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A Denoising Algorithm for CT Image Using Low-rank Sparse Coding

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Abstract

We propose a denoising method of CT image based on low-rank sparse coding. The proposed method constructs an adaptive dictionary of image patches and estimates the sparse coding regularization parameters using the Bayesian interpretation. A low-rank approximation approach is used to simultaneously construct the dictionary and achieve sparse representation through clustering similar image patches. A variable-splitting scheme and a quadratic optimization are used to reconstruct CT image based on achieved sparse coefficients. We tested this denoising technology using phantom, brain and abdominal CT images. The experimental results showed that the proposed method delivers state-of-art denoising performance, both in terms of objective criteria and visual quality.

Keywords

CT; denoising; sparse coding

1. INTRODUCTION

Medical image denoising by means of digital image processing could improve diagnosis potential of CT images. Distinct type of noises like additive Gaussian noise, multiplicative speckle noise and artifacts in different imaging modalities degrade the image quality (1–9). Computer-aided analysis and quantitative measurements become difficult and unreliable due to poor image quality therefore the denoising and enhancement of the images become major requirements for many applications (1, 3, 10–16). By the development of compressive sensing theory (17), sparse-representation-based image restoration algorithms (18, 19) have been considered as a kind of promising methods for image denoising. Generally, this kind of methods utilize a prior knowledge that each image patch can be sparsely and linearly represented under some over-complete domains spanned by a set of bases. However, the

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sparse coefficients of the noisy image patches are generated in degraded patches field, i.e., the regularization optimization for patches' sparse representation are disturbed by the artifact caused by CT imaging system. Thus, these methods often lead to an unstable model and will diminish the denoising performance. In order to solve this problem, in this study, we propose an approach for CT image denoising based on sparse representation error regularization and low-rank sparse coding.

2. METHOD

We assume that the given noisy CT image y is generated by adding an additive noise v to a good standard CT image x , the goal is to reconstruct the x by medical image sparse prior and structural similarity. In order to restore medical images effectively, recent studies (1, 11, 12, 20-23) demonstrate the efficacy of nonlocal self-similarity constraint, i.e., a local image patch can be represented by its nonlocal similar patches. Thus, we first extract overlapping patches and cluster similar patches from noisy image y through k-nearest-neighbor (kNN) (24) searching method. The similarity measurement is set to be inversely proportional to the distance between two patches $x_{i,k}$ and $x_{i,j}$, i.e., $w_{i,j,k} = \exp(-\|x_{i,j} - x_{i,k}\|_2^2 / h) / C$, where h controls the weight and C is the normalization factor. For each of well-clustered similar patches $\{x_{i,1}, x_{i,2}, \dots, x_{i,m}\}$, a data set (matrix) X_i is obtained by concerning the patches as columns: $X_i = \{x_{i,1}, x_{i,2}, \dots, x_{i,m}\} \in R^{n \times m}$. Secondly, the principle component analysis (PCA) dictionary (25, 26) of X_i is initialized by computing the PCA basis for each X_i due to the assumption that similar patches should share common sub-dictionaries, which encourages to admit a similar sparse decomposition. Then, the dictionary is applied for initial sparse representing the patches within X_i by incorporating a group-wise sparse coding noise (GSCN) regularization (12). The regularization parameter is efficiently evaluated by Bayesian interpretation. Thirdly, by initializing the PCA dictionary of X_i and the sparse coefficients, a simultaneous sparse coding method (18) is applied to iteratively refine the adaptive PCA dictionary of similar patch data set X_i and compute the optimized sparse representation of X_i by variable-splitting scheme (27, 28). Simultaneous sparse coding can be regarded as a matrix-based quadratic optimization which can be solved by incorporating low-rank approximation (29–32). Finally, with well-learned PCA dictionary and well-learned sparse representation, the denoised patch is reconstructed by multiplying the dictionary with sparse coefficient and then averaged into the final denoised image. The brief workflow of our proposed method can be seen by Fig. 1.

