoustics simulation process, its applicability to the efficient and routine solution of complex practical problems, the employment of the effective transpiration boundary condition in the computational fluid dynamics solution, and the development and public domain distribution of an associated code are unique features of this paper.

I. Introduction

A RELIABLE evaluation of the aeroelastic-acoustics phenomenon is of critical importance in ensuring the safety of many flight vehicles. For example, in connection with the ongoing NASA Armstrong Flight Research Center's Stratospheric Observatory for Infrared Astronomy flight project (known as SOFIA) [1], accurate determination of the acoustic activities in the open cavity of the Boeing 747SP aircraft that housed the infrared telescope became a critical issue. Extensive wind-tunnel and numerical computational fluid dynamics

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(CFD)-based studies were performed for detection of possible acoustic resonance in the cavity that could damage the telescope, and possibly the aircraft itself.

Earlier work [2] in this connection resulted in development of an analysis methodology and computer software suitable for the solution of practical problems of much complexity that took into account the interaction of flexible structures with the fluids. The present effort extended this development to include the thermal effect on the acoustics solution.

The effect of high temperature plays an important role for the stability of practical high-speed flight vehicles, and their simulation requires the effective integration of a multitude of major individual disciplines as structures, heat transfer, fluids, and controls, among others. This is accomplished by implementing the simultaneous time integration of structural equations of motion with the governing flow equations to yield unsteady aerodynamic forces (Fig. 1). A steady-state CFD analysis is conducted first that computes the heat distribution on structural surfaces due to aerodynamic loading, which is followed by a heat transfer analysis to determine temperature distribution in individual structural elements. Since most structural material characteristics are temperature dependent, the structural stiffness matrices are modified accordingly to reflect these changes. If necessary, the heat conduction analysis can be repeated at additional time steps during unsteady flow computations. In the subsequent time-marching unsteady aerodynamics computation, the CFD mesh typically needs to be updated, as the structure deforms so that it follows the deformed structural configuration. However, this involves significant computational effort, and hence proves to be impractical for common users. An alternative transpiration boundary condition technique [3,4] is adopted at the surface nodal points in which the body normals are rotated in the same directions as they are expected to be in the actual deflected shape. This procedure enables the original aerodynamic mesh to remain unaltered throughout the aeroelastic simulation process. The current procedure has proved to produce accurate results for a wide range of deformations [5]. Once the solution convergence is achieved, the acoustic results [in the shape of sound pressure level (SPL) and acoustic wave frequencies] are directly computed from the unsteady aerodynamic pressure data.

Previous work by various authors in aeroelasticity and aerothermoelasticity involved finite volume CFD idealization for fluids that were coupled to standard finite element (FE) structure models [6–10]. Also, a number of survey papers [11–13] in this area of research is highly relevant to the current subject matter. The common FE discretization for structures and heat conduction, as well as fluids, used herein facilitates the accurate transfer of data between respective disciplines, thereby ensuring high solution accuracy. Furthermore, the use of an unstructured mesh in all involved disciplines enables accurate modeling and simulation of most complex, practical problems. Also, an effective implementation of the transpiration boundary condition effects significant savings in the solution time, as the aerodynamic mesh need not be updated at various solution steps.

II. Numerical Formulation

An aerothermal-elastic-acoustic simulation is multidisciplinary in nature, involving a number of major disciplines. Figure 1 depicts the solution flowchart used by the STARS [14]

analysis code. The related aerodynamics discipline plays a major role in this connection. The associated Navier–Stokes equation can be written as

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{\partial \mathbf{f}_i}{\partial x_i} + \frac{\partial \mathbf{g}_i}{\partial x_i} = \mathbf{f}_b \quad i = 1, 2, 3 \quad (1)$$

where $\mathbf{v}$ is the conservation variable;

$$\mathbf{v} = [\rho \quad \rho u_j \quad \rho E]^T \quad j = 1, 2, 3 \quad (2)$$

