Published in final edited form as:

AIAA J. 2017 December ; 55(12): 4274–4285. doi:10.2514/1.J055944.

Mach 10 Bow-Shock Unsteadiness Modeled by Linear Combination of Two Mechanisms

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Abstract

This paper presents mechanisms to explain, as well as mathematics to model, time-averaged spatially resolved amplitude observations of number density and number density unsteadiness in a Mach 10 flow as it transitions from the freestream, through a bow-shock wave, and into the gas cap created by a blunt-body model. The primary driver for bow-shock unsteadiness is freestream unsteadiness or “tunnel noise.” Primary unsteadiness is bow-shock oscillation. It scales spatially with the number density first derivative and is modeled using a $\text{sech}^2(z)$ term. Secondary weaker unsteadiness begins as freestream unsteadiness and increases linearly in direct proportion to the gas number density across the bow shock and into the gas cap. This is the well-known amplification of the freestream turbulent kinetic energy mechanism and is modeled using a $\tanh(z)$ term. Total unsteadiness [fit using $\tanh(z)$ term + $\text{sech}^2(z)$ term] is expressed as the number density standard deviation and modeled as a linear combination of these two independent, simultaneous, and nonlinear unsteadiness mechanisms. Relationships between mechanism coefficients and various flowfield and wind-tunnel parameters are discussed. For example, bow-shock and gas cap oscillation amplitudes are linearly correlated with stagnation pressure and, by deduction, freestream unsteadiness.

I. Introduction

IN 1914, Wieselsberger [1] concluded that the variation in drag values obtained in different subsonic wind tunnels could be explained by differences in freestream turbulence. Since that time, experiments have shown that the freestream of hypersonic facilities contains high-frequency unsteady pressure oscillations or tunnel noise at frequencies up to 1 MHz [2,3]. Unsteadiness can begin in the stagnation chamber [4], nozzle throat, or turbulent boundary layer on the nozzle wall [5]; and it can enter the freestream [6]. Unsteadiness can be an order of magnitude above what is encountered in flight [6]. Unsteadiness can interact with and significantly perturb the observed offbody flowfield around test articles [7]. This process is called receptivity and creates a disturbance with a given frequency, amplitude, and phase [8]. The resulting flowfield instability can influence laminar and turbulent transition, affect surface heat transfer, and increase stagnation point heating. This, in turn, can influence model downstream vehicle aerodynamics/aerothermodynamics including lift, drag, pitching moment, transition onset, and turbulence [7]. Due to limited understanding of the instability mechanisms and transition processes inherent in a given facility, computational fluid

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dynamic codes have difficulty computing aerodynamic quantities in transitional and turbulent flows. These problems are exacerbated by the aerothermodynamic considerations present in hypersonic flows. After a century of study, the present paper seeks to increase modestly the understanding of these phenomena by measuring the effects of hypersonic freestream unsteadiness as it propagates through and induces oscillations in a Mach 10 bow shock and gas cap (boundary-layer instability) ahead of a blunt body. For clarity, a single flowfield state variable density is measured.

Researchers typically focus on measuring the frequency content of unsteadiness. Instrumentation capable of valid measurements of unsteadiness at high frequencies is limited. Problems with traditional physical probes capable of pitot pressure measurements and hot-wire anemometry have been reviewed [9]. Schlieren imaging is attractive due to its nonintrusive nature and capacity for both frequency and amplitude measurements, but it lacks spatial resolution [10,11]. Focused laser differential interferometry (FLDI) has many desirable characteristics and has demonstrated 1 MHz freestream measurements [9]. FLDI is a point method limited to a 20 mm spanwise spatial resolution. Laser Rayleigh scattering (LRS) has demonstrated freestream density measurements at a single point in space [12] and two points in space, as well as 50 kHz [13] using a continuous-wave laser at 532 nm and atmospheric pressure. This approach suffers from particle interference and produces insufficient signals in low-density hypersonic flows.

There is a small but growing number of experimental studies of hypersonic bow-shock oscillations induced by freestream disturbances [10,11,14,15]. There have been several mathematical analysis and computational studies on freestream-unsteadiness-induced bow-shock oscillations [16–22].

The present paper uses pulsed laser Rayleigh scattering for quantitative density and spatial amplitude unsteadiness measurements in Mach 10 flows at 30 Hz repetition rates. Previous hypersonic results include measurements in the freestream [23–25], supersaturation region [26], wake [27], and gas cap [24,27] of a blunt body. Using planar LRS at 10 Hz repetition rates, Shirinzadeh et al. [28] demonstrated that the density standard deviation and the density standard deviation divided by the density mean (referred to as the relative standard deviation or fluctuating component) were a measure of flow unsteadiness and could be related to flowfield turbulence.

The NASA Langley Research Center's 31 in. Mach 10 (31M10) facility has several unique characteristics relevant to this paper. Quantitative LRS freestream density measurements indicate clusters are not detected in this facility [25]. LRS quantitative density measurements in the supersaturation region indicate the freestream “frozen” vibrational nonequilibrium population is effectively removed by interaction with clusters [26]. By varying the stagnation pressure at a constant stagnation temperature, energy fluctuations per unit stagnation pressure are presumed constant. All these variables are removed as possible contributors to the observed unsteadiness.

Previously, Balla [15] presented a unique Mach 10 bow-shock and gas cap unsteadiness dataset. Using sampling rates of 30 Hz, LRS measured time-averaged density disturbance

amplitudes along a 38.7 mm line. The time-averaged spatial profile of standard deviation unsteadiness as flow transitioned from the freestream, through the bow shock, and into the gas cap produced a distinct and unexpected profile. The gas cap was defined as the shock-heated stagnation zone ahead of a blunt-body model. In supersonic flows, a shock wave can be treated as a discontinuity. In hypersonic flows, rarefactions of the flow cause a spreading of the shock and result in a discernible shock-wave thickness. Assuming amplification of the freestream turbulent kinetic energy (TKE), unsteadiness was expected to increase monotonically from the freestream, across the bow-shock wave, and end with maximum unsteadiness in the gas cap. This was not observed. Instead, unsteadiness was minimum in the freestream, rose to a maximum at densities and a spatial location that were halfway between the freestream density and the gas cap density (center or half-width of shock-wave thickness or ~50% of density maximum between freestream and gas cap or first derivative maximum of bow-shock density profile), and then decreased to levels expected in the gas cap assuming TKE amplification. Results clearly demonstrated two unsteadiness mechanisms were operative. The first and minor unsteadiness was the well-known amplification of freestream TKE (unsteadiness) across a shock wave. The second was bow-shock oscillation. It was concluded that tunnel-noise-induced bow-shock oscillation not only existed but was also the dominant oscillation.

Freestream disturbances enter the boundary layer at different frequencies. Because the laser operates at 30 pulses per second, the LRS measurement technique can provide no information on the frequencies of freestream disturbances or on the frequencies characteristic of the bow-shock amplitude oscillation relative to TKE augmentation unsteadiness.

The present paper analyzes the spatially resolved unsteadiness results. It proposes a linear combination of two simultaneous, nonlinear, and independent mechanisms that spatially have a total shape that can be represented as a sum of a $\tanh(z)$ function and a $\text{sech}^2(z)$ function to explain the spatially resolved standard deviation unsteadiness amplitude results. It provides equations to calculate each mechanism. It correlates spatially averaged flow unsteadiness observed along each line with facility conditions and suggests possible fluid dynamics responsible for creating the observed unsteadiness.

