An Efficient Method for Correcting the Edge Artifact Due to Smoothing

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Abstract: Spatial smoothing is a common pre-processing step in the analysis of functional brain imaging data. It can increase the signal to noise ratio, and specially designed smoothing filters can be used to selectively increase sensitivity to signals of specific shapes and sizes (Rosenfeld and Kak [1982]: Digital Picture Processing, vol. 2. Orlando, Fla.: Academic; Worsley et al. [1996]: Hum Brain Mapping 4:74–90). Also, some amount of spatial smoothness is required if methods from the theory of Gaussian random fields are to be used (Holmes [1994]: Statistical Issues in Functional Brain Mapping. PhD thesis, University of Glasgow). Smoothing is most often implemented as a convolution of the imaging data with a smoothing kernel, and convolution is most efficiently performed using the Convolution Theorem and the Fast Fourier Transform (Cooley and Tukey [1965]: Math Comput 19:297–301; Priestly [1981]: Spectral Analysis and Time Series. San Diego: Academic; Press et al. [1992]: Numerical Recipes in C: The Art of Scientific Computing, 2nd ed. Cambridge: Cambridge University Press). An undesirable side effect of smoothing is an artifact along the edges of the brain, where brain voxels become smoothed with non-brain voxels. This results in a dark rim which might be mistaken for hypoactivity. In this short methodological paper, we present a method for correcting functional brain images for the edge artifact due to smoothing, while retaining the use of the Convolution Theorem and the Fast Fourier Transform for efficient calculation of convolutions. *Hum. Brain Mapping* 6:128–136, 1998. © 1998 Wiley-Liss, Inc.

Key words: functional neuroimaging; smoothing; convolution; artifact; correction

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INTRODUCTION

Spatial smoothing is an important image processing operation commonly performed in the analysis of functional neuroimaging data. Some smoothing is often done during image reconstruction. Further smoothing may be desirable to increase the signal-to-noise ratio (matched filter theorem, Rosenfeld and Kak, 1982; Worsley et al., 1996), and to allow the use of

Gaussian random field theory (Holmes, 1994). Spatial smoothing has the obvious disadvantage of reducing the spatial resolution of the data, which must be weighed against the benefits. The relative costs and benefits of smoothing can vary from data set to data set.

Smoothing is often implemented as a convolution of spatial data with a smoothing kernel. Convolution is essentially a moving average: each voxel is averaged with nearby voxels, with weighting, to obtain the smoothed voxel value. As a result of spatial smoothing, an edge artifact may occur, because brain voxels at the edge of the brain will be averaged with low-intensity voxels outside of the brain. These edge voxels will have artifactually low values after spatial smooth-

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ing; this will in turn result in artifactually low differences in means, and artifactually low variances. This is of special concern in statistical analyses using a pooled estimate of variance (e.g., Fox et al., 1988; Worsley et al., 1992; Chmielowska et al., 1997), where false negatives can occur due to artifactually low local differences in means, unless some sort of correction is done for the edge artifact. In this short methodological paper, we present a method for correcting for this edge artifact due to smoothing.

SIMPLE MODEL OF SMOOTHING

To illustrate both the edge artifact due to smoothing and the proposed method for correcting it, we construct a simple model B of a functional neuroimage, and demonstrate what happens at the edges when it is smoothed.

Simple model of a functional neuroimage

Let R_T be the set of all voxels in a three-dimensional image volume. Let R_B be the subset of voxels which are considered inside the brain, and let R_O be the subset of voxels which are considered outside the brain. Let $M(x, y, z)$ be a binary mask such that

$$\begin{aligned} M(x, y, z) &= 1 \text{ when } (x, y, z) \in R_B \\ &= 0 \text{ when } (x, y, z) \notin R_B, \in R_O \end{aligned}$$

where x, y, z are coordinates in three-dimensional space. Let B_m be the whole-brain mean intensity. Define $B(x, y, z)$ such that

$$B(x, y, z) = (B_m + N(x, y, z)) * M(x, y, z)$$

where $N(x, y, z)$ is random uncorrelated Gaussian noise with zero mean and variance V_N , and $*$ is the multiplication operator. B is a simple model of a functional brain scan which has a mean of B_m within the brain, with zero mean, uncorrelated variation around this value. Outside the brain, B is zero.