ρ , u , and E are, respectively, the density, velocity components, and total energy; f and g are the convection and diffusion terms of the flux vector; and $\mathbf{f}_b$ is the body force vector. A Taylor–Galerkin formulation-based finite element discretization [15,16] and the associated code STARS are used to achieve the CFD solution. The resulting set of equations has the following form:

$$\begin{aligned} \mathbf{M} \Delta \tilde{\mathbf{v}} = & -\Delta t \left[\frac{\partial u_i}{\partial x_i} \mathbf{M} + \mathbf{K} \right] \tilde{\mathbf{v}} - \Delta t (\hat{\mathbf{f}}_1 + \hat{\mathbf{f}}_2) \\ & + \Delta t \hat{\mathbf{R}} + \Delta t [\mathbf{K}_\sigma + \mathbf{f}_\sigma] \end{aligned} \quad (3)$$

in which $\mathbf{M}$ is the consistent mass matrix; $\mathbf{K}$ is the convection matrix; $\hat{\mathbf{f}}_1$ and $\hat{\mathbf{f}}_2$ are the pressure matrices; $\mathbf{K}_\sigma$ is the second-order matrix that includes viscous and heat flux effects; and $\mathbf{f}_\sigma$ is the boundary integral matrix from second-order terms. Then,

$$\begin{aligned} \mathbf{M} &= \int_V \mathbf{a}^T \mathbf{a} \, dV; \quad \mathbf{K} = \int_V \mathbf{a}^T \bar{u}_i \frac{\partial \mathbf{a}}{\partial x_i} \, dV; \\ \hat{\mathbf{f}}_1 &= \int_V \mathbf{a}^T \bar{p}_i \frac{\partial \mathbf{e}_i}{\partial x_i} \, dV; \quad \hat{\mathbf{f}}_2 = \int_V \mathbf{a}^T \bar{e}_i \frac{\partial p_i}{\partial x_i} \, dV; \\ \mathbf{K}_\sigma &= - \int_V \frac{\partial \mathbf{a}^T}{\partial x_j} \mathbf{e} \, j \, \sigma_{ij} \, dV - \int_V \frac{\partial \mathbf{a}^T}{\partial x_j} \mathbf{m} \, j \, q_j \, dV; \\ \mathbf{f}_\sigma &= \int_\Gamma \mathbf{a}^T \mathbf{e} \, j \, \sigma_{ij} \, \hat{\mathbf{n}} \, d\Gamma + \int_\Gamma \mathbf{a}^T \mathbf{m} \, j \, q_j \, \hat{\mathbf{n}} \, d\Gamma \end{aligned}$$

with $\mathbf{a}$ as the shape function; $\bar{p}_i$, $\bar{u}_i$, and $\bar{e}_i$ as the average values; $\mathbf{e}_1 = [0 \ 1 \ 0 \ 0 \ u_1]^T$; $\mathbf{e}_2 = [0 \ 0 \ 1 \ 0 \ u_2]^T$; $\mathbf{e}_3 = [0 \ 0 \ 0 \ 1 \ u_3]^T$; $\hat{\mathbf{R}}$ as the artificial dissipation; and $\mathbf{m}_1 = \mathbf{m}_2 = \mathbf{m}_3 = [0 \ 0 \ 0 \ 0 \ 1]^T$. Equation (3) is solved using a two-step solution process [16] in which the inviscid solution is augmented with the viscous term and stabilized with appropriate artificial dissipation terms.

In this simulation process (Fig. 1), a steady-state CFD analysis is performed first to yield the resulting thermal load on the structural surface due to aerodynamic loading. A heat transfer analysis yields the temperature distribution among all structural elements. Usually, the structural material property is a function of the temperature, and then the structural stiffness matrix is modified accordingly for a subsequent modal analysis. This is followed by the formulation of aerostructural state-space matrices, a solution that yields the generalized displacements and velocities ($\mathbf{q}$ and $\dot{\mathbf{q}}$); the pattern of its convergence is

indicative of aeroelastic or aeroservoelastic stability of the flight system. Thus, the vehicle equation of motion, using NR number of modes, can be written in the generalized coordinate system as

$$\widehat{\mathbf{M}}\ddot{\mathbf{q}} + \widehat{\mathbf{C}}\dot{\mathbf{q}} + \widehat{\mathbf{K}}\mathbf{q} + \mathbf{f}_a(t) + \mathbf{f}_I(t) = 0 \quad (4)$$

