Under-5 Mortality Rates in G7 Countries: Analysis of Fractional Persistence, Structural Breaks and Nonlinear Time Trends

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Abstract

This paper deals with the analysis of the under-5 mortality rate series in the G7 countries by using fractional integration techniques, including structural breaks and potential nonlinearities in the data. Several features were detected in the results: Firstly, we observed that for the neonatal data, the order of integration is equal to or higher than one in all cases, contrary to what happens for the remaining cases ($< 1 - < 5$ years) where mean reversion is found in many cases, especially as we increase the age of death. Thus, shocks affecting the neonatal (< 1 month from delivery) mortality rates will have permanent effects requiring special attention to recover the original trends. As expected, all the time trend coefficients were significantly negative and the highest reduction in the mortality rates was obtained in Japan, which might be related with the 17-year increase in life expectancy for the country. Due to the sensitivity of the methodological approaches, the use of robust time series approaches when analyzing child mortality rates is highly recommended.

Keywords Fractional persistence · G7 countries · Infant mortality rate · Nonlinearity · Under-5 mortality

1 Introduction

Mortality rates are considered to be an important indicator in the analysis of population dynamics of a country; therefore, the United Nations Children Emergency Fund (UNICEF), the World Health Organization (WHO), the World Bank and

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the United Nations Population Division (UNDP) with the United Nations Inter-agency Group for Child Mortality Estimation (UNIGME) formed an ally in 2004 to further the work on monitoring the behavior of child mortality series over the years, as a way of monitoring progress (You et al. 2015). As specified in the Millennium Development Goals (MDG 4) by the UNDP, most countries of the world could not meet up with the target of reducing the under-5 mortality rates (U5MR) by two-thirds between 1990 and 2015, though the global U5MR declined from 90 to 43 deaths per 1000 live births between 1990 and 2015. (You et al. 2015; Liu et al. 2016). This achievement is still not enough to achieve the MDG goal of reducing the U5MR by two-thirds. As part of UNIGME strategies, statistical models are fitted to mortality data, trends are observed and estimated models are extrapolated to target years. Apart from MDG target on U5MR, studying U5MR is preferred to infant mortality rate (IMR), even in the case of developed countries because U5MR series allows for wider age range, from neonate (few days of birth), infant (one month of birth to 12 months) to child (from 12 months of birth to 59 months of birth). During this period, a child is exposed to different diseases and sickness which can cause their lives. Generally, child mortality rates have implications for life expectancy. According to the existing literature, life expectancy in the G7 countries has increased over the last 50 years (see Acsadi and Nemeskeri 1974; Babel et al. 2008; Haleem et al. 2008). For example, this literature has observed a 17-year increase in life expectancy in Japan, 11 years in Germany, while the lowest, of 9 years, has been observed in the USA. Due to the implication of mortality rates in the calculation of most premium and risk capital based on life tables by the life insurance industry, the German Institute of Actuaries (DAV) in 2004 emphasized accurate projection of future mortality rates as one of the most important issues in the German insurance and pension industries (Babel et al. 2008).

Studying child mortality implies studying the under-5 mortality rates, which includes the neonatal (< 1 month from delivery), infants (< 1 year from delivery) and children from age 1 to 5 years. Globally, there has been a general decline in the general level of mortality and this decline is nonlinear (Booth et al. 2002; Booth 2006; Shang et al. 2011). This nonlinearity has actually caused changes in trend, and therefore the authors have queried the linearity assumption of the classical Lee–Carter model (see Lee and Carter 1992). Booth et al. (2002) applied a nonlinear version of the Lee–Carter model and observed an approximate 50% reduction in the forecast error when compared with the results obtained from the classical Lee–Carter model. In order to further prove this, Shang et al. (2011) conducted a multi-country comparison of mortality rates, employing several variants of the Lee–Carter method and found significant differences in the forecasting performance when alternative fitting periods were used. Booth et al. (2006) considered five variants of Lee–Carter model with linear specifications in forecasting mortality rates and compared the results obtained from sex-specific populations of 10 developed countries. The results showed more accuracy for different variants, compared to the original Lee–Carter model. Currie (2013) applied the Lee–Carter model in terms of Generalized Linear Models (GLMs) and smoothing parameters to produce smoothed forecasts.

The UNIGME strategy mentioned above assumes that every five years of vital registration or one set of indirect/direct demographic and health survey (DHS) of mortality are each sufficient to define one trend or slope in the mortality rate. The model included an underlying variable and additional variables measuring time trend as a series of “knot” dates which allows the rate of change of U5MR to change at each knot (see Hill et al. 1999).¹ The model specification described here is actually a third-order nonlinear time trend. Tuljapurkar et al. (2000) showed the opinion that it is difficult to forecast mortality rate since the driving factors for the time series are yet to be understood.

Generally, time-structured data such as mortality rates often possess a certain degree of persistence which determines the degree of integration of the series. Fractional persistence is a case where the time series is neither “pure” stationary $I(0)$ nor nonstationary $I(1)$ but contains a fractional persistence value in either a stationary or a nonstationary range for the time series. Thus, allowing for fractional degrees of integration we allow for a much richer degree of flexibility in the dynamic specification of the series. This fractional persistence evidence might be a consequence of structural breaks. In fact, there are some authors (Diebold and Inoue 2001; Granger and Hyung 2004) that argue that fractional integration might be a consequence of structural breaks or nonlinear patterns not taken into account in the model. Moreover, the presence of breaks may produce abrupt but modest or thin or small changes in the data which are not observed and, in that sense, the use of the Chebyshev (nonlinear) polynomials in time can solve this issue by using smoother approximations. All three cases: fractional persistence, nonlinearities and structural breaks are often experienced in time series and several authors have considered the joint estimation of these properties (see, Cheung 1993; Giraitis et al. 2001; Kapetanios et al. 2003; Granger and Hyung 2004; Mikosch and Starica 2004, among others). Modeling mortality index has therefore proven difficult due to the presence of structural breaks in the data, and this goes a long way to affecting the results of the stationarity level since the standard procedures to check for stationarity will no longer be valid (Perron 1989).² We know that a very serious assumption in time series is the one that implies stationary $I(0)$ errors, and stationary U5MR, for instance, implies shocks will have temporary effects, while there will be a permanent effect of shocks if U5MR is nonstationary $I(1)$, and policy interventions of the governmental and nongovernmental agencies will be required. The time series of U5MR will assist in explaining the degree of stationarity/nonstationarity in the mortality rates of each country, for example, if the rates are stationary, shocks tend to have transitory effects, unlike the case of the U5MR being nonstationary $I(1)$, where shocks tend to be permanent and policy interventions of the government will be required. For

¹ The model is $\ln(q_0)_i = b_0 + b_1(\text{date}) + b_2(\text{postk1}) + b_3(\text{postk2}) + b_4(\text{postk3}) + \dots + e_i$ where “date” is the calendar year; postk1 is date minus the date of the earliest defined knot if positive, or zero otherwise, and picks up any change in trend after the first knot (note that the knots are defined from the present backwards into the past, but the earliest knot is defined to ensure at least five observations between it and the start of the series); postk2 is date minus the date of the second defined knot if positive, or zero otherwise, and picks up any change in trend after the second knot; and so on.