Demonstration of edge artifact

Let us now examine the effects of smoothing at a *particular* voxel in R_B at coordinates (u, v, w) . Define a smoothing function $K(x, y, z)$ which averages the voxel value $B(u, v, w)$ with its neighbors. To ensure that the smoothed value does not have an expectation different

from the original expectation B_m , we require that

$$\sum K(x, y, z) = 1$$

$S(u, v, w)$, the voxel value at (u, v, w) after spatially smoothing $B(x, y, z)$, is a simple weighted average of the voxel values in $B(x, y, z)$, where the weights are given by $K(x, y, z)$:

$$\begin{aligned} S(u, v, w) &= \sum_{R_T} (B(x, y, z) * K(x, y, z)) \\ &= \sum_{R_B} ((N(x, y, z) + B_m) * K(x, y, z)) \end{aligned}$$

Thus, the expected value of $S(u, v, w)$ is

$$E[S(u, v, w)] = B_m \sum_{R_B} K(x, y, z)$$

since $N(x, y, z)$ is zero mean noise. Now, if (u, v, w) were in the center of the brain region R_B , the summation of $K(x, y, z)$ over R_B will be very nearly 1 in the case of a Gaussian smoothing filter; in the case of a simple box car filter, it will in general be exactly 1. Therefore, $E[S(u, v, w)]$ will be approximately B_m , the expected value of $B(x, y, z)$, and no smoothing artifact will be apparent. However, if (u, v, w) were in R_B but at the extreme edge of R_B , the summation of $K(x, y, z)$ over R_B will be drastically reduced. At the edge, approximately half of the signal energy in $K(x, y, z)$ will be outside of R_B , and therefore the summation will be around 0.5. Therefore, at the edge, $E[S(u, v, w)]$ will be much smaller than B_m , on the order of $B_m/2$. This is the edge artifact due to smoothing; voxels along the edge of the brain will tend to have lower intensities than voxels in the center of the brain.

METHOD FOR CORRECTING FOR EDGE ARTIFACT DUE TO SMOOTHING

We propose using the summation

$$\sum_{R_B} K(x, y, z)$$

as a correction factor for the edge artifact at voxel (u, v, w) . Dividing $S(u, v, w)$ by this summation will result in an adjusted value, $A(u, v, w)$:

$$A(u, v, w) = S(u, v, w) / \sum_{R_B} K(x, y, z)$$

in which the expected value at coordinates $(u, v, w) \in R_B$ is corrected to be B_m again. Now, we had defined

$K(x, y, z)$ above specifically for the voxel at coordinates (u, v, w) ; recall that $K(x, y, z)$ was centered on (u, v, w) . To correct not only the smoothed value at (u, v, w) but also at all other voxels in R_B , we would have to define a smoothing filter centered on each voxel in R_B , and then calculate a correction factor for each voxel, generating a map of correction factors for each coordinate, $C(x, y, z)$. But, this is the same as smoothing the mask $M(x, y, z)$ with the smoothing kernel. That is, convolving $M(x, y, z)$ with the smoothing kernel will give $C(x, y, z)$, the map of correction factors.

To obtain an explicit formula for $C(x, y, z)$, first define K_0 to be the smoothing filter centered on the voxel at the origin, $(0, 0, 0)$. For example, K_0 can be a Gaussian smoothing filter whose maximum is at the origin. Then the convolution of $K_0(x, y, z)$ with $M(x, y, z)$, $C(x, y, z)$, is:

$$\begin{aligned} C(x, y, z) &= \sum_{i=1}^{x_{dim}} \sum_{j=1}^{y_{dim}} \sum_{k=1}^{z_{dim}} (K_0(i-x, j-y, k-z) * M(x, y, z)) \\ &= K_0(x, y, z) \otimes M(x, y, z) \end{aligned}$$

where $\otimes$ is the convolution operator, and where x_{dim} , y_{dim} , and z_{dim} are the matrix dimensions of the image volume.

