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Laboratory simulations of the atmospheric mixed-layer in flow over complex topography

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Abstract

A laboratory study of the influence of complex terrain on the interface between a well-mixed boundary layer and an elevated stratified layer was conducted in the towing-tank facility of the U.S. Environmental Protection Agency. The height of the mixed layer in the daytime boundary layer can have a strong influence on the concentration of pollutants within this layer. Deflections of streamlines at the height of the interface are primarily a function of hill Froude number (Fr), the ratio of mixed-layer height (z_i) to terrain height (h), and the crosswind dimension of the terrain. The magnitude of the deflections increases as Fr increases and z_i/h decreases. For mixing-height streamlines that are initially below the terrain top, the response is linear with Fr ; for those initially above the terrain feature the response to Fr is more complex. Once Fr exceeds about 2, the terrain-related response of the mixed layer interface decreases somewhat with increasing Fr (toward more neutral flow). Deflections are also shown to increase as the crosswind dimensions of the terrain increase. Comparisons with numerical modeling, limited field data, and other laboratory measurements reported in the literature are favorable. Additionally, visual observations of dye streamers suggest that the flow structure exhibited for our elevated inversions passing over three dimensional hills is similar to that reported in the literature for continuously stratified flow over two-dimensional hills.

INTRODUCTION

The depth of the daytime well-mixed layer, z_i , has been established as a dominant length scale for characterizing and parameterizing turbulence and diffusion in convectively driven boundary layers (Deardorff, 1970; 1972; and Briggs, 1985). In convective scaling, z_i serves a purpose analogous to that of the Monin-Obukhov length in surface-layer scaling. The mixing layer depth represents the vertical dimension of the dominant energy-carrying buoyant eddies in the often deep convective boundary layer. Large-eddy motions, turbulent velocities, temperature fluctuations, and the vertical profiles of wind and temperature all scale, to some extent, with z_i . The magnitude of z_i and its variation in complex terrain are important in air pollution modeling, since the mixed layer is frequently capped by a stable layer or even an inversion layer that tends to inhibit the vertical diffusion of pollutants. Information in the literature about the mixed layer's variation in complex terrain is fairly limited. Thus, an

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experimental fluid modeling study was performed to examine the response of the streamlines at z_i for varying meteorology and terrain scenarios.

Techniques for estimating z_i in horizontally homogeneous terrain include Holzworth 1964, (Holzworth 1967), Irwin and Paumier (1990), Raynor and Watson (1991), Batchvarova and Gryning (1991), and Weil *et al.*, (1997). The adjustment of z_i with flow over and around irregular terrain has received much less attention. A few studies have addressed this issue from a numerical modeling viewpoint including some comparisons with field data or laboratory scale modeling. For example, Marwitz *et al.* 1969 used a shallow-water model by Lavoie, 1968 to estimate the deflection of the z_i streamline over a model of Elk Mountain in Wyoming. Their results for flow over the mountain top overestimated the adjustment in z_i in comparison with their aircraft measurements. Kitabayashi *et al.* 1971 modeled the same Elk Mountain feature in a stratified wind tunnel and, for the flow over the mountain top, found less adjustment to z_i than that estimated by the Marwitz model. Mahrer and Pielke 1975 used a two-dimensional mesoscale model of airflow over mountains and reported on one case where they explicitly calculated the spatial variation in the depth of a convective boundary layer over a two-dimensional ridge. These cases are compared with our results in Section III. While there is significant literature discussing the characteristics of wind-driven flow over complex topography (e.g., thorough review by Jackson *et al.* 2013) as well as examination of terrain-influenced stratified flow (e.g., Baines 2005), the dynamic response of a mixed layer topped by a stable layer as it passes over complex topography has received relatively little attention.

Hunt 1980 commented on the character of the flow of a neutral layer capped by a stable layer over terrain obstacles. His qualitative descriptions are based primarily on the studies referenced above and give emphasis to lee-side effects. Lamb and Britter 1984 and Smith and Sun 1987 discussed shallow-water models which yield estimates of z_i streamline deflections for two- and three-dimensional terrain features. In addition, Lamb and Britter's modeling was coupled with flow visualization results in a tow tank. Hunt *et al.*, 1988 proposed formulas for calculating the change of the mixing height over low hills (i.e., hill height much less than the horizontal length scale of the hill). For the special case of a two-dimensional, bell-shaped hill, their computations compare well with those of Carruthers and Choularton (1982), who used a three-layer model of flow over low hills. None of these studies focused on determining the adjustment of the mixed layer over a range of conditions.

