Fast Realistic Modeling in Bioelectromagnetism Using Lead-Field Interpolation

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Abstract: The practical use of realistic models in bioelectromagnetism is limited by the time-consuming amount of numerical calculations. We propose a method leading to much higher speed than currently available, and compatible with any kind of numerical methods (boundary elements (BEM), finite elements, finite differences). Illustrated with the BEM for EEG and MEG, it applies to ECG and MCG as well. The principle is two-fold. First, a Lead-Field matrix is calculated (once for all) for a grid of dipoles covering the brain volume. Second, any forward solution is interpolated from the pre-calculated Lead-Fields corresponding to grid dipoles near the source. Extrapolation is used for shallow sources falling outside the grid. Three interpolation techniques were tested: trilinear, second-order Bézier (Bernstein polynomials), and 3D spline. The trilinear interpolation yielded the highest speed gain, with factors better than $\times 10,000$ for a 9,000-triangle BEM model. More accurate results could be obtained with the Bézier interpolation (speed gain $\sim 1,000$), which, combined with a 8-mm step grid, lead to intrinsic localization and orientation errors of only 0.2 mm and 0.2 degrees. Further improvements in MEG could be obtained by interpolating only the contribution of secondary currents. Cropping grids by removing shallow points lead to a much better estimation of the dipole orientation in EEG than when solving the forward problem classically, providing an efficient alternative to locally refined models. This method would show special usefulness when combining realistic models with stochastic inverse procedures (simulated annealing, genetic algorithms) requiring many forward calculations. *Hum. Brain Mapping* 14:48–63, 2001.

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Key words: EEG; MEG; ECG; MCG; inverse problem; forward problem; numerical methods; BEM; realistic head model; dipole localization

INTRODUCTION

Solving the bioelectromagnetic forward and inverse problems in electro- or magneto-encephalography (EEG/MEG) requires models to describe the sources and the conductive media. The sources are usually described as point-like current dipoles, either in small numbers where their position and orientation are op-

timized to best mimic the data [Scherg, 1990], or in high numbers where dipoles have fixed positions and orientations and their amplitude is determined using distributed source techniques [Hämäläinen and Ilmoniemi, 1994; Pascual-Marqui et al., 1994]. Regardless of the source configuration, a geometry model should be chosen to account for conductivity inhomogeneities of the head (EEG/MEG). Classically, a spherical geometry is often used, which provides analytical solutions to the forward problem both in EEG [Ary et al., 1981; Cuffin and Cohen 1977; Rush and Driscoll, 1969] and MEG [Sarvas, 1987], and thus quite simple and fast computations. Because of this model's raw and thus not optimal geometrical approximation,

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however, a surge of new studies has been proposing and evaluating more realistically shaped models, requiring numerical methods to solve the forward problem, such as the boundary element method (BEM) [de Münck 1992; Fergusson et al., 1994; Fuchs et al., 1998; Hämäläinen and Sarvas 1989; Meijs et al. 1987; Mosher et al., 1999; Yvert et al., 1995], the finite element method (FEM) [Buchner et al., 1997; Haueisen et al., 1997; Thevenet et al., 1991; van den Broek et al., 1996; Yan et al. 1991], or the finite difference method (FDM) [Lemieux et al., 1996]. Although the BEM requires surface meshes of each iso-conductivity volume, the FEM and the FDM use volume meshes of the head. For each forward computation, these methods lead to the resolution of important linear systems, which sizes are proportional to the number of mesh nodes.

One major problem of realistic models remains their high computation load, which is responsible, at least in part, for the fact that these models are not yet routinely used when analyzing data in practice. The source model still mostly used in the literature is the first type of model described above, considering a few number of dipoles. Optimization algorithms are used to determine their optimal position and orientation parameters, by minimizing the root-mean-square difference between the measured and the model data. When analyzing data in practice for a given subject, one would typically like to estimate the sources at several latencies (especially when the moving dipole procedure is used), usually for different stimulation conditions, and may often try different digital filter settings or different number of dipoles. For instance, localizing one dipole source at each of 100 latencies, for five conditions and two different filter settings lead to the resolution of 1,000 inverse problems. Using present computers and boundary element models with 6,000 nodes, each inverse procedure would take about 3 min, leading to 50 hr computations for 1,000 inverse problems. Repeating this for 10 subjects would then take 3 weeks continuous computation, making it practically very tedious for the user. Moreover, classical minimization algorithms (Marquardt, Simplex) may fail to reach the optimum solution because of the presence of local minima. Other optimization techniques, such as simulated annealing or genetic algorithms, are promising to overcome this problem [Khosla et al., 1997; McNay et al., 1996; Sekihara et al., 1992; Uutela et al., 1998]. These algorithms, however, require a great number of forward calculations, making them difficult to combine in practice with refined realistic models.

A way to reach reasonable computation time is to consider meshed models with limited number of ele-

ments. This strategy has a drawback, however, because the accuracy of the numerical computation is directly dependent on the mesh density, as has been evaluated using the boundary element method [Fuchs et al., 1998; Menninghaus et al., 1994; Roth et al., 1993; Yvert et al., 1995, 1996; Zanow and Peters, 1995;] or the finite element method [Bertrand et al., 1991].

