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Social Network Spatial Model

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Abstract

Our work is motivated by a desire to incorporate the vast wealth of social network data into the framework of spatial models. We introduce a method for modeling the spatial correlations that exist over a social network. In particular, we model attributes measured for each member of the network as a continuous process over the social space created by their connections. Our method simultaneously models the unobserved locations of network members in social space and the spatial process that exists over that space based on the observed network connections and nodal attributes. The model is evaluated through simulation studies and applied to the importance ranking for a network of emergency response organizations and the physical activity habits of teenage girls. The introduced methods incorporate network data into the spatial framework, expanding traditional models to include this often relevant source of additional information.

Keywords

social network; social space; node attributes

1. Introduction

Social networks explain the interactions among a group of individuals. Individuals are nodes in the network, which we call *characters*, and each character has social connections to some other characters. This leads to a network of interactions that describe social relationships. For each character in the network, more information aside from who they are connected to is usually known. In many instances, we have node attributes for characters, some of which may be continuous. For example, perhaps we have the average daily minutes of moderate-to-vigorous physical activity (MVPA) for each character in a network. As a health outcome, it may be of interest to know how minutes of MVPA changes over a network. Perhaps characters with similar physical activity habits tend to be closer friends than characters with vastly different amounts of MVPA. Additionally, it may be of interest to quantify the level of similarity not only between friends, but also between friends of friends or, more generally, any two characters in a network.

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To describe how a continuous attribute such as daily minutes of MVPA changes over a social network, we aim to incorporate spatial models into the context of social networks. In this paper, we present a social network spatial model that builds upon Hoff et al. (2002) latent representation of a network in *social space* by introducing a spatial-type analysis over this space. In particular, Hoff et al.'s two-dimensional social space provides an abstract space in which a social network can be represented, with the Euclidean distances between members of a network being indicative of the strength of their relationships. Thus, characters who are modeled as being closer in social space are more likely to have a social connection, indicating that characters in close proximity have strong social relationships. We consider only the case of undirected social connections. Since those with strong relationships often have similar attributes that leads to their friendship, we incorporate a spatial modeling of continuous node attributes over social space.

Attributes of members of a network can be incorporated into network models in three main ways: selection models, social influence models, and selection and social influence models. In selection models, connections between members of a network are modeled based on the fixed attributes of each network member. Social influence models use the connections between members to model the attributes observed for each network member. For selection and social influence models, connections and attributes are jointly modeled, with neither being taken as fixed and known. Selection and social influence models best encompass the work we present here, as our goal is to model spatial attributes and a social network.

Snijder's et al. (2007) provide a foundation in selection and social influence models by introducing an actor-driven method, where ties and characteristics are jointly modeled. With this model, the network and behavior of actors are able to co-evolve within the framework of a dynamic network. Fellows and Handcock (2012) introduce a selection and social influence model that allows for the modeling of random nodal covariates. In particular, they put forward a class of exponential-family random network models that logistically model the full network, including both the random connection and random attributes. Fosdick and Hoff (2015) further extend such work, focusing on latent variable models for jointly modeling dependencies between a network and nodal attributes. In their work, Fosdick and Hoff allow node-specific network factors to impact the modeling of the network. Building upon these works, our intention is to incorporate continuous nodal attributes by way of a spatial model, providing a unique application of selection and social influence models to the spatial setting.

Previous work has integrated social network information into spatial models (Zheng et al., 2006; Radil, 2011; Emch et al., 2012) typically by incorporating the social network as an additional effect in a traditional spatial model. Alternatively, spatial-type methods have been integrated into social network models, with Austin et al. (2013) introducing a method for spatially smoothing social locations, allowing for the prediction of locations of characters added to a network based on individual-level covariates. Such approaches, however, have not replaced traditional spatial locations with social locations as we are doing here. In this article, we introduce the social network spatial model, demonstrate its ability to simultaneously model social space and a spatial process through simulation studies, and apply it to a network of emergency-response organizations in Texas and a network of adolescent girls.

1.1. Data

In contrast to traditional spatial models that take locations to be observed, the social network spatial model is appropriate for cases where locations are unobserved and needing to be modeled alongside the spatial process itself. For the social network spatial model, we take the network to be fully observed (i.e. we know all social connections), but the spatial locations of characters in social space are unobserved. Additionally, the spatial attributes of characters must be observed. Our goal is to use the observed network and the observed spatial attributes to simultaneously model both the spatial locations of characters in social space and the spatial process over this space. Such a model can be extended to cases of a partially observed network where the characters whose connections are unobserved have known social space locations. In this case, only part of the social space connections would need to be modeled alongside the spatial process.

2. Spatial Process Specifications

Point-referenced spatial data models are characterized by three interrelated features: stationarity, variograms, and isotropy (Banerjee et al., 2015). Changing the specifications for each of these features will create varying models that are appropriate for different spatial relationships. Let $\mathbf{z}_i$ be the location of site i for which we are interested in modeling a response, $s(\mathbf{z}_i)$. In total, assume there are N sites. All these sites are contained within the fixed space of interest, which we will denote as D . Typically, we take D to be a subset of $\mathbb{R}^2$. When the space of locations is at least two dimensional, we refer to the response's variation over this space as a *spatial process*. The process is used to create a surface representing a characteristic of interest on a continuous scale for the entire space (Schabenberger and Gotway, 2004). We will restrict ourselves to focusing on the two-dimensional case of a spatial process. In a traditional spatial model, the location of sites are fixed and known. In the social network setting, we must estimate a social space representation and simultaneously fit a spatial process over this estimated space. Thus, in the social network setting, the social locations, $\mathbf{z}$, are parameters to be estimated.

