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Large-Eddy Simulation of Thermally Stratified Atmospheric Boundary-Layer Flow Using a Minimum Dissipation Model

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Abstract

A generalized form of a recently developed minimum dissipation model for subfilter turbulent fluxes is proposed and implemented in the simulation of thermally stratified atmospheric boundary-layer flows. Compared with the original model, the generalized model includes the contribution of buoyant forces, in addition to shear, to the production or suppression of turbulence, with a number of desirable practical and theoretical properties. Specifically, the model has a low computational complexity, appropriately switches off in laminar and transitional flows, does not require any ad hoc shear and stability corrections, and is consistent with theoretical subfilter turbulent fluxes. The simulation results show remarkable agreement with well-established empirical correlations, theoretical predictions, and field observations in the atmosphere. In addition, the results show very little sensitivity to the grid resolution, demonstrating the robustness of the model in the simulation of the atmospheric boundary layer, even with relatively coarse resolutions.

Keywords

Atmospheric boundary layer; Large-eddy simulation; Subfilter modelling

1 Introduction

Large-eddy simulation (LES) is an important tool with which to investigate the velocity and turbulence fields in the atmospheric boundary layer (ABL) (Deardorff 1980; Moeng 1984; Nieuwstadt et al. 1993; Mason 1994; Kosovic 1997; Porté-Agel et al. 2000; Beare et al. 2006; Sullivan and Patton 2011; Zhou and Chow 2011; Huang and Bou-Zeid 2013; Sullivan et al. 2016). In the LES approach, turbulent structures larger than a filter scale are resolved, and a subfilter model is used to account for the contribution of the unresolved scales. Applying a spatial filter to the Navier–Stokes equations (here, including buoyancy and Coriolis effects), as well as the transport equation for the potential temperature, leads to

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$$\partial_t \tilde{u}_i = 0, \partial_t \tilde{u}_i + \partial_j (\tilde{u}_i \tilde{u}_j) = -\partial_i \tilde{p} + \partial_j (\nu \partial_j \tilde{u}_i) - \partial_j \tau_{ij} + \delta_{i3} \beta (\tilde{\theta} - \langle \tilde{\theta} \rangle) + f_c \varepsilon_{ij3} \tilde{u}_j$$

(1)

and

$$\partial_t \tilde{\theta} + \partial_i (\tilde{u}_i \tilde{\theta}) = \partial_i (D \partial_i \tilde{\theta}) - \partial_i q_i,$$

(2)

where the tilde denotes spatial filtering, $\tilde{u}_i$ and $\tilde{\theta}$ are the resolved velocity and potential temperature, respectively, $\tilde{p}$ is the kinematic pressure, ν and D are the molecular viscosity and diffusivity, respectively, f_c is the Coriolis parameter, ε_{ijk} is the alternating unit tensor, $\tau_{ij} = \overline{u_i u_j} - \tilde{u}_i \tilde{u}_j$ is the subfilter stress tensor, and $q_i = \overline{u_i \theta} - \tilde{u}_i \tilde{\theta}$ is the subfilter heat flux. In the momentum conservation equations, buoyancy effects are accounted for via the Boussinesq approximation, in which $\beta = g/\theta_0$ is the buoyancy parameter, g is the acceleration due to gravity, θ_0 is the reference potential temperature, δ_{ij} is the Kronecker delta tensor, and the angled brackets represent a horizontal average (Deardorff 1974).

An important class of subfilter models consists of eddy viscosity and diffusivity models, which incorporate the effect of unresolved eddies by increasing the molecular viscosity and diffusivity locally using an eddy viscosity or diffusivity. For this approach, the subfilter turbulent fluxes are evaluated as

$$\tau_{ij}^d = \tau_{ij} - \frac{1}{3} \delta_{ij} \tau_{kk} = -2\nu_e \tilde{S}_{ij}$$

(3)

and

$$q_i = -D_e \partial_i \tilde{\theta},$$

(4)

where $\tilde{S}_{ij} = (\partial_i \tilde{u}_j + \partial_j \tilde{u}_i)/2$ is the resolved strain-rate tensor, ν_e is the eddy viscosity, and D_e is the eddy diffusivity. Here, D_e is related to ν_e by the subfilter Prandtl number Pr_e such that

