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Identity of attached eddies in turbulent channel flows with bidimensional empirical mode decomposition

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Abstract

Bidimensional empirical mode decomposition (BEMD) is used to identify attached eddies in turbulent channel flows and quantify their relationship with the mean skin-friction drag generation. BEMD is an adaptive, non-intrusive, data-driven method for mode decomposition of multiscale signals especially suitable for non-stationary and nonlinear processes such as those encountered in turbulent flows. In the present study, we decompose the velocity fluctuations obtained by direct numerical simulation of channel flows into BEMD modes characterized by specific length scales. Unlike previous works (e.g. Flores & Jiménez, *Phys. Fluids*, vol. 22(7), 2010, 071704; Hwang, *J. Fluid Mech.*, vol. 767, 2015, pp. 254–289), the current approach employs naturally evolving wall-bounded turbulence without modifications of the Navier–Stokes equations to maintain the inherent turbulent dynamics, and minimize artificial numerical enforcement or truncation. We show that modes identified by BEMD exhibit a self-similar behaviour, and that single attached eddies are mainly composed of streaky structures carrying intense streamwise velocity fluctuations and vortex packets permeating in all velocity components. Our findings are consistent with the existence of attached eddies in actual wall-bounded flows, and show that BEMD modes are tenable candidates to represent Townsend attached eddies. Finally, we evaluate the turbulent-drag generation from the perspective of attached eddies with the aid of the Fukagata–Iwamoto–Kasagi identity (Fukagata *et al.*, *Phys. Fluids*, vol. 14(11), 2002, pp. L73–L76) by splitting the Reynolds shear stress into four different terms related to the length scale of the attached eddies.

Keywords

boundary layer structure; turbulent boundary layers

1. Introduction

Wall-bounded turbulence is often regarded as a high-dimensional, nonlinear dynamical system of interacting eddies of different sizes. One prominent theoretical model to describe the multiscale nature of wall-bounded turbulence is the attached eddy hypothesis, first introduced by Townsend (1961, 1976). Inspired by the well-known fact that the mean

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streamwise velocity in turbulent boundary layers exhibits a wall-normal logarithmic dependence, Townsend (1976) hypothesized that the sizes of the energy-containing eddies (or flow motions) in the so-called logarithmic layer (log layer) are proportional to the distance to the wall, y . The roots of these eddies reach into the near-wall region, i.e. they are ‘attached’ to the wall, from where the concept of attached eddy stems.

Similar to the mean streamwise velocity, Townsend’s theoretical analysis of the turbulence intensities predicts a logarithmic dependence with y of the wall-parallel velocity fluctuations, i.e. the streamwise and spanwise velocity fluctuations, while the wall-normal velocity fluctuations and Reynolds shear stress remain constant in y . Hence, a key feature of the turbulence intensities associated with attached eddies is the differentiation between the contribution from the velocity components parallel to the wall, which are significantly large in the near-wall region, and a component related to the wall-normal velocity fluctuations and Reynolds shear stress, whose intensities are small in the region close to the wall. A second prominent feature of attached eddies is their self-similarity with respect to the wall-normal position.

The attached eddy hypothesis was extended by Perry and co-authors (Perry & Chong 1982; Perry, Henbest & Chong 1986; Andelman *et al.* 1987; Perry & Marusic 1995). Perry *et al.* (1986) modelled attached eddies as λ vortices and predicted the one-point statistics and spectra of wall-bounded turbulence (Perry & Chong 1982; Perry *et al.* 1986). They found that the attached eddy model is consistent with a k_x^{-1} scaling of the one-dimensional (1-D) streamwise velocity spectra at high Reynolds numbers, where k_x is the streamwise wavenumber. Recently, a more rigorous framework for the attached eddy hypothesis has been proposed by Marusic & Woodcock (2014) and Woodcock & Marusic (2015).

In the past two decades, additional evidence has emerged suggesting the existence of attached eddies in wall turbulence. The streamwise turbulent intensity in the near-wall region has been shown to increase logarithmically with Re_τ (where Re_τ is the friction Reynolds number) at high Reynolds number (Metzger & Klewicki 2001; Marusic & Kunkel 2003; Hoyas & Jiménez 2006; Hutchins & Marusic 2007), consistently with the deduction of the attached eddy model (Marusic & Kunkel 2003). The wall-normal and spanwise components also agree with the predictions given by the attached eddy theory at lower Reynolds number (Jiménez & Hoyas 2008; Lee & Moser 2015). Numerical and experimental results have also shown that the spanwise integral length of the energy-containing motions grows linearly with the distance to the wall (Tomkins & Adrian 2002; Del Álamo *et al.* 2004; Hoyas & Jiménez 2006; Pirozzoli & Bernardini 2013; Lee & Moser 2015), again in accordance with Townsend’s predictions. Reviews of this controversial but far-reaching model can be found in Smits, McKeon & Marusic (2011), Tardu (2011), Jiménez (2012, 2013, 2018) and Marusic & Monty (2019).

Attached eddies have been traditionally interpreted as statistical entities (Marusic *et al.* 2010; Smits *et al.* 2011), but recent works suggest that they can also be identified as instantaneous features of the flow. In a series of works, Del Álamo *et al.* (2006), Lozano-Durán, Flores & Jiménez (2012), Lozano-Durán & Jiménez (2014b) and Lozano-Durán &

Borrell (2016) studied the spatial organization and temporal evolution of instantaneous eddies responsible for the momentum transfer in turbulent channels up to $Re_\tau = 4200$. They showed that there is a family of eddies that become large enough to get attached to the wall while reaching into the log layer. These eddies are geometrically self-similar, with sizes and lifetimes proportional to their distance from the wall as expected for Townsend's attached eddies. Following a different approach, Hellström, Marusic & Smits (2016) analysed the experimental data of turbulent pipe flow with proper orthogonal decomposition, and concluded that the modes with different wall-normal length scales have self-similar characteristics. Using spectral analysis, Agostini & Leschziner (2017) showed that the attached eddy framework may be extensive to energy eddies in the mesolayer of wall-bounded flows.

In spite of the achievements of previous works, the identification of individual eddies from the continuum of multiscale dynamical processes, and consequently the investigation of their dynamics, still remains a challenging task (Jiménez 2018). Numerical experiments with modified flow equations or constrained domains is an appealing tool to isolate eddies, and numerous simulations have been conducted with the purpose of quantifying the energy-containing motions in the log layer independently of the near-wall cycle or the largest scales (Flores & Jiménez 2006; Flores, Jiménez & Del Álamo 2007). It is known that the energy-containing motions at a given spanwise length scale are self-sustained in the absence of the motions at other scales (Flores & Jiménez 2010; Hwang & Cossu 2010, 2011). Based on this self-sustaining property, Hwang (2015) performed numerical experiments based on overdamped large-eddy simulations (OLES) and domain minimization coupled with explicit filtering. This technique is capable of removing artificially turbulent motions at the designed spanwise length scales and hence of separating the energy-containing eddies hypothesized by Townsend from the rest of the flow. Lozano-Durán *et al.* (2012) and Hwang (2015) concluded that a single attached eddy is composed of two elements. The first is a long streaky structure which mainly carries the streamwise velocity fluctuations and whose streamwise and spanwise length scales (λ_x and λ_z) are

$$y \approx 0.1\lambda_z \text{ and } \lambda_x \approx 10\lambda_z.$$

(1.1a,b)

The second element is a vortex packet with contributions to the three velocity fluctuations. The associated wall-normal position and streamwise length scale of a vortex packet can be roughly described by

$$y \approx (0.5 - 0.7)\lambda_z \text{ and } \lambda_x \approx (2 - 3)\lambda_z.$$

(1.2a,b)

Both the streak and vortex packet are self-similar, with sizes proportional to their spanwise length scale. Based on these scalings, Hwang (2015) further inferred that the smallest attached eddies in wall-bounded turbulence correspond to $\lambda_z^+ \approx 100$ (where $^+$ denotes wall units) coinciding with the streaks and quasi-streamwise vortices observed in the buffer layer. The sizes of the largest attached eddies scale in outer units, $\lambda_z \approx 1.5\delta$ (with δ the channel half-height), and they can be classified as large-scale motions (LSMs) and very-large-scale motions (VLSMs). Recently, Hwang & Bengana (2016) studied the self-sustaining process of attached eddies in the logarithmic and outer layers and found that their dynamical cycle is similar to the well-known near-wall cycle (Jiménez & Moin 1991; Hamilton, Kim & Waleffe 1995; Jiménez & Pinelli 1999; Schoppa & Hussain 2002).

Nonetheless, it is important to remark that many of the aforementioned analyses are based on ‘artificial’ numerical calculations with enforced or imposed scale-based truncation, instead of on naturally evolving turbulence. Moreover, interactions among energy-containing motions are not fully resolved by the OLES technique, and Feldmann & Avila (2018) have recently argued that OLES cannot be used to faithfully investigate VLSMs. Instead, their analysis suggested that the large smooth streaks observed by OLES may be modified near-wall streaks rather than VLSMs.

The following question of practical importance is how attached eddies work individually and cooperatively to generate mean skin-friction drag, which can be regarded as one of the most relevant quantities in aero- and hydrodynamics. Owing to the difficulty of discerning and extracting individual attached eddies, only a small number of works have addressed this issue to date. Hwang (2013) artificially removed the motions with length scales larger than $\lambda_z^+ \approx 100$ and found that these motions play an important role in the generation of wall friction for increasing Reynolds numbers. De Giovanetti, Hwang & Choi (2016) extended the work and showed that removing LSMs and VLSMs at $Re_\tau = 2000$ only results in 5–8 % drag reduction, while estimations using the Fukagata–Iwamoto–Kasagi (FIK) identity-based method (Fukagata, Iwamoto & Kasagi 2002) showed that these eddies are responsible for 20–30% of the total skin-friction drag.