² Conley and Springer (2001), Silva (2007) and Siah and Lee (2015) have considered modeling mortality time series using models that allows for the inclusion of structural breaks.

example, GDP per capita is usually nonstationary and if the mortality rates of these G7 countries are nonstationary, this would suggest potential long-run co-movements. Finally, convergence of U5MR among the G7 countries would be rejected if the order of persistence for mortality rate were different across countries.

2 The Present Study

The present study presents the time series of mortality rates only for the case of the G7 countries, since these are highly developed countries with a large population size. The U5MRs are different across these countries. The global differences are known to depend on the country's development which has a positive transformation on the efficiency of the different health and education policies in each country. Also, the occurrence of wars, droughts, vaccination campaigns, etc., might have caused significant changes in the mortality rates. The relevance of this variable as an indicator of a country's wellbeing and its dynamic response to health policies as it is being affected by other external constraints justifies the study of its dynamics across countries. Although it is positive to every individual as well the society as a whole if life continues, due to health challenges or hazards, lives are shortened. This affects the social and demographic settings and affects the public sector healthcare provision of the country in the long run.

The present study therefore considers the U5MR series of G7 countries and investigates linear time trends and nonlinearities by means of a fractional persistence approach. We tested for structural breaks under the assumption that the degree of persistence will be overestimated here due to the presence of structural breaks in the time series (Diebold and Inoue 2001; Ben Nasr et al. 2014), and with the possibility of nonlinearities in the trend. In testing for nonlinearity in the U5MR series, we use the approach suggested in Cuestas and Gil-Alana (2016) in which nonlinear deterministic (time) trends were employed in testing for fractional nonlinearity. Structural breaks are investigated by using Bai and Perron (2003) multiple structural break tests.

3 Methodology

3.1 Fractional Persistence and Time Trend

The standard framework for modeling time trends in time-dependent data is to assume a linear function of time given as,

$$y_t = \alpha + \beta t + x_t, \quad t = 1, 2, \dots \quad (1)$$

where y_t is the observed time series, that is the U5MRs, in our paper, and x_t is the deviation term. The parameter α is the constant in the model, while parameter β , the slope, measures the average change in y_t per time period. For the U5MR series, we should expect a significant negative value for β , which measures the average yearly reduction in the mortality rate. However, in order to make a valid statistical

inference about β , it is important to determine correctly the structure of the deviation term. For example, if x_t in (1) is an independently drawn random variable obtained from a normal distribution with zero mean and constant variance, then, ordinary least squares (OLS) estimates can be efficiently calculated, and inference is possible based on F and t test statistics (see Hamilton 1994; Draper and Smith 1998; Gil-Alana 2012). It is noted that the detrended data may at times display some degree of dependence, of which such behavior can be captured by different models. One of the most widely used models is the AutoRegressive process of order 1, that is, AR(1), defined for series x_t above as,

$$x_t = \phi_1 x_{t-1} + u_t, \quad t = 1, 2, \dots \quad (2)$$

with $|\phi_1| < 1$ conditioned so as to realize a stationary time series process and allow for meaningful inference, and u_t is the white noise process.³ If one therefore finds that the detrended series is nonstationary, the process is then said to be integrated of order one, that is $I(1)$. Then, x_t is nonstationary and the statistical inference should be based on its first differences, $x_t - x_{t-1}$ which are now stationary $I(0)$. Taking $\phi_1 = 1$, $By_t = y_{t-1}$ where B is the backward shift operator and u_t is an $I(0)$ series defined above, then Eqs. (1) and (2) combined become,

$$(1 - B)y_t = \beta + u_t, \quad t = 1, 2, \dots, \quad (3)$$

and one can easily construct another t test for testing $\beta = 0$ against the one-sided alternative $\beta < 0$.

The $I(0)$ and the $I(1)$ processes described above are two particular cases of a more general class of processes called fractionally integrated or $I(d)$ processes, where d is the number of differences required to obtain $I(0)$ series. Then, we say x_t is integrated of order d if

$$(1 - B)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (4)$$

with $x_t = 0$, $t \leq 0$ and u_t is $I(0)$. The polynomial on the left-hand side of (4) is thus expanded as,

$$(1 - B)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j B^j = 1 - dB + \frac{d(d-1)}{2} B^2 - \frac{d(d-1)(d-2)}{6} B^3 + \dots \quad (5)$$

implying that,

$$(1 - B)^d x_t = x_t - dx_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \frac{d(d-1)(d-2)}{6} x_{t-3} + \dots \quad (6)$$

Thus, the higher the value of d is, the higher the degree of association is between observations at distant time apart. If d is an integer value, x_t will be a function of

³ This model has been widely applied in many disciplines because of its relationship to the first-order stochastic differential equation.

a finite number of past observations, while fractional d implies that all past information matters. Thus, the parameter d plays a very crucial role in determining the degree of persistence in the series. If $d=0$ in (4), then $x_t = u_t$ and the process is short memory, which may be weakly autocorrelated, that is, for example, in the ARMA sense. If d is in the interval $(0, 0.5)$, the process is still covariance stationary but of long memory, that is the autocorrelations will take a longer period of time to decay/disappear than in the previous case of $I(0)$. If d is in the interval $[0.5, 1)$, the process is no longer covariance stationary but mean reverting in the sense that shocks will tend to disappear in the long run.⁴ Finally, if $d \geq 1$, x_t is nonstationary and not mean reverting, with the effect of the shocks persisting forever.

The estimation of d is carried out in the paper first by using the Whittle function in the frequency domain (Dahlhaus 1989) alongside a Lagrange Multiplier (LM) testing procedure (Robinson 1994) for testing the null hypothesis:

$$H_0 : d = d_0, \quad (7)$$

in (1) and (4) for any real value d . In this context, the estimates of d are carried out by choosing the value of d that produces the lowest statistic in absolute value with the tests of Robinson (1994), being in fact the estimates very similar to those obtained with the Whittle method. Nevertheless, this method allows us to compute the confidence intervals for the values of d where H_0 (7) cannot be rejected and it can be performed even in nonstationary contexts ($d \geq 0.5$) unlike other methods. Also, the limiting distribution of the test is standard normal unlike most unit root methods which are based on nonstandard critical values.⁵ Details about the estimating procedure can be found in Robinson (1994) and Gil-Alana (2005).