The impetus of this study was to improve our understanding of boundary-layer flow interactions with complex terrain features and to provide for the development of an algorithm for adjusting the mixed layer depth within applied dispersion models, e.g., the steady-state, Gaussian-based Complex Terrain Dispersion Model, CTDMPLUS (Perry 1992). A large stratified towing tank was used for the study. The data indicate that terrain shape, the magnitude of z_i in the undisturbed region upstream of the hill, and the hill Froude number, Fr , are parameters that scale the dynamic response of z_i to changes in topography. Fr is defined as $U/(Nh)$, where U is the flow speed, h is the terrain height, and N is the Brunt Vaisala frequency $((-g\rho^{-1} \rho / z)^{1/2})$ in the stable layer above the mixed layer. The mixed layer interface shows the greatest response to a two-dimensional (ridge-like) terrain when z_i is initially near or below the hill height, when the long axis of the terrain is perpendicular to

the wind and when the stratification aloft is weak and/or the flow speed is large (i.e., $Fr \gg 2$). Note that the towing-tank measurements do not provide for the simulation of convection processes (in particular convective turbulence) which are primarily responsible for the temporal growth of the daytime convective mixed layer. However, when the height of the mixed layer can be considered quasi-stationary (i.e., the time scale for the convection-induced changes in z_i is large, typically 20–30 min in the atmosphere, compared with the time required for the flow to traverse the terrain), the dynamics of the flow over the terrain, rather than convection, are expected to dominate the changes in the z_i -streamline. In addition, in the convective atmospheric mixed layer, the mean potential temperature and velocity profiles show small vertical variations above the shallow surface layer where shear dominates. This relatively uniform mixed layer might be compared to that in the towing tank where the approach flow is uniform except for a thin turbulent boundary layer that develops over the flat plate supporting the terrain features. In summary, the experiments examined here enhance our understanding of the dynamic effects on the response of the mixed layer depth to flow over topography. This paper contains a description of the experimental approach and a discussion of study findings and conclusions.

EXPERIMENTAL APPROACH

To simulate the mean flow in a daytime well-mixed layer beneath a deep stable layer, the large towing tank (described by Hunt and Snyder 1980) at the U. S. Environmental Protection Agency's Fluid Modeling Facility was used. It should be noted that the laboratory experiments were conducted before the decommissioning in 2005 of the EPA's towing tank facility. The tank was 2.4 m wide and 25 m long with a total working depth of 1.1 m. Stratification was accomplished by filling the tank with many layers of varying salinity. To set up for each tow, the tow-tank filling process began with a layer of fresh water to a depth representing the desired z_i ; then approximately 2.5 cm deep saltwater layers of incrementally increasing salinity (density) were injected, each below the previous layer of salt water in the tank. After filling, this arrangement, when viewed inverted, provided a neutral layer capped by a stable layer with a linear density profile (Figure 1). By towing an inverted terrain model along the water surface (U is the towing speed), the adjustment of the interface (representing the streamline at z_i) between the fresh water and the saltwater was examined. This provided a vertically uniform approach flow to the terrain which is not unlike that in a well-mixed convective layer above the shallow surface layer. The interface movement was documented with measurements of the salinity profile before and during the tow and visually with neutrally buoyant dye streamers. Measurements were initiated at 20% of the tow length and ended at 80% of the tow length to avoid instability in startup of the tow and tank end-wall effects. It should be noted that due to the inverted nature of the experimental setup, the coordinate system for reporting and analysis of the data is such that $z = 0$ is defined as the base of the hill (which is also the water surface) and increases with depth in the tank. All results are reported in this usual atmospheric manner for clarity.

For rigorous similarity between the laboratory and the atmosphere, Snyder (1972; 1981) points out that it is necessary to match the Reynolds number ($Re = Uh/\nu$, where ν is the kinematic viscosity), the Froude number ($Fr = U/Nh$, where N is the Brunt Vaisala frequency), the Peclet number ($Pe = Uh/\kappa$, where κ is the thermal diffusivity), the Rossby

number ($Ro = U/h\Omega$, where Ω is the angular speed of the Earth's rotation) and the boundary conditions (e.g., approach flow turbulence).

Exact matching of Re in the laboratory where models orders of magnitude smaller than in the atmosphere is practically impossible. Townsend 1956 first put forth the hypothesis that geometrically similar flows are similar for sufficiently high Re . While systematic experiments to determine the influence of varying Re on the flow patterns were not conducted in this study, there has been considerable experimental evidence in the literature supporting that variations are minimal for the range of Re included here. For example, Hoydysh *et al.* 1974 found the critical Re to be of the order of 3400 for neutral flow in and around arrays of cubical buildings. Baines and Manins 1989 suggest that independence is achieved for Re in excess of several hundred when modeling the large scale features of stable flows with water as the medium (as in a towing tank). Hunt and Snyder 1980 found no variations in the flow patterns over terrain (similar to that in this study) for $Re > 10\,000$. They go on to point out that numerous authors have noted that flow patterns observed in the atmosphere (e.g., lee waves and hydraulic jumps) can be simulated in the laboratory with Re values even smaller than those in their experiments. In the current study where tow speeds varied between 5 and 50 cm s^{-1} , Re varied between 8000 and 80 000. Hence, Re independence was generally expected.

With Re independence, turbulent diffusion dominates molecular diffusion and the Peclet number may also be neglected as a modeling criterion (Snyder, 1972). Also, for horizontal length scales less than 5 km, the Coriolis accelerations due to the Earth's rotation are small and directional shear in the Ekman layer can be ignored. Therefore, when examining the effects of complex topography on near-field dispersion problems (on the order of a few kilometers), the Rossby number is relatively unimportant.