Encouraged by the above limitations, we aim to utilize a novel data analysis technique to identify attached eddies and quantify their contributions to the mean skin-friction drag generation. Typically, Fourier or wavelet analysis might be tempting tools to carry out such a study. In these approaches, signals are decomposed into modes with a given wavelength. However, these methods rely on *a priori* basis functions and suffer from inflexibility for complex and transient signals. In contrast, adaptive mode decomposition techniques are *a posteriori*, data-driven approaches (Feng, Zhang & Zuo 2017) free from pre-established basis functions and, as such, they adapt to the transient features and local characteristics of the signals. Adaptive decomposition methods represent the original signal as a superposition of several mono-components and a residual, and the resulting instantaneous frequency and amplitude of each mono-component are useful to study the characteristic scales of the problem. In summary, adaptive mode decomposition methods are data-driven techniques where the characteristic wavelengths of the signals are automatically determined.

Among the various adaptive decomposition methods, empirical mode decomposition (EMD), first proposed by Huang *et al.* (1998), is the most extensively used in the field of fluid mechanics. Huang *et al.* (2008) adopted 1-D EMD to study the scaling properties and intermittency of homogeneous turbulence, while Ansell & Balajewicz (2016) employed EMD to analyse the features of large-scale vortex structures in a turbulent mixing layer. Agostini & Leschziner (2014, 2016) used bidimensional empirical mode decomposition (BEMD) to analyse the modulation of LSMs on the small-scale eddies in the near-wall region. Recently, Agostini & Leschziner (2018) adopted BEMD to decompose the streamwise velocity fluctuations and analyse the influence of large-scale outer structures on the near-wall turbulence in a drag-reduction configuration with spanwise-oscillating walls. They found an intermediate mode consistent with the hierarchy of wall-attached eddies. However, the authors did not investigate the self-similar and geometrical properties of the BEMD modes. In the present study, we employ BEMD in turbulent channel flows to decompose the velocity fluctuations in the homogeneous directions (x – z), and provide a comprehensive investigation of wall-attached eddies. Finally, we quantify the contribution of attached eddies to the mean skin-friction drag generation.

The remainder of the article is organized as follows. In § 2, we introduce the numerical algorithms of the direct numerical simulation (DNS). The methodology of the BEMD is provided in § 3. The investigation of attached eddies from BEMD results is presented in § 4. In § 5, we discuss the contribution of the attached eddies to the skin-friction drag generation with the FIK identity. Finally, concluding remarks are summarized in § 6.

2. Numerical experiments

Three DNS of turbulent channel flows are performed. A finite difference code is used to solve the three-dimensional unsteady compressible Navier–Stokes equations. The solver has been extensively validated in previous works (Li, Nonomura & Fujii 2013a,b; Li & Wang 2018). In order to meet low-dissipation and low-dispersion requirements for turbulent flows, a sixth-order compact difference scheme is employed for the discretization of convection terms (Lele 1992). To ensure numerical stability, an eighth-order low-pass spatial filtering (with the optimization parameter $\alpha_f = 0.493$) is applied at each time step to the conservative variables (Lele 1992). The viscous terms are discretized by an eighth-order central difference scheme. A third-order total-variation-diminishing Runge–Kutta scheme (Jiang & Shu 1996) is adopted for the time integration, and the time step is set to ensure a Courant–Friedrichs–Lewy (CFL) number less than unity.

The Mach number based on bulk velocity is set to 0.2 such that the flow can be considered incompressible (Kawai 2016). Three cases are calculated at $Re_\tau = u_\tau h/\nu = 180, 480$ and 810, respectively, where u_τ is the friction velocity and ν is the kinematic viscosity. Details of the numerical parameters of the simulations are listed in table 1. The computational domain is $L_x \times L_y \times L_z = 2\pi \times 2 \times \pi$ normalized by the channel half-height h , where x , y and z stand for the streamwise, wall-normal and spanwise directions, respectively. According to Del Álamo *et al.* (2004) and Lozano-Durán & Jiménez (2014a), the present computational domain is sufficiently large to reproduce healthy turbulent statistics at the current Reynolds numbers. Therefore, our simulations provide an accurate representation of the energy-

containing motions of size $\lambda_z \approx O(1\delta)$ as well as their interactions with the long streaky structures (Lozano-Durán & Jiménez 2014a). Given that the attached eddies are characterized by their spanwise length scales (Flores & Jiménez 2010; Hwang 2015), we can conclude that the present computational domain is suitable for studying Townsend attached eddies. A comparison of results using DNS with different streamwise computational domains can be found in appendix A.

Uniform grids are employed in the streamwise and spanwise directions, while the grid is hyperbolically stretched in the wall-normal direction with points clustered close to the wall. Periodic boundary conditions are used in the spanwise and streamwise directions, and isothermal non-slip conditions are imposed at the walls. To drive the flow, a body force is imposed in the streamwise direction to maintain constant mass flux (Lenormand, Sagaut & Ta Phuoc 2000).

To assess the reliability of the DNS, figure 1 shows the mean streamwise velocity profiles and turbulence intensities for the three cases simulated. The mean profiles of the present DNS agree well with the statistics reported in Del Álamo *et al.* (2004) and with the classic predictions for the viscous sublayer, $U^+ = y^+$, and log layer, $U^+ = 1/0.40 \log(y^+) + 5.2$. The turbulence intensities (figure 1b–d) are compared with those of Kim, Moin & Moser (1987) at $Re_\tau = 180$, Lee & Moser (2015) at $Re_\tau = 544$, and Del Álamo *et al.* (2004) at $Re_\tau = 934$. The Reynolds numbers in the last two cases are slightly different from the previous studies. The results show that the turbulence intensities calculated in the present DNS are consistent with the those reported in the literature.

3. Empirical mode decomposition

EMD is an adaptive mode decomposition method first introduced by Huang *et al.* (1998). In contrast to Fourier and wavelet analysis, EMD does not require a set of basis functions in advance, but directly extracts intrinsic mode functions (IMFs) based on the instantaneous features of the signal. Hence, EMD is a data-driven and *a posteriori* method for data analysis.

To introduce the methodology of the EMD, we first discuss the decomposition of a 1-D signal $f(x)$ that is a function only of x . The steps involved in the calculation of IMFs are as follows:

- (i) Define a residual function $r(x)$, and initialize $r(x)$ to $f(x)$ as the beginning of the calculation.
- (ii) Find all local maxima of $r(x)$ and connect them with a smooth curve (for instance, cubic spline interpolating function) to form the upper envelope $r_{up}(x)$ of the signal. Similarly, the lower envelope $r_{low}(x)$ of the input signal is computed from the local minima.
- (iii) Subtract the average envelope $r_d(x) = 0.5(r_{up}(x) + r_{low}(x))$ from the original data sequence $r(x)$ to obtain the new data sequence $h(x) = r(x) - r_d(x)$.

- (iv) Replace $r(x)$ by $h(x)$ in step (i) and repeat the above steps until a convergence criterion SD is satisfied. The convergence criterion suggested by Huang *et al.* (1998) is

$$SD = \frac{\sum_0^T |r_k(x) - r_{k-1}(x)|^2}{\sum_0^T |r_k(x)|^2} \leq 0.1,$$

(3.1)

where k is the number of iterations, and T is the total number of points. Once the criterion is satisfied, the first IMF is defined as $IMF_1(x) = r_k(x)$.

- (v) After obtaining the first IMF, a new signal is constructed to replace the original one: $f(x) - IMF_1(x) \rightarrow f^*(x)$.
- (vi) Initialize the residual $r(x)$ with the new signal and repeat the steps above to obtain the remaining IMFs.

Following the aforementioned procedure, the original signal $f(x)$ is decomposed as the sum of multiple IMFs plus a final residual,

$$f(x) = \sum_{i=1}^m IMF_i(x) + R(x),$$

(3.2)

where m is the number of IMFs and $R(x)$ is the final residual. The first IMF contains the minimum scale information of the original signal, while the length scale associated with the IMF increases adaptively with m . An example of a 1-D EMD is shown in appendix B.

In the present study, BEMD is used to decompose the velocity fluctuations in wall-parallel planes (x - z). The approach involves radial basis functions (RBFs) based on a thin-plate function (Bookstein 1989; Carr, Fright & Beatson 1997; Nunes *et al.* 2003) to construct the envelope of the two-dimensional (2-D) velocity fields. RBFs avoid distortions near the boundaries of the computational domain compared with those typical of cubic splines thanks to their variational characterization. This makes RBFs capable of interpolating scattered data even with large data-free regions (Carr *et al.* 1997). The reader is referred to Nunes *et al.* (2003) for more details about RBFs.

Instantaneous velocity fluctuations $u'_i(\equiv u', v', w')$ at each x - z plane and different wall-normal distances are used as input signals for the decomposition. For a given velocity component, the BEMD algorithm is terminated when the extremes to construct an envelope on an x - z plane are less than 10. The remaining data are treated as the final residual. It is

worth noting that the BEMD is applied to the three velocity components independently, and modes of u , v and w are not related *a priori*. However, we will show below that velocity modes associated with the same mode number share similar spanwise length scales, and hence can be considered to be related to each other. An alternative approach is to apply BEMD to the velocity vector field such as the three velocity components are coupled in the decomposition. However, this would require the modification of the BEMD algorithm and this is relegated to future work. Following previous studies (Agostini & Leschziner 2014; Agostini *et al.* 2017), the stopping criterion for BEMD is defined as

$$SD = \frac{\sum_{x=0}^{L_x} \sum_{z=0}^{L_z} |r_k(x, z) - r_{k-1}(x, z)|^2}{\sum_{x=0}^{L_x} \sum_{z=0}^{L_z} |r_k(x, z)|^2}, \quad (3.3)$$

and the 2-D signal $f(x, z)$ is decomposed as

$$f(x, z) = \sum_{i=1}^m \text{IMF}_i(x, z) + R(x, z). \quad (3.4)$$

The effect of different stopping criteria is examined in appendix C. It should be noted that the calculations of the BEMD are accompanied by a high computational cost. For the Re810 case, we used more than 95 000 CPU hours to decompose 500 instantaneous flow fields in the Tianhe-2 supercomputer. The amount of BEMD calculations correlates linearly with the grid and the number of snapshots, and more efforts are needed in the future to develop fast algorithms for BEMD.