II. Experimental

Complete descriptions of various components of this experimental setup have been described [15,23–28]. A brief synopsis is presented. LRS was performed during test 466 in the NASA Langley Research Center's 31 in. Mach 10 air wind tunnel. For the light source, a 1.25 mJ 20 ns pulsed Lambda Physik OpTexPro excimer laser was selected to operate in broadband mode on argon fluoride near 193 nm at 30 Hz. The LRS signal generated along a 38.7 mm line in the freestream and gas cap was imaged using a gated double-intensified charge-coupled-device (CCD) camera at 30 Hz. The calibration procedure for each pixel along the laser line has been described [23]. Experiments were performed using a blunt-body model shown in Fig. 1 and based on the Apollo geometry. Figure 1 is a composite representation of three images: a digital reconstruction of a model with sting at a 28 deg angle of attack, an overlay of a 6-in.-diam time-averaged schlieren image of the bow-shock

wave, and an overlay of a 371-image averaged LRS density image within the CCD camera field of view (darkened rectangular region). Stagnation conditions were $P_0 = 350$ psi and $T_0 = 990$ K. The bow shock and laser standoff on the sting centerline were 10.6 and 9.3 mm, respectively. The LRS image was 38.7 mm high.

Figures 1 and 2 are presented only to orient the reader as to the general locations probed. Because Fig. 1 is the lowest stagnation pressure, the bow shock moves furthest away from the model. Therefore, the laser beam (white line in Fig. 1) probes the minimum number of locations of interest at this stagnation pressure. The flow turns sharply near the bottom of the model in Fig. 1, forming an oblique shock [11,29]. Unfortunately, the schlieren image is not properly aligned to view the oblique shock. Its expected location at the higher stagnation pressures is shown as the dotted red line.

For most of the images in this study, Figs. 3 and 4 show that LRS measurements begin near the freestream, traverse the near-normal shock at the top of the images, span the gas cap, traverse the oblique shock near the bottom of the images, and intersect the freestream. This is shown most clearly in Fig. 3e.

Heating of the sting, strut, and model causes the model to move downstream and toward the camera during each run [27]. This paper uses this motion as a convenient and passive method of sampling two different spatial locations in the bow-shock wave as a function of facility runtime. At this high angle of attack, Fig. 1 shows the bow-shock standoff distance is large near the top of the model and monotonically decreases toward the bottom of the model. Beyond this point, shadowgraphs provided by Walpot et al. (figure 16 in [29]) and Fujii et al. (figures 3–5 in [11]) show the shock standoff distance decreases rapidly, the shock turns rapidly, and it becomes more oblique. Flow conditions are calculated using the GASPROPS code [30].

III. Results and Discussion

A. Textbook Hypersonic Bow-Shock-Wave Density Profile

As hypersonic flow transitions between the low-temperature high-velocity freestream and the high-temperature low-velocity gas cap ahead of a blunt body, macroscopic properties of the gas such as translational, rotational, and particularly vibrational temperature are required to change by an order of magnitude over a few molecular mean free paths. The resulting nonequilibrium conditions in the interior of a hypersonic shock wave are equilibrated by kinetic processes that require a finite time, and hence a finite distance, to occur. The result is a thickening of the shock wave [31]. This is in contrast to supersonic flows where equilibration typically requires ~ 10 collisions, equilibration distances are greatly reduced, and shocks can be treated as simple discontinuities.

The x-axis density profile as hypersonic flow begins in the freestream, traverses a hypersonic bow-shock wave, and ends in the gas cap ahead of a blunt body is shown schematically in Fig. 2 [32]. Here, δ_s is the bow-shock-wave thickness derived from the maximum slope of the density profile and λ_1 is the freestream mean free path. The normalized bow-shock thickness is expressed as δ_s/λ_1 , and it is ~ 3.3 for Mach 10 nitrogen

shock [33]. At $P_0 = 350$ psi, $\lambda_1 = 0.023$ mm and δ_s is ~ 0.077 mm. At $P_0 = 1450$ psi, $\lambda_1 = 0.0055$ mm and δ_s is ~ 0.018 mm. The scaled number density rise is defined as $\tilde{n} = (n - n_1)/(n_2 - n_1)$. Also, $\tilde{n} = 0.5$ is the density midpoint or median between the freestream and the gas cap. The x-axis origin ($x/\lambda_1 = 0$) is conventionally chosen where $\tilde{n} = 0.5$. Hereafter, for brevity, δ_s will be written as δ .

The spatial density profile in Fig. 2 is described using a hyperbolic tangent function (equation 93.12 in [34]):

$$\rho(x) = a + b * \tanh(x/\delta)$$

(1)

Values for a and b allow the $\tanh(x)$ function to map its maximum and minimum values of $+1$ and -1 onto the gas cap and freestream densities, respectively, such that $a + b$ is the gas cap density and $a - b$ is the freestream density. The constant δ is the shock thickness in millimeters. It allows for a finite shock thickness δ that may vary with freestream pressure or spatial variations in the local flowfield Mach number.

B. LRS Time-Averaged Density and Density Standard Deviation Spatial Profiles

Number density measurements are obtained along the laser beam (z axis) spanning ~ 38.7 mm at locations shown in Fig. 1. For brevity, they are labeled in the figures as density measurements. For most of the images in this study, Figs. 3 and 4 show that LRS measurements begin near the freestream; traverse the near-normal shock, gas cap, and oblique shock; and end near the bottom of the model in the freestream. $Z = 0$ is the facility and sting centerline. Figure 3 shows line measurements (371-image average) of average density ρ at five stagnation pressures and two spatial locations within the shock [black ($t = 0$ s) and red ($t = 18$ s) curves]. They were discussed previously [27]. Spatial oscillations at $P_0 = 350$ psi (run a) in the postshock (gas cap) number density ($\sim \pm 0.25$ out of ~ 5.25 or $\sim 5\%$) are primarily the measurement uncertainty [27]. The dip near $z = +2$ mm for all $P_0 > 350$ psi may be flow spatial nonuniformity. Higher-fidelity measurements are required to support this speculation. Lines are color coded with black, corresponding to the first dataset taken at the first spatial location (~ 9.3 mm model standoff distance), and red for the second dataset taken at the second spatial location (~ 9.4 mm model standoff distance of 9.44 mm at $P_0 = 350$ psi increasing to ~ 9.6 mm at $P_0 = 1454$ psi). Except for the single $P_0 = 350$ psi case, datasets are recorded and stored in ~ 18 s intervals during each facility run (~ 12 s data acquisition and 6 s storage time).

Figures are arranged vertically as a function of stagnation pressure for rapid visual inspection and comparison of a single measured parameter at different stagnation pressures and spatial locations during each run.

Figure 4 shows the standard deviation σ for each corresponding image in Fig. 3. The results in Fig. 4 are calculated using 371 instantaneous images (hardware limitation) used to

generate the average data in Fig. 3. In Fig. 4e, the maximum unsteadiness indicates $\sigma = 5$ near $z = +12$ mm. For simplicity, values for the standard deviation σ in this paper are written and plotted without the exponent of 1×10^{17} . By comparison with the density at the $z = +12$ mm location in Fig. 3e, time-averaged flowfield unsteadiness is 5 out of 10 or 50% ($\pm 1\sigma$) density unsteadiness. Near the edges of these figures (freestream conditions), σ approaches one. This is primarily the measurement uncertainty due to instrument noise. True freestream unsteadiness is a fraction of this value. Instrument noise has not been subtracted from the Fig. 4 images. Its effect will be computed and discussed following the data analysis in the following. Unfortunately, the locations of interest for many of the unsteadiness phenomena discovered in this paper were not known during the design phase of this experiment. One result is that the length of the imaged line is not sufficiently long to observe all unsteadiness behavior of interest at all stagnation conditions and flowfield locations. Line images are not normalized by pulse-to-pulse laser energy variation (less than 3 %); this effect is negligible. Model motion weakly broadens the profiles of Figs. 3 and 4. Therefore, time-averaged results are a lower limit to true unsteadiness.

