COMMENTARY

Time crystal minimizes its energy by performing Sisyphus motion

Krzysztof Sacha^{a,1} and Peter Hannaford^b

We all know about ordinary space crystals, which are often beautiful objects and also useful in various practical applications. Briefly speaking, they consist of atoms which due to mutual interactions are able to self-organize their distribution in a regular way in space if certain conditions are fulfilled—ideally they form in the lowest-energy state at zero temperature. In 2012 the ideas of time crystals were invented. Shapere and Wilczek proposed classical time crystals and Wilczek alone proposed quantum time crystals (1, 2). Basically, time crystals refer to regular periodic behavior not in the spatial dimensions but in the time domain (3). In the classical case, time crystals are related to the periodic evolution of a system possessing the lowest energy. Motion and minimal energy seem to contradict each other but Shapere and Wilczek (2) showed that if the kinetic energy of a particle on a ring is a quartic function of its velocity, it may happen that the minimal energy corresponds to a particle moving along a ring with non-zero velocity. This counterintuitive situation also seems to be in contradiction to the condition for the minimal value of the Hamiltonian (energy expressed in terms of particle momentum that is commonly used by physicists) because its minimum implies zero velocity. Shapere and Wilczek (2) showed that precisely at the minimum the particle velocity cannot be expressed in terms of the momentum and we indeed can observe periodic motion of a particle even if its energy is the lowest possible. One can ask whether we can gain energy from such a particle if, for example, it hits an obstacle? We cannot because a particle does not have any excess energy to give away. However, there is still the question of how such a particle behaves if it bumps into an obstacle or experiences an external potential on its way. This is the subject of the current paper by Shapere and Wilczek (4), but before we switch to this problem let us also familiarize readers with the quantum versions of time crystals.

In the quantum world the formation of crystals is more complicated than in the classical case because

Fig. 1. (Top) Kinetic and potential energies of a particle. The former is minimized by the particle's motion with the velocity $v = 1$ (or $v = -1$) and the latter by the particle localized at the position $x = 0$. **(Bottom)** To reconcile these 2 contradicting requirements, the particle decides to perform Sisyphus motion depicted schematically. That is, a particle slowly climbs with a constant velocity $v = 1$ (or $v = -1$), crosses the position $x = 0$, and afterward instantaneously rolls back and starts climbing again and so on. Units are the same as in ref. 4.

there are symmetry requirements—ruthless guards who are not easy to mislead. In solid-state physics the energy of the interactions between atoms depends on the relative distances between them and does not change if we translate all atoms by the same distance. Consequently, a solid-state system prepared in the ground state or any other energy eigenstate must correspond to a spatially uniform probability for detection of an atom—no regular crystalline structure in space is visible. In other words, space translation symmetry does not allow us to know where a space crystal is located unless this symmetry is spontaneously broken and a space crystal localizes at a place where we actually see it.

In analogy with the formation of ordinary space crystals, the formation of quantum time crystals relies on self-organization of the motion of a quantum many-body

^aMarian Smoluchowski Institute of Physics, Jagiellonian University, PL-30-348 Kraków, Poland; and ^bCentre for Quantum and Optical Science, Swinburne University of Technology, Hawthorn, Victoria 3122, Australia
Author contributions: K.S. and P.H. wrote the paper.

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¹To whom correspondence may be addressed. Email: krzysztof.sacha@uj.edu.pl.

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system in a regular way in time. Put differently, a quantum many-body system breaks time translation symmetry and spontaneously switches to a periodic motion. Wilczek expected the formation of quantum time crystals in the ground state of a many-body system which turned out to be impossible at least in his original model (1, 5, 6). However, later, quantum time crystals have been discovered in nonequilibrium situations of periodically driven systems (7–9) which have been dubbed discrete time crystals. An isolated quantum many-body system, due to interaction between the particles, is able to self-reorganize its motion and start to move with a period different from the driving period. The formation of such a periodic motion, that is, the formation of a new crystalline structure in time, has already been demonstrated in the laboratory (10, 11).

In the paper published in PNAS, Shapere and Wilczek (4) consider a single particle which from the point of view of its kinetic energy can form a classical time crystal; i.e., the kinetic energy is minimal when the particle is moving with velocity $v = 1$ or $v = -1$ (in the units used in ref. 4). However, the particle also experiences a potential energy which favors its localization at the potential minimum at position $x = 0$. Thus, we are dealing with a situation where a particle that can form a classical time crystal experiences an obstacle along its way. Surprisingly, to satisfy the contradicting requirements of the minimization of the kinetic and potential energies, a particle chooses a “Sisyphus” motion. The total energy is minimal if the particle oscillates around $x = 0$: First it slowly climbs with a constant velocity $v = 1$ (or $v = -1$), crosses the position $x = 0$, and afterward instantaneously rolls back and starts climbing again, and so on, for eternity (Fig. 1).

To perform the analysis of the Sisyphus time crystal, Shapere and Wilczek (4) consider a higher-dimensional problem which

reduces to the original system when a regularization parameter, such as the mass parameter, approaches zero. This strategy allows them also to apply the quantum description of the particle. The higher-dimensional system helps to avoid difficulties in the classical and quantum description of the original problem (12), but it can also be realized experimentally. It indicates that there are experimentally attainable systems where the Sisyphus time crystals can be demonstrated in the laboratory. While the presented results are concerned with the single-particle problem, they give a hint in what direction we should go to realize quantum many-body time crystals where the spontaneous breaking of continuous time translation symmetry and the emergence of periodic motion take place in the lowest-energy state (1). The present PNAS publication (4) has already become an inspiration for further research. It has been shown that the evolution of the Sisyphus time crystal can also be observed in a cosmological model of the oscillating Universe (13).

The discovery of classical and quantum time crystals indicates that periodic behavior in the lowest-energy state and spontaneous breaking of time translation symmetry are possible and thus very important properties of solid-state systems can also be observed in the time domain. It seems that this is not the end and other condensed-matter phenomena can be realized exclusively in the time dimension too (3). Maybe in the not too distant future our everyday life will be based on space–time electronics.

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