A snapshot of the instantaneous streamwise velocity fluctuations for case Re810 is used to illustrate the decomposed modes. Four modes of the streamwise velocity fluctuations are shown in figure 2. The quantities displayed are isosurfaces of $u^+ = u'_d/u_\tau = -2.0$, where u'_d denotes the decomposed streamwise velocity fluctuations. It can be seen that the modes are characterized by different streamwise and spanwise length scales that increase with the mode number. Streaky structures dominate the modes, consistent with previous observations using the full streamwise velocity fluctuations (Flores & Jiménez 2010; Hwang 2015). The results from figure 2 demonstrate that BEMD succeeds in adaptively decomposing the velocity fluctuations into modes with different characteristic length scales. The detailed analysis of the properties of modes is provided in the next section.

4. Identification of attached eddies

With BEMD, the velocity fluctuations u'_i are decomposed into three modes for case Re180, and four modes for cases Re480 and Re810. Note that the final residual $R(x, z)$ is treated as the last mode. The number of identified modes is expected to increase with Re_τ ; however, the difference in Reynolds number between Re480 and Re810 may not be that significant to observe this increment. Hereafter, we denote u'_d , v'_d and w'_d as the decomposed velocity fluctuations in the streamwise, wall-normal and spanwise directions, respectively.

4.1. One-dimensional spectra

Streamwise and spanwise 1-D spectra of the decomposed velocity fluctuations (u'_d , v'_d and w'_d) are performed to statistically quantify the scales of the turbulent motions represented by modes. In order to provide a compact analysis, we report only the results for the two higher-Reynolds-number cases (Re480 and Re810). For case Re180, the spectral content of LSMs overlaps with the near-wall cycle (Marusic *et al.* 2010; Smits *et al.* 2011), and inner and outer scales are not clearly separated. The 1-D spectra for case Re180 can be found in appendix C. They will be used in §§ 4.2 and 5 to investigate the relationship between modes and attached eddies and the Reynolds shear stress carried by eddies with different scales, respectively.

The following discussions are organized around the mode number. For the first two modes, lengths are normalized using inner scaling ($\delta_v = \nu/u_\tau$) denoted by $^+$. For the third and fourth modes, lengths are scaled in outer units ($\delta = h$).

4.1.1. First mode—The spanwise premultiplied 1-D spectra of u'_d , v'_d and w'_d for the first mode are displayed in figure 3. Figure 3(a)–(c) show the spanwise spectra of u'_d , v'_d and w'_d , respectively, for case Re480, and figure 3(d)–(f) show the spanwise spectra of u'_d , v'_d and w'_d , respectively, for case Re810. The contour levels of the spectra shown in figure 3 and the following figures 4–11 indicate 0.17, 0.33, 0.50, 0.67 and 0.84 of their maximum, respectively. The results reveal that the spectra of u'_d and w'_d are located within the near-wall region and peak at $y^+ \approx 12$, whereas the spectra of v'_d are more intense across the buffer layer $y^+ \approx 45$. The spanwise length scale λ_z^+ of all decomposed velocity fluctuations peaks at $\lambda_z^+ \approx 100$, which coincides with near-wall coherent structures reported in previous studies (Smith & Metzler 1983; Kim *et al.* 1987). Similar features are observed in the Re180 case, as shown in appendix C.

The streamwise premultiplied 1-D spectra of u'_d , v'_d and w'_d for the first mode are displayed in figure 4, which are equivalent to figure 3. For u'_d the streamwise spectra span a wide range of scales in the streamwise direction $\lambda_x^+ \approx 200 - 700$ and peak at $y^+ \approx 12$. On the contrary, the spectra for v'_d and w'_d cover a shorter range of scales $\lambda_x^+ \approx 150 - 200$. The statistical

characteristics of u'_d obtained for the first mode are consistent with previous studies (Jiménez & Moin 1991; Jiménez & Pinelli 1999) and the results by Hwang (2013), who conducted a numerical experiment to remove all the turbulent motions wider than $\lambda_z^+ \approx 100$ and found that the streamwise spectra of u' have the largest value at $y^+ \approx 10$ –20 and $\lambda_x^+ \approx 600$.

Therefore, the turbulent motions identified by the first mode are representative of the near-wall coherent structures. The streamwise spectra of u'_d agree with the known properties of the near-wall streaks, whereas the spectra of v'_d and w'_d mainly characterize quasi-streamwise vortices. This will be further discussed in § 4.1.2.

4.1.2. Second mode—The spanwise and streamwise spectra of all decomposed velocity fluctuations for the second mode are shown in figures 5 and 6, respectively. The spectra of the second mode follow a distribution similar to the first mode except for their locations and length scales, which are slightly larger and located further from the wall. The spectra for u'_d peaks at $y^+ \approx 16$ –20 with a wide range of streamwise length scales from $\lambda_x^+ \approx 200$ to 1100, while the spanwise sizes remain shorter, $\lambda_z^+ \approx 150$ –180. The sizes are again consistent with the length scale of the near-wall streaks ($\lambda_x^+ \approx 1000$) (Hwang 2013). The spectra of v'_d and w'_d are more isotropic with peaks located at $\lambda_x^+ \approx 180$ –250, $\lambda_z^+ \approx 150$ –200 and $y^+ \approx 40$ –85, in agreement with the observations of near-wall quasi-streamwise vortices by Kim *et al.* (1987) and Jeong *et al.* (1997), among others. The spectra show a weak dependence on Re_τ , which is acceptable considering, first, that BEMD is a data-driven method which does not contain any explicit information regarding inner or outer units, and, second, the modulation effect of the large scales on the near-wall region for increasing Reynolds number as argued by Mathis, Hutchins & Marusic (2009, 2011) and Samie *et al.* (2018). Although both the first and second modes share similar scale and shape, their spanwise length scales are slightly different, $\lambda_z^+ = 100$ and $\lambda_z^+ = 150$ –200, respectively, which could be indicative of different dynamical structures involved in near-wall cycles within $y^+ < 100$.

The message from the analysis of the first two modes should be that they characterize velocity fluctuations within the buffer layer, and that their sizes scale in wall units. Both the first and second modes represent long streaky structures for u_d and short vortex packets for v_d and w_d . The main difference between the two modes is that the first mode captures the flow structure associated with the near-wall streaks at $y^+ \approx 13$, while the second mode contains larger eddies, although still confined within the buffer layer. The results are consistent with the bimodal behaviour of near-wall velocity components (Jiménez, Del Álamo & Flores 2004).

4.1.3. Third mode—The spanwise and streamwise spectra of u'_d , v'_d and w'_d for the third mode are shown in figures 7 and 8, respectively. The spectral peaks of u'_d and w'_d are still located within the buffer layer at $y^+ \approx 15$ for u'_d and $y^+ \approx 40$ –50 for w'_d . However, the top parts of both spectra penetrate into the log layer ($y^+ > 100$) while the bottom parts remain

attached to the wall ($y^+ < 15$). On the contrary, the spectral peak of v'_d can be found in the outer region ($y \approx 0.25h$) and the v'_d spectra remain detached from the wall. It was argued by Townsend (1976) that the only limiting mechanism of the size of energy-containing structures is the presence of the wall, and that the largest energetic eddies would grow until they were wall-attached. On the other hand, the impermeability condition prevents the wall-normal velocity from extending close to $y/h = 0$, hence remaining detached. Our outcome is consistent with Townsend's argument, and with the work by Jiménez (2013), who showed that the spectra of the wall-parallel velocities and the pressure are attached, while the wall-normal velocity and the tangential Reynolds stress are detached. Hwang (2015) also argued that the penetration of u'_d close to the wall is accompanied by a non-negligible w'_d in the same region due to continuity.

Analogously to the results reported for the first two modes, all decomposed velocity fluctuations share comparable spanwise wavelengths, although in this case the characteristic lengths scale in outer units rather than inner units, i.e. $\lambda_z \approx 0.5h$. Similarly, in spite of the elongated spectral tail of u'_d in the streamwise direction, the spectra of the three velocities exhibit streamwise maxima close to $\lambda_x \approx (0.5-0.67)h$. Therefore, the energy-containing motions represented by the third mode are characterized by $\lambda_x \sim \lambda_z$.

The findings above suggest that the third mode is representative of self-similar Townsend eddies. Our results are consistent with the studies of Lozano-Durán *et al.* (2012) and Dong *et al.* (2017), who showed that the momentum-carrying eddies in the log layer have essentially isotropic sizes, and that these are composed of a cross-flow roll (v, w) associated with a longer streak (u) whose strong part is still comparable to the roll size. The results also agree with Hwang (2015) and de Giovanetti *et al.* (2016), who interpreted the long streaky motions as an 'inactive' part of the attached eddies transporting streamwise momentum to the region close to the wall. Note that only one single mode was identified in the log layer. The total number of the BEMD modes is intimately related to the Reynolds number. Increasing the Reynolds number would increase the number of BEMD modes in the log layer. Recently, Agostini & Leschziner (2018) adopted BEMD to decomposed the streamwise velocity fluctuations at $Re_\tau = 1020$ and six modes were identified.

4.1.4. Fourth mode—Figure 9 shows the spanwise spectra of the decomposed velocity fluctuations for the fourth mode, computed as the final residual of the BEMD. A striking phenomenon observed in figure 9 is that the spanwise spectra of u'_d peak at two wall-normal locations with approximately the same spanwise length scale, i.e. $\lambda_z \approx 1h$, which is characteristic of the LSMs and VLSMs as documented by Del Álamo *et al.* (2004), Tomkins & Adrian (2005) and Hutchins & Marusic (2007). The first peak is located in the near-wall region ($y^+ \approx 15$), whereas the second peak appears in the outer region ($y > 0.2h$). The latter becomes more discernible as the Reynolds number increases, and it is not observed for case Re180 (figure 23g). This may indicate that the energy-containing motions in the outer region become more energetic as the Reynolds number increases. As observed for the third mode, the spectra of u'_d and w'_d reach deep into the near-wall region, but the spectra of v'_d are mostly detached from the wall.