Fletcher et al. [1995] have proposed a variant formulation of the BEM using the reciprocity theorem to increase the speed of realistic models in EEG, which cannot be as efficiently used in MEG. Huang et al. [1999] have proposed an approximation approach for MEG, consisting in estimating an optimal spherical model different for each sensor, yielding the most similar forward solutions at this sensor as the BEM over a grid of dipoles. This overlapping sphere model is based on the fact that the computation of the forward problem in MEG does not require the source to be inside the sphere, because the radius of the sphere does not appear in the analytical formula given by Sarvas [1987]. Obviously, such method cannot be used in the case of EEG, where sources would generally not remain within all spheres.

Here, we propose an original approach allowing for fast use of realistic models. Considering a reasonably time-consuming preliminary computation of a lead-field matrix once for all, subsequent forward calculations encountered when solving inverse problems are directly and quickly interpolated from the pre-calculated lead-fields using analytical polynomials. Although this method is here detailed in the particular case of EEG/MEG and the BEM, it remains equally adapted to electro- and magneto-cardiography (ECG, MCG) and to any other numerical method (FEM, FDM).

DESCRIPTION OF THE METHOD

Basic equations

Given a time-varying primary source distribution $J^P(\mathbf{r}, t)$ in a conductive model of the head, the potential distribution $V(\mathbf{r}, t)$ within and on the outer surface of the head is obtained through the following equation (with appropriate boundary conditions):

$$\nabla \cdot (\sigma(\mathbf{r}) \nabla V(\mathbf{r}, t)) = \nabla \cdot J^P(\mathbf{r}, t), \quad (1)$$

where $\sigma(\mathbf{r})$ is the conductivity tensor at position $\mathbf{r}$ in the medium. The magnetic field can then be calculated with the Ampère and Laplace law:

$B(\mathbf{r}, t)$

$$= \frac{\mu_0}{4\pi} \int \frac{(\mathbf{J}^P(\mathbf{r}', t) - \sigma(\mathbf{r}') \nabla V(\mathbf{r}', t)) \times (\mathbf{r}' - \mathbf{r})}{|\mathbf{r}' - \mathbf{r}|^3} d\mathbf{v}', \quad (2)$$

These general equations are solved either analytically or numerically for spherical or realistic geometries, respectively. In the particular case of a current dipole source located at position $\mathbf{r}_0$ with a time-varying moment vector $\mathbf{Q}(t) = Q(t)\mathbf{n}$, the primary current becomes:

$$\mathbf{J}^P(\mathbf{r}, t) = \mathbf{Q}(t)\delta(\mathbf{r} - \mathbf{r}_0), \quad (3)$$

The set of EEG and MEG forward solutions produced by a dipolar source is called Lead-Field when the source is of unit amplitude. In the following, a Lead-Field corresponding to a source at position $\mathbf{r}$ is noted $\mathbf{L}_0(\mathbf{r}) = \{V_0(\mathbf{r}), \mathbf{B}_0(\mathbf{r})\}$. For a source of arbitrary amplitude, the set of forward solutions is thereafter noted without zero-subscripts: $\mathbf{L}(\mathbf{r}) = \{V(\mathbf{r}), \mathbf{B}(\mathbf{r})\}$.

An important feature of equations (1) and (2) is their linearity with respect to $\mathbf{J}^P(\mathbf{r}, t)$, and thus with respect to $\mathbf{Q}(t)$ in the case of current dipoles. Let us consider two dipole sources (1 and 2) having unit moment amplitudes, who individually produce Lead-Fields $\mathbf{L}_{1,0}(\mathbf{r}) = \{V_{1,0}(\mathbf{r}), \mathbf{B}_{1,0}(\mathbf{r})\}$ and $\mathbf{L}_{2,0}(\mathbf{r}) = \{V_{2,0}(\mathbf{r}), \mathbf{B}_{2,0}(\mathbf{r})\}$, respectively. When these sources are simultaneously active with respective time-varying amplitudes $Q_1(t)$ and $Q_2(t)$, they produce a forward solution that is the following linear combination of the individual Lead-Fields:

$$\mathbf{L}_{1+2}(\mathbf{r}, t) = Q_1(t)\mathbf{L}_{1,0}(\mathbf{r}) + Q_2(t)\mathbf{L}_{2,0}(\mathbf{r}), \quad (4)$$

and, dropping time for simplification:

$$\mathbf{L}_{1+2}(\mathbf{r}) = Q_1\mathbf{L}_{1,0}(\mathbf{r}) + Q_2\mathbf{L}_{2,0}(\mathbf{r}). \quad (5)$$

In particular, the three Lead-Fields $\mathbf{L}_{x,0}(\mathbf{r})$, $\mathbf{L}_{y,0}(\mathbf{r})$, and $\mathbf{L}_{z,0}(\mathbf{r})$, corresponding to unit dipoles at location $\mathbf{r}$ with orientation along positive x, y, and z 3D space cartesian axis, respectively, form a basis of the space spanned by the forward solutions produced by all possible dipoles located at this location. Hence, any dipole at $\mathbf{r}$ with moment vector

$$\mathbf{Q} = \begin{bmatrix} Q_x \\ Q_y \\ Q_z \end{bmatrix}$$

produces $\mathbf{L}(\mathbf{r}) = \{V(\mathbf{r}), \mathbf{B}(\mathbf{r})\}$ satisfying:

$$\mathbf{L}(\mathbf{r}) = Q_x\mathbf{L}_{x,0}(\mathbf{r}) + Q_y\mathbf{L}_{y,0}(\mathbf{r}) + Q_z\mathbf{L}_{z,0}(\mathbf{r}). \quad (6)$$

