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Application of Peukert's Law in Supercapacitor Discharge Time Prediction

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Abstract

This paper studies the application of Peukert's law in predicting the supercapacitor discharge time during a constant current discharge process. Originally developed for lead-acid batteries, Peukert's law states that the delivered charge increases when the discharge current decreases. Recently, it is revealed that this law also applies to supercapacitors when the discharge current is above a certain threshold. This paper examines two application scenarios in which Peukert's law is utilized to predict the supercapacitor discharge time. Extensive experiments are performed using three supercapacitor samples with different rated capacitances from different manufacturers at various voltages. Results show that the prediction error is significantly reduced when the supercapacitor nominal charge and Peukert constant are properly determined and therefore demonstrate the effectiveness of Peukert's law in improving the prediction accuracy.

Keywords

Constant current discharge; discharge time; Peukert constant; Peukert's law; supercapacitor

1. Introduction

Energy storage is becoming an increasingly critical asset in many systems. Among various energy storage technologies, supercapacitors are advantageous in several aspects such as high power density and long cycle life. In fact, supercapacitor-based energy storage systems have been employed by a variety of applications including smart grid [1, 2, 3], electric vehicles [4, 5, 6], and wireless sensor networks [7, 8, 9].

To exploit the supercapacitor technology, a comprehensive and in-depth understanding of its characteristics at the device level is crucial. Therefore, modeling and characterization of supercapacitors have been of great interest. Supercapacitors are usually constructed using porous carbon electrodes [10]. The porous electrode theory suggests that a ladder circuit composed of multiple RC branches with different time constants can be used to capture the

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distributed nature of the supercapacitor capacitance and resistance [11]. Various equivalent circuit models [12, 13, 14, 15, 16, 17, 18, 19] have been proposed to reduce the complexity of the generic ladder circuit model.

Among various aspects of supercapacitor physics, charge redistribution is a process of particular interest. Charge redistribution is a relaxation process originated from the porous structure of the electrodes. Because of this process, the supercapacitor terminal voltage may decay or recover after a charge or discharge process [20]. The physical mechanisms leading to this process have been revealed: the electrode pore sizes are nonuniform. Therefore, the ions in the electrolyte need extra time to penetrate the middle-size mesopores and small-size micropores compared to the large-size macropores [21, 22, 23, 24, 25]. The effects of charge redistribution on supercapacitor terminal voltage behavior [26, 27], supercapacitor energy delivery capability [28, 29], and supercapacitor capacitance characterization methods [30] have been extensively studied.

In particular, the impact of charge redistribution on the amount of charge delivered by a supercapacitor during one or multiple constant current discharge processes has been examined [31, 32]. One of the key findings is that the delivered charge increases when the discharge current decreases if the discharge current is above a certain threshold and the energy loss due to self-discharge is negligible, which is consistent with Peukert's law originally developed for lead-acid batteries [33]. For supercapacitors, Peukert's law applies because of the porous electrode structure and the charge redistribution process [31, 32]. Furthermore, a preliminary study [34] explores the feasibility of predicting the supercapacitor discharge time during a constant current discharge process using Peukert's law and shows that the prediction error can be significantly reduced if the Peukert constant is properly estimated.

This paper studies the application of Peukert's law in predicting the supercapacitor discharge time, which is related to the supercapacitor state estimation problem. Accurate estimation of the supercapacitor state in terms of state of charge (SOC), state of energy (SOE), or state of health (SOH) is important at both the device and system levels. For example, wireless sensor networks need the information of the available charge or energy to schedule tasks such as reading the sensor measurements, processing the collected data, and transmitting the data packets. In fact, energy efficiency is a major concern in wireless sensor networks and a variety of power management strategies [7, 8, 9] have been developed based on the state of the energy storage system. Although the supercapacitor terminal voltage is a natural indicator of its state, accurate state estimation is still challenging because the supercapacitor capacitance and equivalent series resistance (ESR) are affected by multiple factors [35] such as the terminal voltage, operating temperature, and aging condition in a complex manner. Numerous frameworks [36, 37, 38, 39] have been developed to identify supercapacitor parameters and estimate supercapacitor states for various applications. In particular, this paper considers the supercapacitor discharge time prediction problem based on the fact that for the same SOC, the charge that can be released during a constant current discharge process varies depending on the discharge current [31, 32, 34]. The implications of this observation are crucial. For example, a wireless sensor node is usually equipped with three types of energy consumers: wireless transceivers, microcontroller units (MCUs), and

sensors. Their power levels vary significantly depending on the application scenario. Therefore, an accurate prediction of the supercapacitor discharge time can facilitate developing effective power management policies to improve the system energy efficiency.