3.2 Testing NonLinear Time Trends

We test for nonlinear trends in the time series of the U5MR by employing the recently proposed nonlinearity test of Cuestas and Gil-Alana (2016). This test is based on fractional dependence. The nonlinear process is introduced as a Chebyshev polynomial in time to the linear fractionally integrated model of Robinson (1994) discussed earlier above to form a nonlinear deterministic test for testing nonlinearities in $I(d)$ processes. The general framework for the test is:

$$y_t = f(\theta; z_t) + x_t, \quad t = 1, 2, \dots, \quad (8)$$

where y_t is the time series under investigation and x_t follows a fractional $I(d)$ process with $d > 0$. The function $f(\cdot)$ is a nonlinear function that depends on the unknown parameter vector of dimension m , θ , and z_t , which is a vector of deterministic terms, and using the Chebyshev polynomial,

⁴ Such a feature (mean-reversion) is considered by some authors as a misnomer given the nonstationary nature of the series (Phillips and Xiao 1999).

⁵ In other standard large sample testing procedures, Wald and Likelihood Ratio (LR) test statistics against fractional alternatives have the same null and limit theory as the LM test of Robinson (1994). See Velasco and Lobato (2007).

$$y_t = \sum_{i=0}^m \theta_i P_{i,T}(t) + x_t, \quad t = 0, \pm 1, \dots, \quad (9)$$

where the order of the Chebyshev polynomial is m . The Chebyshev polynomial $P_{i,T}(t)$ is defined as,

$$P_{i,T}(t) = \sqrt{2} \cos [i\pi(t - 0.5)/T], \quad t = 1, 2, \dots, T; i = 1, 2, \dots \quad (10)$$

with $P_{0,T}(t) = 1$. Mathematically, whenever $m=0$ in the polynomial, the model in (9) is expressed with an intercept only; if $m=1$, it contains an intercept and a linear trend, and when $m>1$, it becomes a nonlinear process, and the higher m is, the greater the degree of nonlinearity in the polynomial.⁶ The choice of the value for m then depends on the significance of the Chebyshev coefficients.

3.3 Structural Breaks Test

We then carry out structural breaks test using Bai and Perron (2003) structural break tests. This is a multiple structural breaks testing approach, much preferred to Chow structural break tests. The test is based on the linear regression model,

$$y_t = z_t' \beta_i^0 + u_t, \quad i = 1, \dots, l+1, \quad t = T_{i-1}^0 + 1, \dots, T_i^0, \quad (11)$$

where $T_0^0 = 0$ and $T_{m+1}^0 = T$ giving the total number of observations, l denotes the number of breaks and therefore, $l+1$ indicates the number of the break regimes, y_t is the dependent variable, while z_t is a $p \times 1$ vector of exogenous or deterministic regressors which includes the constant term, a linear trend and/or the autoregressive terms, β_i^0 is the vector of regime-dependent coefficients with constant variance σ^2 . Due to unavailable a priori knowledge of the number of breaks to be expected in the series, a search strategy is conducted, and the different estimates of $\beta^* = (\beta_1^{*'}, \beta_2^{*'}, \dots, \beta_{n+1}^{*'})'$ for each break-regime identified are obtained by minimizing the residuals sum of squared,

$$S_N(T_1, \dots, T_n; \beta) = \sum_{i=1}^{n+1} \sum_{t=T_{i-1}+1}^T (y_t - z_t' \beta_i)^2, \quad (12)$$

with respect to $\beta = (\beta_1', \beta_2', \dots, \beta_{n+1}')'$; and we denote these estimates as $\hat{\beta}[(T_i)_{i=1}^n]$. The optimal break dates are then denoted as,

$$(\hat{T}_1, \dots, \hat{T}_n) = \arg \min_{T_1, \dots, T_n} S_T \{T_1, \dots, T_n; \hat{\beta}[(T_i)_{i=1}^n]\}. \quad (13)$$

⁶ Given that the residuals of the auxiliary regression are $I(0)$ stationary by assumption, t -statistics are valid to test for the significance of the nonlinear trends. This solves the problem of choosing the order of the Chebyshev polynomials, which was not clearly defined by Bierens (1997) for the unit root case.

Table 1 Time series data and sample size

Country	Initial	Dates	Sample size
Canada	CAN	1921–2011	91
France	FRA	1816–2013	198
Germany	GER	1956–2011	56
Italy	ITL	1872–2012	141
Japan	JAP	1947–2012	66
UK	UK	1922–2013	92
USA	US	1933–2013	81

The sequential test statistic for testing between the alternative of $n+1$ breaks against n break is given as F test:

$$F_T(n+1/n) = \hat{\sigma}^{-2} \left[S_T(\hat{T}_1, \dots, \hat{T}_n) - \min_{1 \leq i \leq n+1} \inf_{\tau \in \Lambda_i} S_T(\hat{T}_1, \dots, \hat{T}_{i-1}, \tau, \hat{T}_i, \dots, \hat{T}_n) \right], \quad (14)$$

where $S_T(\cdot)$ denotes the sum of squared residuals, $\hat{\sigma}^2$ is a consistent estimator of the disturbance variance and Λ_i is set of all partitions τ within the i th regime defined by $(\hat{T}_1, \dots, \hat{T}_n)$ such that both subsamples $(\hat{T}_\tau - \hat{T}_{i-1})$ and $(\hat{T}_i - \hat{T}_\tau)$ contain at least the minimum fraction ε of the total sample T .⁷

4 Data and Empirical Results

We apply annual under-5 mortality rates for the G7 countries: Canada (CAN), France, Germany, Italy, Japan, UK and the USA. These data were retrieved from Human Mortality Database at www.mortality.org. These data have different start and end dates, and Table 1 presents the data descriptions as well as the sample sizes.

Time plots of these time series are given in Fig. 1. For example, from the legend, CAN_0 represents neonatal mortality rates, CAN_1 represents infant mortality rates and we have CAN_2 to CAN_5 for the remaining under-5 mortality time measures. Due to the magnitude of each data point and for clarity, we have plotted only 0-year (neonatal) mortality series on the left scale, while < 1 year (infant) to < 5 years mortality rates are plotted on the right scale. A clearer look indicates quite a slower reduction of mortality rates in France, while others experience sharp reductions. In fact, the reduction of mortality rates in Japan seems to be sharper than in the remaining countries.⁸

Table 2 focuses on the estimation of the following model,

⁷ A similar approach was conducted by Gil-Alana (2008) for the fractional case. Very similar results in terms of the break dates were obtained to the Bai and Perron's (2003) approach.

⁸ Due to unavailability of longtime series for the case of Germany, we have taken the averages of long-time series for West and East Germany and these are considered as proxies for the German mortality rates.