$D_e = \nu_e Pr_e^{-1}$. The most common way of modelling the eddy viscosity is to use the mixing-length approximation, yielding the well-known Smagorinsky model (Smagorinsky 1963), and an eddy viscosity as $\nu_e = (C_s \Delta)^2 |\tilde{S}|$, where $|\tilde{S}| = \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}}$ and C_s is the Smagorinsky coefficient. In a similar way, the subfilter eddy diffusivity is modelled as $D_e = \Delta^2 C_s^2 Pr_e^{-1} |\tilde{S}|$, where $C_s^2 Pr_e^{-1}$ is a lumped coefficient. In the implementation of eddy-viscosity and diffusivity models into a LES model, one of the main challenges is the specification of the model coefficients. For ABL flows, the model coefficients are traditionally deduced from fundamental theories of turbulence (Lilly 1967; Antonopoulos-Domis 1981; Mason 1994), combined with empirical models for the effect of the wind shear and thermal stratification on the turbulence (Mason and Thomson 1992; Sullivan et al. 1994; Stevens et al. 2000; Redelsperger et al. 2001; Kleissl et al. 2003; Sullivan et al. 2003). Hence, the model coefficients are obtained using predetermined formulations related to flow parameters, such as the Richardson number, the Obukhov length, and the ratio of filter scale to the distance to the ground (Kleissl et al. 2006).

A major advancement in subfilter modelling came with the introduction of dynamic procedures to obtain the appropriate local value of the model coefficients based on comparisons of the eddy dissipation at different filter levels without any ad hoc tuning (Germano et al. 1991; Moin et al. 1991). While the dynamic model removes the need for the prescribed shear and stability dependence, it assumes that the model coefficients are scale invariant. This assumption in the standard dynamic approach has been relaxed through the derivation of the scale-dependent dynamic model (Meneveau and Katz 2000; Porté-Agel et al. 2000; Porté-Agel 2004), while giving much-improved subfilter dissipation characteristics with respect to the traditional model, especially in the surface layer, where the relative contribution of the subfilter scales to the overall turbulent fluxes is very large (Bou-Zeid et al. 2005; Kleissl et al. 2006; Stoll and Porté-Agel 2008; Khani and Waite 2015). However, a perceived disadvantage of this approach lies in its increased computational complexity compared with the traditional model, and in the need for averaging and clipping to obtain numerical stability (Ghosal et al. 1995; Meneveau et al. 1996; Vreman et al. 1997).

A more recent approach is to specify subfilter turbulent fluxes provided by minimum dissipation models, which provide the minimum eddy dissipation required to dissipate the energy of the subfilter scales. The first minimum-dissipation eddy-viscosity model, which is based on the invariants of the resolved rate-of-strain tensor, was developed by Verstappen (2011) for isotropic grids. Later, Rozema et al. (2015) generalized the properties of this model to anisotropic grids by introducing an anisotropic minimum dissipation (AMD) model. Recently, Abkar et al. (2016) extended the AMD approach to modelling the subfilter scalar flux. Of its many desirable practical and theoretical properties, the AMD model has low computational complexity, switches off in laminar and transitional flow, and is consistent with the exact subfilter stress tensor and scalar flux on both isotropic and anisotropic grids. This model has been successfully implemented and tested in simulations of decaying grid turbulence, in simulations of a temporal mixing layer and turbulent channel flow (Rozema et al. 2015), and in the simulation of a high-Reynolds-number rough-wall boundary layer with a constant surface scalar flux (Abkar et al. 2016). However, previous

studies using minimum dissipation models have been limited to the neutrally stratified regime where the effects of buoyancy are ignored. Since buoyancy forces are a key component of turbulent transport in the ABL, we focus here on extending the application of the AMD model to thermally stratified flows.

Below, the AMD model is generalized to include the effects of buoyancy, and tested in the simulation of thermally stratified ABL flows. The derivation of the AMD model is provided in Sect. 2, while the LES framework and the numerical set-up are described in Sect. 3. The results are presented in Sect. 4, with conclusions given in Sect. 5.

2 Anisotropic Minimum Dissipation Model

A minimum dissipation model requires that the energy of the subfilter scales of the LES solution do not increase, so that

$$\partial_t \int_{\Omega_b} \frac{1}{2} \tilde{u}'_i \tilde{u}'_i d\mathbf{x} \leq 0, \quad \tilde{u}'_i = \tilde{u}_i - \frac{1}{|\Omega_b|} \int_{\Omega_b} \tilde{u}_i d\mathbf{x}$$

(5)

and

$$\partial_t \int_{\Omega_b} \frac{1}{2} \tilde{\theta}' \tilde{\theta}' d\mathbf{x} \leq 0, \quad \tilde{\theta}' = \tilde{\theta} - \frac{1}{|\Omega_b|} \int_{\Omega_b} \tilde{\theta} d\mathbf{x},$$