Classical resolution of the forward problem

When using realistic models and the BEM, equations (1) and (2) amount to solve linear systems of the form:

$$\mathbf{V} = \mathbf{A}_{\text{eeg}}^{-1}\mathbf{V}_0 \quad (7)$$

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{A}_{\text{meg}}\mathbf{V}, \quad (8)$$

where:

- $\mathbf{V}$ is the vector of potential values at each of the N nodes of the mesh,
- $\mathbf{A}_{\text{eeg}}$ is a $N \times N$ matrix,
- $\mathbf{V}_0$ is the vector of potential values at the nodes that would be produced by the source in an infinite homogeneous medium of unit conductivity,
- $\mathbf{B}$ is the vector of field components along the normal of each of the N_c sensor coils,
- $\mathbf{B}_0$ is the vector of field components produced by the primary current $\mathbf{J}^P$ alone that can be determined analytically, and
- $\mathbf{A}_{\text{meg}}$ is a $N_c \times N$ matrix ($\mathbf{A}_{\text{meg}}\mathbf{V}$ being the contribution of volume currents).

The matrices $\mathbf{A}_{\text{eeg}}$ and $\mathbf{A}_{\text{meg}}$ have to be computed only once, for they depend only on the geometry and conductivities of the different head compartments, and not on the sources.

The resolution of the first linear system (equation 7) for the potential vector $\mathbf{V}$ is the most time-consuming part. Indeed, although the matrix $\mathbf{A}_{\text{eeg}}$ is usually inverted or LU-decomposed as a preliminary step once for all, each computation of equation (7) still requires N^2 multiplications and N^2 additions, which makes the computation time remain important and directly related to the mesh density.

Proposed approach

Preliminary computations.

While the classical approach requires the preliminary computation of the system matrices $\mathbf{A}_{\text{eeg}}$ and $\mathbf{A}_{\text{meg}}$ and the decomposition or inversion of $\mathbf{A}_{\text{eeg}}$, our approach requires a subsequent preliminary computation, which has to be performed only once for a given subject and a given sensor configuration (electrodes or pickup coils).

This additional step consists in 1) the construction of a discrete grid covering the brain volume (cf. Fig. 1A), and 2) the computation of the Lead-Fields $\mathbf{L}_{x,0}(\mathbf{r})$, $\mathbf{L}_{y,0}(\mathbf{r})$, and $\mathbf{L}_{z,0}(\mathbf{r})$, for each location $\mathbf{r}$ of the grid.

Estimation of the forward problem for an arbitrary source.

Once these preliminary steps are completed, each forward problem is estimated from the pre-computed Lead-Fields. Given an arbitrary dipole source A in the brain volume at position

$$\mathbf{R}_a = \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}$$

with a moment vector

$$\mathbf{Q}_a = \begin{bmatrix} Q_{x,a} \\ Q_{y,a} \\ Q_{z,a} \end{bmatrix},$$

$\mathbf{L}_a = \{\mathbf{V}_a, \mathbf{B}_a\}$ produced by this source are approximated as follows: first, the three Lead-Fields $\mathbf{L}_{x,0}(\mathbf{R}_a)$, $\mathbf{L}_{y,0}(\mathbf{R}_a)$, and $\mathbf{L}_{z,0}(\mathbf{R}_a)$ are interpolated from the Lead-Fields corresponding to the grid sources neighboring the arbitrary source A; second, $\mathbf{V}_a$ or $\mathbf{B}_a$ corresponding to the arbitrary orientation of the source are obtained through equation (6).

We evaluated this method with three different interpolation techniques: the trilinear interpolation, the Bézier interpolation (which is an extension of the trilinear interpolation with higher degree polynomials), and the 3D spline interpolation [Duchon, 1976]. While the first two techniques require a rectangular grid array surrounding the arbitrary source, the third one has the advantage to work for any arrangement of the grid points. The equations driving these 3 types of interpolation are given next.

Trilinear interpolation.

Although electric and magnetic Lead-Fields do not vary linearly with the spatial coordinates of the source, trilinear interpolation was still considered because it yielded very few computations and was thus the best candidate for fastest interpolation.

We assume here that the grid is rectangular, with step sizes equal to Δx , Δy , and Δz , respectively for the three dimensions. We consider two different situations, depending on the location of the arbitrary source A with respect to the grid points.

Figure 1.

A: Schematic representation of a discrete grid covering the brain volume. **B:** The arbitrary source A falls inside a grid cube (trilinear interpolation case) so that interpolation is used. **C:** The arbitrary source A falls outside all grid cubes, although still within the brain volume. Lead-Field extrapolation is used in this case.