For stable flows, the Froude number, Fr , is the most important parameter to match between laboratory and atmosphere (Batchelor, 1953 and Snyder, 1972). Fr^2 represents the ratio of inertial forces to buoyancy forces and is equivalent to the inverse of the Richardson number. The importance of z_i/h and Fr in stable conditions is also supported by Baines and Manins 1989 in their discussions of the general principles of laboratory modeling. For the specific boundary-layer situation where a neutral layer is capped by a constant-density-gradient stable layer in complex terrain, Baines and Manins include the following parameters: U (the towing speed), z_i/h , N , v , the depth of the tank, and the hill shape. Non-dimensional parameters may be combined from these to form Re , Fr , z_i/h . Hill shape is readily matched with a scale model in the tank studies. Tank depth (of approximately 7 h) does not significantly influence the results here since the model terrain heights are relatively low and the blockage ratio is less than 10% for the elongated hill and 5% for the ACCB hill. As mentioned above, Reynolds number independence was expected for the tow speeds and terrain heights in this study. The remaining parameters, Fr and z_i/h , are the indicators against which the behavior of the z_i streamline is characterized. By achieving similarity between tank and atmosphere, the essential dynamics of the corresponding atmospheric flow can be captured in the laboratory.

In the atmospheric boundary layer, Fr generally ranges from about 0.1 (for example, when $U = 1 \text{ ms}^{-1}$, $N = 0.03 \text{ s}^{-1}$, and h is a few hundred meters) to approaching infinity (i.e., high winds, neutral stability). z_i ranges from around 100 m to a few thousand meters with terrain heights generally in the same range. The range of Fr achieved in the laboratory during this study was 0.4 to ∞ , and z_i/h ranged from 0.5 to 1.8. Although the experiments were limited to only two terrain shapes, these results, in conjunction with a few previous field and numerical studies, begin to demonstrate the degree of influence that hill shape has on mixing-height streamline displacement.

The two model terrain features are shown in Figure 2. The first, labeled ACCB, was an axisymmetric, fourth-order polynomial-shaped hill designed as an idealization of Cinder Cone Butte, an actual isolated hill southeast of Boise, Idaho. The hill surface is described by

$$height(r) = \frac{h + c}{1 + \left(\frac{r}{L}\right)^4} - c \quad (1)$$

where r is the radial distance from the hill center, h is the hill height at the center, L is the horizontal dimension from the hill center to surface at half the hill height (i.e., the half-height length), and c (on the order of 1 cm) is a parameter used to truncate the hill surface at a finite value of r . The ACCB model has $h = 15.8 \text{ cm}$ and $L = 38.8 \text{ cm}$ (approximately $2.5h$). The maximum hill slope is 24° . When r exceeds 77.5 cm (a little less than $5h$), the height is equal to zero. A photograph of ACCB is seen in Figure 2(a) with its lateral (crosswind) profile shown in Figure 3. Note that the hill height is approximately 15% of the total tank depth of 105 cm.

The second hill used in the study was an elongated version of ACCB with the same height and along-flow, hill-surface profile but with a crossflow half-height dimension of $5.0h$. The elongated hill was constructed by splitting the ACCB model in half in the along-wind plane and inserting a ridge section between the two halves. The ridge section has the same along-wind profile as the center profile of ACCB. The elongated hill (Figure 2(b)) was oriented in the tank with the long axis perpendicular to the flow. The results for this terrain feature provided an indication of the response of the flow to a more ridge-like obstruction than that of ACCB with all other terrain characteristics the same. The lateral profile of the elongated hill is also shown in Figure 3. While both terrain features in this study have the same half-length, L , in the stream-wise direction, we recognize that L is a significant variable. Additionally, other important influences on flow separation not specifically varied in this study include terrain surface roughness (uniformly smooth in this study) and the along-flow shape of the terrain profile.

Vertical deflections of the streamlines over the crests of the model hills were quantified by comparing the vertical profiles of water density before (static) and during (dynamic) each of the steady-speed tows. Density samples were collected from a vertical sampling rake at the

upstream end of the tank and from a vertical sampling rake located at the crest of the hill. Prior to each tow, static conditions were sampled from each rake providing a complete vertical density profile. Once each tow was initiated and a brief period of time was allowed for the flow to become steady, density samples were collected again from the vertical sampling rake located at the crest of the hill (Figure 1). A vertical profile of the average streamline deflections, indicating vertical shifting of each salinity level in the tank, was obtained when the terrain was towed at constant speed over the length of the test section (approximately 20 m).

