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Correcting image blur in spiral, retraced in/out (RIO) acquisitions using a maximized energy objective

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Abstract

Purpose: Images acquired with spiral k-space trajectories can suffer from off-resonance image blur. Previous work has shown that averaging 2 images acquired with a retraced, in/out (RIO) trajectory self-corrects image blur so long as off-resonant spins accrue less than 1 half-cycle of relative phase over the readout. Practical scenarios frequently exceed this threshold. Here, we derive and characterize a more-robust off-resonance image blur correction method for RIO acquisitions.

Methods: Phantom and human volunteer data were acquired using a RIO trajectory with readout durations ranging from 4 to 60 ms. The resulting images were deblurred using 3 candidate methods: conventional linear correction of the component images; semiautomatic deblurring of the component images using an established minimized phase objective function; and semiautomatic deblurring of the average of the component images using a maximized energy objective function, derived below. Deblurring errors were estimated relative to images acquired with 4 ms readouts.

Results: All 3 methods converged to similar solutions in cases where less than 2 and 4 cycles of phase accrued over the readout in in vivo and phantom images, respectively (<13 ms readout at 3T). Above this threshold, the linear and minimized phase methods introduced several errors. The maximized energy function provided accurate deblurring so long as less than 6 and 10 cycles of phase accrued over the readout in in vivo and phantom images, respectively (<34 ms readout at 3T).

Conclusion: The maximized energy objective function can accurately deblur RIO acquisitions over a wide spectrum of off resonance frequencies.

Keywords

deblurring; double encoding; MRI; off-resonance correction; retraced in/out spirals; spiral

1 | INTRODUCTION

Image blurring remains a substantial barrier to the clinical adoption of spiral k-space acquisition strategies.^{1,2} If the acquisition waveform is too long, phase accrual among off-resonant spins imparts phase modulation to the sampled k-space data. This phase modulation, in turn, causes spatially varying blurring in the image domain.³ When uncorrected, blurring limits the utility of a spiral acquisition strategy to applications where the magnetic field inhomogeneity is quite small. Accordingly, there has been significant interest and effort in deblurring spiral acquisition images, including simulated phase evolution rewinding,² linear correction,^{4,5} image deconvolution,⁶ B0 mapping using rewinding trajectories,⁷ field cameras,⁸ model-based approaches,⁹ and conjugate phase reconstruction.^{10,11}

Many deblurring methods require an accurate field map to correct for blurring, which can be derived either from 2 or more gradient recalled echo images acquired at separate echo times^{4,12–14} or from data acquired while rewinding the moments of the readout scan.⁷ However, field maps can be inaccurate and subject to image artifacts. For example, if acquired using a spiral strategy, the map will be most blurred, and therefore least accurate, in regions where the off-resonant frequencies are most extreme and most necessary to accurately measure. To partially correct these errors, the field map can be smoothed by applying a low-pass filter or performing a polynomial fit.^{4,15,16}

A field map can also be estimated directly from the image^{3,17,18} using a technique termed “automatic deblurring.” In this technique, a field map is estimated by minimizing some objective function over the original, blurred image data. Although computationally more expensive than field mapping, automatic deblurring can capture off-resonant variations in regions where acquired or smoothed field maps may be prone to errors. However, the accuracy of the objective function degrades as the readout length increases,^{19,20} limiting its practical utility to modest field inhomogeneities. In reality, acquired field maps and automatic deblurring strategies are complementary, as demonstrated in references,^{15,17} and combinations of the two are occasionally called “semiautomatic” techniques.

It was recently shown that averaging the 2 component images acquired using a retraced, symmetric spiral in/out (RIO) trajectory will correct modest blurring by transforming the time-dependent phase twist across the sampled k-space data into a more-benign amplitude modulation.²¹ The symmetric nature of this strategy is particularly useful for applications requiring longer echo times such as functional MRI, T2-weighted spin-echo imaging,^{21,22} and MR thermometry.²³ The time symmetry of the sampling pattern ensures that off-resonant phase accrual in 1 sampled data set can cancel out the corresponding accrual in the other data set. Although effective for short readouts, this deblurring property degrades when more than 1 half-cycle of off-resonance phase can accrue over the readout. In these cases, RIO strategies require additional deblurring using field-map based, semiautomatic, or automatic methods.

To our knowledge, a standard method for deblurring a RIO acquisition does not exist. Previous works^{21–23} report applying linear, semiautomatic, and model-based corrections to

the individual spiral-in and spiral-out component images before averaging. However, these methods do not take advantage of the strategy's inherent deblurring capabilities. In search of a more-optimized approach, we derive an objective function specific to semiautomatic deblurring of the average of the component images. We also present theory, phantom, and in vivo data comparing the performance of this technique to the application of linear and semiautomatic deblurring of the individual component images before averaging.