in which $\mathbf{q}$ is the generalized displacement vector ($\Phi^T \mathbf{u}$); $\widehat{\mathbf{M}}$ is the, associated generalized inertial matrix ($\Phi^T \mathbf{M} \Phi$) of order ($NR \times NR$), and similar transformation is adopted for the stiffness $\widehat{\mathbf{K}}$ and damping $\widehat{\mathbf{C}}$ matrices; $\mathbf{f}_a(t)$ is the aerodynamic load vector ($\Phi_a^T P A$); P is the fluid pressure at a fluid node (where A is the effective surface area around the node and Φ_a is the modal vector at fluid grid points on the vehicle structural surface); and $\mathbf{f}_I(t)$ is the generalized structural impulse force vector.

Equation (4) may then be cast in a state-space matrix form as

$$\begin{bmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}} \\ \ddot{\mathbf{q}} \end{bmatrix} - \begin{bmatrix} 0 & \mathbf{I} \\ -\widehat{\mathbf{M}}^{-1}\widehat{\mathbf{K}} & -\widehat{\mathbf{M}}^{-1}\widehat{\mathbf{K}} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix} - \begin{bmatrix} 0 \\ -\widehat{\mathbf{M}}^{-1}\mathbf{f}_a(t) \end{bmatrix} - \begin{bmatrix} 0 \\ -\widehat{\mathbf{M}}^{-1}\mathbf{f}_I(t) \end{bmatrix} = 0 \quad (5)$$

or

$$\dot{\mathbf{x}}_s(t) = \mathbf{A}_{st}\mathbf{x}_s(t) + \mathbf{B}_{st}\mathbf{f}(t) \quad (6)$$

where

$$\mathbf{A}_{st} = \begin{bmatrix} 0 & \mathbf{I} \\ -\widehat{\mathbf{M}}^{-1}\widehat{\mathbf{K}} & -\widehat{\mathbf{M}}^{-1}\widehat{\mathbf{K}} \end{bmatrix}, \quad \mathbf{B}_{st} = \begin{bmatrix} 0 \\ -\widehat{\mathbf{M}}^{-1} \end{bmatrix}, \quad \mathbf{f}(t) = \mathbf{f}_a(t) + \mathbf{f}_I(t), \quad \mathbf{x}_s = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix} \quad (7)$$

and

$$\mathbf{y}_s(t) = \mathbf{C}_{st}\mathbf{x}_s(t) + \mathbf{D}_{st}\mathbf{f}(t) \quad (8)$$

in which $\mathbf{C}_{st} = \mathbf{I}$, $\mathbf{D}_{st} = 0$, matrix $\mathbf{A}_{st}$ is of order $NR2 \times NR2$ ($NR2 = 2 \times NR$), and $\mathbf{B}_{st}$ is the $NR2 \times NR$ matrix.

For the most general aeroservoelastic case, in the presence of sensors, the state-space equations are converted to the zero-order hold discrete time equivalent at the k th step as

$$\mathbf{x}_s(k+1) = \mathbf{G}_s\mathbf{x}_s(k) + \mathbf{H}_s\mathbf{f}(k) \quad (9)$$

$$\mathbf{y}_s(k+1) = \mathbf{C}_s\mathbf{x}_s(k) + \mathbf{D}_s\mathbf{f}(k) \quad (10)$$

where

$$\mathbf{G}_s = e^{\mathbf{A}_s \Delta t} \quad \text{and} \quad \mathbf{H}_s = \left[e^{\mathbf{A}_s \Delta t} - \mathbf{I} \right] \mathbf{A}_s^{-1} \mathbf{B}_s \quad (11)$$

in which $\mathbf{A}_{st}$ and $\mathbf{B}_{st}$, having been modified to include sensors, yield $\mathbf{A}_s$ and $\mathbf{B}_s$, respectively.