Fig. 1 Plots of under-5 mortality rates for the G7 countries

Table 2 Estimates of the coefficients in Eqs. (1) and (4)

Country	Age	d (95% interval)	α (t values)	β (t values)
Canada	0 year	0.95 (0.85, 1.08)	-2.05 (-44.07)	-0.036 (-9.10)
	< 1 year	0.67 (0.58, 0.78)	-4.00 (-43.83)	-0.047 (-16.41)
	< 2 years	0.51 (0.42, 0.64) ^a	-4.76 (-60.59)	-0.044 (-25.61)
	< 3 years	0.38 (0.28, 0.53) ^a	-5.09 (-83.59)	-0.043 (-37.41)
	< 4 years	0.46 (0.31, 0.67) ^a	-5.33 (-68.57)	-0.042 (-26.44)
	< 5 years	0.46 (0.31, 0.68) ^a	-5.56 (-62.44)	-0.040 (-22.11)
France	0 year	0.87 (0.83, 0.92) ^a	-1.54 (-15.30)	-0.020 (-5.33)
	< 1 year	0.83 (0.79, 0.88) ^a	-2.952 (-21.86)	-0.026 (-6.21)
	< 2 years	0.84 (0.79, 0.91) ^a	-3.28 (-26.62)	-0.027 (-6.65)
	< 3 years	0.80 (0.74, 0.87) ^a	-3.64 (-27.63)	-0.026 (-7.25)
	< 4 years	0.84 (0.78, 0.92) ^a	-4.03 (-31.30)	-0.025 (-6.02)
	< 5 years	0.82 (0.75, 0.91) ^a	-4.19 (-30.95)	-0.025 (-6.05)
Germany	0 year	1.02 (0.87, 1.26)	-3.07 (-88.67)	-0.047 (-9.53)
	< 1 year	0.50 (0.36, 0.68) ^a	-5.69 (-103.4)	-0.045 (-24.07)
	< 2 years	0.19 (0.04, 0.41) ^a	-6.36 (-179.3)	-0.042 (-41.31)
	< 3 years	0.12 (0.01, 0.28) ^a	-6.65 (-195.1)	-0.041 (-41.32)
	< 4 years	0.15 (0.02, 0.35) ^a	-6.87 (-147.5)	-0.039 (-28.50)
	< 5 years	0.58 (0.39, 0.84) ^a	-7.08 (-74.98)	-0.035 (-100.9)
Italy	0 year	0.97 (0.92, 1.04)	-1.24 (-18.03)	-0.032 (-6.34)
	< 1 year	0.92 (0.86, 1.00)	-1.94 (-15.09)	-0.046 (-6.09)
	< 2 years	0.76 (0.70, 0.85) ^a	-2.29 (-16.71)	-0.047 (-11.40)
	< 3 years	0.63 (0.56, 0.72) ^a	-2.71 (-19.94)	-0.046 (-17.39)
	< 4 years	0.64 (0.56, 0.75) ^a	-3.17 (-24.55)	-0.043 (-16.78)
	< 5 years	0.66 (0.56, 0.78) ^a	-3.56 (-26.84)	-0.041 (-14.61)
Japan	0 year	0.97 (0.87, 1.11)	-2.33 (-41.51)	-0.057 (-9.27)
	< 1 year	1.15 (1.05, 1.29)	-3.24 (-34.81)	-0.079 (-3.92)
	< 2 years	0.95 (0.85, 1.10)	-4.04 (-35.25)	-0.068 (-5.82)
	< 3 years	0.96 (0.85, 1.13)	-4.40 (-38.42)	-0.063 (-5.25)
	< 4 years	0.90 (0.81, 1.03)	-4.92 (-48.57)	-0.060 (-7.00)
	< 5 years	0.80 (0.71, 0.93) ^a	-5.32 (-49.70)	-0.056 (-8.71)
UK	0 year	0.87 (0.76, 1.03)	-2.46 (-47.75)	-0.033 (-10.51)
	< 1 year	0.78 (0.72, 0.87) ^a	-3.64 (-30.99)	-0.049 (-9.44)
	< 2 years	0.70 (0.62, 0.80) ^a	-4.45 (-39.67)	-0.047 (-12.32)
	< 3 years	0.68 (0.59, 0.79) ^a	-5.06 (-45.68)	-0.044 (-12.41)
	< 4 years	0.76 (0.67, 0.89) ^a	-5.40 (-51.64)	-0.042 (-9.68)
	< 5 years	0.78 (0.69, 0.91) ^a	-5.53 (-51.92)	-0.042 (-8.85)
USA	0 year	1.07 (0.96, 1.24)	-2.76 (-92.39)	-0.028 (-6.54)
	< 1 year	1.02 (0.94, 1.13)	-4.62 (-80.77)	-0.038 (-5.65)
	< 2 years	0.97 (0.87, 1.09)	-5.40 (-99.44)	-0.035 (-6.61)
	< 3 years	0.88 (0.79, 1.00)	-5.74 (-100.6)	-0.034 (-8.72)
	< 4 years	1.00 (0.88, 1.16)	-6.00 (-118.1)	-0.033 (-5.97)
	< 5 years	0.83 (0.73, 0.98) ^a	-6.19 (-107.3)	-0.033 (-9.98)

^aEvidence of mean reversion ($d < 1$)

In bold, significant coefficients

$$y_t = \alpha + \beta t + x_t, \quad (1-L)^d x_t = u_t, \quad t = 1, 2, \dots \quad (15)$$

where y_t is the observed (univariate) time series; α and β are the coefficients corresponding to an intercept and a linear time trend, respectively, and x_t is assumed to be an $I(d)$ process implying that u_t is then $I(0)$. We report the estimated values of d (and the 95% confidence bands of the nonrejection values of d) along with the estimated coefficients for the intercept (α) and the linear trend (β) along with their corresponding t values.

The first thing that we observe is that for the 0-year (neonatal) data the unit root null hypothesis (i.e., $d=1$) is almost never rejected and the degrees of persistence are quite higher than those of infant mortality rates (<1 year), in all countries except for Japan. In fact, the only exception is France with an estimated value of d of 0.87, where mean reversion is statistically significant. However, as we increase the age (from 1 year to <5 years), the number of cases where this hypothesis is rejected in favor of mean reversion ($d < 1$) increases, and for the <5 year case, mean reversion is found in all countries. Except for the neonatal data, the lowest values of d are those obtained for Germany, followed by Canada, while the highest values correspond to Japan and the USA. On the other hand, the coefficients for the deterministic terms are found to be statistically significant in all cases. As expected, the time trends are all significantly negative and the highest reduction is obtained, in every case, for Japan, while the lowest reduction corresponds to France.

Next, to avoid distortions caused by the different sample sizes, and to get better comparisons across countries, we conduct the same type of analysis as in Table 2 but now using the same sample size in all series, i.e., for the time period 1956–2011. The results are presented in Table 3.

Generally speaking, the same conclusions as in Table 2 hold here and though quantitatively there are some differences, qualitatively the values are similar; the estimated values of d seem to decrease as we increase the age of death. Thus, for the neonatal cases, the unit root null hypothesis cannot be rejected in some cases and in the cases where it is rejected, it is against $d > 1$. In other words, we cannot find a single case with mean-reverting behavior. This is just the contrary to what happens for the <5 year case where mean reversion takes place in all cases. As before, Japan displays the highest degrees of persistence and Germany along with Canada and the UK the lowest degrees of persistence for all cases except neonatal. Once more, all the deterministic terms are found to be statistically significant. Again Japan, now closely followed by Italy present the highest reduction in the mortality rates, while the lowest values are those corresponding to the USA.