2 | THEORY

2.1 | Linear and automatic deblurring of non-Cartesian acquisitions

Neglecting relaxation effects, concomitant gradients, and signal noise, the received signal, $s(t)$, during a spiral acquisition can be modeled as (Equation 1):

$$s(t) = \int m(\mathbf{r}) e^{-i2\pi \mathbf{k}(t) \cdot \mathbf{r}} e^{-i\omega(\mathbf{r})t} d\mathbf{r} \quad (1)$$

where $m(\mathbf{r})$ is the magnitude and phase of the object's magnetization at the time of signal acquisition, $\mathbf{k}(t)$ is the time-dependent k-space sampling trajectory, and $\omega(\mathbf{r})$ describes the spatially varying resonant frequency of the object relative to the tuned frequency of the MR receiver. The corresponding image, $\hat{m}(\mathbf{r})$, can be found by calculating (Equation 2):

$$\hat{m}(\mathbf{r}) = \int s(t) W(t) e^{i2\pi \mathbf{k}(t) \cdot \mathbf{r}} dt \quad (2)$$

where $W(t)$ is the sampling density compensation factor appropriate to the k-space trajectory.^{10,11} If $m(\mathbf{r})$ is a point object, then (Equation 3):

$$\hat{m}(\mathbf{r}) = \int W(t) e^{i2\pi \mathbf{k}(t) \cdot (\mathbf{r} - \mathbf{r}_o)} e^{-i\omega(\mathbf{r}_o)t} dt \quad (3)$$

In this case, $\hat{m}(\mathbf{r})$ corresponds to the point spread function (PSF) of the acquisition strategy. For most non-Cartesian trajectories, the off-resonance term spreads the intensity of the image at location $\mathbf{r}_o$ into the surrounding pixels, blurring $\hat{m}(\mathbf{r})$. This is the source of image blurring inherent to a spiral trajectory. Blurring increases with the product $\omega(\mathbf{r}_o)T$, where T refers to the duration of the readout gradient.

If $\omega(\mathbf{r})$ varies relatively slowly across the object, blurring can be corrected using conjugate phase reconstruction^{10,11} (Equation 4):

$$\hat{m}[\mathbf{r}, \hat{\omega}(\mathbf{r})] = \int W(t) e^{i2\pi \mathbf{k}(t) \cdot (\mathbf{r} - \mathbf{r}_o)} e^{-i[\omega(\mathbf{r}_o) - \hat{\omega}(\mathbf{r})]t} dt \quad (4)$$

where $\hat{m}(\mathbf{r})$ is an estimate of $\omega(\mathbf{r})$. When $\hat{\omega}(\mathbf{r}) = \omega(\mathbf{r})$, the PSF is deblurred.

A linear correction assumes that $\omega(\mathbf{r})$ is well approximated by a first-order polynomial. Under this assumption, the conjugate phase reconstruction reduces to demodulating $s(t)$ with a constant offset frequency and correcting both the k-space trajectory and the sampling

density compensation function. Linear corrections can manifest in both field-map-based⁴ and automatic flavors⁵ and remain effective deblurring techniques so long as the higher-order polynomial terms of $\omega(\mathbf{r})$ do not contribute significantly to phase accrual across the readout.

If the linear approximation is insufficient, additional methods are needed to estimate the higher-order terms of the field map. Automatic deblurring is a good candidate for these situations because it can handle a wide variety of nonlinear off resonance fields and is complimentary to linear deblurring.

The automatic deblurring technique introduced in reference³ consists of using Equation 4 to repeatedly reconstruct $\hat{m}(\mathbf{r}, \omega_i)$, where ω_i is a discrete set of spatially constant field maps. The local $\hat{m}(\mathbf{r})$ is chosen to be the particular ω_i that satisfies some objective function evaluated over a finite section of $\hat{m}(\mathbf{r}, \omega_i)$. Assuming that $m(\mathbf{r})$ is real-valued, this function is most frequently taken to be the integral of the absolute value of the image magnitude at $\mathbf{r}$ multiplied by the phase angle at $\mathbf{r}$, all raised to some power between 0 and 1.²⁰ Essentially, a correct ω_i locally minimizes the imaginary component of $\hat{m}(\mathbf{r}, \omega_i)$. For simplicity, we refer to automatic deblurring methods that use this objective function as “minimum phase” methods.

Automatic deblurring fundamentally relies on the assumptions that $\omega(\mathbf{r})$ smoothly varies across the image and that fat is effectively suppressed. Although violations of these assumptions lead to image artifacts, in practice, most deblurring errors arise from violations of the objective-function-specific assumptions that the deblurred PSF is real-valued and circumscribed by the region over which the objective function is evaluated. These 2 violations occur when (1) the readout length exceeds the condition $|\omega_i - \omega(\mathbf{r})| T \gg 8\pi^{20}$ or (2) incidental phase cannot be removed from $\hat{m}(\mathbf{r}, \omega_i)$ before evaluating the objective function. When condition 1 occurs, the energy of the imaginary part of the PSF can spread outside the region over which the objective function is evaluated, creating a spurious minimum in the cost-function surface and an erroneous estimate of $\hat{m}(\mathbf{r})$.^{3,19} Previous works^{15,19} discuss methods to formulate a set ω_i that reduces, but not eliminates, the likelihood of encountering spurious minima. When condition 2 occurs, residual incidental phase translates the objective-function surface, producing an erroneous estimate.