Figure 10 shows the streamwise spectra of u'_d , v'_d and w'_d for the fourth mode. The spectral peaks of v'_d and w'_d are located at $\lambda_x \approx (1.2-1.4)h$, which coincides with the typical sizes of LSMs (Lee *et al.* 2014; de Giovanetti, Sung & Hwang 2017). On the other hand, the streamwise spectra of u'_d have their maxima at $\lambda_x \approx 6h$, which is indicative of long streaky structures carrying streamwise turbulent kinetic energy (Lee *et al.* 2014). This last observation is consistent with VLSMs, which are known to prevail over a wide range of wall-normal locations, from near-wall to outer regions (Kim 1999; Jin, Ahn & Sung 2015). From the above discussions, we may conclude that the fourth mode characterizes LSMs and VLSMs. Finally, some wiggles can be observed in the spectra of streamwise velocity in figure 10. Since the last mode is obtained as the final residual of the decomposition, these wiggles may be the natural outcome of the BEMD procedure.

4.2. Relationship between modes and attached eddies

The velocity fluctuations are decomposed into several modes with typical spanwise length scales. Figure 11 shows the spanwise spectra of u'_d , v'_d and w'_d contained in each mode for Re810, compared with the full spectra calculated from full data. It is evident that the original spectra are covered by modes at different spanwise length scales and wall-normal positions, suggesting that the energy-containing motions are organized into several groups with different characteristic length scales. It is interesting that the decomposed results are consistent with the work of De Silva, Marusic & Hutchins (2016), who proposed that the velocity field at $Re_\tau = 800$ can be synthesized by Λ -eddies with four hierarchies.

The 1-D spectra shown in the present study indicate that the modes of the fluctuating velocities represent a set of flow motions with very similar spanwise sizes scaling in either inner or outer units. For each mode, we have argued that v'_d and w'_d can be assumed to represent the short and tall vortical structures, while u'_d is mainly linked to long streaky structures, although with some contribution from vortex packets. The first two modes characterize the near-wall streaks and quasi-streamwise vortices, and the third and fourth modes represent larger streaky structures and vortex packets spanning the logarithmic and outer regions. These suggest that each BEMD mode represents a typical attached eddy characterized by its spanwise length scale.

Figure 12 shows the root mean square (r.m.s.) of the decomposed velocity fluctuations ($u'_{d,rms}$, $v'_{d,rms}$ and $w'_{d,rms}$) for the three cases. The velocity fluctuations are normalized with their peak values, and the wall-normal distances are rescaled by the value of λ_z where the spectrum peaks for each individual mode. The normalization is consistent with idealized Townsend eddies as illustrated in Townsend (1976, p. 155), which shows that the second-order moments of attached eddies should be self-similar with respect to the wall-normal distance $y^* = y/y_a$, where y_a is the location of the maximum intensity of the eddy. Figure 12(a) shows the profiles of $u'_{d,rms}$. The first two modes collapse for $y \leq 0.2L_z$, but the last mode for case Re180 and the last two modes in the Re480 and Re810 cases do not. This lack of self-similarity for $u'_{d,rms}$ is consistent with the classic inner scaling ascribed to the

influence of viscosity close to the wall. Figure 13 displays $u'_{d,rms}$ scaled by inner units and shows that the profiles collapse below $y^+ \approx 15$, confirming that the near-wall part of $u'_{d,rms}$ in each mode is dominated by viscous effects. However, recent studies have revealed that the near-wall footprint of LSMs scales in inner units (Hwang 2016; Cho, Hwang & Choi 2018; Samie *et al.* 2018). This may not be evident from figure 13, which is probably dominated by smaller-scale contributions.

Figure 12(b,c) display the profiles of $\nu'_{d,rms}$ and $w'_{d,rms}$, respectively. Excluding the last mode of the case Re180 and Re480, all profiles are self-similar within $y \leq 0.5L_z$ for $\nu'_{d,rms}$, and $y \leq 0.6L_z$ for $w'_{d,rms}$. The breakdown of self-similarity beyond these heights could be induced by the interaction between turbulent motions characterized by different length scales. Moreover, the deviation observed for the last mode is probably caused by the fact that it is obtained as the final residual in the BEMD procedure and, therefore, it is prone to mode mixing and statistical errors. Additionally, previous works have shown that large-scale structures are mainly ‘inactive’ (Perry & Chong 1982; Perry *et al.* 1986; Hwang & Sung 2018), which could also contribute to the breakdown of self-similarity reported above.

To conclude the section, it should also be noted that detached flow motions are not energetic enough to be captured by the velocity spectra, although they might be contained in the first EMD mode. As shown by Del Álamo *et al.* (2006) and Lozano-Durán *et al.* (2012), the sizes of detached eddies are roughly 20η , where η is the Kolmogorov length scale, and their streamwise and spanwise length scales are smaller than 100 inner units. They are responsible for the energy cascade and dissipation processes, but they do not carry energy nor contribute to the bulk momentum transfer. The characteristics discussed above support the modes obtained with the BEMD as representative of Townsend attached eddies.

4.3. Geometrical properties of attached eddies

The geometrical properties of attached eddies can be estimated by the location of the maximum in the premultiplied spectra (see appendix D). Considering that a single attached eddy is composed of two elements, i.e. vortex packets and streaky structures, the sizes of the former may be approximated from the spectra of ν'_d as

$$y \approx (0.3 - 0.6)\lambda_z \text{ and } \lambda_x \approx (1 - 2)\lambda_z,$$

(4.1a,b)

where y and λ_x are measured with respect to the peak of λ_z for each eddy. The estimation is roughly consistent with the result of Hwang (2015) ($y \approx (0.5-0.7)\lambda_z$ and $(y \approx (2-3)\lambda_z$). Del Álamo *et al.* (2006) reported a similar relation for the attached clusters ($\lambda_x \approx 2\lambda_z$). Analogously, the geometry of the streaky structures can be represented by

$$\lambda_x \approx (1 - 7)\lambda_z.$$

(4.2)

The geometrical properties of the longest near-wall streaks ($\lambda_x \approx 7\lambda_z$) estimated in the present study are comparable to the results given by Hwang (2015) ($\lambda_x \approx 10\lambda_z$) and nearly identical to the value proposed by Jiménez (2018) ($\lambda_x \approx 8\lambda_z$), providing further evidence for the connection between attached eddies and BEMD modes.

4.4. Near-wall streamwise turbulence intensity

Based on the idea that the attached eddy motions in the logarithmic and outer regions impose a forcing on the buffer and viscous sublayer, Marusic & Kunkel (2003) proposed a new formulation to describe the similarity of the streamwise turbulence intensity for a smooth-wall zero-pressure-gradient turbulent boundary layer. The formulation reads as $u_{rms}/u_\tau = \sqrt{f_1(y^+)f_T(y^+, Re_\tau)}$, where $f_1(y^+)$ is a universal function representing the turbulence fluctuations taking place in the inner region of the boundary layer, and $f_T(y^+, Re_\tau)$ denotes a force function accounting for the influence of the outer layer. The empirical curve for $f_1(y^+)$ is given by

$$f_1(y^+) = \frac{a_1(y^+)^2}{\left(1 + a_2(y^+)^2\right)^{1/2} \left(1 + a_3(y^+)^{2a_4}\right)^{1/2}},$$

(4.3)

where the fitting constants are $a_1 = 0.16$, $a_2 = 0.008$, $a_3 = 0.115$ and $a_4 = 1.6$ (Marusic & Kunkel 2003). By numerically removing turbulent motions wider than $\lambda_z^+ \approx 100$, Hwang (2013) proposed a new set of fitting constants, i.e. $a_1 = 0.113$, $a_2 = 0.008$, $a_3 = 0.10$ and $a_4 = 1.9$.

In the present study, we have shown that the first mode characterizes the smallest attached eddies in the near-wall region with $\lambda_z^+ \approx 100$. Then, $u'_{d,rms}$ associated with the first mode can be used to tune the empirical curve (4.3). Here, we propose new fitting constants based on the BEMD results given by

$$a_1 = 0.122, a_2 = 0.046, a_3 = 0.078, a_4 = 2.213.$$

(4.4a-d)

Figure 14 shows that the fitting curve using the constants in (4.4) is in good agreement with $u'_{d,rms}$ of the first mode over a wide range of wall-normal distances. The amplitude of the curve proposed is smaller than that in Marusic & Kunkel (2003) and Hwang (2013). Note that the curve in Marusic & Kunkel (2003) contains the outer-flow effects, and the curve in Hwang (2013) relies on the ‘artificial’ numerical experiments with enforced scale-based truncation, in which the interaction between energy-containing motions near the cutoff wavelength may not be accurately resolved (de Giovanetti *et al.* 2016).

5. The skin friction generated by attached eddies

5.1. FIK identity and scale-targeted Reynolds shear stress

The FIK identity, first proposed by Fukagata *et al.* (2002), decomposes the mean skin-friction drag coefficient into a laminar and a turbulent component. For turbulent channel flows, it is expressed as

$$C_{f,FIK} = \underbrace{\frac{12}{Re_m}}_{C_{f1}} + \underbrace{6 \int_0^h \left(1 - \frac{y}{h}\right) \left(-\frac{\overline{u'v'}}{U_m^2}\right) dy}_{C_{f2}}, \quad (5.1)$$

where U_m is the bulk velocity, and $Re_m = 2U_m h/\nu$. The first term C_{f1} is the laminar drag at the same Reynolds number, and thus it is referred to as the laminar contribution. The second term C_{f2} accounts for the drag by the turbulent fluctuations, i.e. the integrated Reynolds stress over the boundary layer thickness, and consequently it is termed the turbulent contribution. The FIK identity has been adopted in previous works to investigate the contribution of large-scale structures to the skin-friction generation (Deck *et al.* 2014; de Giovanetti *et al.* 2016).

The mean skin-friction drag coefficient estimated from (5.1) is compared with the exact values and Dean’s empirical correlation ($C_f = 0.073 Re_m^{-0.25}$) (Dean 1978) in figure 15. The exact C_f is directly calculated from the normal gradient of the mean streamwise velocity $\langle U \rangle$ at the wall, $C_f = (2/Re_m) \left(\partial \langle U \rangle / \partial y|_{y=0} \right)$. The agreement with Dean’s empirical correlation is reasonable, and $C_{f,FIK}$ matches the exact C_f within 4 % error.