For the remainder of this paper, we work only with isotropic spatial processes that are intrinsic stationarity, also known as homogenous spatial processes. In this case, we have that the mean response at any two locations contained in our space are expected to be the same, or $E(s(\mathbf{z}_i) - s(\mathbf{z}_{i'})) | \|\mathbf{z}_i - \mathbf{z}_{i'}\| = d) = 0$, and the variance of the difference in responses is a function only of the distance between the two locations, or $var(s(\mathbf{z}_i) - s(\mathbf{z}_{i'})) | \|\mathbf{z}_i - \mathbf{z}_{i'}\| = d) = 2\gamma(d)$ (Banerjee et al., 2015). Note that $\gamma(d)$ is the semi-variogram that specifies the correlation structure between locations that are a distance d apart and can take different forms—e.g. linear, exponential, spherical, Gaussian, wave, Matérn. The semi-variogram is often transformed into a covariance function, $C(\cdot)$, given as $C(d) = \lim_{\|u\| \rightarrow \infty} \gamma(u) - \gamma(d)$, which more intuitively represents the covariance in attributes between two characters distance d apart. We focus primarily on the covariance function representation, noting the natural link to the semi-variogram.

3. Model Specification

3.1. Spatial Model

For a homogeneous spatial process with a specified covariance function structure, we model the response at site $i = 1, \dots, N$ as follows:

$$s(\mathbf{z}_i) = \mu(\mathbf{z}_i) + w(\mathbf{z}_i) + \epsilon(\mathbf{z}_i), \quad i = 1, \dots, N. \quad (1)$$

Note that the response is modeled as having three components, all of which can depend on the unknown social locations: a mean structure, $\mu(\mathbf{z}_i)$, a spatial effect, $w(\mathbf{z}_i)$, and a random error term, $\epsilon(\mathbf{z}_i)$ (Finley et al., 2013). The spatial effect and random error together form the residual portion of the model.

The mean structure, $\mu(\mathbf{z}_i)$, can be expressed in terms of linear predictors, $\mathbf{x}^T(\mathbf{z}_i)\beta$, where $\mathbf{x}^T(\mathbf{z}_i)$ is an $n \times p$ matrix of p predictors and $\beta \in \mathbb{R}^p$. The mean structure is not of direct focus in our research, but covariates can be included in the model if deemed appropriate. We adopt an intercept-only model in which case we assume an overall, site-independent mean response. In this case, we take $\mathbf{x}^T(\mathbf{z}_i)$ to be an $n \times 1$ identity matrix and β to be a scalar

The residual portion of the model is partitioned into two pieces, $w(\mathbf{z}_i)$ and $\epsilon(\mathbf{z}_i)$. The $w(\mathbf{z}_i)$ term captures the residual spatial association and is a realization from a zero-centered stationary Gaussian spatial process (Finley et al., 2013), depending on parameters σ^2 , the partial sill, and $\frac{1}{\phi}$, a parameter relating to the range of the space (in the case of an exponential covariance function, as we will be using in the analyses below, the $\frac{1}{\phi}$ parameter is proportional to the *effective range*, $\frac{-\log(0.05)}{\phi}$), being the distance at which the correlation between attributes at two sites has become less than 0.05. With the spatial influences fully captured by $w(\mathbf{z}_i)$, $\epsilon(\mathbf{z}_i)$ remains as the uncorrelated pure error, or the non-spatial residuals (Finley et al., 2013). Capturing the random deviations of the responses that are not attributable to the spatial process, $\epsilon(\mathbf{z}_i)$ depends on the τ^2 , the nugget. With this specification of the model for $s(\mathbf{z}_i)$, the variability depends only on τ^2 , σ^2 , and ϕ , whereas the expectation is constant but could potentially depend on any covariates of interest.

3.2. Social Network Spatial Model

For each of the N characters in a social network—who have unknown two-dimensional social location $\mathbf{z}_i$ and known social connections y_{ii} —we have a measured attribute, $s(\mathbf{z}_i)$, coming from a spatial process with unknown scalar parameters ϕ , σ^2 , and τ^2 . With this setup and a , b , and β as scalars, our social network spatial model can be presented as follows:

$$s|\beta, \sigma^2, \tau^2, \phi, \mathbf{z} \sim N(\beta, \sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N) \quad (2)$$

$$y_{ii'} | \mathbf{s}, a, b, \mathbf{z} \sim \text{Bernoulli}(p_{adj_{ii'}}), \text{ where } p_{adj_{ii'}} = \frac{e^{a + b\|s(\mathbf{z}_i) - s(\mathbf{z}_{i'})\| - \|\mathbf{z}_i - \mathbf{z}_{i'}\|}}{1 + e^{a + b\|s(\mathbf{z}_i) - s(\mathbf{z}_{i'})\| - \|\mathbf{z}_i - \mathbf{z}_{i'}\|}}. \quad (3)$$