Specifically, this paper extends [34] in three aspects. First, a new application scenario is considered in addition to the one examined in [34]. In the new scenario, the initial voltages of the constant current discharge experiments in the testing set are different from those in the training set. On the other hand, the testing and training sets use the same initial voltages in the scenario examined in [34]. Second, the supercapacitor rated capacitances are extended from 100 F only to three values: 10, 100, and 350 F. Third, three more initial voltages (2, 1.35, and 0.7 V) from which the supercapacitor is discharged are swept while [34] only examines the rated voltage of 2.7 V. This extension is based on the fact that the supercapacitor usually operates at various voltages. These extensions significantly enhance the preliminary results presented in [34] and demonstrate the usefulness of Peukert's law in supercapacitor discharge time prediction for supercapacitor samples with different ratings in different application scenarios.

The remainder of this paper is organized as follows. Section 2 demonstrates the applicability of Peukert's law to supercapacitors. Sections 3 and 4 examine two application scenarios in which Peukert's law is used to predict the supercapacitor discharge time. Section 5 concludes this paper.

2. Applicability of Peukert's Law to Supercapacitors

Originally developed for lead-acid batteries, Peukert's law [33] relates the delivered charge to the discharge current as follows:

$$I^k t = Q_0, \quad (1)$$

where Q_0 is the nominal charge capacity rated at a particular discharge current, I is the actual discharge current, t is the actual discharge time, and k is the Peukert constant. This empirical law states that the delivered charge of a battery depends on the discharge current: the larger the discharge current, the less the delivered charge because $k > 1$.

As elaborated in [31, 32], Peukert's law applies to supercapacitors when the discharge current is above a certain threshold. Specifically, the relationship between the charge delivered during a constant current discharge process and the discharge current is first investigated using four 100 F supercapacitor samples at the rated voltage of 2.7 V in [31]. It is revealed that the delivered charge increases when the discharge current decreases, i.e., Peukert's law applies. This work is developed in [32]: another two supercapacitor samples with different rated capacitances (10 and 350 F) are examined and three more voltages are swept: 2, 1.35, and 0.7 V. The discharge current range is also extended. It is found that Peukert's law only applies when the discharge current is above a certain threshold. When the discharge current is below the threshold, Peukert's law does not apply anymore.

The experiments and results for the applicability study of Peukert's law [31, 32] are summarized as follows. The three supercapacitor samples with different rated capacitances

from different manufacturers listed in Table 1 are tested using an automated Maccor Model 4304 tester at room temperature. For each sample, a set of constant current discharge experiments is performed when the initial voltage of the discharge process is fixed at a particular value and the cutoff voltage is fixed at 0.01 V. The rated voltage is the same for the three samples and the initial voltage is approximately linearly swept: 2.7, 2, 1.35, and 0.7 V.

To illustrate the experiment design, Fig. 1 shows the measured supercapacitor terminal voltage during a 10 A constant current discharge experiment for sample 2 when the initial voltage of the discharge process is 2.7 V. During this experiment, the supercapacitor is first conditioned by ten charging-redistribution-discharging cycles to minimize the effect of residual charge. It is then charged by a constant voltage source of 2.7 V for 3 hours, which is designed to fully charge the supercapacitor. After that, a 10 A constant discharge current is applied and the supercapacitor is discharged to 0.01 V. The discharge termination voltage is set as 0.01 instead of 0 V for safety considerations. Taking 2.7 V as the initial voltage and 0.01 V as the cutoff voltage, the charge delivered during this constant current discharge process is calculated as

$$Q = It, \quad (2)$$

where I is the discharge current and t is the discharge time. For this experiment, the delivered charge is 252.4 C. The experiment continues to estimate the amount of the total charge stored in the supercapacitor, which is beyond the scope of this paper and not considered.

Depending on the supercapacitor sample specifications and the supercapacitor tester capabilities, a set of constant discharge currents is swept for each sample. For sample 2, nine currents are selected: 10, 5, 1, 0.5, 0.1, 0.05, 0.01, 0.005, and 0.0025 A. While the maximum continuous current is not specified in the datasheet for this sample, the value of 10 A is determined according to the datasheets for similar 100 F samples tested in [31]. The discharge current is then scaled down based on the maximum value of 10 A. A relatively wide range of discharge current is swept for two considerations. First, from the theoretical analysis perspective, the effects of supercapacitor physics such as charge redistribution and self-discharge on the delivered charge pattern may vary depending on the discharge current magnitude. As a matter of fact, the effects of self-discharge are negligible when the discharge current is relatively large. For sufficiently low discharge currents, self-discharge results in a significant energy loss, as elaborated in [32]. Second, from the practical application perspective, understanding the supercapacitor behavior over a wide range of discharge current is critical to improve the system energy efficiency. In fact, supercapacitors have been used in both high power applications (e.g., smart grid and electric vehicles) and low power applications (e.g., wireless sensor networks and biomedical devices). Even for the same application, multiple power levels may exist. For instance, the current consumptions of transceivers, MCUs, and sensors vary significantly in wireless sensor networks. Therefore, for all the three samples, an extended range of discharge current is swept.