Example density profiles for static and dynamic conditions for flow over ACCB are shown in Figure 4. These measurements from the sample rake on the crest of ACCB display a general lifting of the horizontal layers from their static levels as the flow moves over the ACCB surface. The amount of deflection occurring at the level of z_i as well as the levels above z_i (in the stably stratified region) was measured. For both the static and dynamic profiles z_i/h is estimated to be the height where a best fit straight line to the density profile in the neutral layer intersects that for the profile in the stratified layer. The initial z_i/h in this case was approximately 1.75. These data show that the top of the mixed layer rose during the tow by approximately $0.55 h$ over its level far upstream of the hill (represented by the value during static conditions). By obtaining static and dynamic density profiles for a range of Froude numbers, static z_i/h values, and the two hill shapes, relationships between these boundary layer parameters, the hill characteristics and the deformation of the z_i streamline were established. Supplemental quantitative and qualitative information about the streamline deflections were obtained by videotaping neutrally buoyant dye streamers released into the undisturbed region upstream of the hill surface (Figure 1).

The upstream (or static) z_i/h was varied by either refilling the tank or by skimming (withdrawing) some of the fresh water from the top of the tank and slowly injecting salt water at the bottom. The Froude number was varied primarily by varying the tow speed. For the majority of the study, the density gradient in the stable layer was fixed at approximately $0.002 \text{ (g cm}^{-3}\text{)/cm}$ yielding $N = 1.35 \text{ s}^{-1}$. However, to additionally support the selection of Fr as an appropriate scaling factor, ten tows were executed with the density gradient established at one-tenth the previous value, i.e., $0.0002 \text{ (g cm}^{-3}\text{)/cm}$ yielding $N = 0.43 \text{ s}^{-1}$. For these additional tows where z_i/h and Fr number were matched with previous tows, the deflections in the z_i streamline matched within a few percent, i.e., within the expected repeatability of the individual experiments.

RESULTS AND DISCUSSION

Seventy-eight individual tows were performed on the model terrain. The qualitative features of the flow, as revealed by the visual observations of tracer dye streamers, are discussed. This is followed by an examination of the quantitative measurements of streamline deflections over the full range of flow conditions and two terrain shapes.

A. Visual observations of flow structure

The primary parameters defining our experiments were Froude number, generally ranging from 0.4 to 2.3, and z_i/h , ranging from 0.5 to 1.8. A few additional experiments were

conducted with larger Froude numbers of 3, 7, and ∞ to examine streamline deflections in near neutral to fully neutral stability. For the infinite Fr case there was no interface (i.e., no definable z_i) between neutral and stable as the entire layer was neutrally stratified.

For the study's lowest Froude number ($Fr = 0.4$) and inversion height ($z_i/h = 0.5$), we estimate that the dividing-streamline height, h_D , is $0.6 h$ based on the energy arguments of (Sheppard, 1956) and the laboratory demonstrations of Snyder *et al.*, (1985). For this case the inversion base is below h_D and thus as a first guess the streamline at $z_i/h = 0.5$ would not be predicted to surmount the hill. A photograph of this case for the elongated hill (Figure 5(a)) shows that the inversion base apparently has difficulty surmounting the hill, as the lowest dye streamer, released at $z_i/h = 0.5$, is seen to virtually disappear at the hill top; flow below the level of this streamer must pass around the sides of the hill. Vertical motions within the neutral region below $z_i/h = 0.5$ are not restricted by stratification. Thus we see that the lowest streamer, positioned at the top of this neutral region, is exhibiting more vertical spread upon approaching the hill while the higher streamlines, released at and above z_i , are much more constrained. This and the associated case for the ACCB hill are the only cases in our matrix where a dividing-streamline exists, i.e., where any part of the elevated stable layer is below the hill top. In the region below the hill top and upwind of the hill, a distinction exists between flows for two- and three-dimensional hills for this low Froude number condition. In the stable layer, streamlines below the dividing-streamline are able to pass (horizontally) around the sides of a three-dimensional hill whereas two-dimensional hills will exhibit the phenomenon of "blocking" (see Turner 1973). Thus, for cases where the mixing-height streamline is below the dividing streamline, differences upwind and near the hill should be expected.

To aid in our understanding of the structure of the flow as it passes over the terrain, it is helpful to consider the lee waves generated by the hill in the stratified portion of the flow. Hunt and Snyder 1980 discussed stratified flow over two-dimensional hills of low to moderate slope. In contrast to our simulations, their stratified layer extended to the surface (i.e., no neutral mixed layer), yet their conclusions concerning stable flows over hills are demonstrated to be applicable here. They suggest that when the Froude number is of order unity, the length of the (lee) waves generated in the stratified flow by the hill is of the same order as the height of the hill, and these waves begin to control the flow. But they go on to suggest that rather than hill height, a more appropriate parameter for controlling the character of the surface boundary layer is the ratio of the wavelength of the lee waves ($2\pi U/N$) to the total length (defined as $2L_0$) of the hill at its base (Hunt and Snyder also point out that the wavelength is modified by the finite depth of the tank, but this effect is rather small for the cases we discuss here). Separation is, in effect, dependent upon how closely the lee wave matches the hill shape. When this ratio ($\Lambda = (2\pi U/N)/2L_0$) is small ($\ll 1$), separation is likely to occur at the lee-wave base, which in this case is rather high up on the lee slope. With separation induced by the lee wave on the lee slope, we refer to this as lee-wave controlled. As Λ increases (increasing wavelength), separation continues to occur at the lee-wave base yet farther down the lee slope of the hill. When Λ is near unity, separation on the lee slope of the hill is reduced to a minimum (or even completely suppressed) and will occur at the downwind base of the hill. As Λ surpasses unity, separation may occur downwind of the base of the hill, forming a rotor or, depending on the

lee slope and nature of the hill surface, separation may occur on the lee slope, i.e., it may be boundary-layer controlled. By boundary-layer controlled we mean that separation is dependent upon the ordinary ability of flow in the boundary layer to follow the curvature of the surface. As Λ grows larger still, separation will be more and more likely to be boundary-layer controlled.