EDGE EFFECTS ON LOCAL VARIANCE

Let us now examine the effects of smoothing and of the proposed edge correction on local variance at a particular voxel in R_B at coordinates (u, v, w) . Using an analysis similar to that performed above (Demonstration of Edge Artifact), we find the estimate of the variance of $S(u, v, w)$ to be

$$\text{var}[S(u, v, w)] = V_N^* \sum_{R_B} K^2(x, y, z)$$

since $N(x, y, z)$ is uncorrelated Gaussian noise with variance V_N . This means that, for most of the brain region, smoothing reduces the variance by a factor equal to the sum of the squared terms of the smoothing filter, a finding consistent with earlier derivations (Poline and Mazoyer, 1993; Worsley et al., 1996). However, like the expectation, variance will be reduced at the edges, again approaching a factor of about one half; this edge effect is implicit in an earlier derivation (Worsley et al., 1996, equation 12). In a paper comparing two important methods of analyzing neuroimages, Arndt et al. (1995) reported finding

lower variance toward the edges of the cortical rim, both inner and outer, which may have been due to the edge effect of smoothing on variance.

After using the method for correcting the edge artifact described in this paper, the estimate of the variance of $A(u, v, w)$ would be

$$\text{var}[A(u, v, w)] = V_N^* \left(\sum_{R_B} K^2(x, y, z) \right) / \left(\sum_{R_B} K(x, y, z) \right)^2$$

While this partially corrects the heteroscedastic edge effect of smoothing on variance, it is not a complete correction. This quantity has nonmonotonic behavior as the edge is approached. For most of the region R_B it is fairly constant, but as the edge is approached this quantity rises, and then falls. The exact behavior will depend on the smoothing filter used and the shape of R_B . Effects of smoothing on variance with and without the edge correction are illustrated below on an actual PET data set (Fig. 3).

In statistical analyses where a within-voxel estimate of variance is used, neither the smoothing edge effect nor the nonmonotonic edge behavior due to the correction is a concern, since all data within-voxel are multiplied by the same scaling constant. On the other hand, in analyses where a pooled estimate of variance is used, smoothing drastically reduces the mean values and variance at the edge, as discussed above. Thus, there will be increased potential for false negatives along the edge. However, after correction, there will be increased potential for false positives in a region just within the edge, where $\text{var}[A(u, v, w)]$ may be underestimated with a pooled estimate of variance. A possible way to avoid these false positives is to simply use a higher estimate of pooled variance (or, equivalently, a higher Z-score threshold) at the cost of sensitivity. Another option would be to assess significance using methods which take into account spatial extent (Friston et al., 1994; Poline et al., 1997), as opposed to peak height only (Friston et al., 1991; Worsley et al., 1992, 1996).

APPLICATION OF CORRECTION USING THE CONVOLUTION THEOREM

As mentioned earlier, smoothing is most often implemented as the convolution of data with a smoothing kernel. The time required to compute the convolution can be drastically reduced using Fourier methods and the Convolution Theorem (Priestley, 1981; Press et al., 1992); if Fourier methods are not used, then smoothing

of large imaging data sets would be impractical. We now demonstrate how the above proposed correction factor for the edge artifact due to smoothing would be implemented using Fourier methods.

Let the operators FT and IFT be the forward Fourier Transform (FT) and the Inverse Fourier Transform (IFT) in three-dimensional space. By the Convolution Theorem, one can convolve B with K_0 by multiplying the FT of B with the FT of K_0 , and then taking the IFT of the result:

$$S = IFT[FT[B]*FT[K_0]]*M$$

The multiplication by M is included so that the smoothed data S is masked; i.e., so that $S(x, y, z) = 0$ for $(x, y, z) \in R_0$. S , the result of smoothing and then masking B , will have edge artifacts because of the reasons argued above.

To correct S for edge artifacts, generate a masked, smoothed version of the mask, C , using the FT and IFT as was done with B :

$$C = IFT[FT[M]*FT[K_0]]*M$$

As before, the voxel-by-voxel multiplication by M is included so that the map of correction factors C is itself masked. Then, define the edge-corrected version of $S(x, y, z)$, $A(x, y, z)$, using the following relation:

$$A(x, y, z) = S(x, y, z)/C(x, y, z)$$

where $/$ is the division operator, applied voxel-by-voxel. To avoid division by zero errors, automatically assign $A(x, y, z)$ a value of 0 if $M(x, y, z) = 0$, which will occur in areas outside of R_B .