(6)

where $\tilde{u}'_i$ and $\tilde{\theta}'$ are the subfilter scales corresponding to the filter box with domain Ω_b applied to the LES solution, and $\mathbf{x} \in \Omega_b$. For a rectangular filter box Ω_b , with dimensions of δ_1 , δ_2 and δ_3 , an upper bound for the subfilter energy can be obtained using the Poincaré inequality, for which different formulations are known, such as the Poincaré-Wirtinger inequality,

$$\int_{\Omega_b} \frac{1}{2} \tilde{u}'_i \tilde{u}'_i d\mathbf{x} \leq \int_{\Omega_b} \frac{1}{2} (\hat{\partial}_i \tilde{u}_j) (\hat{\partial}_i \tilde{u}_j) d\mathbf{x},$$

(7)

and

$$\int_{\Omega_b} \frac{1}{2} \tilde{\theta}' \tilde{\theta}' d\mathbf{x} \leq \int_{\Omega_b} \frac{1}{2} (\hat{\partial}_i \tilde{\theta}) (\hat{\partial}_i \tilde{\theta}) d\mathbf{x},$$

(8)

where $\hat{\partial}_i = \sqrt{C_i} \delta_i \partial_i$ (for $i = 1, 2, 3$) is the scaled gradient operator, C_i is a modified Poincaré constant, $(\hat{\partial}_i \tilde{u}_j)(\hat{\partial}_i \tilde{u}_j)/2$ is the scaled momentum gradient, and $(\hat{\partial}_i \tilde{\theta})(\hat{\partial}_i \tilde{\theta})/2$ is the scaled thermal-energy gradient. The modified Poincaré constant is independent of the size of the filter box, and its magnitude depends only on the accuracy of the discretization method (i.e., order of accuracy) for each direction (Rozema et al. 2015; Abkar et al. 2016).

The Poincaré inequality indicates that the energy of the subfilter scales are confined by imposing a bound on the scaled momentum and thermal-energy gradients. If the eddy viscosity, the eddy diffusivity, and the filter widths are assumed to be constant in the filter box Ω_b , then the evolution equations for the scaled momentum and thermal-energy gradients are expressed as

$$\begin{aligned} \partial_t \left(\frac{1}{2} (\hat{\partial}_i \tilde{u}_j)(\hat{\partial}_i \tilde{u}_j) \right) = & - (\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{u}_j) \tilde{S}_{ij} - (\nu + \nu_e) \hat{\partial}_k (\hat{\partial}_i \tilde{u}_j) \hat{\partial}_k (\hat{\partial}_i \tilde{u}_j) + \delta_{i3} \beta (\hat{\partial}_k \tilde{u}_i) \hat{\partial}_k (\tilde{\theta} - \langle \tilde{\theta} \rangle) \\ & + \hat{\partial}_i f_i \end{aligned}$$

(9)

and

$$\partial_t \left(\frac{1}{2} (\hat{\partial}_i \tilde{\theta})(\hat{\partial}_i \tilde{\theta}) \right) = - (\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{\theta}) \partial_i \tilde{\theta} - (D + D_e) \hat{\partial}_k (\hat{\partial}_i \tilde{\theta}) \hat{\partial}_k (\hat{\partial}_i \tilde{\theta}) + \hat{\partial}_i g_i,$$

(10)

where f_i and g_i are the fluxes of momentum and thermal-energy gradients, respectively. The equations above are obtained by applying the scaled gradient operator on Eqs. 1 and 2, and multiplying the resulting equations by $\hat{\partial}_i \tilde{u}_j$ and $\hat{\partial}_i \tilde{\theta}$, respectively. Upon spatial integration over the filter box Ω_b , the divergence terms, $\hat{\partial}_i f_i$ and $\hat{\partial}_i g_i$, can be rewritten to a boundary integral. Boundary integrals express transport of the momentum and thermal-energy gradients instead of production or dissipation, and are, therefore, ignored in the derivation of the AMD model (Rozema et al. 2015; Abkar et al. 2016; Verstappen 2016). Note that the Coriolis force has no contribution in the above equations. In addition, if there are additional

terms in Eqs. 1 and 2 representing other physical processes, their contributions to the production or suppression of turbulence should be added to the equations above following the same procedure.