$\mathbf{Y}$ is taken to be a symmetric matrix of undirected social connections, where $y_{ii'} = 1$ if a social connection exists between characters i and i' and $y_{ii'} = 0$ otherwise, leading to a Bernoulli model specification as a natural choice, consistent with Hoff et al.'s (2002) latent space model, with the $y_{ii'}$ being conditionally independent. By allowing the probability of a social connection to depend on the similarity of spatial attributes between two characters, we directly allow the social network representation to be impacted by the spatial model. If b is positive, characters with similar spatial effects are more likely to have a social connection. If the spatial effect does not impact the network representation, then $b = 0$, reducing down to the simplified latent space model. Following with the latent space model of Hoff et al. (2002), additional covariates on pairs of characters can also be included as linear predictors in the logistic probability. We consider here the model of only the difference in spatial effects between characters. $M(\phi, \mathbf{z})$ is a correlation matrix with elements $m_{ii'} = \rho(\mathbf{z}_i - \mathbf{z}_{i'}, \phi)$ and ρ being a valid isotropic correlation function arising from the covariance function specification. To determine an appropriate form of the correlation function, we recommend following traditional spatial model practice and examining the empirical semi-variogram based on a social network spatial model fit through two-step inference. In particular, a social network model should be fit independent of the spatial data, following Hoff et al.'s latent method. Then, these locations should be considered fixed and the traditional empirical semi-variogram calculated over this social space. Visual examination of the semi-variogram provides a foundation for an appropriate correlation function. Also note that $\|\cdot\|$ indicates the absolute value for scalar quantities (e.g. $s(\mathbf{z}_i)$) and the Euclidean distance for two-dimensional variables (e.g. $\mathbf{z}_i$). For this social network spatial model, the spatial process is dependent on the social network representation, along with the position of characters in social space being influenced by their spatial effects.

Independent prior distributions are placed on the model parameters, or $P(\beta, \sigma^2, \tau^2, \phi, \mathbf{z}, a, b) = P(\beta)P(\sigma^2)P(\tau^2)P(\phi)P(\mathbf{z})P(a)P(b)$. Natural choices for prior distribution specifications are $\beta \sim N(\lambda, \zeta)$, $\sigma^2 \sim IG(\alpha_1, \alpha_1)$, $\tau^2 \sim IG(\eta_1, \eta_2)$, $\phi \sim \text{Unif}(\nu_1, \nu_2)$, $\mathbf{z}_i \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \delta \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$, $a \sim \text{Unif}(\kappa_1, \kappa_2)$, $b \sim \text{Unif}(\psi_1, \psi_2)$.

Following traditional spatial analysis methods, the hyper-parameters for each of these prior distributions are set to fixed values, as opposed to being directly estimated. Note that in the prior distribution for β , ζ is often taken to be large such that this prior is uninformative. The prior distribution for ϕ should be fairly informative, and thus ν_1 and ν_2 should be reasonably chosen to restrict the range of ϕ . Information relating to hyper-parameter specification can be obtained from the two-step empirical semi-variogram described above. Because of the normal prior on the $\mathbf{z}_i$, the two-dimensional space is not bounded. However, the prior variance, δ , sets the scale of the space, keeping both dimensions on a similar scale.

With this model specification, the spatial distribution $w|s, \mathbf{Y}$ can be obtained through composition sampling based on the posterior samples of $\beta, \sigma^2, \tau^2, \phi, \mathbf{z}, a$, and b .

3.3. Inference

We present here inference for the social network spatial model. The joint posterior distribution of the model parameters is represented as follows:

$$P(\phi, \tau^2, \sigma^2, \beta, \mathbf{z}, a, b | \mathbf{s}, \mathbf{Y}) \quad (4)$$

$$\begin{aligned} &\propto P(\phi)P(\tau^2)P(\sigma^2)P(\beta)P(\mathbf{z})P(a)P(b)P(\mathbf{s} | \beta, \tau^2, \sigma^2, \phi, \mathbf{z})P(\mathbf{Y} | a, b, \mathbf{z}, \mathbf{s}) \\ &\propto (\sigma^2)^{-\alpha_1 - 1} |\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N|^{-\frac{1}{2}} e^{-\frac{\alpha_2}{2} - \frac{1}{2} \mathbf{z}^T \Lambda^{-1} \mathbf{z} - \frac{1}{2} (\mathbf{s} - \beta)^T (\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N)^{-1} (\mathbf{s} - \beta)} \\ &\quad \times \prod_{i=1}^N \prod_{i'=i+1}^N \frac{e^{y_{ii'}(a + b\|s(\mathbf{z}_i) - s(\mathbf{z}_{i'})\| - \|\mathbf{z}_i - \mathbf{z}_{i'}\|)}}{1 + e^{a + b\|s(\mathbf{z}_i) - s(\mathbf{z}_{i'})\| - \|\mathbf{z}_i - \mathbf{z}_{i'}\|}}. \end{aligned}$$

Full conditional posterior distributions for each parameter are given in Appendix A. Aside from β , the full conditional posterior distributions for the parameters of this model do not have convenient forms.

The representation of the posterior samples of character social space locations, $\mathbf{z}$, is not unique since distances in two-dimensional space are invariant to translations, reflections, and rotations. Much in the same way as label switching occurs in Bayesian mixture models (Jasra et al., 2005), there is a potential issue of a rotating posterior distribution due to equivalent representations of the social space (i.e. same distances between characters). In all equivalent representations, the model fit does not change, but we need to keep each sample on the same configuration so as to ensure posterior means are in fact reliable.

To ensure that social space locations are similarly configured across posterior samples, we perform a procrustes transformation (without dilations) at each iteration of the MCMC sampler (Borg and Groenen, 1997), using as a reference the locations $\mathbf{z}_0$ obtained from fitting Hoff et al.'s (2002) latent space model to the social connections only. The procrustes transformation of $\mathbf{z}$ is the representation most similar to $\mathbf{z}_0$ in terms of sum of squared positional deviations, or $\mathbf{z}^* = \arg \min_{R^* \mathbf{z}} \text{tr}(\mathbf{z}_0 - R^* \mathbf{z})^T (\mathbf{z}_0 - R^* \mathbf{z})$, where R^* is the set of all rotations, reflections, and translations (Hoff et al., 2002). This procrustes transformation is unique so long as $\mathbf{z}_0 \mathbf{z}^T$ is nonsingular (Sibson, 1979). The resulting $\mathbf{z}^*$ form the posterior samples for character social space locations.