An unsteady CFD analysis is performed using global time stepping, and the entire process repeated. Once the unsteady aerodynamic forces have converged, the sound pressure level for any desired node may be calculated for a fixed time band ($t = t_j$) by computing the root mean square (rms) of the unsteady pressure. Using n number of sampling points, the related calculations are as follows:

Compute average pressure:

$$P_{\text{avg}} = \frac{\sum_{i=1}^n P_i}{n} \quad (12)$$

Compute the root mean square of pressure:

$$P_{\text{rms}} = \sqrt{\frac{\sum_{i=1}^n (P_i - P_{\text{avg}})^2}{n}} \quad (13)$$

Compute the sound pressure level:

$$\text{SPL(dB)} = 20 \log_{10} \frac{P_{\text{rms}}}{P_{\text{ref}}} \quad (14)$$

The reference pressure is $P_{\text{ref}} = 20 \times 10^{-6}$ Pa for air. Further, the nodal temperature T distribution on the wing structure is calculated from enthalpy H as

$$T = \frac{H}{c_p} \quad (15)$$

where c_p is specific heat capacity at constant pressure. As depicted in Fig. 1, thermal effects may need to be taken into consideration if they modify the modal solution significantly, which will in turn affect the unsteady pressure, and hence the acoustic calculations.

Similarly, the acoustic wave frequencies are derived by performing fast Fourier transform (FFT) on the unsteady pressure data:

$$\begin{aligned} \text{FFT}(P_{\text{unstd}, t}) &= F(\omega) \quad \text{PSD} = \frac{F(\omega) \times F(\omega)^*}{2\Delta f} \\ &= \frac{|F(\omega)|^2}{2\Delta f} \Delta f = \frac{1}{t_j} \end{aligned} \quad (16)$$

where PSD is the power spectral density. Acoustic wave frequencies and amplitudes are obtained from the peaks of the plotted $R(\omega)$, or preferably the PSD curves. All CFD computations are based on the Euler method.

III. Numerical Examples

To demonstrate the effect of thermal loads on acoustics simulation, a number of example problems is presented next. The first example relates to a cantilever three-dimensional (3-D) wing with a NACA 0012 airfoil having temperature-dependent material properties, and its effect on aeroelastic-acoustics instability is demonstrated in detail. The other example relates to a complete aerospace flight vehicle.

A. Three-Dimensional Wing with NACA 0012 Airfoil

The wing has a span of 2.0178 m, with the chord length being 1.0089 m. It consists of shell surfaces supported by a frame made of beams and diaphragms. Table 1 summarizes the flight conditions for the current simulation. Figure 2 depicts the FE CFD and structural meshes on the wing, and the layout of the acoustic computation points are shown in Fig. 3. The structural FE model consists of 486 nodes and 1549 elements (mostly thin shell and line), and it consists of two different aluminum (Al) alloy materials. The CFD mesh consists of 19,286 two-dimensional (2-D) surface triangular elements (9645 nodes) and 352,060 3-D domain tetrahedral elements (65,775 nodes). In this analysis process (Fig. 1), the steady-state CFD solution is performed first, and the solution contours are shown in Fig. 4. The associated temperature distribution plot is further elaborated in Fig. 5. The basic material property data for an aluminum alloy are shown in Table 2, whereas Fig. 6 shows a plot of variation of Young's modulus as a function of temperature, which is used for the formulation of the stiffness matrices of the plate model at this flight condition. The material property data are taken from available standard Web sites. Some higher-temperature values are suitably extrapolated for current analysis. The results of the natural frequency analysis for the temperature-dependent material (TDM) are compared with that for the temperature-independent material (TIM) case at ambient temperature, and they are shown in Table 3.

Significant changes in modal parameters, frequencies, and mode shapes are observed for the TDM case including a switch in mode shapes for mode 7. Modes 1, 2, and 7 are used for the aeroelastic-acoustics analyses for the TIM case, whereas modes 1, 2, and 8 are considered for the TDM and TDM + KG cases. Figure 7 provides the results related to the aerothermoelastic-acoustics simulation in the shape of unsteady pressure, sound pressure level, and acoustic wave frequencies. Table 4 summarizes structural and acoustic wave frequencies. The most significant increases are observed in unsteady pressure, sound wave frequencies, and acoustic SPL values for the TDM cases. Unstable system frequencies also indicate aeroelastic-acoustics instability due to thermal loads.