For further computational speed, the FT and IFT can be implemented as the *Fast* Fourier Transform (Cooley and Tukey, 1965) and its inverse. This will require that x_{dim} , y_{dim} , and z_{dim} (the matrix dimensions of the image volume) all be powers of 2. This is discussed further below.

APPLICATION OF METHOD TO THREE DATA SETS

To illustrate the method, two phantom data sets were generated using MATLAB (the MathWorks, Inc., Natick, MA). Each was then smoothed using the above proposed method to correct for the edge artifact. Also, a small PET data set was used to demonstrate the method in three dimensions. We discuss each data set here, in turn.

TABLE I. Example of correction for smoothing edge artifact

(A)	(B)	(C)	(D)	(E)	(F)
0	0	0.2	0	0	0
0	0	0.2	0	0	0
0	0	0.2	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
102	1	0	53.8	0.6	89.6667
117	1	0	71.4	0.8	89.2500
50	1	0	82.6	1	82.6
88	1	0	80.4	1	80.4
56	1	0	80.6	1	80.6
91	1	0	92.2	1	92.2
118	1	0	103.2	1	103.2
108	1	0	118.8	1	118.8
143	1	0	100.6	0.8	125.7500
134	1	0	77	0.6	128.3333
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0.2	0	0	0
0	0	0.2	0	0	0

Demonstrating the correction for the smoothing edge artifact, in a simple one-dimensional case. (A) Simulated data, after having been masked with the mask, (B). (C) Smoothing kernel. (D) Smoothed data, before correction. (E) Correction factors. (F) Smoothed data after correction for the edge artifact.

One-dimensional phantom data

For a simple example in one dimension, see Table 1. Column (A) is a sequence of 15 random numbers from a uniform distribution in the range (0, 150), after having been masked with the mask, (B). (C) is the moving average smoothing kernel; it is merely the unweighted arithmetic average of a value with the preceding two and following two values. (D) is the result of smoothing the simulated data with the smoothing kernel, and after masking with the mask (B). Note that although the first two values and the last value in (A) were relatively high, in (D), the first two values and the last value are relatively low. This is the edge artifact. (E) is the result of smoothing the mask with the smoothing kernel, and after masking the result with the original mask; values in (E) are the correction factors. (F) is the result of dividing values in (D) by the corresponding values in (E) (restricting the division operation to voxels set to 1 in the original mask), and is the smoothed data after correction for the edge artifact.

With this one-dimensional example, it is apparent that the proposed method for correcting smoothed

data for the edge artifact is closely related to the notion of a “soft mean.” To illustrate this, consider a simple weighted average W_Y of a series of measurements $Y(t)$:

$$W_Y = \left(\sum_t Y(t) * G(t) \right) / \left(\sum_t G(t) \right)$$

where t is the domain over which measurements are obtained, and $G(t)$ is a set of weights for the weighted average. This is a smoothing operation in one dimension. If for some reason some values of $Y(t)$ are artifactually set to zero, the resulting value of W_Y will in turn be artifactually low. To adjust W_Y for zero values in $Y(t)$, these zero values should be in effect excluded from the weighted average. This can be done arithmetically by defining the function $H(k)$ such that

$$\begin{aligned} H(k) &= 1 \text{ if } k \neq 0 \\ &= 0 \text{ if } k = 0 \end{aligned}$$

The “soft mean,” or corrected weighted average, CW_Y , would then be

$$CW_Y = \left(\sum_t Y(t) * G(t) \right) / \left(\sum_t G(t) * H(Y(t)) \right)$$

which is mathematically equivalent to the proposed correction for the edge artifact due to smoothing. However, if the above equation were to be extended to three dimensions and implemented as is, in the spatial domain, it would be extremely inefficient. The method proposed in this paper for performing this “soft mean” retains the use of the Convolution Theorem and the Fast Fourier Transform for efficient calculation of the convolution.