How do these ideas apply to the present situation, where the flow is not continuously stratified and the hills are not two-dimensional? Hunt and Snyder suggest that, as far as lee waves and separation are concerned, the flow structure discussed above is just as applicable to three-dimensional hills as it is to two-dimensional ones. Our flow visualization gives us some insight into the effect of the depth of the neutral layer beneath the inversion and the range of wavelengths wherein stratification suppresses separation on the lee slope.

Looking again at Figure 5(a), waves are generated in the upper levels ($z/h > 0.5$) with wavelength $2\pi U/N (= 2.5 h$ in this case where $Fr = 0.4$) and waves of roughly this magnitude are observed downwind in the stratified layer. The parameter Λ in this case is approximately 0.25, so that the waves are quite short in comparison with the overall length of the hill, and it appears that separation on the lee slope is lee-wave controlled. Also, judging from the “fuzziness” of the streamers downwind, it appears that strong turbulence, indicating overturning, was occurring in the stratified layer to heights well above the hill top. The base of the inversion in the wake is marked by the heavy dye line, which has a trajectory that seems to be controlled by the waves in the stratified layer above. In this case, the flow appears to be quite similar to that described by Hunt and Snyder as if a flat plate were inserted into the flow with a height approximately that of the dividing streamline.

In Figure 5(b), the Froude number is the same (0.4), but the inversion base is now at $z/h = 1.5$. Again, Λ is 0.25, and we again observe waves in the stratified layer that are approximately a quarter of the overall length of the hill. Further, we observe the three lowest streamers dispersing rather freely in the neutral layer, whereas those above (at and above the inversion base) are much more constrained. On the lee side, separation occurs at nearly the same position as in the earlier case, so the separation again appears to be lee-wave controlled; but the nature of the flow downwind of the separation point is quite different, with an elevated rotor in evidence and less turbulence at the higher elevations.

Next, we show a case with a larger Froude number ($Fr = 0.91$, Figure 5(c)) but the same initial inversion height ($z/h = 0.5$) as in Figure 5(a). The lowest streamer sails freely over the hill. Here, Λ is 0.6 and the wavelength is more closely matched to the length of the hill; separation is completely suppressed on the lee slope, but occurs beyond the downwind base (just out of the photograph), forming a rotor beyond the separation point. Very strong downslope winds close to the surface are evidenced by the converging (and narrowing) streamers seen there. Strong turbulence is observed well above the streamers on the lee slope (i.e., fuzziness due to mixing of the layer of different densities is observed to heights in excess of $2 h$). From the standpoint of atmospheric dispersion, it is important to note that the point of closest approach of the streamers to the hill surface is on the lee slope and not at the crest of the hill (at the front of the sampling rake) where the deflections were measured. In fact, at the higher elevations the streamers begin tilting downward well upstream of the crest

of the hill and, above the hill center, the streamers are showing rather strong negative deflections (the deflection of the uppermost streamer from its release elevation is $-0.2 h$).

A case with an even larger Froude number ($Fr = 1.56$) and with z_i/h again at 0.5 is shown in Figure 5(d). Here, $\Lambda = 1.3$, that is, a wavelength somewhat longer than the hill. Separation is evident in a thin layer near the lee slope and, in this case, is boundary-layer controlled. Strong downslope winds are again observed but, because of the separation, the streamlines on the lee side do not approach the surface so closely as those at the Froude number of 0.91 (Figure 5(c)). Nevertheless, the separated region is clearly rather thin. As in the previous case, downward deflections are noted over the central, relatively flat, portion of the hill and at the hill center, with the uppermost streamer exhibiting an even stronger negative deflection ($-0.4 h$).

Finally, Figure 5(e) shows a case with still larger Froude number (1.88) and, again, with $z_i/h = 0.5$. Here, the wavelength is clearly longer than the hill ($\Lambda = 2.0$) and the separated region is much deeper. Separation is clearly boundary-layer controlled. Indeed, the wave would appear to match a hill of considerably longer base length than that of the elongated model of CCB. Note that for this long wavelength, the opposite effect on the upper-level streamers near the center of the hill is in contrast with the previous two cases (Figures 5(c) and 5(d)); i.e., the deflection of the uppermost streamer from its release elevation is positive ($+0.25 h$).