The possibility of nonlinear deterministic terms is also taken into account in the paper. Thus, in Tables 4 and 5 we focused on the following nonlinear specification, proposed by Cuestas and Gil-Alana (2016), and based on the Chebyshev polynomials in time,

$$y_t = \sum_{i=0}^3 \theta_i P_{iT}(t) + x_t, \quad (1-L)^d x_t = u_t, \quad (16)$$

Table 3 Estimates of the coefficients in Eqs. (1) and (4) with equal sample size

Country	Age	d (95% interval)	α (t values)	β (t values)
Canada	0 year	1.17 (1.06, 1.33)	-3.28 (-101.2)	-0.037 (-4.52)
	< 1 year	0.49 (0.35, 0.69) ^a	-6.02 (-102.3)	-0.039 (-20.01)
	< 2 years	0.21 (0.06, 0.41) ^a	-6.54 (-146.2)	-0.039 (-29.81)
	< 3 years	0.20 (0.08, 0.38) ^a	-6.71 (-149.1)	-0.040 (-30.93)
	< 4 years	0.40 (0.23, 0.64) ^a	-6.92 (-87.01)	-0.039 (-16.18)
	< 5 years	0.39 (0.22, 0.63) ^a	-7.06 (-76.64)	-0.038 (-13.73)
France	0 year	1.01 (0.85, 1.25)	-3.40 (-85.49)	-0.039 (-7.21)
	< 1 year	0.53 (0.38, 0.71) ^a	-5.87 (-101.3)	-0.043 (-21.24)
	< 2 years	0.34 (0.15, 0.63) ^a	-6.69 (-156.3)	-0.036 (-28.41)
	< 3 years	0.34 (0.22, 0.50) ^a	-7.03 (-133.1)	-0.034 (-21.73)
	< 4 years	0.49 (0.35, 0.70) ^a	-7.25 (-95.70)	-0.034 (-13.58)
	< 5 years	0.45 (0.30, 0.69) ^a	-7.41 (-91.15)	-0.032 (-12.43)
Germany	0 year	1.02 (0.87, 1.26)	-3.07 (-88.67)	-0.047 (-9.53)
	< 1 year	0.50 (0.36, 0.68) ^a	-5.69 (-103.4)	-0.045 (-24.07)
	< 2 years	0.19 (0.04, 0.41) ^a	-6.36 (-179.3)	-0.042 (-41.31)
	< 3 years	0.12 (0.01, 0.28) ^a	-6.65 (-195.1)	-0.041 (-41.32)
	< 4 years	0.15 (0.02, 0.35) ^a	-6.87 (-147.5)	-0.039 (-28.50)
	< 5 years	0.58 (0.39, 0.84) ^a	-7.08 (-74.98)	-0.035 (-100.9)
Italy	0 year	0.91 (0.75, 1.13)	-2.90 (-67.84)	-0.052 (-12.57)
	< 1 year	0.81 (0.70, 0.96) ^a	-5.16 (-53.52)	-0.059 (-8.89)
	< 2 years	0.67 (0.49, 0.93) ^a	-6.08 (-56.48)	-0.052 (-10.31)
	< 3 years	0.30 (0.16, 0.51) ^a	-6.56 (-83.69)	-0.048 (-20.83)
	< 4 years	0.30 (0.18, 0.49) ^a	-6.82 (-114.6)	-0.045 (-25.62)
	< 5 years	0.47 (0.29, 0.71) ^a	-6.99 (-86.29)	-0.044 (-16.52)
Japan	0 year	1.10 (1.01, 1.21)	-3.16 (-85.19)	-0.053 (-7.41)
	< 1 year	0.96 (0.83, 1.15)	-5.20 (-80.11)	-0.049 (-6.54)
	< 2 years	0.87 (0.72, 1.06)	-5.55 (-61.10)	-0.053 (-6.91)
	< 3 years	0.88 (0.74, 1.07)	-5.80 (-59.54)	-0.050 (-5.93)
	< 4 years	0.82 (0.68, 0.97) ^a	-6.06 (-65.50)	-0.051 (-7.64)
	< 5 years	0.75 (0.63, 0.90) ^a	-6.32 (-63.82)	-0.048 (-8.43)
UK	0 year	1.09 (0.93, 1.34)	-3.69 (-123.9)	-0.033 (-5.96)
	< 1 year	0.28 (0.08, 0.57) ^a	-6.38 (-190.9)	-0.031 (-31.87)
	< 2 years	0.30 (0.12, 0.57) ^a	-6.84 (-172.7)	-0.033 (-28.56)
	< 3 years	0.40 (0.25, 0.71) ^a	-7.36 (-118.8)	-0.033 (-16.69)
	< 4 years	0.45 (0.25, 0.71) ^a	-7.36 (-118.2)	-0.033 (-16.47)
	< 5 years	0.42 (0.26, 0.61) ^a	-7.43 (-129.4)	-0.033 (-18.81)
USA	0 year	1.36 (1.19, 1.64)	-3.54 (-182.2)	-0.026 (-2.77)
	< 1 year	0.57 (0.42, 0.76) ^a	-6.28 (-221.6)	-0.027 (-25.69)
	< 2 years	0.55 (0.37, 0.78) ^a	-6.75 (-266.3)	-0.027 (-29.12)
	< 3 years	0.44 (0.28, 0.64) ^a	-7.01 (-215.9)	-0.027 (-26.80)
	< 4 years	0.68 (0.46, 0.92) ^a	-7.25 (-188.4)	-0.027 (-14.67)
	< 5 years	0.53 (0.35, 0.73) ^a	-7.35 (-180.1)	-0.028 (-19.52)

^aEvidence of mean reversion ($d < 1$)