C. Analysis of Time-Averaged Spatial Density Profiles

With a single CCD pixel flowfield resolution of $\sim 70 \mu\text{m}$, the imaged bow-shock thickness ($\sim 20 \mu\text{m}$) cannot be resolved along the streamwise direction i.e., $\rho(x)$. To compensate as dictated by the experimental optical constraints imposed by the 31M10 facility, vertical line images are acquired normal to the freestream flow (Fig. 1). Comparing the x-axis streamwise (~ 0.02 mm) to the z-axis fit shock thickness (3.5 mm normal shock and 2.5 mm oblique shock) in Table 1, the vertical imaging “stretches” the imaged shock thickness by factors of ~ 175 and ~ 125 for the normal and oblique shocks, respectively. This allows observation of the bow-shock phenomena using the spatial resolution of typical CCD cameras.

The latter average density dataset in Fig. 3d at $P_0 = 1254$ psi (red data points at $t = 18$ s) is shown in Fig. 5. This profile is split into data (solid black circles) spanning the freestream, near-normal shock, and gas cap in Fig. 5a; and data spanning the freestream, oblique shock, and gas cap in Fig. 5b. Each profile has been shifted based on visual inspection (-12 to -18 mm for the normal shock and $+12$ to $+18$ mm for the oblique shock), so the median density ($\bar{n} = 0.5$ in Fig. 1) is centered near $z = 0$ to conform to a $\tanh(z)$ fit. Near-normal shock z-axis coordinates are inverted. The meaning of Z inserts are discussed in Sec. III.D.2.a.

Visual inspection suggests $\rho(z)$ in Fig. 5 and all data in Fig. 3 along the laser line in Fig. 1 are a stretched version of $\rho(x)$ in Fig. 2. To validate this, data in Fig. 5 and all data in Fig. 3 are fit to (Eq. 2) using a nonlinear least-squares algorithm:

$$\rho(z) \sim \rho(x) = a_0 + b_0 * \tanh[(z - c)/\delta_0]$$

(2)

The image length was insufficient to provide data over the entire 30 mm in Fig. 5 required for fit convergence. To compensate, previously measured average freestream densities (figure 3 in [25]) measured at different runtimes during the same facility run were inserted as red circles between $z = -10$ to -15 mm. Each of these experimental densities agreed to better than 6% with GASPROPS-computed freestream values. The constant c allowed the fit to refine the visual shift guess. Solid lines in Fig. 5 are nonlinear least squares fits. Fit parameters a_0 and b_0 ($\times 10^{17}/\text{cm}^3$) and δ_0 (in millimeters) for all datasets in Fig. 3 are given in Table 1. Sequential P_0 listings in Table 1 correspond to sequential datasets acquired during each run ($t = 0$ and $t = 18$ s). Figure 3a contains insufficient data for fit convergence for near-normal shock data at $P_0 = 350$ psi.

Correlation coefficient R^2 values for all fits in Table 1 are not listed but are 99%. The shift guess values for each profile and c-coefficient fit parameters are not listed. All c values were less than 1 mm, indicating good initial shift guesses. Values for $a_0 + b_0$ and $a_0 - b_0$ are not listed but agree within 3 and 10%, respectively, with gas cap and freestream densities computed using the GASPROPS code. All these results strengthen the validity of the fits.

It is concluded in this section and assumed for the remainder of this paper that all $\rho(z)$ data along the laser line shown in Fig. 1 are spatially stretched versions of $\rho(x)$ in Fig. 2. Stated another way, $\rho(z)$ is proportional to or maps onto $\rho(x)$ for near-normal and oblique shocks for the blunt-body configuration in Fig. 1.

D. Analysis of Standard Deviation Unsteadiness Spatial Profiles

1. Unsteadiness Mechanism 1—The time-averaged spatially resolved unsteadiness profile from Fig. 4 at $P_0 = 1254$ psi (second dataset, red, $t = 18$ s) spanning the freestream, near-normal shock, and gas cap from $z = 0$ to $+20$ mm is shown in Fig. 6 (connected data points). Results are expressed as the density standard deviation. For continuity, this dataset will be analyzed and discussed in subsequent subsections. As with the data in Fig. 5, it has been shifted using previous shift guesses and computed c-coefficient fits from (Eq. 2). This results in a standard deviation maximum centered at $z = 0$.

Because no known single mechanism can create the spatial profile of Fig. 6 and all data in Fig. 4, it was concluded that two unsteadiness mechanisms are operative [15]. To deconvolve these mechanisms and calculate their spatially dependent relative magnitudes, four assumptions are required.

All data in this paper are time-averaged and spatially resolved. For the first assumption, time-averaged freestream unsteadiness under all stagnation conditions is spatially isotropic.

The freestream turbulent kinetic energy or unsteadiness is amplified across a shock wave [22]. The physical mechanism is energy transfer from kinetic modes of freestream turbulence energy (vortical, acoustic, and thermal) into higher-amplitude post-shock-wave modes of turbulence energy (vortical, acoustic, and thermal) due to rapid velocity deceleration and pressure/temperature rise. Given a weak freestream disturbance (assumed 1% in [15]), a short distance for disturbance growth (less than 3 mm on the centerline in Fig. 1), and low densities in a Mach 10 freestream, Ribner's linear interaction analysis (LIA) [16]

can be applied to model postshock unsteadiness amplification for any given shock strength, incidence angle, and unsteadiness amplitude [19]. For the second assumption, total unsteadiness can be written as a linear combination of each individual unsteadiness mechanism. In other words, each of the mechanisms operates simultaneously, has independent spatial profiles, and is linearly additive.

Previous data indicate the gas cap unsteadiness (combined instrument and TKE) scales linearly with the gas cap density (figure 10 in [15]), and hence the freestream density and stagnation pressure. Therefore, the LIA is known to apply to TKE unsteadiness passing through the postshock gas cap [22]. For the third assumption, the LIA is assumed to apply to TKE unsteadiness passing through the bow shock. Therefore, as freestream unsteadiness is transported from the freestream, through the bow shock, and into the gas cap, TKE unsteadiness grows in direct linear proportion to gas density in these regions. Figure 5 and Table 1 show the freestream, bow-shock wave, and gas cap spatial density can be modeled as a tanh function [(Eq. 2)]. Therefore, the spatial dependence of TKE amplification in the bow shock can also be modeled as atanh function. As a consequence of this assumption, all freestream TKE unsteadiness ($z \sim -8$ to -10 mm), bow-shock TKE unsteadiness ($z \sim -8$ to $+8$ mm), and gas cap TKE unsteadiness ($z \sim 8$ to 15 mm) in Fig. 6 are the results of a single unsteadiness mechanism and designated as unsteadiness mechanism 1.

Based on these three assumptions, the spatial dependence of unsteadiness mechanism 1 created by TKE amplification is designated $\sigma_1(z)$ and can be written as follows:

$$\sigma_1(z) \sim \rho(z) = a_1 + b_1 * \tanh[z/\delta_1]$$

(3)

The data in Fig. 6 contain insufficient information for fit convergence to (Eq. 3) (solid line fit) with an accurate result for δ_1 . This problem is solved as follows. First, to compensate for a limited image length in Fig. 6, which inadequately samples freestream unsteadiness, previously measured, spatially averaged freestream-unsteadiness data using LRS (figure 2B in [15]) were inserted as red data points between $z = -7.5$ and -10 mm. These results are dominated by instrument noise, and hence are independent of stagnation pressure. All values are 1.05. Errors are $\pm 5\%$ based on the average of 60 spatial data points centered around the facility centerline [15]. The imaged length is sufficiently long for 10 data points near the edges of Fig. 4e to achieve this value. Data in Fig. 6 and all other data in Figs. 4a–4d approach it.

Because Fig. 6 is a superposition of mechanisms 1 and 2, there is insufficient data between $z = -5$ to $+5$ mm for fits using (Eq. 3) to extract an accurate representation of δ_1 . For the fourth assumption, the unsteadiness width for mechanism 1 equals the density shock thickness ($\delta_1 = \delta_0$). Because, in (Eq. 3), $\sigma_1(z) \sim \rho(z)$, δ_0 coefficient values computed from (Eq. 2) fits and listed in Table 1 are used as fixed δ_1 coefficient values in (Eq. 3). Therefore, the data between $z = -5$ and $z = +5$ mm were ignored for the curve fit in Fig. 6.