The above 2 objective-function-specific conditions limit the utility of minimum phase automatic deblurring to a subset of cases where a RIO strategy may be advantageous. Deblurring would be improved if a more-robust objective function could be found. In the next section, we generalize the RIO trajectory²¹ and show that this acquisition scheme is suitable for a new automatic deblurring objective function that is both well-conditioned against spurious minima and invariant with incidental phase.

2.2 | Deblurring RIO acquisitions

We define a retraced trajectory to be an acquisition strategy where each k-space point is sampled at 2 time points symmetric about some other anchor time point. That is, k-space is sampled by 2 trajectories, $k_1(t)$ and $k_2(t)$, that are constrained to be both antisymmetric and mirror images such that, for $t \in \left[-\frac{T}{2}, \frac{T}{2}\right]$ (Equation 5):

$$k_2(t) = k_1(-t) = -k_1(t) \quad (5)$$

Here, we choose the anchor time point to be the echo time. Example retraced spiral trajectories are shown in Figure 1A and 1B.

It was shown by Fielden and Meyer²¹ that, for the retraced trajectory shown in Figure 1A, the average of the 2 received signals, $s_1(t)$ and $s_2(t)$, can be written as (Equation 6):

$$s_c(t) = \frac{s_1(t) + s_2(-t)}{2} = \int_{-T/2}^{T/2} m(\mathbf{r}) \cos[\omega(\mathbf{r})t] e^{-i2\pi k_1(t)r} d\mathbf{r} \quad (6)$$

For modest off-resonance frequencies, the $\cos[\omega(\mathbf{r})t]$ term symmetrically attenuates the highest frequency components of k-space, resulting in a substantially sharper PSF compared to that returned by the phase modulation of a nonretraced acquisition. However, as noted by Fielden and Meyer,²¹ at large off-resonance values, this factor modulates the lower frequency components of k-space and causes substantial blurring.

We extend this theory to conjugate phase reconstruction. Let (Equation 7):

$$s_m(t, \omega_i) = \frac{s_1(t)e^{i\omega_i t} + s_2(-t)e^{-i\omega_i t}}{2} = \int_{-T/2}^{T/2} m(\mathbf{r}) \cos\{[\omega(\mathbf{r}) - \omega_i]t\} e^{-i2\pi k_1(t)r} d\mathbf{r} \quad (7)$$

In this case, each trajectory is demodulated at frequency ω_i and the time reversal of $s_2(t)$ produces a demodulation term that is the complex conjugate of that applied to $s_1(t)$. The effects of off-resonant phase twist are transformed into a cosine term with frequency $[\omega(\mathbf{r}) - \omega_i]$. The corresponding conjugate phase reconstruction is then represented by (Equation 8):

$$\hat{m}(\mathbf{r}, \omega_i) = \int s_m(t, \omega_i) W(t) e^{i2\pi k(t)r} dt \quad (8)$$

The deblurring capability of any given ω_i can be estimated by noting that, as approaches $\omega(\mathbf{r})$, Equation 7 approaches the signal equation for when $\omega(\mathbf{r}) = 0$.

The retraced trajectory poses a very different automatic deblurring problem than that used for traditional spiral readouts. Substituting a point object into Equation 7 and combining with Equation 8, the reconstructed PSF becomes (Equation 9):

$$\hat{m}(\mathbf{r}, \omega_i) = \int W(t) \cos\{[\omega(\mathbf{r}_o) - \omega_i]t\} e^{i2\pi k(t)(\mathbf{r} - \mathbf{r}_o)} dt \quad (9)$$

Unlike Equation 4, the PSF for a retraced readout is real for all ω_i . This nullifies the assumption that only a correctly estimated ω_i produces a real-valued PSF. Meanwhile, Parseval's theorem indicates that the spatial integral of the squared magnitude of the PSF is

maximized when $\omega_j = \omega(\mathbf{r} - \mathbf{r}_o)$. These observations lead us to pose a new deblurring objective function (Equation 10):

$$\xi(\mathbf{r}, \omega_i) = \int h(\mathbf{r} - \mathbf{r}') \hat{m}(\mathbf{r}', \omega_i) \hat{m}(\mathbf{r}', \omega_i)^* d\mathbf{r}' \quad (10)$$

where $\hat{m}(\mathbf{r}, \omega_i)^*$ is the complex conjugate of $\hat{m}(\mathbf{r}, \omega_i)$, and $h(\mathbf{r})$ is a finite convolution kernel. That is, the demodulation frequency, ω_i , that best deblurs some local region of $\hat{m}(\mathbf{r}, \omega_i)$ also maximizes the local integral of the signal energy. For simplicity, we refer to automatic deblurring methods that use this objective function as “maximum energy” methods.