As elaborated in [31, 32], this set of experiments is repeated once. For the 10 A experiment shown in Fig. 1, the second run results in a delivered charge of 248.7 C. The average of these two runs of experiment is then 250.55 C, which is used as the delivered charge corresponding to the 10 A discharge current. For sample 2, Fig. 2(a) shows the relationship between the delivered charge and the discharge current when the initial voltage is 2.7 V. When the discharge current decreases from 10 to 0.01 A, the delivered charge increases from 250.55 to 299.41 C and Peukert's law applies. On the other hand, the delivered charge decreases from 299.41 to 292.62 C when the discharge current decreases from 0.01 to 0.0025 A and Peukert's law does not apply anymore. The results for the other three voltages are shown in Figs. 2(b)–2(d) and the delivered charge patterns are similar to that for 2.7 V. Similar observations hold for samples 1 and 3.

The relationship between the delivered charge and the discharge current is due to the combined effects of the three aspects of supercapacitor physics: porous electrode structure, charge redistribution, and self-discharge, as elaborated in [32]. Specifically, because of the porous electrode structure, or equivalently, the distributed nature of the supercapacitor capacitance and resistance, slow branch capacitors with large time constants are accessed during the extended discharge process when a lower discharge current is applied, which results in an increase in the delivered charge. In the meantime, the unidirectional charge redistribution from slow branches to fast branches decelerates the voltage drop in the main branch with the smallest time constant and prolongs the discharge time, which also contributes to the increase in the delivered charge. The impact of self-discharge on the delivered charge is negligible when the discharge current is relatively large. If the discharge current is sufficiently low, the energy loss due to self-discharge is significant, which results in a drop in the delivered charge.

3. Application Scenario 1: Same Discharge Voltages for Training and Testing Sets

3.1. Application Scenario 1: Problem Statement

Since Peukert's law applies to supercapacitors when the discharge current is above a certain threshold, it follows that the supercapacitor discharge time can be predicted using this relationship, as demonstrated in [34] using four 100 F supercapacitor samples at the rated voltage of 2.7 V. This paper develops this study by extending the rated capacitance (i.e., from 100 F only to three values: 10, 100, and 350 F) and the initial voltage of the constant current discharge experiment (i.e., from 2.7 V only to four values: 2.7, 2, 1.35, and 0.7 V). The supercapacitor samples listed in Table 1 are examined and the results presented in Section 2 are utilized.

The problem considered in this section states as follows. For all the three supercapacitor samples, first determine the current range between which Peukert's law applies when the initial voltage varies. Then use a subset of the experiments within this current range to estimate the Peukert constant. Finally, predict the discharge time for the remaining experiments within this current range and evaluate the prediction accuracy. Take sample 2 for instance. Peukert's law applies for the current range of 10–0.01 A for the initial voltages

of 2.7 and 2 V. The current range is 10–0.005 A for 1.35 and 0.7 V. For consistency, the narrower range of 10–0.01 A is considered for all the four initial voltages. For each of the four initial voltages, four among the seven experiments are used to estimate the Peukert constant (i.e., the 10, 1, 0.1, and 0.01 A experiments form the training set), which is then used to predict the discharge time for the remaining three experiments (i.e., the 5, 0.5, and 0.05 A experiments form the testing set). Note that the training set is selected in a way that its current range covers that of the testing set to ensure that the Peukert constant estimated using the training set is applicable to the testing set. In particular, the 1 A experiment is included in the training set because the charge delivered during this experiment is used as the nominal charge. This setup emulates a typical application scenario in which the Peukert constant determined through a set of constant current discharge experiments at a particular voltage is used to predict the discharge time when a different discharge current is applied to the supercapacitor at the same voltage.