The first and mostly happening case occurs when A falls inside a grid cube (cf. Fig. 1B). We note $(x_{\min}, y_{\min}, z_{\min})$ the corner of the cube with minimum coordinate values along all three axis. Then, any quantity l_a at location $\mathbf{R}_a$ is interpolated from the corresponding quantities l_i ($1 \leq i \leq 8$) known at the eight corners of the cube, using the following relation:

$$l_a = \sum_{i=1}^8 \omega_i l_i, \quad (9)$$

with, according to the notations of Figure 1B and noting $u_a = (x_a - x_{\min})/\Delta x$, $v_a = (y_a - y_{\min})/\Delta y$, $w_a = (z_a - z_{\min})/\Delta z$ the local coordinates of A in the cube (between 0 and 1):

$$\begin{aligned} \omega_1 &= (1 - u_a)(1 - v_a)(1 - w_a) \\ \omega_2 &= u_a(1 - v_a)(1 - w_a) \\ \omega_3 &= (1 - u_a)v_a(1 - w_a) \\ \omega_4 &= u_av_a(1 - w_a) \\ \omega_5 &= (1 - u_a)(1 - v_a)w_a \\ \omega_6 &= u_a(1 - v_a)w_a \\ \omega_7 &= (1 - u_a)v_aw_a \\ \omega_8 &= u_av_aw_a \end{aligned} \quad (10)$$

The other situation occurs when A falls outside all grid cubes (although still remaining inside the brain

volume), that is when it corresponds to a very shallow source (cf. Fig. 1C). In this case, the Lead-Fields are extrapolated from those of the eight grid nodes of the grid cube closest to A using the same formulas (equations 9, 10) in which the u_a , v_a , and w_a coordinates are allowed to be negative or greater than 1.

Bézier interpolation.

Let us here consider that the source A falls within a box made of $(m + 1) \times (n + 1) \times (p + 1)$ grid points, and let $(x_{\min}, y_{\min}, z_{\min})$ be the corner of the box with minimum coordinate values along all three axis. As an extension of the trilinear interpolation, the Bézier interpolation uses a basis of Bernstein polynomials with degrees m , n , and p along the x , y , and z axis, respectively, so that equation (9) becomes:

$$l_a = \sum_{i=0}^m \sum_{j=0}^n \sum_{k=0}^p \omega_{ijk} b_{im}(u_a) b_{jn}(v_a) b_{kp}(w_a), \quad (11)$$

where:

- (u_a, v_a, w_a) are the local coordinates of A in the box: $u_a = (x_a - x_{\min})/m\Delta x$, $v_a = (y_a - y_{\min})/n\Delta y$, and $w_a = (z_a - z_{\min})/p\Delta z$. In case of shallow sources, extrapolation is obtained as for the trilinear interpolation, by letting u_a , v_a , or w_a be negative or greater than one;
- $b_{\alpha\eta}(t) = \frac{\eta!}{\alpha!(\eta - \alpha)!} t^\alpha (1 - t)^{\eta - \alpha}$, $\alpha = i, j, k$; $\eta = m, n, p$.

The unknown coefficients w_{ijk} are determined by:

$$\Omega = B^{-1}L, \quad (12)$$

where:

$$\Omega = \begin{bmatrix} \omega_{000} \\ \omega_{100} \\ \vdots \\ \omega_{mnp} \end{bmatrix},$$

$$B = \begin{bmatrix} b_{0m}(u_0)b_{0n}(v_0)b_{0p}(w_0) & b_{1m}(u_0)b_{0n}(v_0)b_{0p}(w_0) & \cdots & b_{mm}(u_0)b_{nn}(v_0)b_{pp}(w_0) \\ b_{0m}(u_1)b_{0n}(v_0)b_{0p}(w_0) & b_{1m}(u_1)b_{0n}(v_0)b_{0p}(w_0) & \cdots & b_{mm}(u_1)b_{nn}(v_0)b_{pp}(w_0) \\ \vdots & \vdots & \ddots & \vdots \\ b_{0m}(u_m)b_{0n}(v_n)b_{0p}(w_p) & b_{1m}(u_m)b_{0n}(v_n)b_{0p}(w_p) & \cdots & b_{mm}(u_m)b_{nn}(v_n)b_{pp}(w_p) \end{bmatrix},$$

$$L = \begin{bmatrix} l_{000} \\ l_{100} \\ \vdots \\ l_{mnp} \end{bmatrix},$$

and where

$$\begin{bmatrix} u_i = i/m \\ v_j = j/n \\ w_k = k/p \end{bmatrix}$$

are the local coordinates of the grid point located at

$$\begin{bmatrix} x_{\min} + i * \Delta x \\ y_{\min} + j * \Delta y \\ z_{\min} + k * \Delta z \end{bmatrix},$$

and l_{ijk} is the known value at this grid point.

It should be noted that the matrix $\mathbf{B}$ does not depend on the location of the arbitrary source A , so that it can be calculated and inverted only once for all. Moreover, B is a sparse matrix with $(m^2 + 1)(n^2 + 1)(p^2 + 1)$ non zero elements among $(m + 1)^2(n + 1)^2(p + 1)^2$, which enables to spare multiplications when computing equation (12). When $m = n = p = 1$, the matrix B becomes the identity matrix, and the Bézier interpolation reduces to the trilinear interpolation.

In the following evaluations we used $m = n = p = 2$.

3D spline interpolation.