To satisfy the assumptions underlying automatic deblurring, the convolution kernel must be narrow compared to the spatial fluctuations of the field map and yet large enough to encompass the side lobes of the blurred PSF. Previous reports on automatic deblurring utilized a boxcar function as the convolution kernel—mainly because the boxcar function is both conceptually simple and computationally efficient to implement.^{5,20} However, in our experience, we achieve empirically superior results utilizing a Gaussian window function. This is likely because the peak of the Gaussian function can be made narrow compared to the spatial fluctuations of the field whereas the tails can be used to encompass the PSF’s side lobes.

Equation 10 presents a better-conditioned automatic deblurring problem compared to the minimum phase problem. This improvement arises from 2 features. First, unlike the minimum phase objective function, $\xi(\mathbf{r}, \omega_i)$ evaluated over a point object presents an obvious global extremum. This effect can be visualized in Figure 1C, which plots Equation 10 and its minimum phase counterpart, described by Lee et al.,²⁰ over a broad range of ω_i . Both objective functions trend toward a minimum as the PSF extends outside the region of evaluation. For the minimum phase function, it is difficult to distinguish these spurious minima from that produced at the correct demodulation frequency. The maximum energy function, however, contains an obvious global maximum. So long as the search space samples this main lobe, the correct demodulation frequency can be differentiated from the surrounding spurious local maxima. Second, Equation 10 is invariant with image phase, eliminating the need to accurately remove incidental phase. An added benefit is that the global maximum of $\xi(\mathbf{r}, \omega_i)$ is invariant with T_2^* decay, as shown in Appendix A.

In the following sections, we quantitatively and qualitatively assess the deblurring capabilities of the maximum energy objective function when applied to simulated, phantom, and in vivo images acquired with a RIO acquisition strategy. We compare this performance to that obtained using the simple linear trajectory correction described by Irarrazabal et al.⁴ and the minimum phase objective function described elsewhere^{15,20} to deblur the corresponding component images before averaging.

3 | METHODS

3.1 | Bounds and sampling interval of ω_i

An appropriate spacing interval for ω_i can be determined by demodulating the k-space representation of the on-resonance PSF at several candidate intervals and measuring the normalized root mean square error (NRMSE) of the resulting, blurred PSF. We estimated the PSFs for the spiral trajectories used in this paper and computed the NRMSE after blurring by candidate intervals of $1, \frac{1}{16T}, \frac{1}{8T}, \frac{1}{4T}, \frac{1}{2T}, \frac{3}{2T}$ and $\frac{1}{T}$ Hz. In the experiments described below, we used the largest search spacing that produced an average NRMSE of less than 0.01 for all spiral trajectories tested here: $\frac{1}{4T}$ Hz. Additional justification for this spacing can be found by observing that the maxima in the objective function surface plotted in Figure 1C occur at every $\frac{1}{2T}$ Hz.

To interrogate the sensitivity of the objective functions to spurious extrema, a digital phantom (available at <https://big-www.epfl.ch/algorithms/mriphantom/>, downloaded February 17, 2018; described by Guerquin-Kern et al.²⁴) was defined in MATLAB (MathWorks, Natick, MA). The field map for this phantom was uniformly set to 0 Hz. Spiral k-space data were then estimated using a nonuniform Fourier transform approximation²⁵ implemented in the Michigan Image Reconstruction Toolbox (available at <https://web.eecs.umich.edu/~fessler/code/index.html>, downloaded September 18, 2017). The resulting object was then reconstructed using a standard gridding algorithm.²⁶ The field map for this digital phantom was then estimated using the minimum phase and maximum energy objective functions over spans of ω_i ranging from $\frac{1}{2T}$ to $\frac{40}{T}$, sampled at a $\frac{1}{4T}$ Hz interval. The NRMSE between the resulting “deblurred” images and the original image were then computed.

3.2 | Deblurring algorithm

In the following phantom and in vivo experiments, we applied corrections using the linear method and the minimum phase objective function separately to the component images corresponding to $s_1(t)$ and $s_2(t)$ and then averaged the results together. The terms for the linear correction were derived from high-resolution, high-quality field maps acquired before acquisition using the methods described by Irarrazabal et al.⁴ The minimum phase objective function was applied as part of the semiautomatic deblurring method described by Chen et al.¹⁵ with the linear correction constituting the first-stage field map. This included interchanging the order between the demodulation and gridding steps of the component images such that the gridded k-space data were demodulated by a timing mask. The search set, ω_i , then consisted of spatially constant frequency deviations from the first-stage field map. After correction, the 2 component images were then averaged together. The maximum energy objective function was implemented in an almost identical manner to the minimum phase function except that the linearly corrected component images were averaged together before evaluating the maximum energy objective function over ω_i . The span of ω_i was determined to encompass the maximum and minimum differences between the acquired and

estimated linear field maps. Table 1 displays the span of the search space, spacing, and number of demodulation frequencies evaluated for all experiments described here.