The solid line in Fig. 6 is a fit to $\tanh(z)$ using Eq. (3). The a_1 and b_1 coefficients map the $\tanh(z)$ function onto the freestream ($z = -5$ to -10 mm) and gas cap ($z = +10$ to $+15$ mm) unsteadiness with a fixed δ_1 coefficient. In this fit, the δ_1 coefficient represents the unsteadiness width associated with $\sigma_1(z)$. The results of a_1 and b_1 fit parameters, reproduced values of δ_0 from Table 1, and R^2 values for all datasets in Fig. 4 for which fit convergence were obtained are given in Table 2. As in Table 1, sequential P_0 values represent data taken at sequential times ($t = 0$ and $t = 18$ s) during a single facility run. Data in Fig. 4 for $t = 0$ data for $P_0 = 651$ and 902 psi, as well as the near-normal shock, contained insufficient information for fit convergence. Typical correlation coefficient R^2 values are greater than or equal to 0.86 for the higher stagnation pressures (greater than or equal to 902 psi) where the measurements profiles are better defined.

Using fit parameters in Table 2, the results at $P_0 = 1250$ psi (second dataset, $t = 18$ s) for the near-normal shock (Fig. 4d) are presented as the black curve in Fig. 7 [total $\sigma_1(z)$ from Eq. (3)]. This is identical to the fit line in Fig. 6. Because all Table 2 results are a combination of the freestream TKE amplification (bow shock and gas cap) and instrument noise, the results in Table 2 are designated as total $\sigma_1(z)$. A separation procedure is presented.

Because the data mentioned are not shown in Fig. 7 directly, coefficients from Table 1 are used to calculate the corresponding spatial static gas density curve corresponding to $P_0 = 1250$ psi. A previous paper [15] has shown the following equation

$$\sigma(\text{instrument}) = 0.0626 * \rho(\text{static gas}) + 0.60$$

converts static gas density to instrument noise [red curve in Fig. 7 labeled instrument $\sigma_1(z)$]. A statistical analysis dictates that standard deviations cannot be subtracted. Each curve [total $\sigma_1(z)$ and instrument $\sigma_1(z)$] is first converted to its respective variance, the variances are subtracted, and the result is converted to a standard deviation. Using a nonlinear least-square fit, individual data points are fit to a $\tanh(z)$ function using Eq. (3) and presented as the green curve in Fig. 7 [deconvolved $\sigma_1(z)$]. The latter represents the deconvolved freestream TKE amplification across this shock wave. Because the gas cap and freestream unsteadiness at any given stagnation pressure in Fig. 4 are essentially equal for both shocks, only one set is analyzed using normal shock data. Because sequential datasets for a given P_0 are similar, they are used to test reproducibility. The results are presented in Table 3. Correlation coefficient R^2 values are greater than or equal to 0.9. Because total $\sigma_1(z)$ at $z = -5$ mm is within 1.5% of freestream total $\sigma_1(z)$ at $z = -10$ mm (Fig. 6), this small correction to the results was ignored.

Because $\tanh(z > 5 \text{ mm}) = 1$, the $a_1 + b_1$ column in Table 3 represents gas cap unsteadiness with the instrument noise removed. Values for $a_1 + b_1$ increase $\sim 10\%$ over the range of stagnation pressures ($P_0 \sim 50\%$ increase) listed. The errors in $a_1 + b_1$ values are estimated at 17% at $P_0 = 902$ psi and decrease slightly with increasing P_0 . Although not plotted, deconvolved $\sigma_1(z)$ are linear with P_0 . For any given pressure, $a_1 + b_1$ increases $\sim 3\%$ for successive data ($t = 0$ vs $t = 18$ s). This is attributed to model motion causing the laser to probe closer to the bow shock (Fig. 3) and sample higher unsteadiness due to bow-shock oscillation (Fig. 4).

Values for $a_1 - b_1$ represent deconvolved freestream unsteadiness. Errors associated with total $\sigma_1(z)$ and instrument $\sigma_1(z)$ have been discussed [15]. Because instrument errors contribute a large fraction to total $\sigma_1(z)$, the errors in deconvolved freestream turbulence ($a_1 - b_1$) are estimated at 25% ($\pm J\sigma$).

Values for $(a_1 + b_1)/(a_1 - b_1)$ in Table 3 represent the amplification of TKE across the bow shock. McKenzie and Wesphal [18] computed a value of 2.9 for the ratio of unsteadiness amplitudes across a density jump for a sound wave at Mach 11.3 and normal incidence. The average results in Table 3 are 1.62 ± 0.16 , or approximately half the expected value of 2.9. Because the Mach number varies to less than 1.5% [26] over this P_0 range, the Mach number variation cannot explain the discrepancy in ratios. The discrepancy in ratios is attributed to a freestream-unsteadiness measurement, i.e., total $\sigma_1(z)$ with a significant fraction (60%) of instrument noise. If true freestream unsteadiness is ~ 0.4 versus the Table 3 result of ~ 0.7 , good agreement with the expected value would be obtained. The value of ~ 0.4 is within the 50% ($\pm 2\sigma$) errors calculated for the total: instrument $\sigma_1(z)$ or $(a_1 - b_1)$ in the freestream. Hence, Table 3 ratios represent a lower limit to unsteadiness amplification across this shock.

In essence, Table 3 shows freestream-unsteadiness amplification across a shock wave can be measured using LRS. This exploratory investigation is used to show the potential of LRS instrumentation in what is believed to be the first such molecular offbody measurement of the spatial profile of TKE amplification across a shock. Data with a higher signal-to-noise ratio (reduced instrument errors) will be required to achieve more accurate results.

2. Unsteadiness Mechanism 2

a. Mechanism.: Inspection of Fig. 6 indicates a second unsteadiness mechanism $\sigma_2(z)$ is active. To determine the spatial dependence of $\sigma_2(z)$, the computed function for total $\sigma_1(z)$ using Table 2 is subtracted from the total spatial unsteadiness data [$\sigma_{\text{total}}(z)$ in Fig. 6 at $P_0 = 1254$ psi at $t = 18$ s] using Eq. (4). Procedures for subtracting standard deviations were detailed in Sec. III.D.1. Resulting data points $\sigma_2(z)$ are presented in Fig. 8:

$$\sigma_2(z) = \sigma_{\text{total}}(z) - \sigma_1(z)$$

(4)

To determine the mathematics responsible for Fig. 8, the following observations are presented. Figure 8 shows that $\sigma_2(z)$ is maximum at the inflection point of $\tanh(z)$ in Fig. 6 and all data in Fig. 4 where $d(\rho(z))/dz$ is maximum. Also, $\sigma_2(z)$ is both minimal and constant in the gas cap and freestream where $d(\rho(z))/dz = 0$ and $d(\sigma_1(z))/dz = 0$. LRS measures number density. The number density spatial derivative is the only known variable capable of producing Fig. 8 results.

A simplistic visual explanation using Fig. 5b can explain the relative spatial unsteadiness in Fig. 8. Given that the bow-shock number density is described by $\tanh$, assume that a time-averaged freestream disturbance impinging on the bow shock causes the entire $\tanh$ spatial

profile to oscillate over a locally small distance (δz) at all spatial locations. Consider three locations in Fig. 5b to include the gas cap at $z = 10$ mm, the density second derivative at $z = 3$ mm, and the density first derivative at $z = 0$ mm.

For this flowfield, it is assumed that δz is small when compared to the abscissa range of this tanh function. For a blunt-body model with a convex surface at Mach 7, Vashishtha et al. [14] observed δz to be typically small; large δz were observed in $\sim 1\%$ of the data. Large δz were correlated with and attributed to particle strikes. For the current Mach 10 flow, no particles were observed. LRS observed δz was small for greater than 95% of the results [15]. The large δz variations observed were attributed to infrequent large-scale freestream unsteadiness.

