Published in final edited form as:

JP J Biostat. 2018 ; 15(1): 1–20. doi:10.17654/BS015010001.

A DIAGNOSTIC TOOL FOR CHECKING ASSUMPTIONS OF REGRESSION MIXTURE MODELS

Ian Wadsworth¹, M. Lee Van Horn², Thomas Jaki^{1,*}

¹Department of Mathematics and Statistics, Lancaster University, Lancaster, LA1 4YF, United Kingdom

²School of Education, University of New Mexico, Albuquerque, NM 87131, U. S. A.

Abstract

Regression mixture models are becoming more widely used in applied research. It has been recognized that these models are quite sensitive to underlying assumptions, yet many of these assumptions are not directly testable. We discuss a diagnostic tool based on reconstructed residuals that can help uncover violations of model assumptions. These residuals are found by using the posterior probability of class membership to assign, based on a multinomial distribution, a class to each observation. Standard residual checks can be applied to these posterior draw residuals to explore violations of the model assumptions. We present several illustrations of the diagnostic tool.

Keywords

diagnostic tool; model assumptions; posterior draws; regression mixture models; residuals

1. Introduction

Regression mixture models are an extension of finite mixture models (McLachlan and Peel [8]) and have primarily been used in marketing research (Wedel and Desarbo [16], Desarbo et al. [2]) but in recent years gained popularity in the behavioural sciences. (Kaplan [7], Ding [3], Van Horn et al. [12], Schmiede et al. [11]) Regression mixture models are a specific specification of finite mixture models which estimate class specific regression weights which capture unobserved heterogeneity in the effects of predictors on outcomes. Regression mixture models assume that the conditional distribution of a random variable Y , given predictor variables X , can be described by a mixture of K components, each with a conditional distribution described by a linear regression model. Specifically, consider a sample of n individuals measured on a continuous variable, Y , where y_i is the observed value for subject i . The relationship between predictor variables and the outcome, Y , is conditional on membership in latent class k for the set of P observed predictors, as $X = (x_1, x_2, \dots, x_P)$. For class k this is expressed as

*Corresponding author.

$$Y_{i|X,k} = \beta_{0k} + \sum_{p=1}^P \beta_{pk} x_{ip} + \varepsilon_{ik}, \quad (1)$$

where $\varepsilon_{ik} \sim N(0, \sigma_k^2)$, β_{0k} is the intercept and σ_k^2 is the residual variance for class k .

Differential effects are captured by $\boldsymbol{\beta}_k = (\beta_{0k}, \beta_{1k}, \dots, \beta_{pk})$, the $(p+1)$ -dimensional vector of regression coefficients for $\mathbf{X}$ in latent class k .

Regression mixture models are often difficult to utilize due to their sensitivity to underlying assumptions, (Van Horn et al. [13], George et al. [5], George et al. [4]) which makes it challenging to determine whether classes represent meaningful groups or are used to overcome violations of the model assumptions. Estimating a regression mixture model can be performed in the statistical programs Mplus, (Muthén and Muthén [9]) Latent GOLD (Vermunt and Magidson [14]) or with the use of packages such as flexmix (Grün and Leisch [6]) in R. (R Core Team [10]).

Here we describe a method for estimating residuals for each individual in a sample by assigning each individual to a class using posterior draws, (Bandeem-Roche et al. [1], Wang et al. [15]) hence we term these posterior draw residuals. We propose the use of these posterior draw residuals as a diagnostic tool for regression mixture models. Although posterior draw residuals have been proposed before, our goal is to show how they can be used for model diagnostics and how well they work in regression mixture models. We focus on regression mixture models as posterior draw residuals require assignment of individuals to classes with a specific probability and hence they will work best when there is high entropy. Because regression mixtures are often used when class separation is very weak, entropy is also often quite low in these models. Our goal is to evaluate whether this approach can be used with low levels of entropy typically encountered in regression mixture models. Related work includes that of Wang et al. [15] who look at Growth Mixture Models. Unlike in the work of Wang et al. [15], pseudo class residuals only do a good job of addressing some of the issues faced when using regression mixture models, likely because of the generally poor entropy of those models. Additionally, Bandeem-Roche et al. [1] considered pseudo class draws for regression mixture models for multiple discrete outcomes, though not in the context of diagnostics. Other than using information criteria (such as AIC or BIC) and the bootstrap likelihood ratio test to make decisions regarding the number of classes, we are not aware of other tools used for assumption checking for this type of regression modelling.