The dissipation at the scale of a filter box can be approximated by application of the Poincaré inequalities,

$$\int_{\Omega_b} (\partial_i \tilde{u}_j)(\partial_i \tilde{u}_j) d\mathbf{x} \leq \int_{\Omega_b} \hat{\partial}_k (\partial_i \tilde{u}_j) \hat{\partial}_k (\partial_i \tilde{u}_j) d\mathbf{x} \quad (11)$$

and

$$\int_{\Omega_b} (\partial_i \tilde{\theta})(\partial_i \tilde{\theta}) d\mathbf{x} \leq \int_{\Omega_b} \hat{\partial}_k (\partial_i \tilde{\theta}) \hat{\partial}_k (\partial_i \tilde{\theta}) d\mathbf{x}. \quad (12)$$

In the derivation of the minimum dissipation model, it is assumed that the scaled momentum and thermal-energy gradients dissipate at their natural rates (set by the fluid viscosity ν and diffusivity D) (Verstappen 2011, 2016). As a result, the eddy-viscosity and eddy-diffusivity models give sufficient eddy dissipation to cancel the production of scaled momentum and thermal-energy gradients, respectively, so long as the following inequalities hold,

$$\int_{\Omega_b} -(\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{u}_j) \tilde{S}_{ij} d\mathbf{x} + \int_{\Omega_b} \delta_{i3} \beta (\hat{\partial}_k \tilde{u}_i) \hat{\partial}_k (\tilde{\theta} - \langle \tilde{\theta} \rangle) d\mathbf{x} \leq \nu_e \int_{\Omega_b} (\partial_i \tilde{u}_j)(\partial_i \tilde{u}_j) d\mathbf{x} \quad (13)$$

and

$$\int_{\Omega_b} -(\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{\theta}) \partial_i \tilde{\theta} d\mathbf{x} \leq D_e \int_{\Omega_b} (\partial_i \tilde{\theta})(\partial_i \tilde{\theta}) d\mathbf{x}. \quad (14)$$

Taking the minimum eddy dissipation that satisfies these conditions gives

$$v_e = \frac{\int_{\Omega_b} -(\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{u}_j) \tilde{S}_{ij} d\mathbf{x} + \int_{\Omega_b} \delta_{i3} \beta (\hat{\partial}_k \tilde{u}_i) \hat{\partial}_k (\tilde{\theta} - \langle \tilde{\theta} \rangle) d\mathbf{x}}{\int_{\Omega_b} (\partial_l \tilde{u}_m)(\partial_l \tilde{u}_m) d\mathbf{x}}$$

(15)

and

$$D_e = \frac{\int_{\Omega_b} -(\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{\theta}) \partial_l \tilde{\theta} d\mathbf{x}}{\int_{\Omega_b} (\partial_l \tilde{\theta})(\partial_l \tilde{\theta}) d\mathbf{x}}.$$

(16)

By approximating the integrals over the filter box using the mid-point rule for integration, the AMD eddy-viscosity and eddy-diffusivity models are

$$v_e = \frac{-(\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{u}_j) \tilde{S}_{ij} + \delta_{i3} \beta (\hat{\partial}_k \tilde{u}_i) \hat{\partial}_k (\tilde{\theta} - \langle \tilde{\theta} \rangle)}{(\partial_l \tilde{u}_m)(\partial_l \tilde{u}_m)}$$

(17)

and

$$D_e = \frac{-(\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{\theta}) \partial_l \tilde{\theta}}{(\partial_l \tilde{\theta})(\partial_l \tilde{\theta})},$$

(18)

respectively. Hence, as the AMD eddy-viscosity model automatically includes stratification effects, additional corrections for thermal effects are redundant. In addition, the AMD model is reconciled with the exact eddy-dissipation expression on both isotropic and anisotropic grids. Taylor expansion of the subfilter turbulent fluxes gives

$$\tau_{ij} = \overline{u_i u_j} - \tilde{u}_i \tilde{u}_j = (\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{u}_j) + \mathcal{O}(\delta x_i^4)$$

(19)

and

$$q_i = \overline{u_i \theta} - \tilde{u}_i \tilde{\theta} = (\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{\theta}) + \mathcal{O}(\delta x_i^4).$$

(20)

The total eddy dissipation of the exact subfilter turbulent fluxes is

$$-\tau_{ij} \tilde{S}_{ij} + \delta_{i3} \beta (u_i (\overline{\theta - \langle \theta \rangle}) - (\tilde{u}_i \tilde{\theta} - \langle \tilde{\theta} \rangle)) = -(\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{u}_j) \tilde{S}_{ij} + \delta_{i3} \beta (\hat{\partial}_k \tilde{u}_i) \hat{\partial}_k (\tilde{\theta} - \langle \tilde{\theta} \rangle) + \mathcal{O}(\delta x_i^4)$$

(21)

and

$$-q_i \partial_i \tilde{\theta} = -(\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{\theta}) \partial_i \tilde{\theta} + \mathcal{O}(\delta x_i^4).$$

(22)

As shown in these equations, the leading-order terms of these expansions are proportional to the terms in the numerator of the AMD model (i.e., Eqs. 17 and 18). It should be noted that, in the AMD model, the eddy viscosity includes the effect of buoyancy, in addition to shear, on the production and suppression of turbulence. Hence, it is not merely equivalent to the series expansion of the subfilter stress tensor.