In bold, significant coefficients

Table 4 Estimates of d and nonlinear trend coefficients

Country	Age	d (95% interval)	θ_0	θ_1	θ_2	θ_3
Canada	0 year	0.72 (0.59, 0.91) ^a	-3.75 (-48.34)	1.09 (-24.92)	0.02 (1.00)	0.03 (1.38)
	< 1 year	-0.04 (-0.23, 0.22) ^a	-6.30 (-876.5)	1.30 (-172.2)	0.16 (21.71)	0.11 (15.26)
	< 2 years	0.10 (-0.06, 0.31) ^a	-6.84 (-468.3)	1.17 (-90.81)	0.08 (6.70)	0.15 (12.95)
	< 3 years	0.25 (0.11, 0.40) ^a	-7.09 (-249.6)	1.14 (54.07)	0.03 (1.70)	0.15 (9.03)
	< 4 years	0.37 (0.17, 0.60) ^a	-7.27 (-137.7)	1.09 (31.64)	0.01 (0.35)	0.17 (6.65)
	< 5 years	0.37 (0.15, 0.67) ^a	-7.31 (-123.5)	1.06 (26.87)	-0.01 (-0.47)	0.16 (5.45)
France	0 year	0.54 (0.46, 0.64) ^a	-2.74 (-32.65)	1.20 (24.45)	-0.58 (-15.2)	0.19 (6.20)
	< 1 year	0.46 (0.37, 0.56) ^a	-4.54 (-58.38)	1.69 (35.50)	-0.68 (-17.4)	0.15 (4.81)
	< 2 years	0.58 (0.48, 0.70) ^a	-5.15 (-39.98)	1.79 (23.99)	-0.52 (-9.20)	0.06 (1.41)
	< 3 years	0.59 (0.50, 0.70) ^a	-5.53 (-1690.)	0.67 (108.1)	-0.041 (-5.95)	0.06 (8.41)
	< 4 years	0.67 (0.57, 0.79) ^a	-5.83 (-27.95)	1.68 (14.07)	-0.47 (-5.65)	0.06 (1.01)
	< 5 years	0.67 (0.57, 0.80) ^a	-6.01 (-27.01)	1.62 (12.75)	-0.43 (-4.82)	0.08 (1.20)
Germany	0 year	1.07 (0.90, 1.28)	-4.36 (-24.56)	0.80 (7.56)	0.02 (0.58)	0.05 (1.56)
	< 1 year	0.25 (0.05, 0.51) ^a	-7.02 (-328.3)	0.70 (44.69)	0.03 (2.51)	0.12 (9.45)
	< 2 years	-0.08 (-0.30, 0.22) ^a	-7.58 (-1028.)	0.68 (84.24)	-0.01 (-2.13)	0.10 (12.32)
	< 3 years	-0.25 (-0.49, 0.08) ^a	-7.84 (-32.65)	1.20 (24.45)	-0.58 (-15.2)	0.19 (6.20)
	< 4 years	-0.20 (-0.44, 0.09) ^a	-7.99 (-1185.)	0.63 (74.78)	-0.04 (-4.80)	0.04 (4.67)
	< 5 years	0.30 (-0.06, 0.68) ^a	-8.13 (-199.5)	0.67 (23.64)	-0.07 (-3.10)	0.02 (1.17)
Italy	0 year	0.67 (0.55, 0.81) ^a	2.88 (-29.20)	1.34 (23.88)	-0.42 (-10.6)	0.17 (5.79)
	< 1 year	0.70 (0.60, 0.81) ^a	-4.45 (-20.69)	2.18 (17.79)	-0.52 (-6.27)	-0.01 (-0.16)
	< 2 years	0.57 (0.44, 0.72) ^a	-5.24 (-36.84)	2.16 (26.19)	-0.27 (-4.32)	0.04 (0.75)
	< 3 years	0.45 (0.33, 0.60) ^a	-5.71 (-60.16)	2.05 (35.01)	-0.17 (-3.66)	0.06 (1.66)
	< 4 years	0.46 (0.33, 0.63) ^a	-6.01 (-64.75)	1.94 (34.11)	-0.15 (-3.29)	0.07 (1.69)
	< 5 years	0.56 (0.44, 0.72) ^a	-6.26 (-4295)	1.84 (21.71)	-0.14 (-2.25)	0.08 (1.64)

Table 4 (continued)

Country	Age	<i>d</i> (95% interval)	θ_0	θ_1	θ_2	θ_3
Japan	0 year	0.89 (0.19, 1.17)	-4.37 (-25.22)	1.07 (10.75)	-0.18 (-3.31)	0.12 (3.02)
	< 1 year	1.16 (0.99, 1.36)	-5.16 (-6.99)	0.86 (1.89)	0.23 (-1.25)	0.22 (1.86)
	< 2 years	0.70 (0.50, 0.97) ^a	-6.78 (-39.49)	1.16 (12.05)	0.32 (4.93)	0.27 (5.37)
	< 3 years	0.69 (0.50, 0.99) ^a	-7.06 (-42.93)	1.14 (12.35)	0.32 (5.01)	0.24 (4.92)
	< 4 years	0.60 (0.43, 0.83) ^a	-7.40 (-70.96)	1.11 (18.65)	0.28 (6.40)	0.19 (5.55)
UK	< 5 years	0.51 (0.35, 0.73) ^a	-7.61 (-93.26)	1.05 (21.68)	0.23 (6.03)	0.17 (5.38)
	0 year	0.76 (0.61, 0.97) ^a	-3.99 (-37.79)	0.95 (15.99)	-0.00 (-0.10)	0.08 (3.07)
	< 1 year	0.28 (0.16, 0.43) ^a	-6.36 (-197.7)	1.27 (54.96)	0.30 (14.72)	0.25 (13.76)
	< 2 years	0.25 (0.11, 0.45) ^a	-6.94 (-227.1)	1.18 (52.42)	0.21 (10.47)	0.23 (12.53)
	< 3 years	0.45 (0.31, 0.62) ^a	-7.26 (-100.6)	1.15 (25.92)	0.15 (4.27)	0.21 (6.77)
USA	< 4 years	0.58 (0.46, 0.74) ^a	-7.49 (-69.54)	1.13 (18.20)	0.13 (2.83)	0.18 (4.95)
	< 5 years	0.61 (0.49, 0.76) ^a	-7.62 (-62.42)	1.14 (16.32)	0.13 (2.62)	0.17 (4.32)
	0 year	1.08 (0.96, 1.25)	-3.92 (-20.44)	0.72 (6.23)	0.03 (0.65)	0.04 (1.27)
	< 1 year	0.98 (0.88, 1.11)	-6.31 (-25.21)	0.83 (5.63)	0.15 (2.04)	0.18 (3.62)
	< 2 years	0.96 (0.85, 1.10)	-6.84 (-30.35)	0.74 (5.62)	0.08 (1.21)	0.16 (3.57)
	< 3 years	0.88 (0.77, 1.01)	-7.16 (-39.60)	0.74 (7.16)	0.06 (1.11)	0.16 (3.85)
	< 4 years	1.01 (0.88, 1.19)	-7.33 (-28.87)	0.72 (4.79)	0.05 (0.65)	0.14 (2.96)
	< 5 years	0.85 (0.74, 1.00)	-7.56 (-45.76)	0.74 (7.90)	0.04 (0.73)	0.14 (3.67)