Because δz is small, density unsteadiness for the tanh function in Fig. 5 will scale as the local spatial density derivative $d\rho/dz$. In the gas cap at $z = 10$ mm, density is nearly spatially invariant, producing $d\rho/dz \sim 0$, and hence minimum unsteadiness.

For a given small δz and at the locations of the three δz shown in Fig. 5b, the slope of the density tanh(z) function at $z = 0$ mm is greater than the slope at $z = 3$ mm, which is greater than the slope in the gas cap. Because tanh(z) is continuous and can be differentiated at all z , the slope of tanh(z) that corresponds to the local spatial unsteadiness will decrease continuously from the inflection point [$z = 0$ in Fig. 5b, and the density first derivative maximum of $d\rho(z)/dz$] to the gas cap. By symmetry, it will also decrease from the inflection point ($z = 0$ in Fig. 5b) toward the freestream. These results are observed for all stagnation conditions, at all shock locations measured in Fig. 4, and in detail in Fig. 6.

Therefore, the following physical mechanism is responsible for Fig. 8 results. Hypersonic freestream disturbances impact the bow shock, and momentum transfer causes the stagnation region bow-shock density described by tanh(z) in Eq. (2) to oscillate upstream and downstream. The regions of highest-density gradient produce the regions of highest-density unsteadiness. Therefore, the second unsteadiness mechanism, designated $\sigma_2(z)$ in Fig. 8, is attributed to bow-shock-wave motion. The spatial dependence of $\sigma_2(z)$ is fit to the first derivative of $\rho(z)$ from Eq. (2) or $d(\tanh(z))/dz$. This produces a sech²(z) function in Eq. (5):

$$\sigma_2(z) \sim d\rho(z)/dz = a_2 + b_2 * \text{sech}^2[z/\delta_2]$$

(5)

By strict definition, $d\rho(z)/dz$ would produce $a_2 = 0$. In Eq. (5), a_2 is added for two reasons. It allows the $\sigma_2(z)$ fits to account for possible unsteadiness in the freestream and the gas cap not accounted for by $\sigma_1(z)$. It aids Eq. (5) fit convergence by providing a nonzero baseline. This baseline can be created by a poor signal-to-noise ratio in the $\sigma_{\text{total}}(z)$ data and/or nonoptimal $\sigma_1(z)$ fits, which create a baseline when subtracting $\sigma_{\text{total}}(z) - \sigma_1(z)$ to obtain $\sigma_2(z)$. For the near-normal shock data that have the best signal-to-noise ratios and R^2 values of typically 0.99, all a_2 coefficients in Table 4 for $\sigma_2(z)$ are effectively zero. For the oblique

shock data that have poorer signal-to-noise ratios and reduced R^2 values, many of the a_2 coefficients in Table 4 for $\sigma_2(z)$ are nonzero. The b_2 coefficient is the peak unsteadiness at the maximum density gradient, i.e., the inflection point at $z = 0$ mm. In this fit, the δ_2 coefficient represents the unsteadiness width associated with $\sigma_2(z)$. The resulting nonlinear least-squares fit is shown as the solid line in Fig. 8. The correlation coefficient (R^2) value for Fig. 8 is 0.99. The results of fitting all data in Fig. 4 using this approach are presented in Table 4. All R^2 results in Table 4 are greater than or equal to 0.93.

Based on the preceding discussion, it is concluded that all a_2 coefficients for both the near-normal and oblique shocks in Table 4 for $\sigma_2(z)$ are effectively zero. This represents the density gradient induced by unsteadiness far from the bow-shock density inflection point ($z = 0$) in both the freestream and the gas cap, which is expected to be essentially zero. Hence, $\sigma_2(z)$ unsteadiness is confined to the bow-shock wave. Also, $\sigma_2(z)$ makes no contribution to the freestream (less than -10 mm) or gas cap (greater than $+10$ mm) unsteadiness, where the spatial density derivative is essentially zero. Primary unsteadiness contributions to the freestream and gas cap are contained within the $\sigma_1(z)$ mechanism. This was one of the assumptions in Sec. III.D.1. High R^2 values in Table 4 and the lack of any visually apparent spatially unfitted data trends in Fig. 8 indicate no third unsteadiness mechanism is active.

b. Analysis of b_2 and δ_2 Coefficient Values for $\sigma_2(z)$: Fitted b_2 -coefficient values from Table 4 for $\sigma_2(z)$ for the near-normal (squares) and oblique (circles) shock waves are plotted as a function of stagnation pressure in Fig. 9. The b_2 coefficients represent the maximum bow-shock-wave amplitude unsteadiness expressed as standard deviations at the density inflection point or density first derivative maximum for $z = 0$ in Fig. 8. Data at $P_0 = 1454$ psi ($t = 18$ s) are not included in the fits for model motion reasons discussed previously [26]. Slopes and correlation coefficients obtained using a linear least-squares fit are shown in Fig. 9. Each fit was obtained by assuming an intercept of zero because turbulence is defined as zero at zero pressure, and hence zero flow.

Because b_2 coefficients are deconvolved from total unsteadiness, b_2 coefficients are not affected by instrument noise. Because no schlieren images are available for all conditions, b_2 -coefficient values are uncorrected for shock angle variation with increasing stagnation pressure and different shock locations probed due to model motion.

Both near-normal and oblique shock deconvolved b-coefficient amplitudes, and therefore peak $\sigma_2(z = 0)$ amplitudes, are linearly dependent on stagnation pressure, and hence freestream density. Assuming linear wave theory [18], the frequency and amplitude of shock oscillations should correspond to the frequency and amplitude of the incident flow disturbances. It is concluded that each of these amplitudes is a direct measure of, and scales linearly with, time-averaged facility freestream turbulence. Also, freestream turbulence must be increasing linear with stagnation pressure and freestream density.

The ratio of slopes ($b_{2\text{normal}}/b_{2\text{oblique}}$) is 1.39. This is consistent with previous observations that shock unsteadiness amplitude is shock angle dependent, being maximum for the near-normal shock and reduced for the oblique shock [17]. Because the schlieren image in Fig. 2 and the Rayleigh data were not acquired simultaneously, there is uncertainty

regarding the location of the laser beam relative to the bow shock. The result is uncertainty in determining the near-normal shock angle. Figure 1 contains no information on the oblique shock angle. Therefore, this ratio from measured shock angles cannot be obtained to confirm the Fig. 9 slope ratio. Future planar LRS density measurements are recommended to acquire this information.

Increasing the stagnation pressure will modestly flatten the near-normal shock. The correlation coefficients in Table 4 indicate this effect on b coefficients is negligible for the near-normal shock data. Although fit to a straight line, visual inspection of the correlation coefficient for the oblique shock data suggests the data may be curved. If true, this curvature is attributed to significant stagnation pressure-induced bow-shock angle variation. If schlieren images are available to correct b coefficients for shock angle, the author believes all b_2 coefficients for both the near-normal and oblique shocks at each stagnation pressure will be identical.

All data in Table 4 are acquired off the model centerline. Maximum unsteadiness is expected for a normal shock at the sting/model centerline. Therefore, Table 4 represents lower limits to both the facility freestream turbulence and bow-shock oscillation amplitudes.