3.2. Application Scenario 1: Methodology

This section illustrates the supercapacitor discharge time prediction methodology using the 2.7 V dataset of sample 2 shown in Fig. 2(a). As indicated by (1), the Peukert constant and the nominal charge are needed to predict the discharge time. To estimate the Peukert constant, (1) is rewritten as

$$t = \frac{Q_0}{I^k}, \quad (3)$$

which is used as the curve fitting function. The discharge current (10, 1, 0.1, and 0.01 A) and the corresponding discharge time (25.055, 272.01, 2870.9, and 29941 s) in the training set are used to fit the Peukert constant. The charge delivered during the 1 A experiment is used as the nominal charge: $Q_0 = 272.01$ C. Fig. 3 shows the discharge time measurements and the fit determined using MATLAB. The fitted Peukert constant is $k = 1.021$. To quantify the goodness of the fit, the following statistics are extracted from MATLAB: $R^2 = 1$ (coefficient of determination), $SSE = 292$ (sum of squares due to error), and $RMSE = 9.867$ (root mean squared error). The R^2 statistic measures how successful the fit is in explaining the variation of the data. It can take any value between 0 and 1, with a value closer to 1 indicating a better fit. The SSE statistic measures the total deviation of the response values from the fit to the response values. The $RMSE$ statistic is an estimate of the standard deviation of the random component in the data. For the SSE and $RMSE$ statistics, a value closer to 0 indicates a better fit.

The discharge time of the 5, 0.5, and 0.05 A experiments in the testing set is then predicted using (3). The prediction accuracy is evaluated in terms of the relative error:

$$\delta = \frac{|t_p - t_m|}{t_m} \times 100\%, \quad (4)$$

where t_p is the prediction and t_m is the measurement. For the 5, 0.5, and 0.05 A experiments, t_m is 51.786, 552.74, and 5836.0 s, respectively. The predicted discharge time for the 5, 0.5,

and 0.05 A experiments is denoted as t_{p5} , $t_{p0.5}$, and $t_{p0.05}$, respectively. Correspondingly, the prediction error is denoted as δ_5 , $\delta_{0.5}$, and $\delta_{0.05}$, respectively. In addition, the average of the three errors is calculated and denoted as $\bar{\delta}$. The errors associated with the Peukert constant ($k = 1.021$) fitted using the four experiments in the training set are listed in the “Fit” column in Table 2.

While the Peukert constant fitted using the training set serves as a good estimate of the actual value, it is unclear if the fitted value results in the minimum error. Therefore, to determine the optimal Peukert constant that leads to the minimum error in predicting the discharge time of the testing set, Fig. 4 sweeps the Peukert constant and plots the average error. The minimum average error is observed at $k = 1.024$ and the results associated with this value are listed in the “Optimal” column in Table 2.

For comparisons, Table 2 also shows the results for another two setups in which the Peukert constant is set as $k = 1$. In this case, the supercapacitor is assumed to be a constant capacitor and the delivered charge does not vary with the discharge current (i.e., Peukert’s law is not utilized). This assumption does not take into account the supercapacitor physics especially the porous electrode structure and the charge redistribution process. Therefore, it can be anticipated that this assumption leads to significant errors when it comes to predicting the supercapacitor discharge time, which is indeed the case, as shown in Table 2. As for the nominal charge, the “Constant” setup uses the charge delivered during the 1 A experiment (i.e., $Q_0 = 272.01$ C). The “Rated” setup uses the charge calculated based on the supercapacitor rated capacitance (100 F) and the beginning (2.7 V) and termination (0.01 V) voltages of the constant current discharge process: $Q_{\text{Rated}} = CV = 100 \times (2.7 - 0.01) = 269$ C. Table 2 shows that the discharge time prediction error (δ_5 , $\delta_{0.5}$, $\delta_{0.05}$, and $\bar{\delta}$) is significantly reduced when the fitted or optimal Peukert constant is used compared to the cases in which the Peukert’s law is not considered. For practical applications in which an accurate estimate of the supercapacitor runtime is crucial, the discharge time predicted using Peukert’s law can serve as a better “fuel gauge” to indicate the supercapacitor runtime.

3.3. Application Scenario 1: Results

Using the methodology illustrated in Section 3.2, the prediction errors for the other three datasets (2, 1.35, and 0.7 V) of sample 2 are evaluated and the results are summarized in Table 3. For brevity, only the average errors are included. Tables 4 and 5 list the results for samples 1 and 3, respectively. These results show that for all the three samples at all the four initial voltages, the average error associated with the “Fit” or “Optimal” setup is smaller than that resulted from the “Rated” or “Constant” setup, which demonstrates the effectiveness of predicting the supercapacitor discharge time using Peukert’s law within the applicable discharge current range.

To evaluate the conservative and optimistic improvements in terms of average error reduction, the four evaluation setups are examined. Between the “Fit” and “Optimal” setups, the latter results in a smaller error although the difference is not noticeably significant. It can also be observed that in general the fitted Peukert constant is slightly different from the optimal value among the 12 combinations of the three supercapacitor samples and the four initial voltages, which is a natural outcome of the curve fitting approach with a limited

amount of information. In fact, the optimal value is slightly greater than the fitted value in ten combinations, less than in one combination (sample 1, 2.7 V), and equal to in one combination (sample 3, 2 V).