In § 4, we have identified attached eddies by applying BEMD to the full turbulent flow. Based on this scale decomposition of attached eddies, the streamwise and wall-normal velocity fluctuations of the flow fields can be expressed as

$$\left. \begin{aligned} u' &= u'_t + u'_o, \\ \nu' &= \nu'_t + \nu'_o, \end{aligned} \right\}$$

(5.2)

where the subscript t stands for attached eddies with a targeted length scale λ_τ and the subscript o stands for the remaining attached eddies. Then, the Reynolds shear stress can be written as

$$-\overline{u'\nu'} = -\overline{(u'_t + u'_o)(\nu'_t + \nu'_o)}.$$

(5.3)

Using the Reynolds averaging operator, equation (5.3) can be expressed as

$$-\overline{u'\nu'} = \underbrace{-\overline{u'_t\nu'_t}}_{tt} - \underbrace{\overline{u'_o\nu'_o}}_{oo} - \underbrace{\overline{u'_t\nu'_o}}_{to} - \underbrace{\overline{u'_o\nu'_t}}_{ot},$$

(5.4)

splitting the Reynolds shear stress into four different contributions. The first term on the right-hand side of (5.4) represents the Reynolds stress produced by the targeted attached eddy, the second term indicates the Reynolds shear stress produced by attached eddies excluding the targeted one, and the last two terms are the Reynolds shear stress generated by their interactions. The decomposition above allows for the evaluation of the skin friction generated by different scale-targeted Reynolds shear stresses.

5.2. Contribution of near-wall eddies to the skin friction

We assess the contribution to the skin-friction generation of the near-wall attached eddies within the buffer layer. These eddies are characterized by $\lambda_z^+ \approx 100 - 200$ and are represented by the first two modes as shown in § 4. The analysis is relevant since most of the drag-reduction approaches, such as spanwise-oscillating walls (Jung, Mangiavacchi & Akhavan 1992), near-wall blowing and suction (Choi, Moin & Kim 1994), and micro-grooved structures or riblets (Choi, Moin & Kim 1993), attempt to manipulate turbulent structures within the near-wall region.

Figure 16 shows the variation of the four different terms in (5.4). The results are normalized using u_τ . As expected, the sum of the four terms shows excellent agreement with the full Reynolds shear stress. The Reynolds shear stress generated by the near-wall (targeted) attached eddies reaches its maximum at $y^+ \approx 20$, regardless of the Reynolds number. The remaining contribution is dominated by eddies in the logarithmic and outer regions, consistent with the observations of the 1-D spectra in § 4.

The corresponding skin-friction generation obtained by integrating the various Reynolds shear stress components over the boundary layer is illustrated in figure 17. As the Reynolds number increases, the skin friction generated by the smallest attached eddies decreases significantly from 52.4% in case Re180 to 34.3% in case Re810, whereas other attached eddies play a more prominent role, contributing 12%, 40% and 43.7 % of the total skin friction for cases Re180, Re480 and Re810, respectively. This implies that special attention should be paid to the energy-containing motions residing in the logarithmic and outer regions for turbulent drag reduction at high Reynolds numbers. With respect to the contribution of interacting eddies, their values are not negligible and they still account for approximately 10% of the skin-friction generation.

5.3. Contribution of LSMs and VLSMs to the skin friction

As the Reynolds number increases, the thickness of the near-wall layer, and hence the size associated with the structures populating this region, become smaller (Robinson 1991; Gad-el Hak 1994; Abbassi *et al.* 2017). Consequently, the efficiency of near-wall drag-reduction techniques degrades significantly for Reynolds numbers typical of real-world applications (Gatti, Quadrio & Frohnepfel 2015; Encinar, Lozano-Durán & Jiménez 2018). Hwang (2013) pointed out that, for increasing Reynolds numbers, the contribution of the near-wall small-scale structures to the skin friction decays, while the contribution of the LSMs increases due to the amplitude modulation of the small scales and the energy superposition onto the inner region. Thus, evaluating the contribution of the large-scale turbulent motions to the skin-friction generation is necessary to aid the development of new drag-reduction strategies. To that end, we evaluate the drag contribution from LSMs and VLSMs represented by the third BEMD mode in the Re180 case, and the fourth mode in the Re480 and Re810 cases, which are set as targeted eddies.

Figure 18 shows the variation of the four different terms in (5.4). The Reynolds shear stress generated by LSMs and VLSMs reaches its maximum in the outer region ($y \approx (0.3-0.4)h$). The results are consistent with the attached eddy framework; the attached part of the eddies mainly consists of the streamwise and spanwise components, while the wall-normal velocity fluctuations are small due to the constraint of the wall. This makes the attached part of the eddies ‘inactive’, i.e. barely carrying Reynolds stress in the near-wall region (Townsend 1976; Hwang 2015). Recently, de Giovanetti *et al.* (2016) suggested that, even if the attached part is ‘inactive’, the streamwise momentum in the logarithmic and outer regions can still be transported to the near-wall region and interact with the wall-detached wall-normal component. This action results in high values of Reynolds shear stress observed far away from the near-wall region. The evidence presented here regarding the distribution of $-\overline{u'_t v'_t}$ in

figure 18 supports this scenario. The Reynolds shear stress generated by the remaining attached eddies peaks in the buffer layer, i.e. $y^+ \approx 30$ for all cases.

Figure 19 displays the result of integrating the Reynolds shear stresses over the boundary layer. It can be seen that the largest attached eddies are responsible for 12%, 20% and 23 % of the total skin-friction generation for cases Re180, Re480 and Re810, respectively. The present results are in accordance with de Giovanetti *et al.* (2016), who reported that the LSMs and VLSMs are responsible for 20–30% of the total skin-friction drag. On the other hand, the contribution of other attached eddies is roughly 50 % of the total skin friction. Finally, the interaction between eddies accounts for 10 % of the skin-friction generation, which is smaller than previous contributions but cannot be ignored. The origin of this skin-friction generation due to nonlinear interactions can be found in the second component of the FIK identity. The Reynolds stress in C_2 is not only originated from energy-containing motions themselves, but it is also related to the nonlinear interactions among them. Recently, Cho *et al.* (2018) investigated the turbulent kinetic energy transfer thoroughly and showed that this process is highly associated with the skin-friction generation, consistent with the present results. Additionally, other works such as Mathis *et al.* (2009) and Mathis *et al.* (2011) have highlighted the interaction of large-scale with small-scale structures near the wall via amplitude modulation.

5.4. Discussion

The combination of the BEMD and FIK identity provides a new framework to assess the contribution of attached eddies to the mean skin-friction generation. The method can also be adopted for flow control. For example, several studies have investigated the variation of Reynolds shear stress ($-\overline{u'v'}$ in (5.3)) for different drag-reduction approaches at different Reynolds numbers (Choi *et al.* 1994; Chang, Collis & Ramakrishnan 2002; Karniadakis & Choi 2003; Toubert & Leschziner 2012), but few of them have established a link between drag reduction and attached eddies populating the logarithmic region or outer layer. Furthermore, future works may utilize BEMD synchronously with DNS to remove or modify one or more attached eddies. This will help to clarify whether the attached eddies are self-sustained, or to analyse their relationship with the mean skin-friction generation.

6. Concluding remarks

In the present work we have utilized bidimensional empirical mode decomposition (BEMD) to investigate the multiscale nature of turbulent channel flows. The key findings are summarized below.

- (i) The velocity fluctuations u'_i ($\equiv u', v', w'$) of the DNS database were decomposed into several BEMD modes with different characteristic length scales. The 1-D spectra and statistics of the decomposed velocity fluctuations suggested that: (1) all decomposed velocity fluctuations are associated with a spanwise length scale λ_z within a given mode; (2) self-similarity is observed for the wall-normal and spanwise velocity fluctuations, but the streamwise component exhibits broken self-similarity; and (3) the second-order moment of each mode is consistent with

the attached eddy theory by Townsend (1976). These findings provide direct support of the existence of attached eddies in wall-bounded flows.

- (ii) We have estimated the geometrical properties of attached eddies. By considering the spanwise wavelength λ_z as the representative scale, the sizes of ν_d' follow $y \approx (0.3 - 0.6)\lambda_z$ and $\lambda_x \approx (1 - 2)\lambda_z$, similarly to the scales and positions of vortex packets, while the sizes of u_d' are characterized by $\lambda_x \approx (1-7)\lambda_z$, which are reminiscent of streaky structures. These geometrical properties are consistent with the results given by Del Álamo *et al.* (2006), Hwang (2015) and Jiménez (2018). The range of Reynolds numbers in the present work are too low to exhibit a well-developed log layer, and further investigations at higher Re_τ are necessary in order to convincingly assess the self-similar scaling of BEMD modes in the log layer.
- (iii) By combining the BEMD and the FIK identity, we have evaluated the contribution of attached eddies of size λ_z to the mean skin-friction generation. Our analysis suggests that the skin friction generated by the near-wall attached eddies with $\lambda_z^+ \approx 100 - 200$ decreases significantly as the Reynolds number increases. At high Reynolds numbers, the largest attached eddies, mostly representing the LSMs and VLSMs with $\lambda_z \approx O(1\delta)$, are responsible for 20–30% of the total skin-friction drag generation. The interaction between the targeted attached eddies and others accounts for approximately 10 % of the skin-friction generation, which cannot be ignored.

This study provides a systematic approach to identify attached eddies from DNS data by using BEMD. The geometrical and statistical properties of attached eddies obtained in the present work may aid the development of the attached eddy model and improve its predictability (Woodcock & Marusic 2015; De Silva *et al.* 2016, 2018). The BEMD method emerges as a promising tool for understanding the multiscale nature of wall-bounded turbulence, naturally connecting the mean skin-friction drag generation with attached eddies and providing additional physical insights for flow control.

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Appendix A.: Comparison of spectra between modes in short and long streamwise domains

Here, we compare the decomposed results of BEMD in the short streamwise domain ($L_x = 2\pi$, case Re480 in the present study) with those in the long streamwise domain ($L_x = 4\pi$). The grid size used in the long-streamwise-domain computation is $402 \times 201 \times 201$. Other simulation parameters are identical to the Re480 case. As mentioned in the main body of

this paper, the streamwise and wall-normal velocity fluctuations well describe the different elements of attached eddies, so we mainly compare the statistics of these two velocity components for brevity.