For both the minimum phase and maximum energy objective functions, the convolution kernel, $h(\mathbf{r})$, was chosen to be a circularly symmetric Gaussian window with a standard deviation of 9 pixels, computed over a support space of 90 to 100 pixels. For the high-resolution coronal data set, the standard deviation was increased to 27 pixels. This parameter was determined by empirically observing the standard deviation that jointly produced the best results for both the minimum phase and maximum energy methods. In our implementation, a single-coil image could be gridded, subjected to linear correction, and deblurred using the maximum energy function within approximately 1.5 seconds using a personal computer with a 2.5-GHz processor and 16 GB of memory. Finally, as a safeguard against spurious errors in the image background, the background of the final images was suppressed using a mask derived from the original, blurred image. The source of these errors will be discussed later.

3.3 | Image acquisition

All images were acquired using a 3T MR scanner (MR750; General Electric, Waukesha, WI). Pulse programming and control were performed using the RTHawk framework (HeartVista, Los Altos, CA). A cylindrical phantom was imaged using a 2D, spin-echo sequence with RIO trajectories matching the general form in Figure 1A and designed with uniform sampling density. A spin-echo sequence was chosen to avoid severe T_2^* losses while scanning with very long readouts. The body coil was used for both radiofrequency (RF) transmission and signal reception. Excitation was performed using a 90° sinc RF pulse. The refocusing pulse was a non-selective 180° hard pulse bracketed by crusher gradients. For all experiments, the sequence was held to a TE of 90 ms and a TR of 4.1 seconds. The spiral readout gradients had durations of either 4, 7, 14, 28, 38, or 60 ms, and an image was acquired with either 64, 32, 16, 8, 6, or 4 interleaves, respectively. The theoretical resolution, computed from the k-space trajectory was 1.1 mm, and the field of view was 21 cm. Scanning was preceded by automatic frequency and RF calibration. The long axis of the phantom was purposefully placed oblique to the direction of the magnetic field, producing a nonuniform field map. Imaging with the phantom was repeated 3 times with 3 different linear gradients applied across the object. The combined effects of the phantom's orientation and the linear gradients produced resonance offsets spanning approximately 250 to 450 Hz. During the deblurring process, the linear correction produced an actual span of ω_i of 255 to 275 Hz.

After passing a safety screening process as well as providing informed consent according to our institutional policy, axial and sagittal slices of the head of a volunteer were imaged using the same 2D, spin-echo sequence as in the phantom experiments, but with a spectral-spatial pulse replacing the sinc excitation pulse for fat suppression. Data were received by an 8-channel receive-only head coil. The spiral readouts had durations of 4, 8, 13, 26, 34, or 60 ms, and each image was acquired with 64, 32, 16, 8, 6, or 4 interleaves, respectively. The field of view was 20 cm, and the theoretical resolution was 1.2 mm. To demonstrate the utility of the maximum energy objective function at higher resolutions, a coronal orientation

scan with 0.5-mm theoretical resolution was also acquired using a spiral readout with 30 ms duration readouts and 32 interleaves. Scanning was preceded by automatic calibration and gradient shimming as well as a field mapping sequence. Individual coil images were combined using the sum of squares method.

4 | RESULTS

4.1 | Simulations

Our simulations demonstrate that, for a given sampling interval of ω_p , the different spiral trajectories used in this paper produced less than 2% variation in NRMSE. Furthermore, sampling intervals of $\frac{1}{4T}$ Hz or less produced an NRMSE of less than 0.01. These results lead us to conclude that an interval of $\frac{1}{4T}$ is sufficient for all the experiments reported in this paper. In Figure 2A, we plot the average NRMSE for each candidate spacing interval.

Reconstructing the digital phantom over various spans of ω_i indicated that the maximum energy objective function is less susceptible to spurious extrema than the minimum phase objective function so long as ω_i spans less than $\frac{30}{T}$ Hz. Additionally, the majority of errors produced by the maximum energy function occurred in regions of low signal and low contrast such as the background surrounding the object and the simulated white matter tracts. Figure 2B plots the computed NRMSE of the phantom image as a function of the span of ω_i . The object is shown in Figure 2C, and “deblurred images” reconstructed using the minimum phase and maximum energy objective functions over a span of $\frac{40}{T}$ Hz are shown in Figure 2C and E, respectively, and are examples of the potential failure modes of this method. We also found that the errors in the simulated white matter tracts of Figure 2E could be mitigated by using a larger convolution kernel. The practical trade-offs of using a larger kernel are addressed in the Discussion section.