On the other hand, between the “Rated” and “Constant” setups, in general the latter results in a smaller error except in two combinations (sample 1, 2.7 V and sample 1, 2 V). Therefore, the conservative improvement is estimated using the ratio of the larger error between the “Fit” and “Optimal” setups to the smaller error between the “Rated” and “Constant” setups. The optimistic improvement is estimated using the ratio of the smaller error between the “Fit” and “Optimal” setups to the larger error between the “Rated” and “Constant” setups. The conservative ratio is between 9.0% (sample 1, 1.35 V, 0.67% associated with “Fit” versus 7.46% associated with “Constant”) and 40.4% (sample 3, 0.7 V, 1.08% associated with “Fit” versus 2.67% associated with “Constant”). The optimistic ratio is between 0.3% (sample 1, 2 V, 0.02% associated with “Optimal” versus 6.83% associated with “Constant”) and 21.6% (sample 3, 2 V, 0.37% associated with “Fit” or “Optimal” versus 1.71% associated with “Rated”). In summary, the average error of the discharge time prediction is significantly reduced even in terms of the conservative ratio when Peukert’s law is utilized.

4. Application Scenario 2: Different Discharge Voltages for Training and Testing Sets

4.1. Application Scenario 2: Problem Statement

While Section 3 considers a scenario in which the initial voltage from which the supercapacitor is discharged is the same for the experiments in the training and testing sets, this section examines another scenario in which the initial voltages are different in the training and testing sets. For the three supercapacitor samples, the experiments at the four voltages (i.e., 2.7, 2, 1.35, and 0.7 V) are used as the training set. The nominal charge and Peukert constant extracted from these experiments are utilized to predict the discharge time of experiments at other voltages. In particular, this section considers three voltages in the testing set: 2.4, 1.7, and 1 V, which are approximately linearly interpolated based on the training set. At each voltage, the discharge current is swept and the prediction error is evaluated for all the experiments in the testing set. This scenario is typical in practical applications because the supercapacitor voltage varies significantly during its normal operation and it may be discharged from a voltage different from those already examined.

4.2. Application Scenario 2: Methodology

To illustrate the methodology, sample 2 is considered again. As in the first scenario, the nominal charge and Peukert constant are needed to predict the discharge time. Fig. 5 shows different approaches to estimate these two parameters using the training set. Specifically, Fig. 5(a) includes the nominal charge measurements at the four voltages in the training set (i.e., Q_0 in the “Fit” setup in Table 3), which are fitted using a linear function Q_{0L} and a quadratic function Q_{0Q} , respectively:

$$Q_{0L} = 106.1V - 18.16, \quad (5)$$

$$Q_{0Q} = 7.583V^2 + 80.33V - 03924. \quad (6)$$

The nominal charge estimates at the three voltages in the testing set are then determined using these two fits.

On the other hand, Fig. 5(b) shows the optimal Peukert constants determined using the training set. Note that at each of the four voltages, the optimal Peukert constant is determined using all the seven experiments. Therefore, their values are slightly different from those listed in Table 3, which are based on four of the seven experiments. For example, the optimal Peukert constant is 1.023 at 2.7 V in Fig. 5(b) while it is 1.024 in Table 3. The Peukert constants in Fig. 5(b) are fitted using a linear function k_L and a piecewise linear function k_P , respectively:

$$k_L = -0.006307V + 1.04, \quad (7)$$

$$k_P = \begin{cases} -0.009231V + 1.042, [0.7, 1.35) \\ -0.004615V + 1.036, [1.35, 2), \\ -0.005714V + 1.038, [2, 2.7]. \end{cases} \quad (8)$$

As a benchmark, the nominal charge takes the third value, which is measured during a 1 A experiment at the three testing voltages. The third Peukert constant value is determined using four experiments (10, 1, 0.1, and 0.01 A) at the three testing voltages. In total, nine combinations of nominal charge and Peukert constant are examined.