The spanwise spectra of u'_d and ν'_d obtained in the two cases are compared and shown in figure 20. The shapes and peaks of the spectra are almost the same, which indicates that the spanwise scales of decomposed modes are not affected by the streamwise length of the computational domain. Figure 21 shows the streamwise spectra of u'_d and ν'_d . Differences exist only in the streamwise spectra of u'_d due to the distinct streamwise length of the computational domain. The streaky structures with streamwise scale $\lambda_x \geq 6.2h$ are not well resolved in the case with $L_x = 2\pi$. However, the maxima of the spectra are not changed with the increase of L_x , justifying the use of the short streamwise domain for the study of attached eddies.

Appendix B.: Outline of empirical mode decomposition

An example is shown here to exhibit the process of EMD. The 1-D EMD is adopted to decompose an artificial signal, $f(x) = \sin(\frac{1}{2}\pi x) + \sin(\pi x) + \sin(2\pi x)$. Figure 22(a) shows the principles of EMD by reference to an x -dependent signal. The upper envelope $r_{up}(x)$ and lower envelope $r_{low}(x)$ of the input signal are formed from the local extreme points. Then the averaged envelope is calculated by $r_d(x) = 0.5(r_{up}(x) + r_{low}(x))$. The decomposed results are displayed in figure 22(b). It is apparent that the frequencies in $f(x)$ are captured by EMD and the signal is decomposed into three IMF modes.

Appendix C.: One-dimensional spectra of the Re180 case

The 1-D spectra of the Re180 case are displayed here. Figures 23 and 24 show, respectively, the spanwise and streamwise premultiplied 1-D spectra of the decomposed velocity fluctuations, which provide statistical measurements of the decomposed scales. Inner length scale $\delta_v = \nu/u_\tau$ is used to normalize the scales.

The spanwise premultiplied 1-D spectra of u'_d , ν'_d , and w'_d in the first mode are displayed in figure 23, in which panels (a)–(c) respectively show the spanwise spectra of u'_d , ν'_d , and w'_d for mode 1, panels (d)–(f) respectively show the spanwise spectra of u'_d , ν'_d , and w'_d for mode 2, and panels (g)–(i) respectively show the spanwise spectra of u'_d , ν'_d , and w'_d for mode 3.

With regard to the spanwise length scale λ_z^+ , the most important feature is that the spectra of all decomposed velocity fluctuations peak at $\lambda_z^+ \approx 100$ for mode 1, at $\lambda_z^+ \approx 200$ for mode 2, and at $\lambda_z^+ \approx 400$ for mode 3. The streamwise premultiplied 1-D spectra of u'_d , ν'_d , and w'_d in the first mode are displayed in figure 24, in which the panels have the same meanings as in figure 23. Detailed discussion of the relationship between modes and coherent structures is given in § 4, in comparison with the other two cases.

What is more, the influence of different stopping criteria is examined. Figures 23 and 24 show the effect of stopping criterion on the spectra. All the spectra coincide with each other, and the wall-normal positions and length scales of the peaks are not changed by varying the stopping criterion, indicating that the stopping criterion $SD = 0.1$ adopted in the present study is suitable.

Appendix D.: Summary of the wall-normal locations and length scales of the peaks in each mode

Table 2 shows the wall-normal locations of spectral peaks. Tables 3 and 4 display the streamwise and spanwise length scales of the peaks, respectively.

REFERENCES

- ABBASSI MR, BAARS WJ, HUTCHINS N. & MARUSIC I. 2017 Skin-friction drag reduction in a high-Reynolds-number turbulent boundary layer via real-time control of large-scale structures. *Intl J. Heat Fluid Flow* 67, 30–41.
- AGOSTINI L. & LESCHZINER M. 2014 On the influence of outer large-scale structures on near-wall turbulence in channel flow. *Phys. Fluids* 26 (7), 075107.
- AGOSTINI L. & LESCHZINER M. 2016 Predicting the response of small-scale near-wall turbulence to large-scale outer motions. *Phys. Fluids* 28 (1), 339–352.
- AGOSTINI L. & LESCHZINER M. 2017 Spectral analysis of near-wall turbulence in channel flow at $Re_\tau = 4200$ with emphasis on the attached-eddy hypothesis. *Phys. Rev. Fluids* 2, 014603.
- AGOSTINI L. & LESCHZINER M. 2018 The impact of footprints of large-scale outer structures on the near-wall layer in the presence of drag-reducing spanwise wall motion. *Flow Turbul. Combust* 100, 1–25.
- AGOSTINI L, LESCHZINER M, POGGIE J, BISEK NJ & GAITONDE D. 2017 Multi-scale interactions in a compressible boundary layer. *J. Turbul* 18, 1–21.
- ANDELMAN D, CATES ME, ROUX D. & SAFRAN SA 1987 Structure and phase equilibria of microemulsions. *J. Chem. Phys* 87 (12), 7229–7241.
- ANSELL PJ & BALAJEWICZ MJ 2016 Separation of unsteady scales in a mixing layer using empirical mode decomposition. *AIAA J.* 55 (2), 419–434.
- BOOKSTEIN FL 1989 Principal warps: thin-plate splines and the decomposition of deformations. *IEEE Trans. Pattern Anal. Mach. Intell* 11 (6), 567–585.
- CARR JC, FRIGHT WR & BEATSON RK 1997 Surface interpolation with radial basis functions for medical imaging. *IEEE Trans. Med. Imaging* 16 (1), 96–107. [PubMed: 9050412]
- CHANG Y, COLLIS SS & RAMAKRISHNAN S. 2002 Viscous effects in control of near-wall turbulence. *Phys. Fluids* 14 (11), 4069–4080.
- CHO M, HWANG Y. & CHOI H. 2018 Scale interactions and spectral energy transfer in turbulent channel flow. *J. Fluid Mech* 854, 474–504.
- CHOI H, MOIN P. & KIM J. 1993 Direct numerical simulation of turbulent flow over riblets. *J. Fluid Mech* 255, 503–539.
- CHOI H, MOIN P. & KIM J. 1994 Active turbulence control for drag reduction in wall-bounded flows. *J. Fluid Mech* 262, 75–110.
- DE SILVA CM, KEVIN K, BAIDYA R, HUTCHINS N. & MARUSIC I. 2018 Large coherence of spanwise velocity in turbulent boundary layers. *J. Fluid Mech* 847, 161–185.
- DE SILVA CM, MARUSIC I. & HUTCHINS N. 2016 Uniform momentum zones in turbulent boundary layers. *J. Fluid Mech* 786, 309–331.
- DEAN RB 1978 Reynolds number dependence of skin friction and other bulk flow variables in two-dimensional rectangular duct flow. *J. Fluids Engng* 100 (2), 215–223.

- DECK S, RENARD N, LARAUFIE R. & WEISS P. 2014 Large-scale contribution to mean wall shear stress in high-Reynolds-number flat-plate boundary layers up to $Re_\theta = 13\,650$. *J. Fluid Mech* 743, 202–248.
- DEL ÁLAMO JC, JIMÉNEZ J, ZANDONADE P. & MOSER RD 2004 Scaling of the energy spectra of turbulent channels. *J. Fluid Mech* 500, 135–144.
- DEL ÁLAMO JC, JIMÉNEZ J, ZANDONADE P. & MOSER RD 2006 Self-similar vortex clusters in the turbulent logarithmic region. *J. Fluid Mech* 561, 329–358.
- DONG S, LOZANO-DURÁN A, SEKIMOTO A. & JIMÉNEZ J. 2017 Coherent structures in statistically stationary homogeneous shear turbulence. *J. Fluid Mech* 816, 167–208.
- ENCINAR MP, LOZANO-DURÁN A. & JIMÉNEZ J. 2018 Reconstructing channel turbulence from wall observations In *Procs. CTR Summer School, Stanford University*, (in press).
- FELDMANN D. & AVILA M. 2018 Overdamped large-eddy simulations of turbulent pipe flow up to $Re_\tau = 1500$. *J. Phys.: Conf Ser* 1001, 012016.
- FENG Z, ZHANG D. & ZUO MJ 2017 Adaptive mode decomposition methods and their applications in signal analysis for machinery fault diagnosis: a review with examples. *IEEE Access* 5, 24301–24331.
- FLORES O. & JIMÉNEZ J. 2006 Effect of wall-boundary disturbances on turbulent channel flows. *J. Fluid Mech* 566 (566), 357–376.
- FLORES O. & JIMÉNEZ J. 2010 Hierarchy of minimal flow units in the logarithmic layer. *Phys. Fluids* 22 (7), 071704.
- FLORES O, JIMÉNEZ J. & DEL ÁLAMO JC. 2007 Vorticity organization in the outer layer of turbulent channels with disturbed walls. *J. Fluid Mech* 591 (591), 145–154.
- FUKAGATA K, IWAMOTO K. & KASAGI N. 2002 Contribution of Reynolds stress distribution to the skin friction in wall-bounded flows. *Phys. Fluids* 14 (11), L73–L76.
- GATTI D, QUADRIO M. & FROHNAPFEL B. 2015 Reynolds number effect on turbulent drag reduction. In *15th European Turbulence Conference (ETC15) University of Delft*.
- DE GIOVANETTI M, HWANG Y. & CHOI H. 2016 Skin-friction generation by attached eddies in turbulent channel flow. *J. Fluid Mech* 808, 511–538.
- DE GIOVANETTI M, SUNG HJ & HWANG Y. 2017 Streak instability in turbulent channel flow: the seeding mechanism of large-scale motions. *J. Fluid Mech* 832, 483–513.
- GAD-EL HAK M. 1994 Interactive control of turbulent boundary layers – a futuristic overview. *AIAA J.* 32 (9), 1753–1765.
- HAMILTON JM, KIM J. & WALEFFE F. 1995 Regeneration mechanisms of near-wall turbulence structures. *J. Fluid Mech* 287, 317–348.
- HELLSTRÖM LHO, MARUSIC I. & SMITS AJ 2016 Self-similarity of the large-scale motions in turbulent pipe flow. *J. Fluid Mech* 792, R1.
- HOYAS S. & JIMÉNEZ J. 2006 Scaling of the velocity fluctuations in turbulent channels up to $Re_\tau = 2003$. *Phys. Fluids* 18 (1), 011702.
- HUANG YX, SCHMITT F, LU ZM & LIU YL 2008 An amplitude–frequency study of turbulent scaling intermittency using empirical mode decomposition and Hilbert spectral analysis. *Europhys. Lett* 84 (4), 40010.
- HUANG NE, SHEN Z., LONG SR, WU MC, SHIH HH, ZHENG Q., YEN NC, TUNG CC & LIU HH 1998 The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. *Proc. R. Soc. Lond. A* 454, 903–995.
- HUTCHINS N. & MARUSIC I. 2007 Evidence of very long meandering features in the logarithmic region of turbulent boundary layers. *J. Fluid Mech* 579, 1–28.
- HWANG Y. 2013 Near-wall turbulent fluctuations in the absence of wide outer motions. *J. Fluid Mech* 723 (5), 264–288.
- HWANG Y. 2015 Statistical structure of self-sustaining attached eddies in turbulent channel flow. *J. Fluid Mech* 767, 254–289.
- HWANG Y. 2016 Mesolayer of attached eddies in turbulent channel flow. *Phys. Rev. Fluids* 1 (6), 064401.