To generalize, the behavior of these upper-level streamers was relatively independent of z_i/h but strongly dependent upon the Froude number (or, equivalently, upon the wavelength of the lee waves). At small Froude numbers (short wavelengths), separation was lee-wave controlled and occurred near the upper portion of the lee slope of the hill, and the upper-level streamers were, for the most part, roughly horizontal. For Froude numbers in the range of 0.9 to 1.5 ($0.6 < \Lambda < 1.3$), separation was suppressed on the lee slope and the upper-level streamlines exhibited negative deflections over the hill top. At larger Froude numbers, lee waves considerably longer than the hill base were observed, so that separation was boundary-layer controlled and occurred on the lee slope. The separated layer grew in depth as the lee waves grew longer and the deflections of the upper-level streamers turned positive. For any given case, it was observed that the magnitude of the deflections decreases with an increase in release height.

In summary, within the stratified towing tank, the flow structure exhibited by an elevated inversion passing over an elongated three-dimensional hill seems to be reasonably well-described in a qualitative sense using the arguments of Hunt and Snyder, (1980) for continuously stratified flow over two-dimensional hills. This comparison holds true even though our experiments contained a neutral layer beneath the inversion in contrast to the continuous stratification of Hunt and Snyder. Our discussion has been centered on the elongated hill but our experiments suggest that the arguments of Hunt and Snyder also hold for the ACCB hill.

B. Quantitative observation of the flow structure

Having examined some of the qualitative features of the streamline deflections over the model hills, we turn next to quantifying the amount of deflection of the specific streamline

that is centered horizontally on the hill and at a height (z_i) defining the interface between the stable layer and the neutral layer below it. Deflection in this discussion is the change in height of the z_i -streamline from its upwind, undisturbed level to that at the hill top (hill center). The experimental results for both hills are summarized in Figures 6 and 7. Figure 6 shows the deflections of z_i as a function of both the initial z_i level and the Fr for the ACCB hill. Not surprisingly, as the upstream height of z_i increases, the influence of the hill decreases and the deflections in z_i are reduced. For example, with $Fr = 1.55$, a doubling of the initial z_i level from one to two hill heights results in a 52% decrease in the z_i deflection at hill center.

For a given initial z_i , an increase in Fr is generally associated with an increase in z_i deflections. Since $(Fr)^2$ represents the ratio of inertial to buoyancy forces in the tank, large Fr are associated with either large kinetic energy of the approach flow or a weakness in the stability (density gradient) aloft. In either case, increasing Fr increases the ability of the flow in the mixed layer to lift the stratified fluid aloft in response to the presence of the hill.

The relationship between streamline deflections, Fr , and initial z_i/h is shown for the elongated hill in Figure 7. Comparisons with Figure 6 show that in practically every case the magnitude of the deflection for a given Fr and initial z_i is somewhat greater in comparison with that for ACCB. Since the elongated hill is more ridge-like, there is less lateral divergence of the streamlines approaching the hill top and an associated increase in the vertical deflections.

The more direct comparison of the differences between streamline deflections by the two hills is displayed in Figure 8. Here, the ratio of the z_i for the elongated hill and that for ACCB is shown for Froude numbers of 0.4 and 2.2. The difference is most pronounced for the lower Froude number and when the initial z_i is significantly above the hill top. In particular, for $Fr = 0.4$ and initial z_i between 1.5 and 1.8 times the hill height, the elongated terrain provides almost twice the deflection as that with ACCB. Although not demonstrated in this laboratory study, if the elongated feature was lengthened further in the lateral dimension (toward a truly two dimensional feature), it is reasonable to surmise that the deflections would increase even further.

An interesting feature of the relationships in Figures 6 and 7 can be seen by regrouping these data by initial z_i/h and plotting Fr on the abscissa. Figure 9 recasts the data of Figure 6 in this manner for the ACCB results. Similar relationships are found for the elongated hill (not shown here). We note that, generally speaking, deflections increase with Froude number but, as we expect, decrease with z_i/h . This is in accordance with our observations of flow structure: as we stated earlier, the behavior of the upper-level streamers was relatively independent of z_i/h , but strongly dependent upon the Froude number; also, we found that, in any given case, observed streamline deflections decreased with an increase in height.

When the inversion base is at half the hill height ($z_i/h = 0.5$), the deflections of the z_i -streamline appear to increase almost linearly with the Froude number over the range $0.5 < Fr < 2.2$. At larger values of z_i/h , the sensitivity to Froude number decreases in the range of $0.9 < Fr < 1.5$, with little variation when z_i/h is 1.47 and 1.84. Several repetitions of these cases

verified this interesting result. These “less-sensitive” regions in the curves appear to be related to the separation on the lee slopes of the hills; this is the same range of Fr where the lee wavelengths generated in the stratified portion of the flow roughly match the overall length of the hills. When z_i/h was small, the deflections of the z_i -streamlines, being closer to the hill surface, were less influenced by the presence or absence of separation on the lee slope of the hill. For the cases where the undisturbed z_i/h was made larger, however, the deflections become more influenced by the separation on the lee slope, and in the range of conditions where separation on the lee slope was suppressed (i.e., $0.9 < Fr < 1.5$), the flow structure remained the same, so that deflections were more nearly the same, that is, independent of Froude number.