Inference on model parameters is performed through the use of Just Another Gibbs Sampling (JAGS), where the posterior distribution for β is sampled from via a Gibbs

sampler and the remaining non-conjugate posterior distributions are sampled from using slice samplers (Plummer, 2013).

After calculating the posterior distributions of model parameters, we then perform composition sampling to recover the posterior distribution of the random spatial effects, $\mathbf{w}$. See Appendix A for the full conditional posterior distribution of the random spatial effect $\mathbf{w}$. At the j^{th} iteration of the composition sampler, model parameters are evaluated at the j^{th} values of the sampled marginal posterior distributions of σ^2 , τ^2 , ϕ , and $\mathbf{z}$. The samples drawn from this posterior distribution collectively form the marginal posterior distribution of $\mathbf{w}$.

4. Simulations

To evaluate the social network spatial model presented above, we perform a series of simulations. In particular, for each simulation, we generate a social network of $N = 100$ characters. For each character in the network, we simulate an attribute according to a spatial model, using the positions of the characters in social space as the spatial locations. We use an exponential covariance function with parameters $\sigma^2 = 0.1$, $\tau^2 = 0.02$, $\phi = 0.5$, and $\beta = 1$ to simulate character attributes. Our goal is to use the simulated adjacency matrix and attributes as data for the social network spatial model to determine how well we are able to recover spatial variations and character locations in social space.

Since location of characters in social space are estimated, the values of parameters σ^2 , τ^2 , ϕ , a , and b will be dependent on the scaling of the social space. In particular, if social space is estimated to be on a large scale, the estimates of these parameters will also be larger than if the space were on a smaller range. Because of this, for the spatial portion of the model, we are interested in the proportion of the variability attributable to the spatial process, or $\frac{\sigma^2}{\sigma^2 + \tau^2}$.

Regardless of the range on which the social space is estimated to be on, the proportion of variability attributable to the spatial effect is comparable. Higher values of this proportion indicate that the spatial process is explaining more of the observed variability in the response. In the simulations below, we therefore assess the ability of the social network

spatial model to properly recover $\frac{\sigma^2}{\sigma^2 + \tau^2} \cdot \frac{1}{\phi}$ is not of direct inferential interest in these

simulations, but it has a reasonable interpretation relating to the effective range (e.g. $\frac{-\log(0.05)}{\phi}$) in the context of any given social space representation. The overall mean effect

should not be affected by the social space representation, and thus β remains of direct inferential interest in the simulations. a is a nuisance parameter and is not of direct interest. While the magnitude of b will depend on the scale of the social space, we do pay attention to its direction and significance.

In performing inference using the social network spatial model, we jointly model the social network and spatial correlations using the marginalized model where the spatial effects, $\mathbf{w}$, are recovered via composition sampling. We simulate 100 such spaces and repeat the inferential process for each, determining the average estimates of the spatial parameters σ^2 , τ^2 , ϕ , and β , along with the social network parameters $\mathbf{z}$, a , and b . For the estimation of the

network, we specify the prior distributions for the logistic parameters as $a \sim \text{Unif}(-10,10)$ and $b \sim \text{Unif}(-20,20)$ and the two-dimensional network locations as $\mathbf{z}_i \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}\right)$. A flat prior distribution is used for β , along with informative priors of $\sigma^2 \sim \text{IG}(2,0.1)$, $\tau^2 \sim \text{IG}(2,0.02)$ and $\phi \sim \text{Unif}(0.01,1)$. The inverse-gamma prior distributions for σ^2 and τ^2 have means equal to the simulated values.

In drawing samples from the posterior distributions via JAGS, we assess convergence of the Markov chain based on a multivariate effective sample size criterion applied to parameters σ^2 , τ^2 , ϕ , β , a , and b (Flegal et al., 2016). Summary results for the parameter estimates are presented in Table 1.

Overall, the social network spatial model performs relatively well in recovering the true values of the model parameters. For the proportion of variability attributable to the spatial effect, the coverage is reasonably high at 90%, indicating that the posterior intervals frequently cover the truth and are thus well estimating the ratio. A slight negative bias is noticed, which suggests that the social network spatial model is attributing a bit more of the variability to random error. Since the social locations are estimated, there will be additional variability captured by τ^2 , leading to the observed result. The overall spatial effect, β , is generally well estimated with high coverage and a posterior mean that closely matches the simulated value.

Although we are not directly interested in the values of a and b , we do pay attention to the sign of their estimates, particularly that of b . Consider a situation of three characters who are equidistant in social space. If b is negative, the two characters with the most similar attributes will be the most likely to have a connection. Alternatively, if b is positive, the two characters with the most dissimilar attributes will have the highest probability of a connection. To further investigate this, we simulated data and fit the social network spatial model with two sets of prior distribution specifications: the uninformative case listed above and for $a \sim \text{Unif}(0,10)$ and $b \sim \text{Unif}(-20,0)$. Under both prior distribution specifications, similar results were reached in estimating all model parameters, indicating that either specification of the prior distributions can be appropriate, if the correct knowledge is used to inform the sign of a and b .

To assess performance in estimating social locations, we look at the sum of squared deviations from the truth. After drawing all the samples from the posterior distribution of the character locations, we allow for a procrustes transformation of each sample, using the truth as our reference locations. Additionally, we fit an independent social network model that does not include any spatial information, similarly applying a procrustes transformation, and compare the sum of squared deviations to those for the social network spatial model. From this comparison, we see that the social network spatial model performs better than the independently fit social network model with-out spatial information in 80% of the simulations. The social network spatial model generally gains information by incorporating the spatial attributes of each character, providing a well-estimated social space representation.