To illustrate the evaluation framework, Table 6 lists the results associated with three setups of nominal charge and Peukert constant: “Rated”, “Constant”, and “Fit”, as in Section 3. The leftmost block lists the measured initial voltage V , discharge current I , and discharge time t_m for the experiments in the testing set. The rightmost block is for the “Fit” setup using Peukert’s law: the nominal charge uses the linear fit Q_{0L} in (5), the Peukert constant uses the linear fit k_L in (7), and δ_F is the prediction error calculated using (4). This table also lists the errors associated with the “Rated” and “Constant” setups in which the Peukert constant is set as $k = 1$ and the nominal charge is determined using the same way as in Section 3. In terms of individual error, the “Fit” setup outperforms both the “Rated” and “Constant” setups in seven of the nine experiments. In the 2.4 V/0.1 A experiment, the “Fit” setup results in a slightly larger error than the “Rated” setup: 2.48% versus 2.19% and still outperforms the “Constant” setup: 2.48% versus 3.22%. A similar observation holds for the 1.7 V/0.1 A experiment: “Fit” is better than “Constant” but worse than “Rated”. For all the three setups, the average of the nine individual errors is calculated and denoted as $\bar{\delta}$, which is used as the metric to evaluate the overall prediction accuracy. In terms of average error, the prediction accuracy is improved when Peukert’s law is utilized: 4.75% in “Fit” versus 10.20% in “Rated” or 10.27% in “Constant”.

4.3. Application Scenario 2: Results

Note that Table 6 only shows one of the nine combinations of nominal charge and Peukert constant in the “Fit” setup: both use the linear fit. For all the nine combinations, Table 7 summarizes the average errors for sample 2. In the rightmost block corresponding to the “Fit” setup, the nominal charge Q_0 determined using the measurements, the linear fit, and the quadratic fit is denoted as “M”, “L”, and “Q”, respectively. The Peukert constant k estimated using the measurements, the linear fit, and the piecewise linear fit is denoted as “M”, “L”, and “P”, respectively.

As in Section 3, other than the Peukert constant directly fitted using the experiments, the optimal value is also determined. Therefore, in the three combinations involving the measured Peukert constant “M” (i.e., $Q_0 = M/k = M$, $Q_0 = L/k = M$, and $Q_0 = Q/k = M$), the fitted value is used in the $\bar{\delta}_F$ column while the optimal value is used in the $\bar{\delta}_O$ column. In the remaining six combinations, the Peukert constant is determined using the fits (i.e., “L” and “P”). Therefore, the results are only listed in the $\bar{\delta}_F$ column and the entries are marked as “-” in the $\bar{\delta}_O$ column. In fact, the average error of the “Fit” setup in Table 6 corresponds to the combination of $Q_0 = L/k = L$ in Table 7: $\bar{\delta}_F = 4.75\%$. Since the Peukert constant is determined using the linear fit “L” (i.e., “M” determined using the measurement is not involved), the entry in the $\bar{\delta}_O$ column is not applicable and denoted as “-”. On the other hand, in the $Q_0 = L/k = M$ combination involving the measured Peukert constant, the fitted and optimal Peukert constant values are different, which result in different average errors: $\bar{\delta}_F = 4.33\%$ and $\bar{\delta}_O = 4.50\%$.

Moreover, Table 7 lists the average errors associated with another two setups: $\bar{\delta}_R$ for “Rated” and $\bar{\delta}_C$ for “Constant”. Note that the nominal charge in “Rated” is denoted as “R”, which has only one entry because it is calculated using the measured voltages. The nominal charge in “Constant” uses the same values as in “Fit” and has three entries: “M”, “L”, and “Q”. The results for samples 1 and 3 are summarized in Tables 8 and 9, respectively.

The results in Tables 7 through 9 show that the prediction accuracy is improved in terms of average error for all the three samples when Peukert’s law is used. Compared to the “Rated” and “Constant” setups in which Peukert’s law is not considered, the “Fit” setup always results in a smaller average error for all the nine combinations of nominal charge and Peukert constant. Therefore, this section demonstrates the usefulness of Peukert’s law in predicting the supercapacitor discharge time in an application scenario different from the one considered in Section 3.

The results in Tables 7 through 9 also show that in the “Fit” setup, the nominal charge has a more significant impact on the average error than the Peukert constant. Specifically, the same nominal charge results in an approximately equal average error when the Peukert constant varies. On the other hand, the average error is noticeably different for the same Peukert constant when the nominal charge varies. Take Table 7 for instance. The errors are the smallest (2.75, 2.82, and 2.69%) in the $Q_0 = M$ block and the largest (4.33, 4.75, and

4.62%) in the $Q_0 = L$ block. The quadratic fit is better than the linear fit but worse than the measurements, which results in the medium errors (3.45, 3.60, and 3.49%) in the $Q_0 = Q$ block. Within each of the three blocks, the differences between the average errors are relatively insignificant.