It was demonstrated earlier [17] that, if the material is changed to titanium (Ti)-Al, the wing becomes stable and aerothermoelastic-acoustics simulation results become convergent.

B. X-43 Hypersonic Flight Vehicle

A finite element structural model of the vehicle [18,19] is shown in Fig. 8, having 11,685 nodes and 11,245 elements of various forms and materials, with the latter being variants of about 35% Al alloy, 57% of Ti-Al alloy, and the rest 2.6% of Carbon–Carbon (C–C) composites. The CFD surface mesh is depicted in Fig. 9a, which consists of 54,388 2-D triangular surface elements (27,198 nodes); whereas Fig. 9b shows the solution domain, consisting of 1,241,779 3-D tetrahedral elements (228,776 nodes) around the X-43 vehicle. Figure 10 presents the surface Mach number and temperature distribution for the steady-state analysis for Mach 5.0 at 70,000 ft at an angle of attack of 10 deg; the vehicle flight profile is shown in Fig. 11. A plot of the temperature-dependent material properties for the C-C composites is shown in Fig. 12, such a data for the other materials as Al alloy and Ti-Al alloy, are taken from the available standard literature (ASME materials handbook).

The first few natural frequencies are shown in Table 5, for both the temperature-dependent and -independent materials (TDM and TDM + KG, TIM); similarly, the corresponding frequency values obtained from the ground vibration test [18] are shown in parentheses. The CFD simulation capability was verified from actual flight-test aerodynamic pressure data [18,19]. Table 6 presents such data. The results of the associated aeroelastic analyses, in the shape of generalized displacement, are depicted in Fig. 13 using the first four elastic modes. Figure 14 shows the aerothermoelastic-acoustics plots at a typical nodal point (horizontal tail: 10% from leading edge, 50% of root) on the surface of the vehicle. The thermal effects on the acoustic response are found to be moderate due to the prudent choice of material. The total single CPU time for the solution, using a direct integration technique, is about 60 h for each altitude, whereas the autoregressive moving average (ARMA) method takes about 4 h and 30 min for such an analysis [19].

IV. Conclusions

This paper presents the details of a novel finite element CFD-based aerothermoelastic-acoustics analysis methodology and the associated code STARS [14], which enables the computation of acoustic data derived from unsteady aerodynamics, generated from interaction of flexible structure with fluid flow. At high-speed flow, a flight vehicle is subjected to very high-temperature loading that may render it unstable from an aeroelastic-acoustics point of view. The methodology and the code presented herein will routinely perform accurate and economical solution of complex practical problems.

Two example problems are presented in the paper in this connection. The first one is a cantilever wing, which at high-speed flow shows aerothermoelastic-acoustics instability. The second example relates to a full hypersonic vehicle: the X-43. The analysis is routinely performed, and acoustic data are computed using modest computing resources. Also presented are verification data for both fluid and structure disciplines by comparing the analysis with test data; such data for acoustics loads are given in [2]. From Fig. 1, it is noted that the natural frequencies are recalculated for each time step if a large variation of temperature occurs in subsequent time steps.

The associated code [14] is freely available for public use.

Nomenclature

A_{st}, B_{st}	structural state-space matrices
A_s, B_s, C_s, D_s	modified system state-space matrices that include sensors
$\hat{C}$	generalized damping matrix
E	total energy in fluids
$f_a(t)$	generalized aerodynamic load vector
f_b	body force vector
f_i	convection term in flux vector
$f_l(t)$	generalized impulse function
G_s, H_s	transformed state-space matrices in the presence of sensor
g_i	diffusion term in flux vector
K	structural stiffness matrix
K_E	elastic stiffness matrix
K_G	geometrical stiffness matrix
$\hat{K}$	generalized stiffness matrix
M	inertia matrix
$\hat{M}$	generalized mass matrix
$q, \dot{q}$	generalized displacements and velocities
u	structural deformation
ρ	freestream density
Φ	matrix of structural eigenvectors
ω	structural frequencies

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Fig. 1.
Aerothermoelastic-acoustics analysis flowchart.

a) CFD surface mesh of the wing

b) Structural grid of the wing

Fig. 2.
FE models of a cantilever wing.