- HWANG Y. & BENGANA Y. 2016 Self-sustaining process of minimal attached eddies in turbulent channel flow. *J. Fluid Mech* 795, 708–738.
- HWANG Y. & COSSU C. 2010 Self-sustained process at large scales in turbulent channel flow. *Phys. Rev. Lett* 105 (4), 044505.
- HWANG Y. & COSSU C. 2011 Self-sustained processes in the logarithmic layer of turbulent channel flows. *Phys. Fluids* 23 (6), 061702.
- HWANG J. & SUNG HJ 2018 Wall-attached structures of velocity fluctuations in a turbulent boundary layer. *J. Fluid Mech* 856, 958–983.
- JEONG J, HUSSAIN F, SCHOPPA W. & KIM J. 1997 Coherent structures near the wall in a turbulent channel flow. *J. Fluid Mech* 332, 185–214.
- JIANG G. & SHU C. 1996 Efficient implementation of weighted ENO schemes. *J. Comput. Phys* 126 (1), 202–228.
- JIMÉNEZ J. 2012 Cascades in wall-bounded turbulence. *Annu. Rev. Fluid Mech* 44 (1), 27–45.
- JIMÉNEZ J. 2013 Near-wall turbulence. *Phys. Fluids* 25 (10), 97–120.
- JIMÉNEZ J. 2018 Coherent structures in wall-bounded turbulence. *J. Fluid Mech* 842, P1.
- JIMÉNEZ J, DEL ÁLAMO JC & FLORES O. 2004 The large-scale dynamics of near-wall turbulence. *J. Fluid Mech* 505, 179–199.
- JIMÉNEZ J. & HOYAS S. 2008 Turbulent fluctuations above the buffer layer of wall-bounded flows. *J. Fluid Mech* 611, 215–236.
- JIMÉNEZ J. & MOIN P. 1991 The minimal flow unit in near-wall turbulence. *J. Fluid Mech* 225, 213–240.
- JIMÉNEZ J. & PINELLI A. 1999 The autonomous cycle of near-wall turbulence. *J. Fluid Mech* 389, 335–359.
- JIN L, AHN J. & SUNG HJ 2015 Comparison of large- and very-large-scale motions in turbulent pipe and channel flows. *Phys. Fluids* 27 (2), 431–442.
- JUNG WJ, MANGIAVACCHI N. & AKHAVAN R. 1992 Suppression of turbulence in wall-bounded flows by high-frequency spanwise oscillations. *Phys. Fluids A* 4 (8), 1605–1607.
- KARNIADAKIS GE & CHOI K. 2003 Mechanisms on transverse motions in turbulent wall flows. *Annu. Rev. Fluid Mech* 35 (35), 45–62.
- KAWAI S. 2016 Direct numerical simulation of transcritical turbulent boundary layers at supercritical pressures with strong real fluid effects. In 54th AIAA Aerospace Sciences Meeting, p. 1934.
- KIM KC 1999 Very large-scale motion in the outer layer. *Phys. Fluids* 11 (2), 417–422.
- KIM J, MOIN P. & MOSER R. 1987 Turbulence statistics in fully developed channel flow at low Reynolds number. *J. Fluid Mech* 177 (177), 133–166.
- LEE J, LEE J, CHOI J. & SUNG H. 2014 Spatial organization of large- and very-large-scale motions in a turbulent channel flow. *J. Fluid Mech* 749, 818–840.
- LEE M. & MOSER RD 2015 Direct numerical simulation of turbulent channel flow up to $Re_\tau \approx 5200$. *J. Fluid Mech* 774, 395–415.
- LELE SK 1992 Compact finite difference schemes with spectral-like resolution. *J. Comput. Phys* 103 (1), 16–42.
- LENORMAND E, SAGAUT P. & TA PHUOC L. 2000 Large eddy simulation of subsonic and supersonic channel flow at moderate Reynolds number. *Intl J. Numer. Meth. Fluids* 32 (4), 369–406.
- LI W, NONOMURA T. & FUJII K. 2013a Mechanism of controlling supersonic cavity oscillations using upstream mass injection. *Phys. Fluids* 25 (8), 086101.
- LI W, NONOMURA T. & FUJII K. 2013b On the feedback mechanism in supersonic cavity flows. *Phys. Fluids* 25 (5), 056101.
- LI W. & WANG L. 2018 Geometrical structure analysis of a zero-pressure-gradient turbulent boundary layer. *J. Fluid Mech* 846, 318–340.
- LOZANO-DURÁN A. & BORRELL G. 2016 Algorithm 964: an efficient algorithm to compute the genus of discrete surfaces and applications to turbulent flows. *ACM Trans. Math. Softw* 42 (4), 34:1–34:19.

- LOZANO-DURÁN A, FLORES O. & JIMÉNEZ J. 2012 The three-dimensional structure of momentum transfer in turbulent channels. *J. Fluid Mech* 694, 100–130.
- LOZANO-DURÁN A. & JIMÉNEZ J. 2014a Effect of the computational domain on direct simulations of turbulent channels up to $Re_\tau = 4200$. *Phys. Fluids* 26 (1), 011702.
- LOZANO-DURÁN A. & JIMÉNEZ J. 2014b Time-resolved evolution of coherent structures in turbulent channels: characterization of eddies and cascades. *J. Fluid Mech* 759, 432–471.
- MARUSIC I. & KUNKEL GJ 2003 Streamwise turbulence intensity formulation for flat-plate boundary layers. *Phys. Fluids* 15 (8), 2461–2464.
- MARUSIC I., MCKEON BJ, MONKEWITZ PA, NAGIB HM, SMITS AJ & SREENIVASAN KR. 2010 Wall-bounded turbulent flows at high Reynolds numbers: recent advances and key issues. *Phys. Fluids* 22 (6), 065103.
- MARUSIC I. & MONTY JP 2019 Attached eddy model of wall turbulence. *Annu. Rev. Fluid Mech* 51, 49–74.
- MARUSIC I. & WOODCOCK JD 2014 Attached eddies and high-order statistics In *Progress in Wall Turbulence: Understanding and Modelling*, pp. 47–60. Springer.
- MATHIS R, HUTCHINS N. & MARUSIC I. 2009 Large-scale amplitude modulation of the small-scale structures in turbulent boundary layers. *J. Fluid Mech* 628, 311–337.
- MATHIS R, HUTCHINS N. & MARUSIC I. 2011 A predictive inner–outer model for streamwise turbulence statistics in wall-bounded flows. *J. Fluid Mech* 681, 537–566.
- METZGER MM & KLEWICKI JC 2001 A comparative study of near-wall turbulence in high and low Reynolds number boundary layers. *Phys. Fluids* 13 (3), 692–701.
- NUNES JC, BOUAOUNE Y, DELECHELLE E, NIANG O. & BUNEL P. 2003 Image analysis by bidimensional empirical mode decomposition. *Image Vis. Comput* 21 (12), 1019–1026.
- PERRY AE & CHONG MS 1982 On the mechanism of wall turbulence. *J. Fluid Mech* 119 (119), 173–217.
- PERRY AE, HENBEST S. & CHONG MS 1986 A theoretical and experimental study of wall turbulence. *J. Fluid Mech* 165, 163–199.
- PERRY AE & MARUSIC I. 1995 A wall-wake model for the turbulence structure of boundary layers. Part 1. Extension of the attached eddy hypothesis. *J. Fluid Mech* 298 (298), 361–388.
- PIROZZOLI S. & BERNARDINI M. 2013 Probing high-Reynolds-number effects in numerical boundary layers. *Phys. Fluids* 25 (2), 319–346.
- ROBINSON SK 1991 The kinematics of turbulent boundary layer structure. NASA ST//Recon Technical Report No. 91.
- SAMIE M, MARUSIC I, HUTCHINS N, FU MK, FAN Y, HULTMARK M. & SMITS AJ 2018 Fully resolved measurements of turbulent boundary layer flows up to $Re_\tau = 20000$. *J. Fluid Mech* 851, 391–415.
- SCHOPPA W. & HUSSAIN F. 2002 Coherent structure generation in near-wall turbulence. *J. Fluid Mech* 453, 57–108.
- SMITH CR & METZLER SP 1983 The characteristics of low-speed streaks in the near-wall region of a turbulent boundary layer. *J. Fluid Mech* 129, 27–54.
- SMITS AJ, MCKEON BJ & MARUSIC I. 2011 High-Reynolds number wall turbulence. *Annu. Rev. Fluid Mech* 43, 353–375.
- TARDU S. 2011 *Statistical Approach to Wall Turbulence*. ISTE, John Wiley.
- TOMKINS CD & ADRIAN RJ 2002 Spanwise structure and scale growth in turbulent boundary layers. *J. Fluid Mech* 490 (490), 37–74.
- TOMKINS CD & ADRIAN RJ 2005 Energetic spanwise modes in the logarithmic layer of a turbulent boundary layer. *J. Fluid Mech* 545, 141–162.
- TOUBER E. & LESCHZINER MA 2012 Near-wall streak modification by spanwise oscillatory wall motion and drag-reduction mechanisms. *J. Fluid Mech* 693, 150–200.
- TOWNSEND AA 1961 Equilibrium layers and wall turbulence. *J. Fluid Mech* 11 (1), 97–120.
- TOWNSEND AA 1976 *The Structure of Turbulent Shear Flow*, 2nd edn Cambridge University Press.
- WOODCOCK JD & MARUSIC I. 2015 The statistical behaviour of attached eddies. *Phys. Fluids* 27 (1), 97–120.