Initially described by Duchon [1976], volume splines have recently been applied to estimate the EEG scalp current density maps over a realistic scalp surface model derived from MRIs [Babiloni et al., 1996]. Using Babiloni's notations, given an arbitrary set of N points in 3D space, any quantity l_a at location

$$\mathbf{R}_a = \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}$$

is interpolated from the corresponding quantities l_i ($1 \leq i \leq N$) known at the N point locations $\mathbf{r}_i$, using the following relation:

$$l_a = \sum_{i=1}^N t_i |\mathbf{R}_a - \mathbf{r}_i|^{2m-3} + \sum_{d=0}^{m-1} \sum_{k=0}^d \sum_{g=0}^k r_{dkg} x_a^{d-k} y_a^{k-g} z_a^g,$$

where the coefficients t_i and r_{dkg} are obtained by solving the following linear system:

$$\begin{bmatrix} \mathbf{H} & \mathbf{F} \\ \mathbf{F}' & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{T} \\ \mathbf{R} \end{bmatrix} = \begin{bmatrix} \mathbf{L} \\ \mathbf{0} \end{bmatrix}$$

and where the known matrices and vectors $\mathbf{H}$, $\mathbf{F}$, and $\mathbf{L}$ are:

$$\mathbf{H} = \{h_{ij} = |\mathbf{r}_i - \mathbf{r}_j|^{2m-3}\}_{1 \leq i \leq N, 1 \leq j \leq N}$$

$$\mathbf{F} = \begin{bmatrix} 1 & x_1 & y_1 & z_1 & x_1^2 & x_1 y_1 & \cdots & z_1^{m-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_N & y_N & z_N & x_N^2 & x_N y_N & \cdots & z_N^{m-1} \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} l_1 \\ \vdots \\ l_N \end{bmatrix}$$

and the unknown vectors $\mathbf{T}$ and $\mathbf{R}$ are:

$$\mathbf{T} = \begin{bmatrix} t_1 \\ \vdots \\ t_N \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} r_{000} \\ r_{100} \\ \vdots \\ r_{m-1, m-1, m-1} \end{bmatrix}$$

In the following evaluations, we used $m = 3$ and the $N = 80$ grid points closest to the arbitrary source A . Note that in case of a shallow source A outside the cloud of the N grid points, extrapolation is obtained using the very same equations.

Evaluation procedure.

For the simulations presented below, spherical geometry models were considered with an external radius $R = 90$ mm, and brain and skull radii of $0.87R$ and $0.92R$, respectively. Conductivities were set to 0.45, 0.45/80, and 0.45 S/m for the brain, the skull, and the scalp, respectively. Spherical isotropic grids were used with 6 different step sizes: 6, 8, 10, 12, 15, and 20 mm. The respective number of points were 9315, 3887, 2007, 1189, 619, and 251. First, to account for the intrinsic errors inherent to the interpolation approximation, the method was tested with analytically pre-computed Lead-Fields. Next, it was evaluated with Lead-Fields pre-computed using the linear BEM [de Münck, 1992; Ferguson et al., 1994]. For this purpose, a uniformly meshed 3-shell spherical model was used, having a total of 9,000 triangles, and the isolated problem approach was considered [Hämäläinen and Sarvas, 1989]. The method was evaluated for both forward and inverse problems, and both in EEG and in

Figure 2.

A: 63-Electrode (white circles) and 143-coil (black rings) montages used in the present simulations are shown with the outer surface of the 3-shell spherical meshed model (3,000 triangles per layer). **B:** Representation of the fine 4.3-mm step grid (pluses at each point of which the Lead-Fields are interpolated from those

pre-computed over a 10-mm step grid (filled circles). **C:** The 261 dipole locations used for the inverse problem evaluation are evenly spread over 9 axis of the upper right quadrant of the inner sphere (see Table I for precise coordinates).

MEG. Figure 2A shows the 63-electrode EEG montage (including electrodes of the international 10-20 system) and the 143-channel MEG array (first-order gradiometer arrangement from CTF) used in these simulations.

The accuracy of the forward problem was evaluated at each point of a fine 4.3-mm-step grid covering the whole spherical brain volume (25,341 points). For each point A of this grid, we considered three unit dipole orientations (along the positive x, y, and z cartesian axis). For each location, the three forward solutions interpolated from the pre-computed Lead-Fields of the coarser grids were compared with the true analytical solutions, using the following RMS error criterion:

$$\text{RMS} = \sqrt{\frac{\sum_{\text{ori}=1}^3 \sum_{c=1}^{\text{Nbchannels}} (I_{c,\text{ori}}^{\text{analytical}} - I_{c,\text{ori}}^{\text{interpolated}})^2}{\sum_{\text{ori}=1}^3 \sum_{c=1}^{\text{Nbchannels}} (I_{c,\text{ori}}^{\text{analytical}})^2}}$$

where $I_{c,\text{ori}}$ stands for either the potential or the magnetic field value at the cth channel for a given orien-

tation. Figure 2B shows an example of a 10-mm-step grid (filled circle symbols), for which Lead-Fields were pre-computed, and which was used to interpolate the Lead-Fields at each point of the fine grid (plus symbols)