Two-dimensional phantom data

Figure 1 shows the application of the method to a two-dimensional phantom data set. Figure 1a is a phantom of random, uncorrelated, Gaussian distributed numbers (mean 128, standard deviation 254) generated in MATLAB according to the simple model $B(x, y, z)$ described above (simple model of a functional neuroimage). Figure 1b is the binary mask defining the region of analysis, R_B . Figure 1c is the result of smoothing the simulated data with a Gaussian filter of size 5 “pixels” full width at half-maximum (FWHM), and after masking with the mask (Fig. 1b). Note the edge artifact, visible as a dark rim. Less easily seen is that it is somewhat greater at sharp corners, such as the vertices of the triangle. Figure 1d is the result of

smoothing the mask with the same Gaussian filter, and after masking the result with the original mask; values in Figure 1d are the correction factors. Figure 1e is the result of dividing values in Figure 1c by the corresponding values in Figure 1d, restricting the division operation to voxels set to 1 in the original mask, and is the smoothed data after correction for the edge artifact.

For examples of the use of “soft mean” smoothing in the literature, see the use of the map $n(x, y, z)$ in an earlier paper (Worsley et al., 1992), and of the map $N(x)$ in a more recent paper (Worsley et al., 1996). See also the discussion of kernel estimation in a recent book (Bailey and Gatrell, 1995). Another name for this sort of smoothing is “boundary truncation smoothing” (Holmes, 1994).

Three-dimensional PET data

To demonstrate the method in three dimensions, consider the following analysis of a small PET data set. Two 15-slice PET scans were acquired during each of three conditions: a control condition, during which a normal subject lay in the scanner at rest with his eyes closed; a self-paced sequential motor task on the left hand at about 2 Hz, where the fingertips of the second through fifth digits were tapped against the thumb; and an analogous self-paced sequential finger-tapping on the right hand. PET scans were acquired on a Scanditronix PC 2048-15B tomograph (Uppsala, Sweden) every 12 minutes, with 50 mCi H_2O^{15} injected for each scan. The data were of relative tissue activity, since no arterial blood sampling was done. Images were reconstructed using filtered back-projection, and the resolution in-plane and along the transverse axis was approximately 6.5 mm full-width at half-maximum. Slices were 6.5 mm thick and were acquired in the axial orientation. An attenuation correction using a transmission scan was done. A scatter correction was performed (Bergstrom et al., 1983), estimations of random coincidences were subtracted out, and corrections for detector normalization and deadtime (Daube-Witherspoon and Carson, 1991) were made.

PET scans were registered using an iterative method (Woods et al., 1992). A common-element mask defining R_B was generated by averaging the six PET scans, thresholding the result at 33% the maximum value, deleting from the mask voxels which were zero in any of the original PET scans, and then hand-editing to remove any remaining extra-cerebral voxels. Data were masked and then smoothed with the edge correction described in this paper. A pooled estimate of variance was generated (Fox et al., 1988; Worsley et al., 1992); the square root of this quantity yielded a pooled

estimate of standard deviation. A mean scan was generated for each of the three conditions. A difference map for right-hand movement vs. control was generated by subtracting the mean control scan from the

mean right-hand movement scan. A Z-score map for the contrast right-hand movement vs. control was generated by dividing the difference map by the pooled estimate of standard deviation. The analysis was repeated, but smoothing was then performed without the edge correction. The analyses with and without the described edge correction are compared in Figure 2, in which the two most superior slices of the processed PET volume still containing brain data are shown. Motor cortex is well represented in these slices; the left side of these images represents the subject's left hemisphere. After registration, the very most superior slice (not shown) was blank due to out-of-plane movements, and was therefore not included in the region of analysis, R_B .

Figure 2a shows the last two slices containing brain data of one of the control scans smoothed with the edge correction described in this paper. Compare this with Figure 2b, which shows the same data, but smoothed without the edge correction; note the smoothing edge artifact, which appears as a dark rim along the edge of the brain. More subtly, the last slice with brain data (the right-hand slice in the figure) is on the whole artifactually slightly darker than the same slice in Figure 2a. This is because this slice has been averaged with the very most superior slice, which is blank. Figure 2c shows the Z-score map for the right-hand movement minus control comparison, with the edge correction. The red dots show the locations of local maxima in three-dimensional space; the local maxima ranged in value from 3.95 to 5.72, with a mean of 5.08 and a standard deviation of 0.72. Compare this with Figure 2d, which shows the Z-score map for the same comparison, but without the edge correction. Superficially, Figure 2d appears nearly identical to Figure 2c, but there are subtle differences. Again, the red dots show the locations of local maxima; the distribution of values are similar to those in Figure 2c (they ranged in value from 2.95 to 5.79, with a mean of 4.58 and a standard deviation of 1.18). However, note that most of the local maxima have been moved away from the last slice with brain data into the preceding slice. This is another effect of the smoothing edge artifact. As stated above (Edge Effects on Local Vari-

Figure 1.