4.2 | Phantom images

Example reconstructions of the phantom using various spiral readout lengths and deblurring techniques are shown in Figure 3. For convenience, the number of interleaves used for each readout and the span of ω_i —divided by the readout length and presented in units of $\frac{Hz}{T}$ —are also displayed. Although the linear and minimum phase deblurring methods could deblur readouts up to 14 ms, the maximum energy method was able to provide excellent deblurring for readouts up to 38 ms.

The structural similarity indices (SSIM)²⁷ of the reconstructed phantom images—referenced to the uncorrected image acquired with a 4 ms readout—are plotted in Figure 4A as a function of the span of ω_i divided by the readout lengths used to acquire each image. These were computed on only the magnitude of the reconstructed images.

For acquisitions where less than 2 cycles of phase accrued across the readout gradient, all 3 deblurring methods converged to similar results. However, the linear correction could not as accurately deblur images acquired with longer readouts and suffered decreases in SSIM.

Methods using the minimum phase and maximum energy functions still converged to similar SSIM values for readouts that could accrue up to 4 cycles of phase. For longer readouts, the maximum energy function produced the largest SSIM. However, the maximum energy function produced severe decreases in SSIM when the readouts were long enough to accrue more than 10 cycles of phase. All acquisitions produce some loss in SSIM relative to the control image, which is attributed to thermal noise and residual delays at high spatial frequencies. The increase in threshold from 4 to 10 cycles of phase accrual represents a substantial improvement in deblurring capability.

Example residual field maps after linear correction as well as after deblurring using both objective functions are also displayed in Figure 4B–4E. In this figure, data were acquired using a 28 ms readout. A 0-Hz residual field indicates accurate deblurring. The linear correction was able to remove the linear terms of the map, but the higher-order components remained. The minimum phase and maximum energy functions were able to correctly estimate the field map in the center of the phantom. However, they both produced errors in regions with little high-frequency information. Further, the area over which the maximum energy method produced errors is smaller than that produced by the minimum phase method and does not encroach into the hypointense resolution elements.

4.3 | In vivo images

Deblurring the axial images required ω_i to span approximately 80 Hz. This frequency span could only generate 4.8 cycles of phase during the longest readout tested here. As a result, the maximum energy function was able to provide excellent deblurring for all readout lengths. However, images acquired with readouts longer than 13 ms retained blurring artifacts when deblurred with the linear and minimum phase methods. Figure 5 displays axial images acquired with a 60 ms readout produced in the volunteer using all 3 deblurring methods as well as the original, blurred image.

The field map was more heterogeneous for the sagittal in vivo data sets, spanning approximately 150 Hz. Deblurring these images using the linear correction and both objective functions produced almost identical deblurred images for readouts up to 13 ms in duration. For longer readouts, both the linear correction and the minimum phase function failed to accurately deblur the data. However, the maximum energy function performed very well for readouts up to 34 ms. These readouts were long enough to accrue up to 5.7 cycles of phase. Sagittal reconstructions acquired with readouts of 34 ms and deblurred with the linear, minimum phase, and maximum energy methods are displayed in Figure 6. For these scans, concomitant fields translated the surfaces of both objective functions in the cortex of the frontal lobe. An arrow in the figure indicates the resulting defect.

For the coronal orientation data set, the search space spanned approximately 270 Hz. The 30 ms readouts were long enough to accrue 8.1 cycles of phase. The maximum energy objective function produced excellent deblurring at the base of the skull, where the field map deviated the most from center frequency. This reconstruction is displayed in Figure 7.

5 | DISCUSSION

We observed that a semiautomatic off-resonance correction method utilizing the maximum energy objective function, when coupled with an RIO trajectory, provided accurate deblurring over a wider range of readout durations, off-resonance values, and image features than either the conventional linear correction or a semiautomatic method utilizing the minimum phase objective function. Because of sensitivity to spurious extrema, the minimum phase objective function for a PSF is usually limited to readout durations less than 4 times the period of the largest off-resonance frequency.²⁰ For in vivo cases, residual incidental phase can considerably further constrain this limitation. In comparison, the maximum energy objective function, paired with a retraced trajectory, can deblur in vivo and phantom images with readout durations up to 6 or 10 times the period of the largest off-resonance frequency, respectively.

Although the retraced trajectory requires one to double the number of sampled data points, longer readout durations afforded by effective deblurring can potentially offset or even negate this increase in scan time while simultaneously providing an increase in signal-to-noise ratio. For example, the time required to sample the image in Figure 7 using a nonretraced, 48-interleave, spiral in/out trajectory with 10 ms readout gradients can also be acquired by a *retraced*, 48-interleave, spiral in/out trajectory using 20 ms readout gradients, producing a $\sqrt{2}$ increase in SNR without an increase in total acquisition time, assuming the same TR. Using a 32-interleave retraced readout with 30 ms readouts further reduces the total scan time by 33% while simultaneously preserving the $\sqrt{2}$ SNR benefit.