The localization error of the inverse problem was calculated for 261 dipole locations distributed along nine axis of the upper right quadrant of the inner sphere (Fig. 2C). Along each axis, 29 locations were equally spaced every 2.61 mm with depth below the brain shell ranging from 2.7–75.7 mm. Table I gives the locations of these dipoles in spherical coordinates. At each location, three dipole orientations along x, y, and z were considered in the EEG case (total of 783 dipoles), and two tangential orientations (θ and ϕ standard spherical directions) were considered in the MEG case (522 dipoles). For each dipole, forward simulated data were generated analytically, and inverse localizations were performed using either the classical BEM, or the proposed approach with grid Lead-Fields computed either analytically or using the BEM. In the MEG case, because the meshed model was spherical, the dipole orientation was constrained to remain tangential during the inverse procedure. The localization and orientation errors between the original and esti-

TABLE I. Dipole locations used in the inverse problem evaluation

Track	1	2	3	4	5	6	7	8	9
(θ , φ)	(5,5)	(45,−80)	(45,−45)	(45,−13)	(45,45)	(45,75)	(83,−80)	(76,−13)	(87,75)

* Locations are given in classical spherical coordinates (r , θ , φ). For each of the (θ , φ) angle pairs given in the table, a track of 29 dipole locations was considered with radial coordinate values regularly ranging from 2.6 to 75.6 mm.

TABLE II. Time required for the pre-computation of the Lead-Fields for the different grids used in this study*

Grid type	Step (mm)	Number of points	Computation time for EEG (hr:min)	Computation time for MEG (hr:min)	Number of points in a realistic model
Non-cropped	4.3	25341	28:08	34:27	20183
	6	9315	10:07	12:16	7444
	8	3887	4:18	5:05	3144
	10	2007	2:14	2:34	1608
	12	1189	1:18	1:31	944
	15	619	0:40	0:47	475
	20	251	0:16	0:19	199
Cropped	6	6667	7:21	8:40	5156
	8	2801	3:06	3:40	2161
	10	1365	1:31	1:47	1110
	12	847	0:54	1:06	635
	15	437	0:28	0:36	330
	20	179	0:11	0:15	132

* In the last column, the number of grid points obtained in a realistic volume conductor model are indicated for similar step sizes. It can be seen that a realistic grid would contain about 20% less points than a 78.3-mm-radius sphere model.

mated dipoles were averaged across the nine different axis and the two or three orientations to get mean error values for each depth.

Effect of grid cropping.

It is known that shallow sources produce high numerical errors on the forward problem, especially in EEG when no mesh refinement is used [Yvert et al., 1995, 1996]. The use of locally refined models is rather tedious in practice, however, because models should be modified as the source position changes during the inverse procedure, each time leading to the re-computation and inversion or decomposition of the system matrices A_{eeg} and A_{meg} . For this reason, most authors using realistic head models in the literature have considered uniformly meshed models. Using our proposed approach, shallow grid points should intuitively not be used, because they lead to erroneous pre-computed Lead-Fields. We tested whether cropped grids would bring any improvement on the accuracy of the interpolated forward solutions and the inverse localization procedure, by removing shallow grid points having a depth smaller than the mean length of all triangles (8 mm in our case). As a consequence, these grids lead to more extrapolations than non cropped grids.

RESULTS

Computation time

When using realistic models, the total computation time can be split into the time used for the preliminary

computations and that used when solving each forward problem. In practice, it is not much a drawback that the preliminary computations take a few hours, because they usually have to be done only once for each subject. By contrast, the time used for each forward calculation is critical because it defines the time the user has to wait in front of his/her computer screen when analyzing data. In practice, it should thus be as small as possible.

Preliminary computations associated with the construction of matrices A_{eeg} and A_{meg} and to the decomposition or inversion of the matrix A_{eeg} are common to the classical and to the proposed approaches. For the simulations presented here, these computations took 61 min on a 333-MHz Pentium II with 256 Mb RAM (using the LU-decomposition of the matrix A_{eeg}).

The proposed approach leads to an additional preliminary computation corresponding to the pre-computed Lead-Fields over the grid, which is proportional to the number of grid points. Table II gives this computation time for all grids used in these simulations. It is to be noted that for a given step size, the number of grid points is larger in a 78.3-mm-radius spherical “brain” than in a realistically shaped one. For this reason, we considered a realistic model and built grids with similar step sizes than those used in the present study. Table II provides the numbers of grid points obtained in this realistic volume conductor model, which were about 80% those corresponding to a spherical model. Hence, with a realistic volume conductor model the additional preliminary computation would only last about 80% of the time shown in Table

Figure 3.

Computation time needed to compute the 3 Lead-Fields at all 25,341 points of the fine 4.3-mm grid. Although the values corresponding to the proposed approach were obtained for a grid step of 10 mm, they were independent of the grid size. Computation time gain factors in EEG and MEG, respectively, are of 20,000 and 13,700 for the trilinear interpolation, 1,200 and 750 for the Bézier interpolation, and 40 and 23 for the 3D spline interpolation.

II. For example, with a realistic model and a grid step of 8 mm, an additional preliminary computation time of about 3–4 hr would be required for the proposed approach on a Pentium II 333 MHz (256 Mb RAM).