2.1 Adaptive Regularization

After well-clustered similar CT patches and computing the initial PCA dictionary, the sparse coefficients of similar patch group $X_i = \{x_{i,1}, x_{i,2}, \dots, x_{i,m}\}$ can be estimated by optimizing the following GSCN regularization:

$$\alpha_i = \underset{\alpha_i}{\operatorname{argmin}} \sum_{j=1}^m \left(\|x_{i,j} - U_i \alpha_{i,j}\|_2^2 + \tau_i \|\alpha_{i,j} - \beta_{i,j}\|_1 \right) \quad (1)$$

where α_{ij} denotes the sparse coefficients of x_{ij} , U_i is the PCA dictionary, β_{ij} denotes the good standard of x_{ij} which can be obtained by mean of $\alpha_i = \{\alpha_{i,1}, \alpha_{i,2}, \dots, \alpha_{i,m}\}$, τ_i represents the regularization parameter which can be solved by Bayesian interpretation. In contrast to classical sparse representation, Eq. (1) use l_1 regularization of $\alpha_{ij} - \beta_{ij}$ i.e., difference between sparse representation and good standard, to improve the performance the sparse representation, due to the fact that the difference term $\alpha_{ij} - \beta_{ij}$ should be minimized for the image quality enhancement of denoising CT image patch $x_{ij} = U_o \alpha_{ij}$. The proposed regularization of Eq. (1) utilizes the non-local similarity of CT patches under the assumption that similar patches can be sparsely represented in similar coefficients by a PCT dictionary, takes both sparse prior constraint and sparse coding error minimization into consideration to diminish sparse coding error. Thus, it will potentially enhance the robustness and stability of denoising model. The efficiency and effectiveness of Eq. (1) is demonstrated by (12).

2.2 Simultaneous Sparse Coding

With initialized PCA dictionary and sparse representation, in order to build the highly connection between sparse coefficients and denoised patches with higher quality and less artifact, the simultaneous sparse coding is applied to further improve the informative and relevant of sparse coefficients. The objective function can be formulated as:

$$(\hat{x}, \hat{U}_i, \hat{\alpha}_i) = \arg \min_{x, U_i, \alpha_i} \|y - x\|_2^2 + \gamma \sum_i \sum_{j=1}^m (\|R_{i,j}x - U_i \alpha_{i,j}\|_2^2 + \tau_i \|\alpha_{i,j} - \beta_{i,j}\|_1) \quad (2)$$

where $\hat{\alpha}_i$ is the concatenation of α_{ij} , γ is an adaptive regularization parameter, R_{ij} denotes the patch extraction operator. The first term $\|y - x\|_2^2$ of Eq. (2) is the loss penalty computed by quadratic difference between denoised CT x and input noisy CT image y . The second term $\|R_{i,j}x - U_i \alpha_{i,j}\|_2^2$ of Eq. (2) denotes the reconstruction penalty computed by squared l_2 - norm for each group of similar patches. The third term $\|\alpha_{ij} - \beta_{ij}\|_1$ of Eq. (2) represents the group-wise sparse coding noise regularization which is described in last subsection. Thus Eq. (2) integrate the CT image patches' sparsity regularization and the sparse coding error minimization into an image denoising model which is a unified multi-variable framework for optimization.

Generally, Eq. (2) can be solved by the iterative shrinkage algorithm (33). However, the major disadvantage of iterative shrinkage algorithm is the computational complexity. Thus, we apply a variable-splitting scheme that separates the image reconstruction and sparse representation to alleviate the problem and significantly reduce the computational complexity. First, the sparse coefficients α_{ij} can be solved similar as Eq. (1) with evaluated PCA dictionary in last iteration. Then, we use low-rank approximation to refine the PCA dictionary in next iteration. Let $A_i = \{\alpha_{i,1}, \alpha_{i,2}, \dots, \alpha_{i,m}\}$ and $B_i = \{\beta_{i,1}, \beta_{i,2}, \dots, \beta_{i,m}\}$, and suppose σ_{ij} denotes the standard deviation of the sparse coefficients A_i in the i -th row, then we have the transformation of Eq. (2):

$$(\hat{U}_i, \hat{A}_i) = \arg \min_{U_i, A_i} \|X_i - U_i A_i\|_F^2 + \tau_i \sqrt{\sum_{j=1}^m \left(1 + \sum_{k=1}^m w_{i,j,k}^2\right)} \sum_{l=1}^n \sigma_{i,l} \quad (3)$$

where $w_{i,j,k}$ denotes the measurement of similarity between CT patch $x_{i,j}$ and $x_{i,k}$. Low-rank approximation Eq. (3) can be solved by thresholding singular value decomposition (SVD) algorithm. At last, with well-estimated $\hat{U}_i$ and $\hat{A}_i$ by optimizing Eq. (3), the denoised CT image x is computed by solving the quadratic optimization:

$$\hat{x} = \frac{y + \gamma \sum_i (R_i x - \hat{U}_i, \hat{A}_i)}{I + \gamma \sum_i R_i^T R_i} \quad (4)$$

where $R_i = [R_{i,1}, R_{i,2}, \dots, R_{i,m}]$ denotes a set of patch extraction operators and I denotes the unit matrix with same size of y .