In Eq. (5), the δ_2 coefficient represents the unsteadiness width associated with $\sigma_2(z)$. Table 4 shows the computed unsteadiness width (δ_2 coefficient) as a function of stagnation pressure for both the normal and oblique shocks. Based on results spanning 650–1254 psi,

$$\delta_2(\text{normal shock}) = 4.08 \pm 0.21 (\pm 1\sigma)$$

$$\delta_2(\text{oblique shock}) = 3.44 \pm 0.84 (\pm 1\sigma)$$

and

$$\delta_2(\text{normal})/\delta_2(\text{oblique}) = 1.19 \text{ for } \sigma_2(z)$$

To the first order, unsteadiness widths are independent of the stagnation pressure and shock angle within the errors of this experiment. To the second order, inspection of δ_2 coefficients in Table 4 shows a weak linear decrease with increasing stagnation pressure for both the near-normal and oblique shocks. LRS is observing weak stagnation pressure-dependent changes in the shock shape, which compress the oblique relative to the normal shock δ_2 coefficients along the z axis.

Previous results obtained with physical probes measuring unsteadiness behind a bow shock in a gas cap conclude the freestream unsteadiness (due presumably to TKE amplification) exponentially decreases with increasing stagnation pressure [35,36]. Offbody molecular-based measurements in Fig. 9 based on peak bow-shock oscillation measurements ($z = 0$) show the freestream unsteadiness linearly increases with increasing stagnation pressure. A similar, albeit weak, linear trend is observed for the deconvolved unsteadiness amplification

coefficients for $\sigma_1(z)$ in Table 3 in the gas cap with increasing stagnation pressure. Further study is required to resolve these discrepancies.

3. Total Unsteadiness: Mechanisms One and Two

a. Data and Fits.: Based on the results in this paper, the time-averaged spatial unsteadiness profiles in Fig. 4 spanning the freestream, bow-shock wave, and gas cap consist of two mechanisms. The computed fits for each mechanism [total $\sigma_1(z)$ from Table 2 and $\sigma_2(z)$ for Table 4 at $P_0 = 1254$ psi and $t = 18$ s] are shown in Fig. 10a. The total unsteadiness $\sigma_{\text{total}}(z)$ is written as a linear combination of two independent and simultaneous unsteadiness mechanisms given by Eq. (6):

$$\begin{aligned}\sigma_{\text{total}}(z) &= \sigma_1(z) + \sigma_2(z) \\ &= a_1 + b_1 * \tanh[z/\delta_1] + b_2 * \text{sech}^2[z/\delta_2]\end{aligned}$$

(6)

Experimental data points and the computed $\sigma_{\text{total}}(z)$ line from fit parameters in Tables 2 and 4 are shown in Fig. 10b. The R^2 value is 0.99. Similar R^2 fit results are obtained for all data in Fig. 4. This indicates good agreement between the experimental data and the assumptions stated to analyze these data. Procedures for combining standard deviations were detailed in Sec. III.D.1.

Both unsteadiness mechanisms [$\sigma_1(z)$ and $\sigma_2(z)$] derive from the time-averaged spatial density profile $\rho(z)$ as flow transitions from the freestream through the bow shock and into the gas cap. Note that $\rho(z)$ is given as $\tanh(z)$ in Eq. (2).

Figure 10a shows the primary stronger unsteadiness $\sigma_2(z)$ is bow-shock-wave location oscillation. The spatially dependent magnitude of unsteadiness of the bow-shock-wave oscillation is determined by the local gradient in the mean density profile, i.e., $d\rho(z)/dz$ or $d(\tanh(z))/dz$. The shock location unsteadiness is independent of the local density unsteadiness, but the effect of the shock location unsteadiness on the density field is determined by the local gradient in the mean density profile. This dominates across the bow shock from $z = -3$ to $+2.5$ mm, where density derivatives are maximum. The fractional contribution of $\sigma_2(z)$ to the freestream ($z < -10$ mm) and gas cap ($z = +10$ to $+15$ mm) unsteadiness is essentially zero; density gradients are negligible at these locations.

Figure 10a shows that secondary weaker unsteadiness $\sigma_1(z)$ is the result of freestream unsteadiness being amplified as it transitions from the freestream, through the near-normal shock, and into the gas cap. Its effect scales in direct proportion to the local density across the bow shock and has the same spatial profile as the density spatial profile given by $\tanh(z)$.

It is an unfortunate and confusing coincidence that the spatial profile of mechanism 2 [i.e., $\text{sech}^2(z)$] appears to be the derivative of the spatial profile of mechanism 1 [i.e., $\tanh(z)$].

This is not true. In fact, each mechanism is spatially independent of the other and occurs simultaneously.

For comparison with the data, Fig. 10a uses total $\sigma_1(z)$, for which the freestream is dominated by instrument noise. Deconvolved $\sigma_1(z)$ reduces total $\sigma_1(z)$ in Fig. 10a by nearly a factor of two. When compared to deconvolved $\sigma_1(z)$, $\sigma_2(z)$ unsteadiness will dominate across nearly the entire bow-shock wave in Fig. 10b and all data in Fig. 4 from $z = -5$ to $z = +5$ mm.

Figure 10a displays significant mathematical symmetry. Note that $\text{sech}^2(z)$ or $\sigma_2(z)$ is symmetric about $z = 0$, whereas $\tanh(z)$ or $\sigma_1(z)$ is antisymmetric about $z = 0$. Because $\sigma_2(z)$ is dominant, the total bow-shock unsteadiness has significant symmetry.

Tables 2 and 4 allow comparison of the fitted bow-shock unsteadiness widths (δ coefficient values) for each mechanism for the near-normal and oblique shocks. For the normal shock, $\delta(\sigma_1(z)) = 3.60 \pm 0.18$ and $\delta(\sigma_2(z)) = 4.08 \pm 0.21$. For the oblique shock, $\delta(\sigma_1(z)) = 2.50 \pm 0.14$ and $\delta(\sigma_2(z)) = 3.44 \pm 0.84$. Errors represent $(\pm 1\sigma)$. Comparing $\delta(\sigma_1(z))$ to $\delta(\sigma_2(z))$ at each shock angle, these values are identical within errors of $\pm 2\sigma$. Therefore, with the listed assumptions, the bow-shock unsteadiness width is independent of the stagnation pressure and unsteadiness mechanism at the locations probed within the measurement uncertainties.

Freestream unsteadiness at hypersonic speeds impacts the bow-shock wave and forces it toward the model. The incompressible and subsonic gas cap relieves this local and essentially instantaneous pressure increase by moving fluid in all directions by collisional transfer, but mostly away from the model. The observed time-averaged shape and symmetry of mechanism 2 is the result of these two forces. In contrast, turbulent kinetic energy transfer to the density field by mechanism 1 near the bow-shock density inflection point ($z = 0$) is much weaker. Mechanism 1 cannot force fluid upstream. Mechanism 2 contributes a small fraction of the total unsteadiness observed in the gas cap, which decays exponentially with the extent of penetration.

Figure 10a shows that unsteadiness mechanism 2 [$\sigma_2(z)$ or bow-shock unsteadiness] created by freestream unsteadiness can contribute to time-averaged bow-shock-wave thickness. Because all bow-shock thickness measurements in the literature were performed using time-averaged methods, all bow-shock thickness measurements should be considered upper limits to true bow-shock thickness. Statistically, unsteadiness results in this study are identical within experimental errors. However, assuming $\delta(\sigma_1(z)) = 3.60$ (i.e., the unsteadiness width from mechanism 1 was assumed equal to the shock thickness from density curves), $\delta(\sigma_2(z)) = 4.08$ (shock unsteadiness thickness from sech^2 fits), and near-normal shock results represent the true values, the results crudely suggest ~10–0% of the time-averaged bow-shock thickness is due to freestream unsteadiness in the 31M10 facility.

b. Relative Mechanism Coupling Efficiencies of Freestream Unsteadiness.: Table 5 compares the coupling efficiency of freestream-unsteadiness energy into each individual mechanism at the location of maximum unsteadiness for each individual mechanism. This occurs in the gas cap for $\sigma_1(z)$ and the bow-shock number density inflection point at $z = 0$

for $\sigma_2(z)$. Using Eqs. (3) and (5), maximum unsteadiness is given as deconvolved σ_1 (gas cap) = $a_1 + b_1$ and $\sigma_2(z=0) = b_2$. Data are taken from Tables 3 and 4.