In practical applications of the AMD model, the size of the filter box is set equal to a grid cell ($\delta_i = \Delta_i$), and the corresponding Poincaré constant is chosen according to the accuracy of the discretization method for each direction. In particular, the Poincaré constant is 1/12 for a spectral method, and equal to 1/3 for a second-order accurate scheme (Verstappen et al. 2010, 2014). Also, to prevent numerical instability, the eddy viscosity and eddy diffusivity are constrained to non-negative values to avoid any backwards cascade induced by the model (Abkar et al. 2016). Note that while the computational complexity of the AMD model is comparable with traditional models (e.g., the S_σ model proposed by Stevens et al. (2000)),

the AMD model does not require any of the ad hoc stability and shear corrections found in traditional models.

3 Numerical Simulation

Equations 1 and 2 are solved numerically by discretizing the computational domain into N_x , N_y , and N_z uniformly spaced grid points with the resolution of x , y , and z in the streamwise, spanwise, and wall-normal directions, respectively. In the horizontal directions, pseudo-spectral discretization and periodic boundary conditions are used, whereas the wall-normal direction is discretized with a second-order accurate method. Hence, in the AMD model, we adopt $C_x = C_y = 1/12$ and $C_z = 1/3$ for the modified Poincaré constant. The equations are integrated in time using a second-order-accurate Adams-Bashforth scheme. The non-linear terms are dealiased using the 3/2 rule (Canuto et al. 1988). Since the Reynolds number is high, no near-wall viscous processes are resolved, and the viscous terms are neglected in the governing equations. The upper boundary condition for the momentum conservation equations is set to be stress free, with a zero vertical velocity component. The temperature gradient at the top of the boundary is prescribed as a constant lapse rate Γ , which is defined in the initial conditions. At the wall, the instantaneous surface shear stress and heat flux are related to the velocity and potential temperature, respectively, at the first vertical grid point through application of Monin-Obukhov similarity theory (Moeng 1984), using

$$\tau_{i3} \Big|_w = -u_*^2 \frac{\tilde{u}_i}{\tilde{u}_r} = - \left(\frac{\tilde{u}_r \kappa}{\ln(z/z_o) - \Psi_M} \right)^2 \frac{\tilde{u}_i}{\tilde{u}_r} \quad (23)$$

and

$$q_3 \Big|_w = \frac{u_* \kappa (\theta_s - \tilde{\theta})}{\ln(z/z_{ot}) - \Psi_H}, \quad (24)$$

where $\tau_{i3}|_w$ and $q_3|_w$ are the local wall shear stress and heat flux, respectively, u_* is the friction velocity, z_o and z_{ot} are the roughness lengths for momentum and heat, respectively, κ is the von Kármán constant, $\tilde{u}_r$ is the local filtered horizontal velocity at the first level $z = z/2$, and θ_s is the surface potential temperature. Here, Ψ_m and Ψ_H represent the surface-layer stability corrections for momentum and heat expressed as (Businger et al. 1971; Stull 1988)

$$\Psi_M = \begin{cases} -4.8z/L & \text{for } z/L \geq 0, \\ 2\ln\left[\frac{1}{2}(1 + \zeta)\right] + \ln\left[\frac{1}{2}(1 + \zeta^2)\right] - 2\tan^{-1}[\zeta] + \pi/2 & \text{for } z/L < 0 \end{cases}$$

(25)

and

$$\Psi_H = \begin{cases} -7.8z/L & \text{for } z/L \geq 0, \\ 2\ln\left[\frac{1}{2}(1 + \zeta^2)\right] & \text{for } z/L < 0, \end{cases}$$

(26)

respectively, where $L = -\left(u_*^3 \theta_0\right) / (\kappa g q_3 l_w)$ is the local Obukhov length, and $\zeta = (1 - 15 z/L)^{1/4}$. Note that these wall models parametrize the unresolved near-surface fluxes occurring at a scale below the first vertical grid point and involve a series of uncertainties (Piomelli and Balaras 2002; Bou-Zeid et al. 2005; Stoll and Porté-Agel 2006b).