Steps of smoothing with edge correction, demonstrated on two-dimensional phantom data. (a) Phantom data before smoothing. (b) Mask defining region of analysis. (c) Smoothed phantom data, before correction for edge artifact. Note dark rim, which is the artifact. (d) Smoothed mask. (e) Smoothed phantom data, after correction for edge artifacts. Compare with (c).

Figure 2.

Results of smoothing and edge correction demonstrated on three-dimensional PET data. **(a)** Most superior two slices of a rest scan which still contain brain data, smoothed with the edge correction. **(b)** Same slices, smoothed without the edge correction. **(c)** Most superior two brain-containing slices of the Z-score map of the right-hand movement minus control comparison, where smoothing was performed with the edge correction. Red dots indicate local maxima. **(d)** Same slices of the Z-score map of the right-hand movement minus control comparison, where smoothing was performed without the edge correction.

ance), the edge artifact due to smoothing will be factored out in statistical maps made with within-voxel estimates of variance. However, it will still not be factored out in maps showing a measure of an effect size, such as a map of a difference in means or a map of regression coefficients.

Finally, Figure 3a shows a map of the ratio of the local value of

$$\text{var}[A(u, v, w)] = V_N * \left\{ \sum_{R_B} K^2(x, y, z) \right\} / \left\{ \sum_{R_B} K(x, y, z) \right\}^2$$

to the global mean of this quantity, averaged across R_B , in the same two slices. This is therefore a map of the ratio of the local estimate of variance to the pooled variance, which ideally should equal 1; variability in this map reflects side effects of both the smoothing edge artifact and the edge correction proposed in this paper. Given the assumption that the variance was spatially homogeneous with a value of V_N before smoothing, this map gives an indication of how well the assumption of homogeneity of variance holds after performing spatial smoothing with the edge correction. For most of the region shown, an intermediate gray value representing values near 1 appear, although a slight increase in intensity is seen at the edges. This indicates that, along the edge, the local variance will be slightly underestimated by a pooled estimate of variance. This effect is small, however, compared to the edge effect seen in Figure 3b, which is the map of the ratio of the local value of

$$\text{var}[S(u, v, w)] = V_N * \sum_{R_B} K^2(x, y, z)$$

to the global mean of this quantity, averaged across R_B , in the same two slices; this map is intensity scaled to the same level as Figure 3a. This is a map of the ratio of the local estimate of variance to the pooled variance, and variability in this map is secondary to the edge artifact of smoothing alone. Given the assumption that the variance was spatially homogeneous with a value of V_N before smoothing, this map gives an indication of how well the assumption of homogeneity of variance holds after performing spatial smoothing *without* the edge correction. The last slice with brain data in Figure 3b (the right-hand slice) is noticeably darker than the slices in Figure 3a, indicating that the local variance in the last slice is slightly overestimated by the pooled variance. On the other hand, the next to last slice with brain data is brighter than the slices in Figure

Figure 3.

Effects of smoothing and of the edge correction on local variance, demonstrated on the same slices of the same PET data as in Figure 2. **(a)** Ratio of the local estimate of variance to pooled variance, when smoothing is performed with the edge correction. **(b)** Ratio of the local estimate of variance to pooled variance, when smoothing is performed without the edge correction.

3a, indicating that local variance is underestimated in this slice by the pooled variance. Also note the dark rims in both slices with brain data in Figure 3b, indicating a consistent overestimation of local variance along the edges, when a pooled estimate of variance is used.

DISCUSSION AND CONCLUSION

Smoothing is most often implemented as a moving average filter applied to data. This is the same as convolving the data with a filter kernel. Convolution of large data sets is most efficiently done using Fourier Transforms (FTs) and the Convolution Theorem (Priestley, 1981; Press et al., 1992). Fourier Transforms of large data sets such as functional neuroimages are most efficiently computed using the *Fast* Fourier Transform, or FFT (Cooley and Tukey, 1965). The goal of this paper was to provide a method which corrects the edge artifact due to smoothing, while still permitting the use of the Convolution Theorem and the FFT for efficient computation.