Finally, several experiments were conducted on the ACCB feature with high Fr numbers (i.e., 3.1 and 7.1) by increasing the tow speed up to the maximum capable in the tank. Additionally, two experiments were conducted without stratification in the tank ($N=0$ and $Fr=\infty$). For these latter two cases with no density stratification, streamline deflections were estimated by examining videotapes of dye plumes released in an x-z plane centered laterally on the hills. Interestingly, the results for $Fr=3.1$, 7.1, and ∞ were indistinguishable from each other and the z_i deflections for these near neutral cases were actually less than those for $Fr=2.24$.

C. Comparisons of tow tank results with other published data

Figure 10 shows a comparison of our laboratory results with those from previous numerical modeling and wind tunnel studies where flow over three-dimensional terrain features has been examined and the response of the mixing-height streamline can be estimated. Marwitz *et al.* 1969 used a shallow-water model to examine the flow over Elk Mountain in Wyoming, USA. Elk Mountain is a roughly axisymmetric hill with the major and minor axes having half-height dimensions of about 4 km and 3 km, respectively. For their only reported case of mixing-height streamline deflection over the hill center (initial z_i/h of 0.68, Fr approximately 1), Marwitz found a deflection of approximately 0.55 h. Agreement with our findings for ACCB is quite reasonable (Figure 10). The wind direction in the Marwitz case was such that the flow paralleled the longer axis of the hill. Kitabayashi *et al.*, (1971) also modeled Elk Mountain with the same orientation in a stratified wind tunnel. With an initial z_i/h of 1.25 and Fr of 1.0, Kitabayashi measured a streamline deflection of 0.28, that is, about 20% below our findings for ACCB. The reason for the difference in mixing height deflection is not obvious. But as mentioned above, Elk Mountain is only approximately axisymmetric and both the Marwitz and Kitabayashi results are for an approach flow along the major axis. Perhaps having an approach flow along a more-gentle slope than with the ACCB hill can be considered one reason for the differences. Further investigation with other hill shapes is desirable.

A case of mixing-height streamline deflection reported by Mahrer and Pielke 1975 is also included in Figure 10. Their numerical modeling results represent the change in the height of a simulated convective boundary layer as it passed over a two dimensional ridge of significant horizontal scale (the half-height length scale in the along wind direction is about 16 km and hill height is about 1.9 km). The initial z_i/h is 0.97 and the Fr is 0.72. Their model

results indicate a deflection in z_i from its well-upstream value to that at the mountain top of about $0.52 h$. This matches fairly well with our tank-study results for the elongated hill with $z_i/h = 1.0$ and $Fr = 0.9$ where the z_i streamline was found to deflect by $0.49 h$. With the lower Froude number (0.72 versus 0.9), we would have expected the Mahrer and Pielke model to show a somewhat lower deflection than with the elongated hill. However, the mesoscale model included surface heating that would likely increase the vertical position of the mixed layer top over that expected from dynamic effects alone as the flow passes over the extensive mountain surface. This in combination with the truly two-dimensional nature of their terrain feature may explain the observed differences with our laboratory measurements.

Finally, we examine the streamline deflections at all heights for a neutral flow throughout the depth of the towing tank. For this particular experimental condition, there is no defined mixing layer since no stable layer aloft exists. Streamline deflections were estimated from dye streamers released upstream of the hills. Figure 11 displays streamline-height deflections for ACCB and the elongated hill. For comparison, we have also included the same streamline deflections observed by Hunt and Snyder 1980 for neutral flow over a bell-shaped hill (the profile of which is also the reciprocal of a fourth-order polynomial but with a much steeper slope than the hills in our study). Their experiments were conducted in the same towing tank as ours. Lastly, we included streamline deflections predicted by potential flow theory over a hemispherical hill shape.

Figure 11 shows, as we expect, that streamline deflections decrease with increases in initial height. More importantly, it clearly shows that streamline deflections increase markedly with an increase in the crosswind dimension of the hill. The “Hunt hill” shape was closest to that of a hemisphere, and the data are quite close to those suggested by potential flow. The progressively larger crosswind dimensions of the ACCB and elongated hills result in correspondingly larger streamline deflections over the hill tops. By comparing these deflections with those from moderately stratified flows (e.g., Figure 10, where $Fr = 0.9$), it is clear that the stratification is very effective in suppressing streamline deflections.

DISCUSSION

A laboratory fluid modeling study of the influence of terrain on the interface between a well mixed-layer and a stratified layer was conducted in an effort to understand the analogous effects of complex topography on the height of the convective atmospheric mixed layer. A database has been established from measurements during 78 individual “tows” in a stratified towing tank that can serve to develop and evaluate modeling approaches for estimating terrain-induced changes in this important boundary layer scaling parameter.