4 Results

The performance of the AMD model is tested here for two thermally stratified ABL flows. For the first case, a stably stratified boundary layer following the LES intercomparison studies of the GEWEX Atmospheric Boundary Layer Study (GABLS) (Beare et al. 2006) is considered, which is a typical quasi-equilibrium moderately stable boundary layer commonly observed over polar regions and in equilibrium nighttime conditions over land at mid-latitudes. For the second case, a convective boundary layer similar to that used by Moeng and Sullivan (1994) is simulated, representing a buoyancy-dominated flow with relatively little wind shear. For both cases, three different spatial resolutions of $48 \times 48 \times 48$, $72 \times 72 \times 72$ and $96 \times 96 \times 96$ are used to assess the grid-resolution sensitivity of the simulation results. The simulated (bulk) boundary-layer parameters are compared with ones reported in Stoll and Porté-Agel (2008), Huang and Bou-Zeid (2013) and Sullivan et al. (2016) for the stable case, and in Moeng and Sullivan (1994) and Sullivan et al. (1994, 2016), the two-part subfilter model is used, which utilizes the transport equation for subfilter kinetic energy and an eddy-viscosity approach in the parametrization of the subfilter turbulent fluxes. In contrast, Stoll and Porté-Agel (2008) and Huang and Bou-Zeid (2013) parametrized the subfilter turbulent fluxes with the Lagrangian scale-dependent dynamic model. Through this method, the model coefficients (i.e., C_s and $C_s^2 Pr_e^{-1}$) are computed dynamically, and averaged over fluid pathlines (the Lagrangian approach). We note that the computational complexity of the aforementioned models is much higher than the AMD

model. Specifically, the two-part subfilter model requires solving an additional equation for the subfilter turbulent kinetic energy, and some model constants must be specified. For a detailed explanation of the two-part subfilter model, see Deardorff (1980), Moeng and Sullivan (1994) and Sullivan et al. (1994). The Lagrangian scale-dependent dynamic model also has a much higher computational complexity compared with the AMD model resulting from the application of two test-filter operations for both the momentum and scalars (Porté-Agel et al. 2000; Porté-Agel 2004), as well as performing the Lagrangian-averaging procedure (Meneveau et al. 1996). More details about the Lagrangian scale-dependent dynamic model are found in Porté-Agel et al. (2000), Porté-Agel (2004), Bou-Zeid et al. (2005) and Stoll and Porté-Agel (2006a).

4.1 Large-Eddy Simulation of the Stable Boundary Layer

Following the GABLS description, the flow is driven by a constant geostrophic wind speed of 8 m s^{-1} , the Coriolis parameter is set to $f_c = 1.39 \times 10^4 \text{ rad s}^{-1}$ (corresponding to the latitude 73°N), and the domain size is $800 \text{ m} \times 800 \text{ m} \times 400 \text{ m}$. Note that the horizontal domain is chosen as twice that used in Beare et al. (2006) to resolve a larger range of scales in excess of the boundary-layer height (Stoll and Porté-Agel 2008). The simulations are initialized with a constant stream wise velocity component of 8 m s^{-1} and zero velocity for the span wise and vertical velocity components. The initial potential temperature is prescribed to be 265 K up to 100 m , above which a constant lapse rate of 10 K km^{-1} is imposed. The surface potential temperature decreases at a constant rate of -0.25 K h^{-1} . While a very small perturbation is imposed within the first 50 m above the ground, the initial flow is otherwise laminar. The surface roughness length for momentum and heat is 0.1 m , and the reference potential temperature is $\theta_0 = 263.5 \text{ K}$. As the simulation attains a quasi-steady state within $8\text{--}9 \text{ h}$ (Beare et al. 2006), all statistics are calculated during the final hour of the simulation. A summary of the mean simulated results, including the boundary-layer height h (i.e., $(1/0.95)$ multiplied by the height at which the total momentum flux reaches five percent of the surface value (Kosovic and Curry 2000)), the surface friction velocity u_* , and the surface buoyancy flux Q_* are provided in Table 1. As can be seen, the results are in good agreement with those reported previously (Stoll and Porté-Agel 2008; Huang and Bou-Zeid 2013; Sullivan et al. 2016)), with very little sensitivity to the grid resolution. Note that to maintain numerical stability in the simulation of the stable regime, around 20% of the obtained values for the eddy viscosity and eddy diffusivity are clipped.