Peak unsteadiness and relative unsteadiness ratios [i.e., $\sigma_2(z=0)/\sigma_1$ (gas cap)] are listed in Table 5 for both near-normal and oblique shocks. It is noted that all flowfield state variables (T, ρ, V, P) are different at the point of maximum unsteadiness for each mechanism. Because

$$\text{sech}^2(z=0) = \tanh(\text{gas cap}) = 1$$

the amplitude ratio is independent of the mathematical functions associated with each mechanism.

Because LRS data represent time-averaged results, this analysis assumes time-averaged three-dimensional isotropic freestream turbulence. Assuming LIA theory, computed unsteadiness amplitudes at any given point in the bow shock and gas cap should each be proportional to freestream turbulence. As concluded in Sec. III.D.2.b, freestream unsteadiness increases in direct proportion to freestream pressure. Therefore, the unsteadiness amplitude at the location of maximum unsteadiness should increase linearly with stagnation pressure for both the near-normal and oblique shock data.

The data in Table 5 are not consistent with these expectations. They show the ratio increasing with increasing P_0 . This is attributed to errors in σ_1 (gas cap) due to fit errors in the a coefficient, which is heavily affected by instrument noise. Therefore, because unsteadiness is maximum at $P_0 = 1454$ psi and results are expected to have the least error at $P_0 = 1454$ psi, the only conclusion from Table 5 is that the ratio at $P_0 = 1454$ psi represents lower limits to the ratio of $\sigma_2(z=0)/\sigma_1$ (gas cap) at the locations stated for each shock location probed. It also supports the assumption that each mechanism is independent.

To further understand these independent unsteadiness amplification mechanisms, consider the unsteadiness for each mechanism at $z=0$. At a single point in space, all flowfield state parameters are common to both mechanisms. This includes all flowfield state variables (T, ρ, V, P) and the freestream unsteadiness of a given value. Because $\tanh(z=0) = 0$, $\sigma_1(z=0) = a$. Computed values for a that represent $\sigma_1(z=0)$ are taken from Table 3. Essentially, this value is 0.9, independent of stagnation pressure. Hence, ratios in Table 5 are only increased by $\sim 1.1/0.9$ or $\sim 20\%$ at $z=0$. As previously mentioned, this is due to a large instrument error for freestream unsteadiness. Assuming the TKE amplification ratio of 2.9 discussed previously is correct, the freestream unsteadiness would be 0.38 at 1254 psi, $\sigma_1(z=0) = 0.74$, and $\sigma_2(z)/\sigma_1(z) = 5.7$. This ratio is only $\sim 50\%$ greater than the results in Table 5. Overall, this suggests that the momentum transfer from freestream turbulence to the bow shock (bow-shock oscillation) at the location of the maximum density gradient produces $\sim 5 \pm 1$ times greater unsteadiness than TKE amplification at the $z=0$ spatial location.

IV. Conclusions

This study demonstrates the first application of laser Rayleigh scattering to observe, quantify, and provide mechanisms along with mathematics to explain and model tunnel-

noise-induced bow-shock and gas cap oscillations at Mach 10. Time-averaged spatially resolved density and density unsteadiness are observed along a line. This approach samples flow in the freestream, through the bow shock, and in the gas cap ahead of a blunt-body model. Both near-normal shock and oblique shock data are acquired simultaneously. Line data are acquired vertically along the z axis. Fits of the z -axis spatial density profile to a $\tanh(z)$ function demonstrate the density profile along the z axis is a stretched version of the standard streamwise or x -axis density profile. Z -axis imaging effectively expands the bow-shock wave and allows observation of relevant features within the spatial resolution of typical intensified camera instrumentation.

Unsteadiness is expressed as the density standard deviation. Amplitude and not frequency results are presented. Two different unsteadiness mechanisms are observed as flow passes from the freestream, through the bow shock, and into the gas cap. Both observed unsteadiness mechanisms are initiated by freestream turbulence. The first mechanism and lower-amplitude effect is the well-known amplification of turbulent kinetic energy as flow transitions from the ~ 1500 m/s freestream to the stagnation region. It increases in direct linear proportion to gas density in the bow shock and gas cap. It is described by a $\tanh(z)$ function. The second mechanism and higher-amplitude effect is the physical oscillation of the bow shock. It scales with the spatial first derivative of density and is described by a $\text{sech}^2(z)$ function. Both mechanisms are shock angle dependent. The total unsteadiness [fit using $\tanh(z) + \text{sech}^2(z)$] is expressed as the number density standard deviation and modeled as a linear combination of the latter two independent, simultaneous, and nonlinear unsteadiness mechanisms.

Because both mechanisms are driven by common freestream turbulence, the ratio of energy coupling into each mechanism at any given spatial location is constant. Total amplitudes at any spatial location will increase with increasing freestream turbulence, freestream density, and stagnation pressure. Still, different results are observed as a function of spatial location. This indicates each mechanism is spatially independent of the other, occurs simultaneously, is modeled using different mathematical functions, increases linearly with freestream turbulence, and is linearly additive. At the bow-shock location where the spatial density derivative is maximum, and hence mechanism 2 is maximum, the ratio of their standard deviation amplitudes (mechanism 2/mechanism 1) is estimated to be 5 ± 1 . Laser Rayleigh scattering permits quantification of spatially resolved unsteadiness, unsteadiness amplification across the shock, and dual mechanisms, along with the amplitude and width of unsteadiness associated with each mechanism.

This paper quantifies the relationships between unsteadiness mechanism coefficients and various flowfield and tunnel parameters. The bow-shock oscillation amplitude is linearly correlated with freestream turbulence, freestream density, and stagnation pressure. Assuming a linear amplification model of transition and turbulence, the time-averaged bow-shock oscillation amplitude is a new, nonintrusive, offbody, and direct measure of time-averaged freestream turbulence. Instantaneous images show instantaneous turbulence levels.

Assuming time-averaged isotropic freestream unsteadiness, each streamline impinging on a hypersonic bow shock has an unsteadiness spatial profile described by the sum of a sech^2

and tanh function. The maximum oscillation in this flowfield is due to bow-shock oscillation, i.e., sech^2 function. The unsteadiness spatial profile amplitude across the bow-shock wave is driven by the local shock angle, which affects the density spatial derivative. Therefore, a curved bow-shock wave exhibits a unique localized unsteadiness amplitude, which varies spatially with shock angle along the entire shock front. For time-averaged unsteadiness, the bow shock converts isotropic freestream unsteadiness into anisotropic bow-shock unsteadiness. At hypersonic speeds, different parts of the bow shock cannot communicate via collisions; this maintains the bow-shock spatial anisotropic unsteadiness.

Logically, one would expect the nonuniform bow-shock spatial unsteadiness to move into the gas cap to create a time-averaged unsteadiness profile that varies spatially with the shock angle. Counterintuitively, time-averaged gas cap unsteadiness within the errors in this study is nearly spatially uniform, approximately independent of the bow-shock angle and local oscillation amplitude, and increases linearly with local density. Hence, the gas cap to the first order recreates the time-averaged isotropic unsteadiness of the freestream. The magnitude of gas cap unsteadiness is attributed primarily to amplification of the freestream turbulent kinetic energy across the shock.

The deconvolved spatial profiles of unsteadiness for mechanism 2 [i.e., $\sigma_2(z)$] show that the time-averaged bow-shock oscillation amplitude decays exponentially with increasing gas cap penetration. Therefore, time-averaged bow-shock oscillation is a spatially localized phenomena. Given a sufficient distance behind the bow shock, time-averaged bow-shock unsteadiness has essentially no effect or, at best, a minimal secondary effect on time-averaged gas cap unsteadiness. The effect decreases exponentially from the bow-shock maximum density derivative toward the model surface and becomes zero at locations in the gas cap where the density derivative is zero.