Visual observations of dye streamers released “upwind” of the modeled terrain reveal several features of the flow and specifically the mixing height streamline. For example, the dividing-streamline concept (Snyder et al., 1985) appears to apply to the flow within the elevated inversion in spite of the well-mixed or neutral layer below. The behavior of the streamers in the stable layer was found to be approximately independent of the height of the neutral layer below but strongly dependent upon the Froude number. Separation of the flow on the lee

slopes of the hills was controlled by the lee-waves when the wavelength was short compared to the length scale of the terrain; that separation was controlled more by the flow following the shape of the terrain. The strongest downslope flow was observed when the lee-wave length was close to the terrain length. In this case, the streamlines in the stratified layer were swept down the lee slope very close to the surface. It is interesting that these features of the flow that we observed in our experiments appear very similar to those discussed by Hunt and Snyder (1980) for a two-dimensional terrain.

Deflections in the mixing-height streamline are shown to be primarily related to the shape of the hill feature and to two dimensionless parameters, the Froude number and the ratio of the upstream mixing height to the hill height. The relationship of flow response to Fr and upstream mixing height coincides with findings by Baines and Manins (1989) when simulating elevated stable layers in complex terrain. In the current study, the magnitude of the deflections was found to increase with increasing Fr (up to $Fr = 2.21$) and with decreasing z_i/h . Measurements were taken for Fr values as high as 7.1 (a nearly neutral layer). These, in combination with estimates from video recordings of streamers in purely neutral flows ($Fr = \infty$), suggest that little sensitivity to Fr exists for values of about 3 or greater. The data also show that the mixing height response to changing Fr is nearly linear when the layer is shallower than the hill. For initial z_i above the hill top, the response to changing Fr is nonlinear. In particular, for initial z_i at $1.2 h$ and higher, the deflection showed little response to increasing Fr within the range of 0.9 and 1.5. These “weak-response” areas were seen for both terrain features tested.

Terrain shape is clearly an important factor in influencing the flow distortions. Deflections over a more ridge-like feature (i.e., the elongated hill) are generally greater than those over an axisymmetric feature particularly for high initial z_i/h . As shown by Mahrer and Pielke 1975, deflections over a fully two-dimensional feature are even greater. Several previous studies are mentioned that address, for limited cases, the issue of z_i variations over terrain. In the cases where comparisons with the results of the current study were practical, good agreement was shown. These kinds of comparisons over a wide range of conditions for a variety of both actual and idealized terrain features are important for both understanding these complex flows and for providing a basis for improving algorithms for simulating flow and dispersion in the atmospheric boundary layer.

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FIG. 1. Schematic diagram of experimental setup in stratified towing tank. The tow direction from left to right yields an effective flow direction from right to left.

FIG. 2. Photographs of terrain features used in study. (a) ACCB hill, (b) elongated hill. Sample rakes for collecting vertical profiles of salinity during the tows are seen on the top of each hill; dye release rakes are seen at the leading edge of the terrain platforms.

FIG. 3. Cross-flow profiles of the two hills used in this study. Both hills have the same centerline profile in the stream-wise direction and both have same height, $h = 15.9$ cm.

FIG. 4.

Example vertical profiles of specific gravity as measured from the sampling rake on the crest of the model hill. The static profiles were taken immediately before the tow began and the dynamic profiles are sample averages over the duration of the tow.

FIG. 5.

Photographs of streamers over elongated hill. “Flow” from left to right. (a) $Fr = 0.4$, $z_i/h = 0.5$; (b) $Fr = 0.4$, $z_i/h = 1.5$; (c) $Fr = 0.91$, $z_i/h = 0.5$; (d) $Fr = 1.56$, $z_i/h = 0.5$; (e) $Fr = 1.88$, $z_i/h = 0.5$. Dye streamers in each case originate upwind of the hill at $z/h = 0.5, 1.0, 1.3, 1.6, 1.8, 2.1, 2.3$, and 2.6 .

FIG. 6.

Deflections of the z_i -streamline (from its undisturbed upwind position to its position over the hill center) as a function of its initial height and hill Froude number for the ACCB hill. Lines are included here and in Figure 7 simply to aid the eye.

FIG. 7.

Deflections of the z_i -streamline (from its undisturbed upwind position to its positions over the hill center) as a function of its initial height and hill Froude number for the elongated hill.

FIG. 8.

Ratio of z_i - streamline deflections over the elongated hill to those over ACCB for two different Froude numbers. Lines are included to simply aid the eye.

FIG. 9.

Deflections of z_i -streamlines as a function of Froude number and upstream height for hill ACCB. Note that data at $Fr = 3, 7$, and ∞ were indistinguishable from each other and are all represented by points at $Fr = 3.0$.

FIG. 10.

Effects of hill shape on deflection of z_i – streamlines. Current experiments, $Fr = 0.9$; Marwitz *et al.* and Kitabayashi *et al.*, Elk Mtn, $Fr = 1$; Mahrer and Pielke, long ridge, $Fr = 0.72$. Inset shows lateral (cross-flow) hill profiles. Lines are added to aid the eye.

FIG. 11.

Effects of hill shape on deflection of streamlines in neutral flow. Hunt hill and potential flow data from Hunt and Snyder 1980. Inset shows lateral hill profiles. Lines are added to aid the eye.