The vertical profiles of the mean wind speed and potential temperature averaged over the final hour of the simulation plotted in Fig. 1a, b, respectively, show a super-geostrophic nocturnal jet and a positive curvature in the potential temperature near the top of the boundary layer, respectively, in agreement with the theoretical model of Nieuwstadt (1985) and previous LES studies of the stationary stable boundary layer (Kosovic and Curry 2000; Beare et al. 2006; Basu and Porté-Agel 2006; Stoll and Porté-Agel 2008). To evaluate the performance of the AMD model in reproducing the surface-layer similarity profiles under different spatial resolutions more rigorously, one may examine the values of the non-dimensional shear and temperature gradient,

$$\Phi_M = \left(\frac{\kappa z}{u_*} \right) \sqrt{(\partial_z U)^2 + (\partial_z V)^2}$$

(27)

and

$$\Phi_H = \left(\frac{\kappa z}{\theta_*} \right) \partial_z \theta,$$

(28)

respectively, as a function of the local stability parameter z/L , as shown in Fig. 2a, b, and compared with the empirical formulations

$$\Phi_M = 1 + 4.7z/L$$

(29)

and

$$\Phi_H = 0.74 + 4.7z/L$$

(30)

proposed by Businger et al. (1971), and

$$\Phi_M = 1 + 6.1 \frac{z/L + (z/L)^a (1 + (z/L)^a)^{-1 + 1/a}}{z/L + (1 + (z/L)^a)^{1/a}}$$

(31)

and

$$\Phi_H = 1 + 5.3 \frac{z/L + (z/L)^b (1 + (z/L)^b)^{-1 + 1/b}}{z/L + (1 + (z/L)^b)^{1/b}},$$

(32)

by Brutsaert (1982), where $a = 2.5$ and $b = 1.1$. As indicated, the results correspond well to the empirical relations, with very little sensitivity to the grid resolution.

Figure 3a, b displays the total (resolved plus subfilter) values of the momentum and wall-normal buoyancy fluxes, respectively. In the GABLS intercomparison study (Beare et al. 2006), a significant spread is evident in the total momentum and buoyancy-flux profiles resulting from the different subfilter models and spatial resolutions employed. In particular, the mean momentum and buoyancy fluxes at the surface vary in the range from -0.06 to $-0.08 \text{ m}^2 \text{ s}^{-2}$ and -3.5 to $-5.5 \times 10^{-4} \text{ m}^2 \text{ s}^{-3}$, respectively. However, the results of the AMD model fall within these ranges, with a very weak grid-resolution dependence.

The theoretical model of Nieuwstadt (1985) predicts that the total momentum flux, normalized by its surface value, follows a $3/2$ power law with z/h , with the total normalized buoyancy flux a linear function of z/h . Figure 4a, b compares the normalized momentum and buoyancy fluxes with the theoretical predictions and field observations of Nieuwstadt (1984), where our results are in accordance with the theoretical model and observational data. As pointed out by Basu and Porté-Agel (2006), the deviation in the buoyancy-flux profile near $z/h = 1$ results from h defined in terms of the momentum flux rather than the buoyancy flux. As shown in Fig. 3, the later predicts a slightly higher value for the boundary-layer height.

4.2 Large-Eddy Simulation of the Convective Boundary Layer

Following Moeng and Sullivan (1994), the boundary layer is driven by a constant geostrophic wind speed of 10 m s^{-1} , the surface heat flux is set to 0.24 K m s^{-1} , the roughness length is set to 0.16 m , and the Coriolis parameter is $f_c = 1 \times 10^4 \text{ rad s}^{-1}$. The domain size is $5 \text{ km} \times 5 \text{ km} \times 2 \text{ km}$. To initialize the simulations, a constant streamwise velocity component of 10 m s^{-1} and zero velocity for the spanwise and wall-normal velocity components is used. The initial potential-temperature profile is 300 K up to 937 m , increases by a total of 8 K across 126 m above, and then with a lapse rate of 3 K km^{-1} . The reference potential temperature is $\theta_0 = 301.78 \text{ K}$. Table 2 provides a summary of the mean simulated results, including the boundary-layer height z_i (i.e., the height where the buoyancy flux is minimum Sullivan et al. 1994), the Obukhov length L , the friction velocity u_* , and the convective velocity ($w_* = (g/\theta_0)q_3/w_{zi})^{1/3}$). Note that the simulations are run for 9000 s , which is about 17 large-eddy turnover times ($\tau_* = z_i/w_*$), and the statistics are computed during the last five large-eddy turnover times. The values of the same parameters reported in Moeng and Sullivan (1994) and Sullivan et al. (1994) are also provided for comparison where, again, the bulk quantities representing the simulated boundary layer are in good

agreement with the previous numerical experiments. That our results are insensitive to the spatial resolution, which is a very desirable behaviour, is a testament to the veracity of the subfilter model. Note that in the simulation of the convective condition, around 16% of the obtained values for the eddy viscosity and eddy diffusivity are clipped.