Assuming time-dependent anisotropic unsteadiness, each stream-line impinging on the hypersonic bow shock will have a unique unsteadiness character. Localized instantaneous strikes push the bow shock toward the model surface and create instantaneous gas cap unsteadiness. In the stagnated subsonic gas cap flow, there is a damping mechanism whereby localized unsteadiness is partially dispersed by gas collisional energy transfer. This process will reduce localized anisotropic unsteadiness (density waves) induced in the gas cap from bow-shock oscillations but not eliminate time-dependent unsteadiness.

Phenomenologically, this study shows freestream turbulence impacts the bow shock and creates a physical movement. A simple momentum transfer argument from the 1500 m/s freestream to the near-zero velocity gas cap supports the experiment result. Therefore, freestream turbulence impacts contribute to time-averaged bow-shock thickness. All literature measurements of bow-shock thickness are based on time-averaged measurements. Bow-shock thickness measurements at a given Mach number in different facilities will be affected by the facility-dependent level of freestream turbulence. Therefore, all current measurements of bow-shock thickness are upper limits to true bow-shock thickness. In the future, all bow-shock-wave thickness measurements should be obtained in freestreams with minimal unsteadiness using high-spatial-resolution instantaneous molecular-based methods.

Acknowledgments

The author thanks Mark Kulick for providing the expert mechanical design and assembly of structures to support the laser, camera, and optics. The author thanks Steve Jones, Principal Investigator Paul Danehy, and his research group for providing the schlieren image. The author is indebted to Peter Gnoffo for many helpful discussions. The use of trademarks or names of manufacturers in this report is for accurate reporting and does not constitute an official endorsement, either expressed or implied, of such products or manufacturers by NASA.

Nomenclature

a, b, c	density and density unsteadiness mechanism fitting coefficients, ($\times 10^{17}/\text{cm}^3$) molecules/ cm^3
$\tilde{n}$	density midpoint or median between freestream and gas cap
P_0	stagnation pressure, psi
R^2	correlation coefficient
T	temperature, K
T_0	stagnation temperature, K
t	time, s
x	streamwise distance along model centerline, mm
z	vertical distance from model centerline, mm
δ	bow-shock-wave thickness, mm
λ	freestream mean free path, mm
$\rho(x), \rho(z)$	density, molecules/ cm^3
$\sigma_{\text{total}}(z)$	total flowfield standard deviation, molecules/ cm^3 ; $\sigma_1(z) + \sigma_2(z)$
$\sigma_1(z)$	mechanism 1 standard deviation (turbulent kinetic energy amplification), molecules/ cm^3
$\sigma_2(z)$	mechanism 2 standard deviation (bow-shock oscillation), molecules/ cm^3

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Fig. 1.
Composite image: model, bow-shock schlieren image, and LRS image.

Fig. 2.
Hypersonic bow-shock-wave density profile.

Fig. 3.
LRS bow-shock and gas cap density $\rho(z)$ profiles.

Fig. 4.
LRS density standard deviation $\sigma(z)$ profiles.

a) Near-normal shock wave

b) Oblique shock wave

Fig. 5.
LRS density data and $\tanh(z)$ fit for $P_0 = 1254$ psi.

Fig. 6.
Total unsteadiness data and mechanism 1 $\sigma_1(z)$ fit (TKE amplification) for $P_0 = 1254$ psi.

Fig. 7.
Total, instrument, and total-instrument standard deviation amplification for $\sigma_1(z)$ for $P_0 = 1254$ psi.

Fig. 8. Unsteadiness mechanism 2 data and computed $\text{sech}^2(z)$ fit (bow-shock oscillation) for $P_0 = 1254$ psi.

Fig. 9.
Bow-shock peak unsteadiness versus stagnation pressure.

Fig. 10. Representations of a) unsteadiness mechanisms fit, and b) total unsteadiness data and $\tanh(z) + \text{sech}^2(z)$ fit.

Table 1Near-normal and oblique shock-wave fit coefficients to $\tanh(z)$ function

Near-normal shock				Oblique shock			
P_0 , psi	a_0	b_0	δ_0	P_0 , psi	a_0	b_0	δ_0
350	— —	— —	— —	350	3.066	2.211	2.6831
651	5.498	4.052	3.5398	651	5.726	4.165	2.2366
651	5.559	4.014	3.6390	651	5.692	4.145	2.5278
902	8.196	5.849	3.9277	902	8.159	5.772	2.4913
902	8.001	5.651	3.5448	902	7.943	5.604	2.5511
1254	10.75	7.652	3.4483	1254	10.64	7.553	2.5833
1254	10.71	7.672	3.4758	1254	10.62	7.544	2.6281
1454	11.68	8.116	3.0497	1454	11.41	7.864	2.4201
1454	11.36	7.967	3.3014	1454	10.96	7.723	2.5947

Table 2Fit parameters for total $\sigma_1(z)$

P_0 , psi	Near-normal shock				Oblique shock			
	a_1	b_1	δ_1	R^2	a_1	b_1	δ_1	R^2
651	—	—	—	—	3.54	—	—	—
651	1.14	0.094	3.64	0.68	1.14	0.09	2.53	0.68
902	—	—	—	—	3.93	—	—	—
902	1.42	0.419	3.54	0.91	1.44	0.39	2.55	0.87
1254	1.56	0.511	3.45	0.92	1.56	0.51	2.58	0.86
1254	1.56	0.521	3.48	0.98	1.57	0.43	2.63	0.94
1454	1.66	0.529	3.05	0.92	1.58	0.47	2.42	0.84
1454	1.62	0.574	3.30	0.88	1.57	0.53	2.59	0.91

Table 3

$\sigma_l(z)$ unsteadiness amplification coefficients (total minus instrument)

P_{00} psi	a_1	b_1	δ_1	R^2	$a_1 + b_1$	$a_1 - b_1$	$(a_1 + b_1)/(a_1 - b_1)$
902	0.905	0.162	2.49	0.90	1.07	0.743	1.44
902	0.901	0.217	3.54	0.99	1.12	0.684	1.63
1254	0.896	0.206	3.45	0.99	1.10	0.690	1.60
1254	0.904	0.223	3.46	0.99	1.13	0.682	1.65
1454	0.980	0.198	3.05	0.99	1.18	0.783	1.50
1454	0.944	0.290	3.30	0.99	1.23	0.654	1.89

Table 4Fit parameters for $\sigma_2(z)$

P_0 , psi	Near-normal shock				Oblique shock			
	a_2	b_2	δ_2	R^2	a_2	b_2	δ_2	R^2
651	— —	— —	— —	— —	0.069	1.87	4.89	0.95
651	-0.018	2.32	4.14	0.99	0.473	1.54	3.86	0.95
902	— —	— —	— —	— —	0.452	2.26	2.91	0.95
902	0.000	3.24	4.04	0.99	0.409	2.30	2.96	0.95
1254	0.055	4.21	3.81	0.99	0.546	2.87	2.57	0.97
1254	0.010	4.25	4.31	0.99	0.391	2.91	3.46	0.98
1454	0.017	4.90	3.77	0.99	0.051	3.35	2.90	0.97
1454	-0.135	4.67	5.25	0.93	0.150	3.88	3.93	0.95

Table 5

Unsteadiness coefficient peak amplitudes and lower-limit ratios

P_0 , psi	Near-normal shock			Oblique shock	
	σ_1 (gas cap)	σ_2 ($z = 0$)	Ratio	σ_2 ($z = 0$)	Ratio
	$a_1 + b_1$	b_2	σ_2/σ_1	b_2	σ_2/σ_1
902	1.07	— —	— —	2.26	2.12
902	1.12	3.24	2.90	2.30	2.06
1254	1.10	4.21	3.82	2.87	2.60
1254	1.13	4.25	3.77	2.91	2.58
1454	1.18	4.90	4.16	3.35	2.84
1454	1.23	4.67	3.78	3.98	3.23