Figure 3 shows the computation time needed to compute the Lead-Fields at all points of the 4.3-mm-step grid, using both the classical approach, and the proposed one with the three different interpolation methods and a grid step of 10 mm. This time was independent of the preliminary grid used for interpolation, for only the time to load the precomputed grid from disk changed, which was actually negligible. As expected, the trilinear interpolation was the fastest method, leading to a gain factor in computation time of the order of 20,000 in EEG and 13,700 in MEG, with respectively 5 and 9 sec instead of 28 hr 8 min and 34 hr 27 min with the classical approach. The Bézier interpolation lead to gain factors of 1,200 in EEG and 750 in MEG, and the 3D spline only to gain factors of 40 in EEG and 23 in MEG. It should be noted that the gain factor is inversely related to the number of sensors (here: 63 in EEG and 143 in MEG). Indeed, the time required by the classical BEM approach depends strongly on the size of the mesh and only slightly on the number of channels, whereas the time required by our method is directly proportional to the number of channels. Hence the approach proposed here is all the more powerful as the number sensors is small. As shown here with realistic sensor configurations, it brings appealing improvements even for large sensor arrays.

Intrinsic precision of the proposed approach (analytical lead-fields)

The intrinsic precision of the method was evaluated in the case of analytical Lead-Fields, so that no errors could here be attributed to numerical approximation. Figure 4A reports the RMS errors on the forward problem obtained in EEG and MEG for the 3 different interpolation methods and the six different grid steps (6, 8, 10, 12, 15, and 20 mm). Figure 4B gives the localization and orientation errors obtained when solving the inverse problem. These curves indicate that intrinsic errors well below 0.5 mm and 0.2 degrees can be easily achieved with a sufficiently small grid step (6–8 mm) and either Bézier or 3D spline interpolations for any dipole depth. When deep dipoles are considered, larger grid step (10–12 mm) can be used to obtain the same accuracy. The trilinear interpolation method, although faster than the 2 others, lead to less accurate results, especially in MEG and especially at the center of the model where interpolated values were small. Localization and orientation errors below 0.5 mm and 0.2 degrees, however, could be achieved for almost any dipole depth using grid steps of 6 or 8 mm.

Precision of the proposed method with respect to the classical BEM

Forward and inverse errors were also evaluated in a spherical numerical model using the BEM, and compared with errors obtained by the classical approach. Here only non-cropped grids were considered with the same 6 different grid steps as before. Figures 5A and B give the forward and inverse errors, respectively. It can be seen that, irrespectively of the dipole depth, grid steps of about 6–8 mm lead to very comparable localization errors than the classical BEM. In other words, errors due to the interpolation are negligible with respect to numerical errors inherent to the BEM. For deep dipoles (depth >20 mm), larger grid steps (10–12 mm) might even be used to obtain a good accuracy. Regarding orientations, errors are large in EEG for shallow sources, which can be attributed to the fact that uniformly meshed models were considered. These results are in accordance with previously published ones [Yvert et al., 1995, 1996]. In MEG, orientation errors are large for very deep sources, which is to be expected in a spherical geometry. It should be noted, however, that these orientation errors obtained for deep dipoles with the proposed approach remain comparable to those obtained with the classical approach when 6–8-mm grid steps are considered, and even with larger grid steps (10–12 mm).

A) Forward Problem

B) Inverse Problem

Figure 4.

Intrinsic precision of the proposed approach evaluated in the case of analytical Lead-Fields. Forward RMS (A) and inverse localization and orientation errors (B) are plotted for the three different interpolation techniques and for the six grid steps ranging from 6–20 mm. In (A), error curves are ordered with respect to the step size, hence only the two extreme cases (6 and 20 mm) are labeled on the first plot.

A) Forward Problem **B) Inverse Problem** **Figure 5.**

Results of the evaluation of the proposed methods in the case of BEM Lead-Fields. **A:** Forward RMS errors. **B:** Localization and orientation errors are given for each step size in a separate plot. Note that it is hard to separate the curves between the original

and the proposed approaches for grid steps of 6 or 8 mm. Results using the original BEM approach (star symbols) are superimposed for comparison.

when using the Bézier interpolation (Fig. 5B). In general, the Bézier interpolation shows superiority with respect to the trilinear interpolation and lead to more stable results than the 3D spline interpolation. It should thus be preferred to the two others.

Effect of grid cropping

Figures 6A and B respectively give forward and inverse errors obtained for cropped grids. It shows up that using cropped grids improves the accuracy of the

Figure 6.

Results of the evaluation of the proposed methods in the case of BEM Lead-Fields using cropped grids: shallow grid points with depth below 8 mm (one average triangle edge length) have been removed. Results using the original BEM approach (star symbols)

are superimposed for comparison. Note that the dipole orientation is much better retrieved in EEG using the proposed approach (and any kind of interpolation) than the original BEM.

forward problem for shallow sources, especially in EEG (Fig. 6A). The accuracy gain is not, however, so obvious in MEG. Regarding the inverse problem, whereas localization errors are not smaller with cropped grids, orientation errors are dramatically decreased in EEG for any interpolation method used and

any grid step between 6 and 15 mm (Fig. 6B). In MEG, only very slight improvements on the orientation errors could be observed for the shallowest sources and steps of 6–8 mm when using the Bézier interpolation. Hence, grid cropping should be considered in EEG when the dipole orientations have to be retrieved with

good accuracy. In this case it is a rather “cheap” and efficient alternative to locally refined models.