Moreover, the results in Sections 3 and 4 lead to an observation on the Peukert constant. For the same supercapacitor sample, the fitted or optimal Peukert constant increases when the initial voltage decreases (e.g., Table 3 and Fig. 5(b) for sample 2). In fact, the dependence of the supercapacitor Peukert constant on its terminal voltage, aging condition, and operating temperature has been investigated [40]. Experimental results show that the Peukert constant increases when the initial voltage of the constant current discharge process is lower, the supercapacitor is more heavily aged, or the operating temperature is lower. The physical mechanisms accounting for the Peukert constant dependence are illustrated by analyzing an RC ladder circuit model. When the supercapacitor terminal voltage is higher, the aging condition is lighter, or the operating temperature is higher, more charge is stored in the supercapacitor. Consequently, when the same discharge current is applied, the discharge time is longer and the branch capacitors are more deeply discharged. Therefore, the relaxation effects of the slow branches are reduced and the supercapacitor behaves more like a single capacitor rather than a distributed capacitor network, which ultimately leads to a lower Peukert constant.

Finally, the results in Sections 3 and 4 also suggest that the Peukert constant is smaller when the supercapacitor rated capacitance is larger (e.g., Tables 3 through 5 for samples 2, 1, and 3, respectively). To examine if this observation is generally applicable and furthermore to reveal the physical mechanisms leading to this observation, extensive experiments with a much wider range of supercapacitor samples and a systematic investigation of the supercapacitor electrochemical processes are needed. In fact, a relevant observation on the discharge current threshold above which Peukert's law applies has been reported in [32]: the lower the rated capacitance, the smaller the current threshold. An ongoing work is being conducted to further investigate these two observations.

5. Conclusion

This paper shows that Peukert's law can be used to predict the supercapacitor discharge time during a constant current discharge process. Two application scenarios are examined using three supercapacitor samples with different rated capacitances from different manufacturers. In the first scenario, the experiments in the testing and training sets use the same initial voltage from which the super-capacitor is discharged. In the second scenario, the experiments in the testing set use initial voltages different from those in the training set. In both scenarios, the prediction error is significantly reduced when Peukert's law is utilized. In addition to demonstrating the usefulness of Peukert's law in supercapacitor discharge time prediction, this paper also stimulates further research on Peukert's law and the Peukert constant dependence on device ratings, which will help better utilize the supercapacitor technology in a variety of applications.

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Figure 1:
A 10 A constant current discharge experiment for sample 2.

Figure 2:
Relationship between delivered charge and discharge current for supercapacitor sample 2.
(a) Initial voltage is 2.7 V. (b) Initial voltage is 2 V. (b) Initial voltage is 1.35 V. (d) Initial voltage is 0.7 V.

Figure 3:
Measured and fitted discharge time for supercapacitor sample 2.

Figure 4:
Optimal Peukert constant for supercapacitor sample 2.

(a)

(b)

Figure 5:
Nominal charge and Peukert constant for supercapacitor sample 2. (a) Nominal charge. (b) Peukert constant.

Table 1:

Supercapacitor Samples.

Sample	1	2	3
Manufacturer	Eaton	AVX	Maxwell
Model	HV1030-2R7106-R	SCCV60B107MRB	BCAP0350
Capacitance (F)	10	100	350
Voltage (V)	2.7	2.7	2.7

Table 2:

Scenario 1: Evaluation Setups for Supercapacitor Sample 2.

	Rated	Constant	Fit	Optimal
k	1	1	1.021	1.024
Q_0 (C)	269	272.01	272.01	272.01
t_{p5} (s)	53.8	54.402	52.59	52.34
$t_{p0.5}$ (s)	538	544.02	552.00	553.15
$t_{p0.05}$ (s)	5380	5440.2	5793.44	5845.74
δ_5 (%)	3.89	5.05	1.56	1.07
$\delta_{0.5}$ (%)	2.67	1.58	0.13	0.07
$\delta_{0.05}$ (%)	7.81	6.78	0.73	0.17
$\bar{\delta}$ (%)	4.79	4.47	0.81	0.44

Table 3:

Scenario 1: Results for Supercapacitor Sample 2.

Voltage		Rated	Constant	Fit	Optimal
2.7 V	k	1	1	1.021	1.024
	Q_0 (C)	269	272.01	272.01	272.01
	$\bar{\delta}(\%)$	4.79	4.47	0.81	0.44
2 V	k	1	1	1.025	1.028
	Q_0 (C)	199	189.86	189.86	189.86
	$\bar{\delta}(\%)$	5.91	5.29	0.93	0.62
1.35 V	k	1	1	1.028	1.030
	Q_0 (C)	134	122.77	122.77	122.77
	$\bar{\delta}(\%)$	8.37	6.41	1.51	1.27
0.7 V	k	1	1	1.034	1.036
	Q_0 (C)	69	59.17	59.17	59.17
	$\bar{\delta}(\%)$	16.70	8.76	2.76	2.52

Table 4:

Scenario 1: Results for Supercapacitor Sample 1.