When the maximum energy function did fail to deblur an image, it tended to fail in regions demonstrating low amplitude or little contrast such that the evaluation window was composed of almost uniform signal. When reconstructed at the correct offset frequency, these regions contained very little high-frequency content, desensitizing the objective function to an accurate result. For modest readout durations, this condition alone was usually insufficient to generate a spurious extremum on the cost-function surface. However, for long readouts, and at the edges of the search set ω_r , distant, high-amplitude signal sources could be blurred into the evaluation window, producing a spurious maximum. Figures 2E and 4D are excellent examples of this phenomenon: The majority of deblurring errors reside in the nearly uniform simulated white matter tracts, blank background surrounding the object, and uniform regions within the phantom.

A secondary failure mode occurred when the spatial variations of the magnetic field existed on a scale smaller than the convolution kernel—a potential error first described by Noll et al.³ Conjugate phase reconstruction assumes a spatially smooth and temporally constant field map. If the field fluctuates too rapidly in space or time, the objective function surface will produce erroneous extrema. This failure mode produced errors in the mouth regions shown in Figure 6. To partially mitigate these artifacts, the ideal kernel should be large enough to always include some high-frequency content, but remain small compared to the spatial fluctuations of the field map. We found a Gaussian window to maintain a reasonable compromise between these 2 constraints.

Finally, this paper does not address the effects of signal noise or concomitant fields. For example, the described objective function is not rigorously optimized against noise contamination. A possible approach could be a convolution kernel that adapts to the noise characteristics of the image. However, the averaging effect of even a simple convolution kernel provides further protection against noise contamination. And, as mentioned above, a retraced spiral acquisition inherently provides a $\sqrt{2}$ increase in SNR. We anticipate the maximum energy objective function to provide effective deblurring over a broad range of SNRs.

Fielden et al.²¹ describes how a retraced spiral acquisition can provide some inherent protection against concomitant field-induced phase accruals of less than 1 cycle. Additionally, semiautomatic deblurring can provide partial correction for concomitant phase.¹⁶ However, for long, slew-rate limited readouts, concomitant fields will still alter the surface of the objective function. For example, concomitant fields produce the blurring artifact highlighted by the arrows in Figure 6. Additional correction methods will be necessary to guard against these effects. For example, the proposed objective function could easily be incorporated into the joint inhomogeneity and concomitant field correction algorithm presented by Chen et al.¹⁶

6 | CONCLUSION

We have presented theoretical and experimental evidence that the proposed maximum energy method is appropriate and effective for deblurring RIO acquisitions. The surface of the proposed objective function is resistant against spurious extrema and independent of incidental image phase. Additionally, this method produces superior deblurring results when compared to using a linear correction or semiautomatic deblurring method on the acquisition's individual component images. In extreme cases, a failure mode can occur in regions of low signal levels and low signal contrast. A careful choice of objective function parameters, including search width, search spacing, and convolution kernel can safeguard against these errors.

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APPENDIX A

Fielden and Meyer²¹ addressed the effects of T_2^* decay on an RIO acquisition using a first-order approximation. Here, we perform a general derivation. Including decay, the signal equation can be expressed as (Equation A1):

$$\begin{aligned}
s_m(t) &= \frac{1}{2} \int_{T/2}^{-T/2} m(\mathbf{r}) \left[e^{-i\{2\pi k_1(t)\mathbf{r} + [\omega(\mathbf{r}) - \omega_i]t\}} e^{-t/T_2^*} \right. \\
&\quad \left. + e^{-i\{2\pi k_1(t)\mathbf{r} - [\omega(\mathbf{r}) - \omega_i]t\}} e^{t/T_2^*} \right] d\mathbf{r} \\
&= \int_{T/2}^{-T/2} m(\mathbf{r}) e^{-i2\pi k_1(t)\mathbf{r}} \cos\{[\omega(\mathbf{r}) - \omega_i]t\} \cosh(t/T_2^*) d\mathbf{r} - i \int_{T/2}^{-T/2} m(\mathbf{r}) \\
&\quad e^{-i2\pi k_1(t)\mathbf{r}} \sin\{[\omega(\mathbf{r}) - \omega_i]t\} \sinh(t/T_2^*) d\mathbf{r}
\end{aligned} \tag{A1}$$

The spatial integral of the squared magnitude of the resulting PSF is still maximized when $\omega_i = \omega(\mathbf{r} - \mathbf{r}_o)$. This can be shown by evaluating Equation A1 for a point object at location $\mathbf{r}_o$ (Equation A2):

$$s_m(t) = e^{-i2\pi k_1(t)\mathbf{r}_o} \left[\cos\{[\omega(\mathbf{r}_o) - \omega_i]t\} \cosh(t/T_2^*) - i \sin\{[\omega(\mathbf{r}_o) - \omega_i]t\} \sinh(t/T_2^*) \right] \tag{A2}$$

We then multiply $s_m(t)$ by its complex conjugate and simplify to yield (Equation A3):

$$s_m(t)s_m(t)^* = \cos^2\{[\omega(\mathbf{r}_o) - \omega_i]t\} + \sinh^2(t/T_2^*) \tag{A3}$$

The $\cos^2\{[\omega(\mathbf{r}) - \omega_i]t\}$ term attenuates the distribution of energy across the k-space trajectory for any $\omega_i \neq \omega(\mathbf{r}_o)$. Thus, by invoking Parseval's theorem, the integrated squared magnitude of the PSF is maximized when $\omega_i = \omega(\mathbf{r}_o)$ for any given T_2^* .