In bold, significance of the test at 5% level

Table 5 Estimates of d and nonlinear trend coefficients with equal sample sizes

Country	Age	d (95% interval)	θ_0	θ_1	θ_2	θ_3
Canada	0 year	0.88 (0.32, 1.22)	-4.47 (-52.66)	0.65 (13.45)	0.13 (4.80)	0.01 (0.76)
	< 1 year	0.21 (-0.09, 0.68) ^a	-7.22 (-330.8)	0.64 (37.52)	0.06 (4.16)	0.05 (3.63)
	< 2 years	-0.28 (-0.51, 0.03) ^a	-7.65 (1763.)	0.64 (106.4)	-0.01 (-2.33)	0.02 (2.76)
	< 3 years	-0.04 (-0.22, 0.21) ^a	-7.87 (-733.3)	0.67 (59.15)	0.04 (3.68)	0.03 (3.32)
	< 4 years	0.19 (-0.10, 0.56) ^a	-8.01 (-260.2)	0.66 (27.12)	-0.05 (-2.56)	0.03 (1.46)
	< 5 years	0.09 (-0.28, 0.48) ^a	-8.11 (-335.5)	0.66 (30.68)	-0.05 (-2.50)	0.01 (2.25)
France	0 year	0.84 (0.56, 1.17)	-4.63 (-50.61)	0.69 (13.49)	0.06 (2.18)	0.06 (2.74)
	< 1 year	0.46 (0.21, 0.76) ^a	-7.179 (-140.4)	0.68 (22.03)	0.06 (2.33)	0.11 (5.08)
	< 2 years	0.32 (0.00, 0.70) ^a	-7.72 (-251.5)	0.60 (28.50)	-0.01 (-0.33)	0.03 (1.87)
	< 3 years	-0.05 (-0.28, 0.25) ^a	-7.99 (-914.4)	0.57 (61.57)	-0.05 (-5.63)	0.02 (1.86)
	< 4 years	0.03 (-0.27, 0.42) ^a	-8.19 (-635-4)	0.59 (48.19)	-0.08 (-6.56)	0.01 (0.86)
	< 5 years	-0.55(-0.86, -0.11) ^a	-8.29 (-4675.)	0.58 (20.03)	-0.08 (-24.34)	0.05 (1.36)
Germany	0 year	1.07 (0.90, 1.28)	-4.37 (-24.56)	0.80 (7.56)	0.03 (0.58)	0.05 (1.56)
	< 1 year	0.25 (0.05, 0.51) ^a	-7.03 (-328.3)	0.70 (44.69)	0.04 (2.51)	0.12 (9.45)
	< 2 years	-0.08 (-0.30, 0.22) ^a	-7.58 (-1028.)	0.68 (84.24)	-0.02 (-2.13)	0.11 (12.32)
	< 3 years	-0.25 (-0.49, 0.08) ^a	-7.84 (-32.65)	1.20 (24.45)	-0.58 (-15.2)	0.20 (6.20)
	< 4 years	-0.20 (-0.44, 0.09) ^a	-7.99 (-1185.)	0.64 (74.78)	-0.04 (-4.80)	0.04 (4.67)
	< 5 years	0.30 (-0.06, 0.68) ^a	-8.14 (-199.5)	0.67 (23.64)	-0.07 (-3.10)	0.03 (1.17)
Italy	0 year	0.55 (0.26, 0.94) ^a	-4.42 (-121.7)	0.92 (43.51)	0.03 (1.92)	0.06 (4.50)
	< 1 year	0.36 (0.09, 0.68) ^a	-7.18 (-174.6)	0.95 (34.73)	0.18 (7.85)	0.17 (8.19)
	< 2 years	0.36 (0.01, 0.80) ^a	-7.70 (-145.5)	0.82 (23.31)	0.09 (3.21)	0.14 (7.49)
	< 3 years	0.02 (-0.22, 0.34) ^a	-7.96 (-411.1)	0.74 (39.65)	0.06 (3.31)	0.13 (7.49)
	< 4 years	-0.17 (-0.43, 0.20) ^a	-8.14 (-1134.)	0.71 (81.11)	0.07 (7.22)	0.09 (9.64)
	< 5 years	0.35 (0.13, 0.62) ^a	-8.27 (-169.6)	0.70 (21.63)	0.03 (1.08)	0.11 (4.56)

Table 5 (continued)

Country	Age	d (95% interval)	θ_0	θ_1	θ_2	θ_3
Japan	0 year	1.04 (0.88, 1.22)	-4.72 (-27.02)	0.79 (7.65)	0.15 (2.92)	0.12 (3.62)
	< 1 year	0.96 (0.72, 1.23)	-6.57 (-27.52)	0.69 (5.00)	0.09 (1.30)	0.13 (2.77)
	< 2 years	0.78 (0.52, 1.09)	-7.19 (-39.36)	0.78 (7.64)	0.14 (2.15)	0.16 (3.40)
	< 3 years	0.80 (0.59, 1.06)	-7.41 (-36.17)	0.77 (6.76)	0.15 (2.11)	0.14 (2.69)
	< 4 years	0.67 (0.46, 0.92) ^a	-7.73 (-60.93)	0.79 (11.12)	0.16 (3.26)	0.13 (3.49)
	< 5 years	0.62 (0.43, 0.83) ^a	-7.90 (-67.86)	0.77 (11.75)	0.14 (2.92)	0.11 (3.11)
UK	0 year	1.07 (0.79, 1.32)	-4.57 (-29.60)	0.56 (6.16)	0.02 (0.61)	-0.00 (-1.12)
	< 1 year	0.04 (-0.26, 0.38) ^a	-7.28 (-827.7)	0.52 (62.16)	-0.01 (-0.67)	0.02 (3.38)
	< 2 years	0.13 (-0.19, 0.50) ^a	-7.79 (-524.1)	0.54 (43.30)	-0.02 (-1.99)	0.04 (3.85)
	< 3 years	0.28 (0.02, 0.55) ^a	-8.07 (-233.3)	0.56 (22.85)	-0.04 (-1.78)	0.02 (1.18)
	< 4 years	0.14 (-0.09, 0.44) ^a	-8.26 (-490.5)	0.57 (40.76)	-0.03 (-1.97)	0.01 (0.85)
	< 5 years	-0.21 (-0.45, 0.11) ^a	-8.38 (-1949.)	0.57 (105.1)	-0.01 (2.34)	0.00 (0.57)
USA	0 year	1.20 (0.92, 1.50)	-4.39 (-28.83)	0.47 (5.03)	0.08 (2.11)	0.02 (0.9)
	< 1 year	0.41 (0.21, 0.66) ^a	-7.09 (-384.2)	0.45 (39.16)	0.02 (1.87)	0.04 (5.33)
	< 2 years	0.06 (-0.34, 0.58) ^a	-7.52 (-1586.)	0.45 (103.4)	-0.01 (-2.46)	0.03 (7.67)
	< 3 years	0.09 (-0.14, 0.39) ^a	-7.79 (-1015.)	0.46 (67.52)	-0.01 (-2.17)	0.03 (4.90)
	< 4 years	0.36 (0.11, 0.69) ^a	-8.00 (-439.8)	0.47 (39.27)	-0.01 (-0.93)	0.02 (3.15)
	< 5 years	0.35 (0.19, 0.54) ^a	-8.14 (-380.3)	0.49 (34.19)	0.01 (-1.19)	0.02 (2.26)

In bold, significance of the test at 5% level

assuming again that u_t is a white noise process.