In addition to the simulation conditions specified above, we also seek to investigate the sensitivity of results to the choice of prior distributions, specifically on σ^2 and τ^2 . In contrast to the informative priors used above, we now investigate simulation results when an uninformative inverse-gamma prior distribution, specifically $IG(0.0001, 0.0001)$, is used for both σ^2 and τ^2 . We keep an inverse-gamma specification to align with our general prior recommendations, and by setting the hyper-parameters equal to 0.0001, we get an inverse-gamma distribution that is uninformative, even though its moments are undefined. Summary results are presented in Table 2. We note that it took 50,000 iterations to reach convergence compared to the 10,000 iterations used in the informative simulations above.

In Table 2, we see that the overall mean effect, β , is still well estimated, regardless of the choice of prior distributions. We also note that the estimates of $\mathbf{z}$, a , and b are not noticeably affected by the choice of prior distributions on σ^2 and τ^2 . In this case of uninformative prior distributions, the proportion of variability explained by the spatial process is not well estimated. With the uninformative prior distribution, larger values of σ^2 and τ^2 were sampled. In particular, τ^2 is more positively biased, leading to less of the variability being attributed to the spatial process. Since the true value of τ^2 is small, the influence of sampling larger values when using this uninformative prior distribution has a sizable impact on its estimation.

Based on these simulation results, we thus recommend that fairly informative prior distributions be chosen for σ^2 and τ^2 , as opposed to using the uninformative inverse-gamma specification put forth above. In particular, inverse-gamma prior distributions should be used with means equal to plausible values for σ^2 and τ^2 . To determine plausible values for the mean of each prior distribution, we recommend an empirical Bayesian approach. We examine the empirical semi-variogram fit through two-step inference and use the resulting estimates of σ^2 and τ^2 as the prior means. This will lead to informative prior distributions, in which case the posterior means are generally well estimated.

4.1. Model Selection

The above simulations demonstrate the ability of the social network spatial model to effectively recover both the spatial and the social network parameters that were simulated under this dependence model. In practice, though, we must determine whether a spatial effect does in fact exist over a social network. If the social network is unrelated to an observed attribute, and thus the observed attribute is being independently generated under some regression model (Banerjee et al., 2015), the social network spatial model (known hereafter as the dependence model) is not appropriate and the independent social network and intercept-only regression models (known hereafter as the independence model) should be fit to describe the space. To determine whether the dependence model is more appropriate than the independence model, we rely upon the deviance information criterion (DIC) (Spiegelhalter et al., 2002).

We evaluate the effectiveness of using DIC to properly choose when the dependence model is appropriate through a series of simulations. First, we simulate data based on the dependence model, in which case there is a relationship between the social network and the spatial effect, and then fit both the dependence and independence models. The DIC is

calculated for each model fit, with the expectation that the social network spatial model will yield the smaller value of DIC and thus be chosen as the optimal model. In this particular setup, we simulate networks of $N=50$ characters, with social network parameters $a=1.5$ and $b=-5$ and generate character attributes, $\mathbf{s}$, based on a spatial model parameters $\sigma^2=0.1$, $\tau^2=0.02$, $\phi=0.5$ and $\beta=1$ over the social space. 100 of such datasets were simulated.

In addition to the simulations generated under the dependence model, we simulate a scenario where the social network is independent of the character attributes and then fit both the dependence and independence models. The DIC is calculated in each case, and in this scenario, we expect that the dependence model will not yield a value of DIC higher than the independence model, and thus the independence model will be preferred. 100 datasets are simulated and fit under such conditions. For these simulations, we again generate networks of $N=50$ characters, setting the social network parameter for the independence model at $a=1.5$. Node attributes in this intercept-only model are generated by a normal distribution with mean 0 and standard deviation 0.1.

Regardless of which model we are using to simulate the data (dependence or independence), we use the following prior distribution specifications: $\sigma^2 \sim IG(2,0.1)$, $\tau^2 \sim IG(2,0.02)$, $\phi \sim \text{Unif}(0.01,1)$, $\beta \sim N(0,10000)$, $\mathbf{z}_i \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}\right)$, $a \sim \text{Unif}(-10, 10)$ and $b \sim \text{Unif}(-20,20)$. The results for the simulations are presented in Table 3. When the data are simulated from the dependence model, the fitted dependence model yields the smaller mean DIC. In fact, in all 100 simulations, the dependence model is chosen as better compared to the independence model. When the independence model is used for simulating data, both the dependence and independence models perform fairly similarly in terms of DIC. Since the independence model is more parsimonious, the independence model should thus be chosen as optimal in this case.

As a general rule, the DIC can be used to determine whether the dependent social network spatial model is appropriate to fit to a dataset. If the DIC for the fitted dependence model is much smaller than the DIC for a fitted independence model, then the dependence model should be fit. If the DIC for both the fitted dependence and independence models are similar, then the more parsimonious independence model should be chosen over the dependence model.

5. Data Analysis

5.1. Emergency Organizations

Drabek et al. (1981) have compiled data on the multi-organizational networks of search and rescue activities in Texas. Organizations that interact during search and rescue efforts are considered to have a connection and otherwise are unconnected. Additionally, we have the average reverse rank in decision-making processes during responses to search and rescue initiatives, as judged by objective informants, for each organization. Organizations with higher ranks are, on average, more influential in the decision-making process. We model the natural logarithm of this rank score in order to reduce the skewness in the untransformed values. A total of twenty organizations have ranks and are included in this network.