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FIGURE 1.

Two example RIO trajectories. (A), k-space is sampled along the path X-O-Y. On the next TR, k-space is sampled along the path Y-O-X. (B), k-space is sampled along the trajectory X-O-X in a single TR. In both cases, k-space is sampled twice, at time points symmetric about the gradient recalled echo. (C), A plot of the maximum energy and minimum phase objective functions evaluated for a point object

FIGURE 2.

(A) Average NRMSE of the PSFs of the spiral trajectories used in this paper when reconstructed at various candidate sampling intervals of ω_i . (b) The NRMSE of a digital phantom deblurred using a ω_i with various spans. The x -axis reports the span as the number of cycles that could possibly accrue during the readout time (cycles/T). (C) displays the digital phantom, and (D) and (E) show the “deblurred” versions using the minimum phase and maximum energy objective functions, respectively, with a span of $\frac{40}{T}$

FIGURE 3.

Reconstructions of a phantom acquired with a RIO acquisition using increasingly longer readouts. For convenience, the figure displays the readout duration, the span of ω_i used for semiautomatic deblurring, and number of interleaves used for the acquisition. The readout duration increases from left to right. Each image was reconstructed without deblurring (None), deblurred using only a linear correction on the component images (Linear), deblurred by applying the minimum phase objective function to the component images after linear correction (Min Phase), or deblurred by applying the maximum energy function on the average of the linearly corrected component images (Max Energy)

FIGURE 4.

(A) The SSIM of the reconstructed phantom images—referenced to the uncorrected image acquired with a 4 ms readout—are plotted as a function of the span of ω_j divided by the readout lengths used to acquire each image. The maximum energy method produces a consistently larger SSIM compared to the other methods until the readouts acquired more than 10 cycles of phase throughout the readout. This is a significant enhancement in deblurring capability. (B) Field map of the phantom spanning ~ 350 Hz. (C) The residual field map after linear correction. (D) Error in the field map produced after using the minimum phase objective function. (E) Error in the field map produced after using the maximum energy objective function. (C–E) are derived from a 28 ms readout acquisition and an ω_j with a span of 10 cycles of phase accrual. The maximum energy method produced a field map with the fewest errors

FIGURE 5.

Reconstructions of the axial data set acquired with a 60 ms readout. (A) Uncorrected image. (B) Linear correction. (C) Minimum phase objective function. (D) Maximum energy objective function. The maximum energy objective function provides much better deblurring performance across the entire slice

FIGURE 6.

Reconstructions of the sagittal data set acquired with a 34 ms readout. (A) Uncorrected image. (B) Linear correction. (C) Minimum phase objective function. (D) Maximum energy objective function. The maximum energy objective function provides much better deblurring performance across the entire slice. The cerebellum is highlighted by an open arrow. Blurring artifacts caused by concomitant gradients are marked by the solid yellow arrows. Rapid temporal and spatial variations in the field map in the mouth introduce additional aliasing and blurring

FIGURE 7.

Reconstructions of the coronal data set acquired with a 30 ms readout. (A) Uncorrected image. (B) Correction using the maximum energy objective function. The maximum energy function resolves structures in the brainstem where the field map deviates most strongly from the center frequency as indicated by the yellow arrow

TABLE 1

Parameters used for semiautomatic deblurring

Object	Readout length (ms)	Interleaves	Search range (Hz)	Spacing (Hz)	Levels
P	4	64	255 to 275	62.5	4 to 5
	7	32	255 to 275	35.7	8
	14	16	255 to 275	17.9	15 to 16
	28	8	255 to 275	8.9	29 to 31
	38	6	255 to 275	6.6	39 to 42
V	60	4	255 to 275	4.2	62 to 66
	30	32	270 (c)	8 (c)	42 (c)
	4	64	150 (s) 90 (a)	62.5	3 (s) 2 (a)
	8	32	150 (s) 90 (a)	31.3	4 (s) 3 (a)
	13	16	150 (s) 90 (a)	19.2	8 (s) 6 (a)
	26	8	150 (s) 90 (a)	9.6	15 (s) 10 (a)
	34	6	150 (s) 90 (a)	7.4	20 (s) 14 (a)
	60	4	150 (s) 90 (a)	4.2	35 (s) 24 (a)

A description of the readout lengths, interleaves, span of ω /search spacing, and the number of levels used for semiautomatic deblurring for all experiments reported here, including phantom (P) and volunteer (V) images in sagittal (s), coronal (c), and axial (a) orientations.