One important concerning of solving Eq. (3), which will potentially affect the denoising performance, is the appropriate choice of regularization parameter τ_i . Inspired by the study of previous wavelet denoising algorithms (2, 34) which utilize a Bayesian-interpretation-based maximum a posteriori estimation (MAP) (35, 36) method for the regularization parameter, we apply this method to our CT image denoising task. Denoting σ_n as the standard variance of the additive noise, and denoting σ_i as the standard deviation of the image patches locally estimated variance at the position i , the regularization parameter τ_i is well-estimated by $\tau_i = 2\sqrt{2}\sigma_n^2 / \sigma_i$. Recent studies proved the efficiency of the proposed estimating method (2, 12, 37). Moreover, σ_i can be approximately obtained by maximum likelihood estimation (38), i.e., $\sigma_i = \sqrt{\max(\lambda_i^2 / m - \sigma_n^2)}$, where λ_i denotes the singular value generated from the matrix X_i .

3. RESULTS

In order to test our denoising method, we applied the proposed method to phantom, head and neck, and abdomen CT images. All CT data were acquired using a Siemens CT scanner with the voxel size was $0.8 \times 0.8 \times 1.0 \text{ mm}^3$ for phantom and head and neck images $1.0 \times 1.0 \times 2.0 \text{ mm}^3$ for abdomen images. We compared our proposed method with a state-of-art denoising method – block-matching 3D filtering method (BM3D) (13, 14). The main idea of BM3D-based denoising method is to first group similar image patches in 3-D groups and then shrinkage the spectrum of a 3-D transform applied on such group. We used the peak signal-to-noise ratio (PSNR) (39, 40) and feature similarity indexes (FSIM) (41) to get a quantitative evaluation of this comparison.

We successfully performed the denoising method for all CT images. As demonstrated in Fig. 2, the proposed method works well for all CT images and achieved similar results as compared to BM3D. Table 1 shows PSNR and FSIM of two methods for each image.

Overall the PSNR of our proposed method was 42.511dB, 48.500dB and 36.163dB for the phantom, head and neck, and abdomen CT images, while the PSNR of the state-of-art BM3D method was 39.876dB, 46.683dB and 35.335dB, respectively. The FSIM of our proposed method was 0.980, 0.997 and 0.973 for the phantom, head and neck, and abdomen CT images, while the FSIM of the state-of-art BM3D method was 0.952, 0.994 and 0.935, respectively. These results demonstrated the proposed method can achieve much better results than the state-of-art BM3D method.

4. DISCUSSION AND CONCLUSION

In this paper, we propose a CT image denoising method based on low-rank sparse coding model and simultaneous sparse coding framework. The adaptively learned dictionary is utilized to remove the noises in CT image by incorporating the nonlocal self-similarity constraint. The sparse coding error is diminished by incorporating the regularization of group-wise sparse coding noise. The parameters of group-wise sparse coding noise regularization are automatically estimated using the Bayesian interpretation. The variable-splitting method is used to ease the solving of unified variational framework. The novelty of our approach is the integration of the spatial correlation and nonlocal self-similarity into the patch-based sparse representation image denoising framework. This approach has 3 distinctive strengths: 1) Instead of using the fixed dictionary for sparse representation, we introduced a low-rank sparse coding model for dictionary learning and sparse representation. 2) Group-wised sparse coding noise minimization is taken in to concern in this denoising framework. 3) Contrary to the classical regularization-based image denoising method, which uses fixed regularization parameter, we used the Bayesian interpretation to learn the adaptive parameter. Finally, in order to solve the multi-variable optimization, the scheme of variable-splitting is utilized to simultaneously sparsely represent the CT image patches, automatically learn the dictionary and restore the entire denoised CT image. The denoised results of phantom and patient CT images have demonstrated that the proposed method delivered state-of-art denoising performance, both in terms of objective criteria and visual quality.

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Figure 1.
The brief architecture of the proposed method.

Figure 2.

Comparison between the proposed and the BM3D denoising methods for the phantom CBCT, head and neck, and abdominal CT images. The images from left to right are the original CT images, the denoised images using the BM3D method, the difference between original and denoised images using the BM3D method, the denoised images using the proposed method, the difference between original and denoised images using the proposed method.

Table 1.

PSNR and SSIM for the phantom, head and body CT images using the both methods.

Method	Phantom		Head and Neck		Abdominal	
	PSNR (dB)	FSIM	PSNR (dB)	FSIM	PSNR (dB)	FSIM
BM3D	39.8761	0.9524	46.6830	0.9938	35.3352	0.9354
The proposed	42.5114	0.9804	48.5001	0.9966	36.1625	0.9730