We fit an independent social network model without spatial information and present the resulting social space in Figure 1. This provides a preliminary representation of how the organizations are related. With the social space representation in place, we then color each node based on the organization's observed influence score. From preliminarily examining the social space, it is clear that, in general, similarly important organizations also happen to be socially close together.

To additionally explore the data, we look at correlations in the log-rank score for connected pairs ($r = -0.05$) and for unconnected pairs ($r = -0.11$). In both cases, there is not a significant correlation between the log-rank scores in such a preliminary analysis. However, this is treating all connections equally and not taking into consideration the strength of the relationships (i.e. the closeness of a connected pair in social space). Alternatively, the social network spatial model will provide a method for uncovering relationships by taking into account the relative strength of connections.

We apply the social network spatial model to this dataset, jointly modeling the social network and the spatial effects of the emergency organizations and their log rank scores. Based on visually examining the empirical semi-variogram for a fixed social network in Figure 2, we adopt an exponential covariance structure for our model, as this appears to reasonably describe the covariance of the observed response with increasing distance.

When the data are fit with a dependent social network spatial model, the DIC is calculated as 463.2. In comparison, the DIC when the data are fit with independent social network and regression models is 533.0. In the case of this emergency organizations data, the DIC indicates that the social network spatial model is appropriate to use for analysis.

We draw 100,000 samples from the posterior distributions of σ^2 , τ^2 , ϕ , β , $\mathbf{z}$, a , and b . Prior distributions are set as $\sigma^2 \sim IG(2, 0.5)$, $\tau^2 \sim IG(2, 0.01)$, $\phi \sim \text{Unif}(0.1, 1)$, $\beta \sim N(0, 10000)$, $\mathbf{z}_i \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}\right)$, $a \sim \text{Unif}(-5, 5)$, and $b \sim \text{Unif}(-20, 20)$. The sill parameters are chosen such that the prior means are based on the empirical semi-variogram estimates, which in this case are $\tau^2 = 0$ and $\sigma^2 = 0.47$. The multivariate effective sample size diagnostic (Flegal et al., 2016) indicates convergence of the posterior distributions, and trace plots (presented in Appendix B) visually support convergence. Our resulting mean parameter estimates, along with 95% posterior intervals, are presented in Table 4.

As described in section 3.2, we transform each sample of the posterior distribution of character social space locations, $\mathbf{z}$, using a procrustes transformation with an independent social network fit as the reference. Based on the posterior distributions of these parameters, composition sampling is used to determine the posterior distribution of the spatial effects, $\mathbf{w}$. The mean spatial effects for each of the twenty organizations are graphically displayed in Figure 3. This provides a visual representation of how log decision rank varies over this social space, based on our exponential covariance model. We can see that a few highly connected organizations are also influential in the decision-making process. Organizations on the fringe of the social space are less influential, and moderately influential organizations are socially close and interact with each other. Overall, important emergency response

organizations are relatively similar to each other. Highly influential organizations tend to be socially close together, indicating that they often respond together. Organizations that work together are typically similar in their influence.

In examining Figure 3, there is a noticeable organization in the upper-left corner of the social space that is relatively socially far from all other organizations. Additionally, this organization has a low influence score. Upon further examination, this organization is found to be the Pipe Creek Volunteer Fire Department. Comparatively, the Pipe Creek Volunteer Fire Department is geographically far away from many other departments located in Texas. The geographic separation leads to the Pipe Creek Volunteer Fire Department not having as many connections, with its social connections being to the Emergency Medical Services in Kendall County (19.8 miles away) and the Bandera County Civil Defense Emergency Management (13.8 miles away). These organizations are two of the closer geographic locations to Pipe Creek. In this case, the geographic orientation of organizations directly impacts the social network connections, and since the Pipe Creek Volunteer Fire Department has few social connections and is not very influential, its estimated social location is relatively far away from all other emergency organizations.

From an inspection of our mean posterior estimates of the model parameters, we notice a few interesting conclusions. First, 97.6% of variability in the model is spatial in nature as opposed to random variability. Thus, the spatial model accounts for much of the variability in the space, indicating that there is indeed a strong social-spatial effect. This is visually supported by the results in Figure 3, where a spatial effect is noticeable. For this spatial effect, we have ϕ estimated at 0.551, leading to an effective range estimate of $\frac{-\log(0.05)}{\phi} = 5.437$. Inspecting Figure 3, this effective range seems reasonable for this social space representation. In general, as we move 5.437 units away from a character, the maximal variability is nearly reached. Characters with similar attributes tend to fall nearby within a few units of each other, whereas those with dissimilar attributes are generally separated by distances greater than 5.437 units.

Additionally, it should be noted that b is estimated to be positive. This positive estimate of b indicates that for organizations that are the same distance apart, those who have similar influence scores are estimated to have a lower probability of a connection than two similarly spaced organizations that have dissimilar influence scores. In the context of emergency organizations, though, this perhaps is not unreasonable. For organizations that are equally socially close together, it is plausible that those with dissimilar influence scores will more often respond to calls together, as there perhaps is less of a need for multiple highly influential organizations who are socially all the same distance apart to respond to the same emergency.

5.2. Physical Activity

As a secondary example of modeling a social attribute over a network, we investigate the relationship between a social network and physical activity. In particular, we fit a social network spatial model to a group of eleventh-grade girls for whom we have the average number of minutes per day spent on moderate-to-vigorous physical activity (MVPA). This

data was collected by Young et al. (2014). Friends often have an influence on how physically active an individual is (Estabrooks et al., 2008; Shelton et al., 2011; Yu et al., 2011), so this social support should lead to closely-related friends having similar daily minutes of MVPA.