As in the linear case, the evidence of mean reversion is higher as we increase the age of death and mean reversion takes place in all cases for the <5 year case. With respect to the nonlinearities, we observe strong evidence of nonlinearities in the majority of the cases. Thus, in Table 4, of the 42 cases presented (6 cases for 7 countries), we found strong evidence of nonlinearities (i.e., both θ_2 and θ_3 -coefficients being statistically significant) in 25 cases, and partial evidence of nonlinearity (with either θ_2 or θ_3 being statistically significant) in 14 cases. In fact, the only 3 cases with clear evidence of linearity correspond to the neonatal data in Canada, Germany and the USA. The corresponding proportions in Table 5 (with equal sample size in all cases) are 25, 15 and 2. These two cases with evidence of linearity are now Germany and the UK again with neonatal data.

Due to the fact that fractional persistence and nonlinearities exist in the time series of the mortality rates, we then proceed to check for structural breaks as well as the timings of the breaks within each country mortality rates, with the results presented in Table 6. Due to the longtime series involved, we have identified five significant breaks in the mean of the series, each for the U5MR. For Canada, the three breaks that are consistent across the neonatal (0 year), infant (<1 year) and child (1–5 years) mortality rates are found in 1934, 1947 and 1960. France experienced consistent breaks for mortality ages 1–5 years in 1862, 1892, 1921 and 1950. Consistent break in German mortality is found in 1964 for mortality ages 0–4 years. For Italy, from mortality ages 2–5 years, we identified consistent breaks in 1921, 1947, 1948 and 1989. In Japan, across the U5MR, we identified consistent breaks in 1956, 1965, 1974 and 1983. In the UK, these breaks are found across the U5MR in 1935, 1948 and 1961, while in the USA, the consistent breaks are found in 1945 and 1957. The fact that there are consistent structural breaks in the U5MR series is an indication of the response of neonatal, infant and child mortality rates to health policies put in place by the government as well as the effects of other external constraints being felt at the same time in the country.

5 Conclusion and Policy Implications

In this paper, we have examined the statistical properties of the under-5 mortality rate series in the G7 countries. For this purpose, we have used $I(d)$ techniques combined with structural breaks and nonlinear structures. The main result obtained in the paper is that for the neonatal data the orders of integration are equal to or higher than 1 in all cases, contrary to what happens in the remaining cases (<1 – <5 years) where mean reversion ($d < 1$) is found in many cases, especially as we increase the age of death. This implies that shocks affecting the neonatal series will have permanent effects requiring strong measures to recover the original trend unlike the remaining cases where the effects of the shocks will recover by themselves. As expected, all the time trend coefficients are negative and statistically significant, and the highest reduction in the mortality rates are obtained in Japan while the lowest takes place in the USA.

Table 6 Results of Structural breaks

Country	Age	<i>F</i> test	Five break points
Canada	0 year	480.81	1934, 1947, 1960 , 1973, 1986
	< 1 year	227.79	1934, 1947, 1960 , 1973, 1987
	< 2 years	270.90	1934, 1947, 1960 , 1977, 1991
	< 3 years	323.95	1934, 1947, 1960 , 1976, 1990
	< 4 years	260.03	1934, 1947, 1960 , 1974, 1989
France	< 5 years	244.19	1934, 1947, 1960 , 1974, 1987
	0 year	911.16	1845, 1893, 1922, 1951, 1980
	< 1 year	767.47	1860, 1895, 1924, 1953, 1982
	< 2 years	830.59	1862, 1892, 1921, 1950 , 1979
	< 3 years	912.61	1862, 1892, 1921, 1950 , 1979
Germany	< 4 years	849.80	1862, 1892, 1921, 1950 , 1982
	< 5 years	790.54	1862, 1892, 1921, 1950 , 1984
	0 year	247.90	1964, 1972 , 1980, 1988, 1997
	< 1 year	226.41	1964, 1972 , 1981, 1989, 1999
	< 2 years	311.84	1964, 1972 , 1982, 1992, 2000
Italy	< 3 years	289.79	1964, 1974, 1983, 1992, 2002
	< 4 years	272.14	1964, 1972 , 1981, 1989, 1998
	< 5 years	243.81	1965, 1975 , 1983, 1993, 2004
	0 year	994.79	1896, 1922, 1948, 1969, 1990
	< 1 year	479.04	1893, 1922, 1947, 1948, 1989
Japan	< 2 years	396.04	1896, 1921, 1947, 1968, 1989
	< 3 years	344.35	1897, 1921, 1947, 1968, 1989
	< 4 years	338.74	1896, 1921, 1947, 1968, 1989
	< 5 years	333.06	1893, 1921, 1947, 1968, 1989
	0 year	113.05	1956, 1965, 1974, 1983 , 1996
UK	< 1 year	26.40	1956, 1965, 1974, 1983 , 1999
	< 2 years	55.77	1956, 1965, 1974, 1983 , 1999
	< 3 years	72.53	1956, 1965, 1974, 1983 , 1996
	< 4 years	103.56	1956, 1965, 1974, 1983 , 1998
	< 5 years	97.35	1956, 1965, 1974, 1983 , 1996
USA	0 year	757.83	1935, 1948, 1961 , 1976, 1990
	< 1 year	202.22	1935, 1948, 1961 , 1975, 1992
	< 2 years	191.36	1935, 1948 , 1962, 1975, 1992
	< 3 years	327.77	1935, 1948, 1961 , 1976, 1992
	< 4 years	359.19	1935, 1948, 1961 , 1974, 1990
	< 5 years	346.85	1935, 1948, 1961 , 1977, 1992
	0 year	318.39	1945, 1957 , 1971, 1983, 1995
	< 1 year	102.48	1945, 1957 , 1969, 1981, 1996
	< 2 years	108.25	1945, 1957 , 1970, 1982, 1996
	< 3 years	118.83	1945, 1957 , 1971, 1983, 1997
	< 4 years	126.59	1945, 1957 , 1970, 1982, 1996
	< 5 years	138.28	1945, 1957 , 1972, 1984, 1998

Critical value for the sequential *F* tests at 5% significant level for one break as given in Bai and Perron (2003) is 3.91, and the significance is in bold. The same break dates across the mortality rates are also given in bold

As noted in Babel et al. (2008), the 17-year increase in the life expectancy in Japan results in the highest reduction observed in the U5MR, while the nine-year increase in the life expectancy in the USA confirms the low reduction of U5MR in the country. Finally, we suggest using nonlinear trend approaches while studying mortality rates, and the computation of life tables should account for modeling these nonlinear trends of mortality. A robust time series approach is therefore necessary to investigate and explain the trend dynamics in child mortality rates.

Though the analysis of the G7 countries is of great interest, the study of the IMR in the developing countries is even more interesting to study their dynamic behavior, in spite of the shortness and the potential poor quality of the data. Alternative forms of nonlinear terms including, for example, Hermite polynomials can also be employed. Work in these directions is now in progress.

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