Improvements for MEG by interpolating only of the magnetic field produced by secondary currents

Previous results show that interpolation works better for EEG than MEG. This being likely due to steepest MEG than EEG solutions, we tested whether interpolating only the magnetic field due to secondary currents and compute analytically the part created by primary currents (B_0 in equation 8) would increase the precision of our method in the case of MEG. Only trilinear and Bézier interpolations were considered here. Figure 7 gives localization and orientation errors obtained this way, for full and cropped grids pre-computed with the BEM. For comparison, errors obtained using the classical BEM are superimposed. It shows up that errors are now smaller than when interpolating the whole magnetic field (cf. Figs. 5, 6), and that cropped grids now yield much improvements for shallow sources. With an 8-mm grid step, errors were below 0.1 mm and 0.1 degrees for any source located at more than 20 mm from the center of the model. Even with coarser grids (step up to 15 mm), errors below 0.2 mm and 0.1 degrees were achieved.

DISCUSSION

Among the parameters tested here, a grid step size of 8 mm and the Bézier interpolation provided a good compromise between preliminary computation time and accuracy. That the 3D spline interpolation yielded larger errors than the Bézier interpolation for BEM pre-calculated grids for dipoles located at $r > 40$ mm (cf. Fig. 5, unstable results are obtained for a 15 mm grid step in the EEG case) might be attributed to the fact that 3D spline used Lead-Field values pre-calculated over a larger region (80 points around the source) than the Bézier interpolation. As a consequence, numerical errors present in BEM Lead-Fields corresponding to shallow sources influenced forward solution estimation for deeper sources at $r \sim 40$ mm. This is further supported by the fact that when no errors are made on the pre-calculated Lead-Fields (analytical case, cf. Fig. 4), 3D spline lead to better results than the Bézier interpolation, and by the fact that when cropped grids are considered, unstable results yielded by 3D spline disappear (Fig. 5 vs. Fig. 6).

Intrinsic errors were found to be smaller in EEG than in MEG when, in the later case, the whole magnetic field was interpolated. Errors were also smaller

for deep than for shallow sources (except very deep sources in MEG). This could find an intuitive explanation in the fact that interpolation works best where Lead-Field gradients are smoothest, which is the case for EEG and deep sources. MEG results could be strongly improved by interpolating only the field produced by secondary currents (Fig. 7). This was expected because the coils were oriented so as to mostly record the radial component of the magnetic field, which is known to be sensitive only to primary sources in spherical geometries. In realistic geometries, where secondary currents have a more important contribution, one would still obtain better results by interpolating only the contribution of secondary currents, with effects somewhat smaller than for the spherical case. The fact that errors were high in MEG for deep sources should be considered as an artifact of using spherical geometries for which a dipole at the center creates no magnetic field. In any case, intrinsic errors (forward and inverse) found in the present study remained small in any case with respect to classical numerical errors reported in the literature [Fuchs et al., 1998; Roth et al., 1993; Yvert et al., 1995, 1996; Zanow and Peters, 1995]. Even combined to more accurate BEM formulations [Fischer et al., 1999; Frijns et al., 2000; Mosher et al., 1999] or other numerical methods such as FEM, or considering more refined models, the errors induced by the proposed approach (about 0.2 mm and 0.2 degrees) would remain negligible with respect to other sources of errors occurring in practice, such as the geometry and conductivity approximations, or the registration errors, the sum of these being commonly estimated to be of the order of 3–5 mm. Finally, our interpolation method based on polynomial functions is more accurate than previously proposed methods aiming at approximating realistic Lead-Fields with optimal spherical Lead-Fields, which have been shown to yield errors up to 5–10 mm [Huang et al., 1999].

Cropped grids yielded better results than the classical BEM for shallow sources when estimating the dipole orientation in EEG or the dipole position and orientation in MEG when only the field produced by secondary currents was interpolated. Hence, we suggest that cropped grids should be used in practice. Such a strategy would lead in all cases to small inverse errors even for shallow sources without the need of using locally refined meshes.

The proposed method lead to a gain in computation time on each forward calculation of 1,200 in EEG and 750 in MEG when using the second order Bézier interpolation. This gain was reduced by 20% in MEG when only the contribution of secondary currents was

Figure 7.

Inverse localization and orientation errors with the proposed method in the MEG case, where only the contribution of secondary sources is interpolated, and the field $\mathbf{B}_0$ produced by primary currents directly computed analytically. Full and cropped grids

with BEM-computed Lead-Fields are considered. Results using the original BEM approach are superimposed for comparison. Dipole positions and orientations are better retrieved using the proposed approach with cropped grids than with the original BEM.

interpolated and the field B_0 due to primary sources computed analytically. Much faster (gain of more than 10,000), the trilinear interpolation did not lead to results as accurate, although errors could generally be acceptable for practical use.

These improvements are of course only of interest when many forward calculations have to be performed (typically more than three times the number of grid points) to compensate for the additional preliminary computation time (about 3 hr for a 8-mm-step in realistic geometries on a Pentium II 333 MHz). This would certainly be the case when searching for sources from experimental data with several conditions and several latencies for a given subject. Furthermore, the proposed approach would also open to the practical use of realistic models combined with stochastic approaches to search for global minima when solving the inverse problem, such as simulated annealing [Khosla et al., 1997; Sekihara et al., 1992] or genetic algorithms [McNay et al., 1996; Uutela et al., 1998]. These methods indeed require a much greater amount of forward calculations than needed by conventional simplex or gradient-based minimization algorithms.

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