We fit a social network spatial model to this network of adolescent girls, using the average daily minutes of MVPA as the continuous attribute of interest. Based on the empirical semi-variogram fit through two-step inference in Figure 4, we model spatial variations using an exponential covariance function. Prior distributions are set as $\sigma^2 \sim IG(2, 175)$, $\tau^2 \sim$

$IG(2, 120)$, $\phi \sim \text{Unif}(0.01, 1)$, $\beta \sim N(0, 10000)$, $\mathbf{z}_1 \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}\right)$, $a \sim \text{Unif}(-50, 50)$. and $b \sim$

$\text{Unif}(-50, 50)$. The convergence is met based on the multivariate effective sample size criterion. Trace plots (presented in Appendix B) visually support convergence as well. The resulting mean parameter estimates, along with 95% posterior intervals, are presented in Table 5.

The mean spatial effects are presented in Figure 5. From this, we can see that there does not appear to be a clear spatial effect. Alternatively, it appears that most people have relatively similar physical activity habits, with only a few individuals being highly active. These highly active individuals generally do not tend to be socially close, but instead have strong social relationships with less physically active girls. Examination of the DIC indicates that a spatial model is not, in fact, appropriate for use here (DIC is calculated as 11849 when the social network spatial model is fit, and 11397 when independent social network and intercept-only regression models are fit). In this case, we have a social network and were able to measure physical activity for each member of this network, but this attribute is not significantly socially determined. Physical activity levels do not tend to influence social relationships of adolescent girls. In particular, these girls usually do not have the strongest social relationships with others girls who have similar levels of physical activity to them.

6. Discussion

In this paper, we jointly modeled a social network and the spatial correlations of nodal attributes over this network, providing a comprehensive social network spatial model. By simultaneously updating the social space and spatial correlations, the presented model provides a loop of information in which characters' locations in social space are used as the spatial locations in modeling spatial correlations and observed spatial attributes are used to better inform characters' locations in social space. Our introduced method draws upon traditional spatial models in which site locations are fixed, but by incorporating the social network as the new basis for locations, additional variability is present. By jointly modeling the social network and spatial correlations, we are able to reasonably describe how an attribute varies over a social network and better model the resulting social space. The social network spatial model is built upon the implicit belief that social networks can influence human behavior. With this model, individual behaviors are not influenced only by their friends, but by all members of the network, at varying degrees of severity based on their social distances. We presented an illustration of an attribute varying over a social space through the examination of a network of emergency response organizations in Texas, finding that similarly important organizations tend to be socially close together. We additionally

illustrated that social networks do not influence all behaviors, as physical activity for a group of adolescent girls did not have a social relationship.

The introduced model marks the first comprehensive unification of spatial modeling and social network analyses where social locations are used as the basis for modeling spatial correlations. The social network spatial model performs well in describing a spatial effect over an entire social network, with all members of the network having some influence on all other individuals. Spatial variations are generally well estimated, and the resulting social space closely matches the true orientation of characters.

With this foundation set, the next step is then to extend more sophisticated traditional spatial modeling techniques to the context of social networks. For example, spatiotemporal modeling of attributes over a social network is of interest. Both spatial attributes and social networks have the potential to change with time. By incorporating temporal spatial attributes and dynamic networks into a comprehensive model, we could perhaps better describe changes to individuals over time.

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Appendix A. Conditional posterior distributions

The full conditional posterior distribution of β can be derived as:

$$P(\beta | s, Y, \sigma^2, \tau^2, \phi, \mathbf{z}) \propto e^{-\frac{1}{2}(\beta - \hat{\beta})^T V_{\beta}^{-1}(\beta - \hat{\beta})}. \quad (\text{A.1})$$

Note that:

$$\begin{aligned} \hat{\beta} &= \left(J_N^T (\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N)^{-1} J_N \right)^{-1} J_N^T (\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N) \mathbf{s} \\ V_{\beta} &= \left(J_N^T (\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N)^{-1} J_N \right)^{-1}. \end{aligned}$$

Additionally, J_N is a length N vector with all entries equal to $\frac{1}{N}$. Thus $\beta \sim N(\hat{\beta}, V_{\beta})$. The mean of this distribution is a weighted average of the data, with each character's attribute being weighted by the average variability.

For the remaining parameters in this marginalized model, we have the following full conditional posterior distributions:

$$P(\sigma^2 | \mathbf{s}, Y, \tau^2, \phi, \beta, \mathbf{z}) \propto (\sigma^2)^{-\alpha_1 - 1} |\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N|^{-\frac{1}{2}} e^{-\frac{1}{2}(\mathbf{s} - \beta)^T (\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N)^{-1} (\mathbf{s} - \beta) + \alpha_2 (\sigma^2)^{-1}}$$

$$P(\tau^2 | \mathbf{s}, Y, \sigma^2, \phi, \beta, \mathbf{z}) \propto (\tau^2)^{-\eta_1 - 1} |\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N|^{-\frac{1}{2}} e^{-\frac{1}{2}(\mathbf{s} - \beta)^T (\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N)^{-1} (\mathbf{s} - \beta) + \eta_2 (\tau^2)^{-1}}$$

$$P(\phi | \mathbf{s}, Y, \sigma^2, \tau^2, \beta, \mathbf{z}) \propto |\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N|^{-\frac{1}{2}} e^{-\frac{1}{2}(\mathbf{s} - \beta)^T (\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N)^{-1} (\mathbf{s} - \beta)}$$