Voltage		Rated	Constant	Fit	Optimal
2.7 V	k	1	1	1.032	1.031
	Q_0 (C)	26.9	26.18	26.18	26.18
	$\bar{\delta}(\%)$	3.83	5.29	0.40	0.37
2 V	k	1	1	1.036	1.039
	Q_0 (C)	19.9	17.58	17.58	17.58
	$\bar{\delta}(\%)$	5.47	6.83	0.53	0.02
1.35 V	k	1	1	1.039	1.042
	Q_0 (C)	13.34	10.85	10.85	10.85
	$\bar{\delta}(\%)$	13.77	7.46	0.67	0.14
0.7 V	k	1	1	1.046	1.050
	Q_0 (C)	6.93	5.1	5.1	5.1
	$\bar{\delta}(\%)$	23.67	8.99	1.07	0.42

Table 5:

Scenario 1: Results for Supercapacitor Sample 3.

Voltage		Rated	Constant	Fit	Optimal
2.7 V	k	1	1	1.006	1.008
	Q_0 (C)	941.5	987.72	987.72	987.72
	$\bar{\delta}(\%)$	4.32	1.29	0.50	0.37
2 V	k	1	1	1.008	1.008
	Q_0 (C)	696.5	689.23	689.23	689.23
	$\bar{\delta}(\%)$	1.71	1.35	0.37	0.37
1.35 V	k	1	1	1.009	1.011
	Q_0 (C)	469	439.89	439.89	439.89
	$\bar{\delta}(\%)$	7.19	1.84	0.66	0.52
0.7 V	k	1	1	1.012	1.015
	Q_0 (C)	244.3	215.56	215.56	215.56
	$\bar{\delta}(\%)$	14.51	2.67	1.08	1.00

Table 6:

Scenario 2: Evaluation Setups for Supercapacitor Sample 2.

Measurement			Rated	Constant	Fit
V (V)	I (A)	t_m (s)	δ_R (%)	δ_C (%)	δ_F (%)
2.3998	10	20.96	14.02	12.81	6.54
2.3998	0.1	2443.35	2.19	3.22	2.48
2.4001	0.01	25597.66	6.63	7.61	*3.60
1.7000	10	13.93	21.32	16.45	8.86
1.7000	0.1	1680.28	0.58	3.46	3.27
1.6999	0.01	17770.91	4.91	8.73	4.45
1.0091	10	7.39	35.20	20.31	11.34
1.0068	0.1	951.54	4.76	6.82	0.68
1.0037	0.01	10158.53	2.18	13.05	1.54
		$\bar{\delta}(\%)$	10.20	10.27	4.75

Table 7:

Scenario 2: Results for Supercapacitor Sample 2.

Q_0	k	$\bar{\delta}_R(\%)$	Q_0	k	$\bar{\delta}_C(\%)$	Q_0	k	$\bar{\delta}_F(\%)$	$\bar{\delta}_O(\%)$
R	1	10.20	M	1	10.65	M	M	2.75	2.66
						M	L	2.82	-
						M	P	2.69	-
			L	1	10.27	L	M	4.33	4.50
						L	L	4.75	-
						L	P	4.62	-
			Q	1	10.52	Q	M	3.45	3.42
						Q	L	3.60	-
						Q	P	3.49	-

Table 8:

Scenario 2: Results for Supercapacitor Sample 1.

Q_0	k	$\bar{\delta}_R(\%)$	Q_0	k	$\bar{\delta}_C(\%)$	Q_0	k	$\bar{\delta}_F(\%)$	$\bar{\delta}_O(\%)$
R	1	6.40	M	1	12.74	M	M	0.40	0.39
						M	L	0.76	-
						M	P	0.74	-
			L	1	9.80	L	M	3.05	3.17
						L	L	3.43	-
						L	P	3.46	-
			Q	1	11.65	Q	M	1.48	1.59
						Q	L	1.47	-
						Q	P	1.48	-

Table 9:

Scenario 2: Results for Supercapacitor Sample 3.

Q_0	k	$\bar{\delta}_R(\%)$	Q_0	k	$\bar{\delta}_C(\%)$	Q_0	k	$\bar{\delta}_F(\%)$	$\bar{\delta}_O(\%)$
R	1	6.58	M	1	3.32	M	M	1.09	1.08
						M	L	1.09	-
						M	P	1.14	-
			L	1	3.70	L	M	2.94	2.93
						L	L	2.93	-
						L	P	2.94	-
			Q	1	3.30	Q	M	1.74	1.74
						Q	L	1.74	-
						Q	P	1.74	-