Figure 5a, b shows the vertical profiles of the mean wind speed and potential temperature, respectively. The well-mixed layer throughout the interior of the boundary layer is clearly observed, which demonstrates the effectiveness of turbulent mixing in the convective boundary layer (Sullivan et al. 1994). The non-dimensional shear and temperature gradient are also plotted in Fig. 6a, b, respectively, and compared with the empirical formulations

$$\Phi_M = (1 - 15z/L)^{-1/4}$$

(33)

and

$$\Phi_H = 0.74(1 - 9z/L)^{-1/2}$$

(34)

proposed by Businger et al. (1971), where agreement with these well-established empirical correlations is quite good, with very little sensitivity to the grid resolution.

The vertical profiles of the total momentum and buoyancy fluxes are shown in Fig. 7a, b, respectively, where the magnitudes of the turbulent fluxes decrease linearly with height to serve as confirmation of stationarity. A similar behaviour has been reported by Nieuwstadt et al. (1993) and Moeng and Sullivan (1994) in the simulation of buoyancy-driven ABL flows. The variances of the horizontal and vertical velocity fluctuations as a function of z/z_i are presented in Fig. 8a, b, respectively, together with observations obtained from atmospheric field experiments (Lenschow et al. 1980) for comparison, where excellent agreement with the measurements and a very weak dependence on the spatial resolutions is observed.

5 Conclusion

Minimum dissipation models are a new class of subfilter models that provide the minimum eddy dissipation needed to dissipate the energy of subfilter scales. A generalized form of the recently developed anisotropic minimum dissipation (AMD) model (Abkar et al. 2016) is proposed to include the effect of buoyancy, in addition to shear, on the production and suppression of turbulence. Of its many desirable properties, the proposed model has a low computational complexity, appropriately switches off in laminar and transitional flows, removes the need for prescribed shear and stability dependencies, and is consistent with the

theoretical subfilter turbulent fluxes. In simulations of stable and convective boundary-layer flows, very good agreement with well-established empirical correlations, theoretical predictions, and field observations is demonstrated. In particular, the proposed model is capable of accurately reproducing the expected surface-layer similarity profiles for both the velocity and potential temperature, which is particularly important since the relative contribution of the subfilter scales to the overall turbulent fluxes is very large in the surface layer, and the gradients are also larger in that region. The results of simulations are only very weakly dependent on the grid resolution, indicating the robustness of the proposed model. The good performance of the AMD model, despite its simplicity and its low computational complexity, makes it a viable alternative to the traditional approaches in the simulation of thermally stratified ABL flows.

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Fig. 1.
Vertical profiles of mean, **a** wind speed, and **b** potential temperature in the stable boundary layer

Fig. 2.

Non-dimensional, **a** shear, and **b** temperature gradient as a function of the stability parameter z/L . The *dash-dotted* and *dashed lines* correspond to the expressions proposed by Businger et al. (1971) and Brutsaert (2005), respectively

Fig. 3.

Vertical profiles of the total, **a** momentum, and **b** buoyancy fluxes

Fig. 4.

Vertical profiles of the normalized, **a** momentum, and **b** buoyancy fluxes. Mean observations of Nieuwstadt (1984) are shown as crosses. The theoretical profile of Nieuwstadt (1985) is shown as a *dash-dotted* line

Fig. 5.
Vertical profiles of mean, **a** wind speed, and **b** potential temperature in the convective boundary layer

Fig. 6. Non-dimensional, **a** shear, and **b** temperature gradient as a function of the stability parameter z/L . The *dash-dotted lines* correspond to the expressions proposed by Businger et al. (1971)

Fig. 7.

Vertical profiles of the total, **a** momentum, and **b** buoyancy fluxes

Fig. 8.
Vertical profiles of the normalized, **a** horizontal, and **b** vertical velocity variances

Table 1

Basic characteristics of the simulated stable boundary layer

Grid points	h (m)	u_* (m s ⁻¹)	$Q_* \times 10^{-4}$ (m ² s ⁻³)
$48 \times 48 \times 48$	168	0.253	-3.92
$72 \times 72 \times 72$	165	0.254	-3.94
$96 \times 96 \times 96$	169	0.257	-3.99
Stoll and Porté-Agel (2008)	173	0.262	-4.03
Huang and Bou-Zeid (2013)	158	0.247	-3.60
Sullivan et al. (2016)	<i>n/a</i>	0.255	-3.59

Table 2

Basic characteristics of the simulated convective boundary layer

Grid points	z_i (m)	L (m)	u_* (m s ⁻¹)	w_* (m s ⁻¹)
$48 \times 48 \times 48$	1021	-59.6	0.570	2.00
$72 \times 72 \times 72$	1014	-58.9	0.568	1.99
$96 \times 96 \times 96$	1032	-59.2	0.569	2.00
Moeng and Sullivan (1994)	1030	-57.2	0.556	2.02
Sullivan et al. (1994)	1028	-58.7	0.570	2.00