$$P(a | \mathbf{s}, Y, \mathbf{z}, b) \propto \prod_{i=1}^N \prod_{i'=i+1}^N \frac{e^{y_{ii'}(a + b\|s(\mathbf{z}_i) - s(\mathbf{z}_{i'})\| - \|\mathbf{z}_i - \mathbf{z}_{i'}\|)}}{1 + e^{a + b\|s(\mathbf{z}_i) - s(\mathbf{z}_{i'})\| - \|\mathbf{z}_i - \mathbf{z}_{i'}\|}}$$

$$P(b | \mathbf{s}, Y, \mathbf{z}, a) \propto \prod_{i=1}^N \prod_{i'=i+1}^N \frac{e^{y_{ii'}(a + b\|s(\mathbf{z}_i) - s(\mathbf{z}_{i'})\| - \|\mathbf{z}_i - \mathbf{z}_{i'}\|)}}{1 + e^{a + b\|s(\mathbf{z}_i) - s(\mathbf{z}_{i'})\| - \|\mathbf{z}_i - \mathbf{z}_{i'}\|}}$$

$$P(\mathbf{z} | \mathbf{s}, Y, \sigma^2, \tau^2, \phi, \beta, a, b) \propto$$

$$|\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N|^{-\frac{1}{2}} e^{-\frac{1}{2} \mathbf{z}^T \Lambda^{-1} \mathbf{z} - \frac{1}{2} (\mathbf{s} - \beta)^T (\sigma^2 M(\phi, \mathbf{z}) + \tau^2 I_N)^{-1} (\mathbf{s} - \beta)} \\ \times \prod_{i=1}^N \prod_{i'=i+1}^N \frac{e^{y_{ii'}(a + b\|s(\mathbf{z}_i) - s(\mathbf{z}_{i'})\| - \|\mathbf{z}_i - \mathbf{z}_{i'}\|)}}{1 + e^{a + b\|s(\mathbf{z}_i) - s(\mathbf{z}_{i'})\| - \|\mathbf{z}_i - \mathbf{z}_{i'}\|}}.$$

The full conditional posterior distribution of the random spatial effects is $\mathbf{w} | \mathbf{s}, Y, \sigma^2, \tau^2, \phi, \mathbf{z} \sim N(\mu_w, \Sigma_w)$, where the mean and variance are given as:

$$\mu_w = ((\sigma^2 M(\phi, \mathbf{z}))^{-1} + n(\tau^2 I_N)^{-1})^{-1} (n(\tau^2 I_N)^{-1} \bar{s}) \\ \Sigma_w = ((\sigma^2 M(\phi, \mathbf{z}))^{-1} + n(\tau^2 I_N)^{-1})^{-1}.$$

In this notation, $\bar{s}$ is the overall mean response.

Appendix B. Trace plots for data analysis examples

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Figure B.6:
Trace plots of posterior samples for model parameters in emergency organization influence ranking data application indicate convergence

Figure B.7:

Trace plots of posterior samples for model parameters in adolescent girl MVPA data application indicate convergence

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Figure 1: Mean of spatial attributes overlaid on the estimated organizational social space that was independently estimated without consideration of spatial effects. High values of organizational influence are seen to be close together, indicating a pre-existing social-spatial effect.

Figure 2: Exponential semi-variogram overlaid on the empirical semi-variogram for network emergency response organizations' log decision rank scores.

Figure 3: Mean posterior spatial effects of organizational influence with observed edges for a network of emergency response organizations shows that the high values are spatially close and the least influential organizations are socially furthest away.

Figure 4:
Exponential semi-variogram overlaid on empirical semi-variogram for network of adolescent girls' daily minutes of MVPA

Figure 5: Mean posterior spatial effects of MVPA with observed edges for a network of adolescent girls shows that there is not a clear spatial effect over this social space.

Table 1:

Posterior mean and coverage of spatial model parameters when informative priors are used on the partial sill and nugget.

	Truth	Mean	Coverage
$\frac{\sigma^2}{\sigma^2 + \tau^2}$	0.83	0.802	90%
β	1.00	1.007	99%

Table 2:

Posterior mean and coverage of spatial model parameters when uninformative priors are used on the partial sill and nugget.

	Truth	Mean	Coverage
$\frac{\sigma^2}{\sigma^2 + \tau^2}$	0.83	0.778	83%
β	1.00	1.008	100%

Table 3:

Results from evaluating DIC as a measure of model fit. Simulating data both under a dependence and independence model, the DIC resulting from fitting both the dependence and independence models are show, indicating the appropriateness of DIC as a measure of model fit.

	Fitted Model	
Simulating Model	Dependence	Independence
Dependence	363.74	837.06
Independence	2497.82	2513.77

Table 4:

Posterior summary statistics for social network spatial model parameters fit to an emergency organization network based on each organization's influence ranking.

	Mean	95% Posterior Interval
$\frac{\sigma^2}{\sigma^2 + \tau^2}$	0.976	(0.874, 0.997)
ϕ	0.551	(0.130, 0.976)
β	1.746	(0.874, 2.501)
a	1.067	(0.477, 1.680)
b	2.145	(1.351, 3.173)

Table 5:

Posterior summary statistics for social network spatial model parameters fit to a network of adolescent girls based on their MVPA.

	Mean	95% Posterior Interval
$\frac{\sigma^2}{\sigma^2 + \tau^2}$	0.804	(0.339, 0.958)
ϕ	0.654	(0.201, 0.984)
β	24.616	(11.960, 40.435)
a	0.818	(0.684, 0.960)
b	0.010	(0.001, 0.018)