These two crucial advantages, use of the Convolution Theorem and use of the FFT, will often require padding the data with blank zero slices, which we will

now discuss at length. The choice of the number of zero slices with which to pad will depend in part on the amount of smoothing to be done in the Z-dimension. Using the Convolution Theorem results in a *circular* convolution. This means that voxels at the right edge will be smoothed with voxels at the left edge, and voxels at the top edge will be smoothed with voxels at the bottom edge. For most brain scans this is not a problem, since there is usually adequate blank non-brain space along those edges, which are not included in the analysis region R_B anyway. But a potential problem arises in the Z-dimension. Often, the field of view does not cover the entire brain along that dimension, so that the first and last slices both contain brain data. Therefore, if circular convolution is done along the Z-dimension, brain voxels in the first slice will be smoothed with brain voxels in the last slice, an undesirable side-effect. Padding with zero slices will avoid the wrap-around smoothing effects along the Z-axis. Extending the mask with the same number of zero slices and then using the proposed method for edge effect correction will ensure that voxels in the first and last slices will not then be averaged with voxels from the padding zero slices.

An adequate number of padding zero slices will also depend on the fact that the FFT requires the length of the data along each axis be a power of 2. This is usually the case within-plane: the X and Y dimensions are usually 64 by 64, or perhaps 128 by 128, which are both powers of 2. However, the length along the Z-dimension is often not a power of 2. In that case, the Z-dimension will need to be padded with enough blank “zero” slices during smoothing to bring it up to a power of 2, allowing the FFT to be performed.

Thus, it is desirable to pad with enough zero slices to both avoid wrap-around effects and have a number of slices equal to a power of 2. Here is a rule-of-thumb to choose an adequate number of slices to pad with, when smoothing with a Gaussian filter. Let $zdim$ be the number of slices in a three-dimensional volume to be smoothed. Let $FWHM_z$ be the amount of smoothing to be done in the Z-dimension, *in units of voxel length along the Z-dimension*; e.g., if the voxel length along the Z-axis (or slice thickness) were 5 mm, and 10 mm smoothing FWHM were to be done along the Z axis, then $FWHM_z = 10/5 = 2$ voxel lengths. If no smoothing is to be done along the Z-axis, $FWHM_z = 0$. At a distance of 3 FWHMs from the center of a Gaussian smoothing kernel, the kernel values are near zero; since $FWHM = 2.35 \times$ (standard deviation), 3 FWHMs would equal more than 7 standard deviations, far off into the tails of the normal curve where it is nearly zero. So padding with $3 \times FWHM_z$ zero slices should be

adequate to prevent significant wrap-around effects, since contributions from wrap-around effects will be weighted with near-zero values. Calculate the quantity

$$N_{\text{wrap}} = \text{zdim} + (3 * \text{FWHMz})$$

which is the original number of slices plus the minimum number of blank slices needed to avoid significant wrap-around smoothing effects. Let N_2 be the smallest power of 2 which is equal to or greater than N_{wrap} . Calculate the difference

$$N_{\text{pad}} = N_2 - \text{zdim}$$

N_{pad} is the minimum number of padding zero slices necessary to avoid significant wrap-around smoothing effects *and* to have a number of slices equal to an even power of 2.

Finally, we suggest a possible extension of the edge correction proposed in this paper to deal with partial volume effects due to smoothing. During smoothing, voxels of one tissue type, e.g. gray matter, will be averaged with voxels of another tissue type, e.g. white matter. This results in a partial volume effect in which, after smoothing, some voxels have contributions from both gray and white matter. It may be desirable to perform spatial smoothing without averaging these two sorts of tissue together, that is, to somehow segregate the two sorts of tissue while smoothing. A possible approach would be to define two masks, one defining gray matter vs. non-gray matter, and the other defining white matter vs. non-white matter. Using these two masks and the method described in this paper, one could perform edge-adjusted smoothing on gray and white matter separately, generating a map of smoothed gray matter and a map of smoothed white matter. The sum of the two smoothed maps will be a smoothed image volume in which gray matter and white matter have not been averaged together. This approach can be extended to other arbitrary segmentations of neuroimages.

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