FIGURE 1.

(a) Mean streamwise velocity profiles for all cases (circles, Del Álamo *et al.* (2004)) and turbulence intensities of (b) the Re180 case (circles, Kim *et al.* (1987)), (c) the Re480 case (circles, Lee & Moser (2015)), and (d) the Re810 case (circles, Del Álamo *et al.* (2004)).

FIGURE 2.

(Colour online) Isosurfaces of the decomposed streamwise velocity fluctuations in (a) the first, (b) the second, (c) the third, and (d) the fourth mode for Re810. (Top view, $u^+ = -2.0$.)

FIGURE 3.

The spanwise premultiplied 1-D spectra of the decomposed velocity fluctuations in the first mode: (a–c) spectra of u'_d , v'_d , and w'_d , respectively, for the Re480 case; (d–f) spectra of u'_d , v'_d , and w'_d , respectively, for the Re810 case.

FIGURE 4.

The streamwise premultiplied 1-D spectra of the decomposed velocity fluctuations in the first mode: (a–c) spectra of u'_d , ν'_d , and w'_d , respectively, for the Re480 case; (d–f) spectra of u'_d , ν'_d , and w'_d , respectively, for the Re810 case.

FIGURE 5.

The spanwise premultiplied 1-D spectra of the decomposed velocity fluctuations in the second mode: (a–c) spectra of u'_d , ν'_d , and w'_d , respectively, for the Re480 case; (d–f) spectra of u'_d , ν'_d , and w'_d , respectively, for the Re810 case.

FIGURE 6.

The streamwise premultiplied 1-D spectra of the decomposed velocity fluctuations in the second mode: (a–c) spectra of u'_d , ν'_d , and w'_d , respectively, for the Re480 case; (d–f) spectra of u'_d , ν'_d , and w'_d , respectively, for the Re810 case.

FIGURE 7.

The spanwise premultiplied 1-D spectra of the decomposed velocity fluctuations in the third mode: (a–c) spectra of u'_d , v'_d , and w'_d , respectively, for the Re480 case; (d–f) spectra of u'_d , v'_d , and w'_d , respectively, for the Re810 case.

FIGURE 8.

The streamwise premultiplied 1-D spectra of the decomposed velocity fluctuations in the third mode: (a–c) spectra of u'_d , ν'_d , and w'_d , respectively, for the Re480 case; (d–f) spectra of u'_d , ν'_d , and w'_d , respectively, for the Re810 case.

FIGURE 9.

The spanwise premultiplied 1-D spectra of the decomposed velocity fluctuations in the fourth mode: (a–c) spectra of u'_d , v'_d , and w'_d , respectively, for the Re480 case; (d–f) spectra of u'_d , v'_d , and w'_d , respectively, for the Re810 case.

FIGURE 10.

The streamwise premultiplied 1-D spectra of the decomposed velocity fluctuations in the fourth mode: (*a-c*) spectra of u'_d , v'_d , and w'_d , respectively, for the Re480 case; (*d-f*) spectra of u'_d , v'_d , and w'_d , respectively, for the Re810 case.

FIGURE 11.

(Colour online) The spanwise spectra of u'_d , v'_d , and w'_d in modes together with the full spectra calculated from full data for Re810: (a) spectra of u'_d , (b) spectra of v'_d , and (c) spectra of w'_d . Black line, full spectra; red line, mode 1; green line, mode 2; blue line, mode 3; magenta line, mode 4.

FIGURE 12.

Profiles of the normalized root mean squares of the decomposed velocity fluctuations as a function of wall-normal distances rescaled with the spanwise length L_z of each mode: (a) streamwise component, (b) wall-normal component, and (c) spanwise component.

FIGURE 13.

Profiles of the normalized root mean squares of the streamwise velocity fluctuations as a function of wall-normal distances rescaled with the inner scale δ_v of each mode. Lines and symbols are as in figure 12.

FIGURE 14.
Streamwise velocity fluctuations in the first mode and fitting curves.

Figure 15.

Comparison of $C_{f,FIK}$ with exact C_f and Dean's empirical correlation.

FIGURE 16.

Variation of the four contributing terms in (5.4) when the near-wall attached eddies are set as the targeted eddy: (a) Re180; (b) Re480; (c) Re810.

FIGURE 17.
(Colour online) Decomposition of C_f when the near-wall eddies are set as the target eddy:
(a) Re_{180} ; (b) Re_{480} ; (c) Re_{810} . (d) The contributions of each term as a function of Re_τ .

FIGURE 18.

Variation of the four contributing terms in (5.4) when the largest attached eddy is set as the target eddy: (a) Re180; (b) Re480; (c) Re810.

FIGURE 19.
 (Colour online) Decomposition of C_f when the largest attached eddy is set as the target eddy:
 (a) Re_{180} ; (b) Re_{480} ; (c) Re_{810} . (d) The contributions of each term with respect to Re_τ .

FIGURE 20.

The spanwise premultiplied 1-D spectra of the decomposed u'_d and v'_d in two cases: (a,b) spectra of u'_d and v'_d respectively for mode 1; (c,d) spectra of u'_d and v'_d respectively for mode 2; (e,f) spectra of u'_d and v'_d respectively for mode 3; and (g,h) spectra of u'_d and v'_d for mode 4. Shaded, $L_x = 4\pi$; solid, $L_x = 2\pi$.

FIGURE 21.

The streamwise premultiplied 1-D spectra of the decomposed u'_d and v'_d in two cases: (a,b) spectra of u'_d and v'_d respectively for mode 1; (c,d) spectra of u'_d and v'_d respectively for mode 2; (e,f) spectra of u'_d and v'_d respectively for mode 3; and (g,h) spectra of u'_d and v'_d for mode 4. Shaded, $L_x = 4\pi$; solid, $L_x = 2\pi$.

FIGURE 22.

(a) The principles of 1-D EMD. (b) IMFs decomposed by EMD.

FIGURE 23.

(Colour online) The spanwise premultiplied 1-D spectra of the decomposed mode fluctuations in case Re180: (a–c) spectra of u'_d , ν'_d , and w'_d for mode 1; (d–f) spectra of u'_d , ν'_d , and w'_d for mode 2; and (g–i) spectra of u'_d , ν'_d , and w'_d for mode 3. Black dashed line, $SD = 0.3$; shaded region, $SD = 0.1$; red dashed line, $SD = 0.05$.

FIGURE 24.

(Colour online) The streamwise premultiplied 1-D spectra of the decomposed mode fluctuations in case Re180: (a–c) spectra of u'_d , v'_d , and w'_d for mode 1; (d–f) spectra of u'_d , v'_d , and w'_d for mode 2; and (g–i) spectra of u'_d , v'_d , and w'_d for mode 3. Black dashed line, $SD = 0.3$; shaded region, $SD = 0.1$; red dashed line, $SD = 0.05$.

TABLE 1.

Details of the parameter setting in the DNS. Here L_x , L_y and L_z are the streamwise, wall-normal and spanwise dimensions of the computational domain, respectively; N_x , N_y and N_z are the number of collocation points in the three coordinate directions, respectively; x^+ and z^+ are used to denote the streamwise and spanwise resolutions in inner units; Δy_1^+ and Δy_{max}^+ are the finest and coarsest resolution along the wall-normal direction, respectively; and Tu_{τ}/h is the total time used for statistics after convergence of the flows to a statistically steady state, in eddy turnovers.

Case	Re_{τ}	$L_x(h)$	$L_y(h)$	$L_z(h)$	$N_x \times N_y \times N_z$	x^+	z^+	Δy_1^+	Δy_{max}^+	Tu_{τ}/h
Re180	180	2π	2	π	$81 \times 141 \times 81$	14.1	7.1	0.21	6.5	21.7
Re480	480	2π	2	π	$201 \times 201 \times 201$	15.1	7.6	0.48	10.1	19.2
Re810	810	2π	2	π	$301 \times 251 \times 301$	16.8	8.4	0.72	14.4	13.9

TABLE 2.

The wall-normal locations of spectral peaks of each mode in inner units. Mode 4 of u'_d contains two peaks. Here, M1 is the first mode, M2 is the second mode, M3 is the third mode, and M4 is the last mode.

Case	Component	M1	M2	M3	M4
Re480	u'_d	11	16	15	15/144
Re480	ν'_d	45	80	120	144
Re480	w'_d	12	35	48	72
Re810	u'_d	12	20	15	16/160
Re810	ν'_d	50	85	200	242
Re810	w'_d	16	45	50	55

TABLE 3.

The streamwise length scales of spectral peaks of each mode. The first two modes are in inner scale and the last two are in outer scale. Here, M1 is the first mode, M2 is the second mode, M3 is the third mode, and M4 is the last mode.

Case	Component	M1	M2	M3	M4
Re480	u'_d	420	650	$0.67h$	$6.2h$
Re480	ν'_d	140	180	$0.67h$	$1.2h$
Re480	w'_d	180	200	$0.67h$	$1.2h$
Re810	u'_d	480	650	$0.5h$	$6.2h$
Re810	ν'_d	160	230	$0.5h$	$1.2h$
Re810	w'_d	200	250	$0.5h$	$1.2h$

TABLE 4.

The spanwise length scales of spectral peaks of each mode. The first two modes are in inner scale and the last two are in outer scale. Here, M1 is the first mode, M2 is the second mode, M3 is the third mode, and M4 is the last mode.

Case	Component	M1	M2	M3	M4
Re480	u'_d	90	150	$0.5h$	$1.2h$
Re480	ν'_d	80	160	$0.5h$	$1.1h$
Re480	w'_d	90	160	$0.5h$	$1.2h$
Re810	u'_d	100	180	$0.42h$	$0.9h$
Re810	ν'_d	100	200	$0.42h$	$0.9h$
Re810	w'_d	100	200	$0.42h$	$0.9h$