Fig. 3.
Layout of the acoustic computation points.

Fig. 4.
CFD steady-state solution of a cantilever wing.

Fig. 5.
CFD steady-state solution: temperature distribution data on wing surface.

Fig. 6.
Temperature-dependent elastic modulus: Al alloy.

Fig. 7.

AI alloy: unsteady pressure and aerothermoelastic-acoustics plots at C-50.

Fig. 8.
FE Structural Mesh of the X-43

Fig. 9.
CFD Mesh of the X-43.

Fig. 10.
CFD steady solutions of the X-43.

Fig. 11.
X-43 flight profile.

Fig. 12.
Temperature-dependent elastic modulus, C-C alloy.

Fig. 13.

Aeroelastic and aerothermoelastic plots: generalized displacements plots of the X-43.

Fig. 14.

Unsteady pressure and aerothermoelastic-acoustics plots of the X-43.

Table 1

Flight condition

Parameter	Value
Mach number	5.0
Angle of attack	0 deg
Altitude	13,716 m 45,000 ft
Air density	0.23713 kg/m ³ 0.00046 slug/ft ³
Speed of sound	295.0728 m/s 968.0750 ft/s
Temperature	216.649 <i>K</i> (−56.501 °C) 389.969 °R(−69.701 °F)

Table 2

Material property: aluminum alloy

Density	2700 kg/m ³	5.2371 slug/ft ³
Elastic modulus (ambient)	68.95 GPa	10000 kpsi
Poisson's ratio	0.33	0.33
Coefficient of thermal expansion (CTE) (linear)	25.6 $\mu\text{m}/\text{m} - ^\circ\text{C}$	14.2 $\mu\text{in.}/\text{in.} - ^\circ\text{F}$
Thermal conductivity	200 W/m-K	1390 BTU-in./hr-ft ² -°F

Table 3
Free vibration analysis results comparison for TIM, TDM, and TDM + KG: Al alloy

	TIM	TDM	TDM + KG
Mode 1
	3.5204	3.0825	3.0509

Mode 2
	14.7064	12.1911	10.8226

Mode 7	58.8896	49.4082	48.8733

Table 4

Al alloy: summary of acoustic wave and structural natural frequencies

Frequencies, Hz						
Structural			Aeroelastic-acoustics (C-50) ^a			
Mode	TIM	TDM	TDM + KG	TIM	TDM	TDM + KG
1	3.5204	3.0825	3.0509	4.7294	7.6853 ^b	N/A ^b
2	14.7064	12.1911	10.8226	13.0059	15.3706 ^b	N/A ^b
7 (8)	58.8896	52.9155	52.4663	59.1178	23.6471 ^b	N/A ^b

^a N/A denotes “not applicable.”

^b Unstable system frequencies.

Table 5Natural frequencies of the X-43^{a,b}

Elastic mode		TIM ^c	TDM	TDM + KG
1	HT-1B-S	39.7857 (40.03)	39.2083	38.9618
2	HT-1B-AS	41.1018 (41.90)	40.7827	40.3182
3	F-1B w/HT-B-S	45.3783 (46.62)	44.5274	44.4409
4	F-1T	82.5246	75.2560	74.9162
5	HT-1B-S-IP ^a	88.3260	86.9657	86.9457
6	HT-1B-AS-IP ^a	88.9623 (77.29)	87.4078	87.3990

^aModes (5 and 6) switch between TIM and TDM, and TDM + KG cases.^bHT denotes horizontal tail, F denotes fuselage, S denotes symmetry, and AS denotes antisymmetry.^cGround vibration test results are shown in parentheses in the TIM column.

Table 6

Comparison of computed and flight measured pressure data on the X-43

Sensor point	Pressure, psi	
	Flight test	CFD computed
001	1.6900	1.7297
003	1.7800	1.6890
007	−0.2419	−0.1400
085	−0.1567	−0.3886
090	0.07	−0.08