2. Posterior Draw Residuals

Regression mixture models make several assumptions:

- (1) $\text{Var}(Y|X, k) = \sigma_k^2 \forall$ values of $\mathbf{X}$.
- (2) $\varepsilon_{ik} \sim N(0, \sigma_k^2)$, for some constant σ_k^2 .
- (3) $E[Y|X, k] = \beta_{0k} + \sum_{p=1}^P \beta_{pk} x_{ip}$.

We will look at using what we term posterior draw (Wang et al. [15]) residuals to validate these assumptions. As the residuals are for a particular K class solution, we illustrate the approach with a two-class solution ($K = 2$), although extensions to more classes are straightforward. The following steps can be used to obtain the posterior draw residuals:

- (1) Fit a 2-class regression mixture model to the data.
- (2) Extract the parameter estimates for class 1 ($\hat{\beta}_{01}, \hat{\beta}_{11}, \dots, \hat{\beta}_{P1}$) and the parameter estimates for class 2 ($\hat{\beta}_{02}, \hat{\beta}_{12}, \dots, \hat{\beta}_{P2}$).
- (3) Using these parameter estimates calculate two residuals for each observation. In particular, the two residuals for observation i are found as,

$$r_{i1} = Y_i - \hat{\beta}_{01} - \sum_{p=1}^P \hat{\beta}_{p1} x_{ip},$$

$$r_{i2} = Y_i - \hat{\beta}_{02} - \sum_{p=1}^P \hat{\beta}_{p2} x_{ip}.$$

- (4) Find the posterior probabilities of class membership for each observation.
- (5) Use the posterior probabilities to randomly assign a class to each observation using a binomial random variable.
- (6) Create a new set of posterior draw residuals for each class by taking the corresponding residuals from either r_{i1} or r_{i2} depending on the class assigned to each observation.

For more complex cases with more classes, a multinomial random variable can be used together with the posterior probabilities to assign a class to each observation. Step 5 randomly assigns a class to each observation, as simply assigning class according to the maximum posterior weight truncates the tail of the residual distribution where the probability of class assignment is 0.5. Simple standard graphical checks for residuals such as Q-Q plots, histograms and plots of residuals against X can be used to assess the fit.

3. Illustrative Examples

3.1. No violation of assumptions

First we illustrate the diagnostic tool in the situation where the model assumptions are not violated. We simulate a dataset with 6000 observations that are split equally between two classes. The values of a single covariate, X , are generated as uniform random variables on -3 to 3 while the responses are generated using the following models:

$$Y_{i1} = 0.7X_i + \varepsilon_{i1}, \quad i = 1, \dots, 3000,$$

$$\varepsilon_{i1} \sim \text{Normal}(0, 1 - 0.7^2),$$

$$Y_{i2} = 0.2X_i + \varepsilon_{i2}, \quad i = 3001, \dots, 6000,$$

$$\varepsilon_{i2} \sim \text{Normal}(0, 1 - 0.2^2).$$

Note that we are taking $\beta_{01} = \beta_{02} = 0$ i.e., no intercept in this model for simplicity. The Bayesian information criterion (BIC) can reliably identify the correct number of classes in this set-up (for 100 out of 100 simulations, the 2-class solution has the lowest BIC, one iteration failed to fit 3-classes). Focusing on a single dataset, we can obtain the posterior draw residuals by fitting a 2-class regression mixture model and subsequently explore the model fit and the distribution of the residuals.

Visual inspection of the scatter plot for one simulated dataset (Figure 1) does not reveal any violations of the assumptions of regression mixture models. For this dataset, the 2-class model gives parameter estimates (standard errors) $\hat{\beta}_{01} = 0.024(0.021)$, $\hat{\beta}_{11} = 0.698(0.021)$, $\hat{\sigma}_1^2 = 0.495(0.035)$, $\hat{\beta}_{02} = -0.001(0.026)$, $\hat{\beta}_{12} = 0.211(0.032)$ and $\hat{\sigma}_2^2 = 1.014(0.037)$. These estimates were very close to the values used to generate the data ($\beta_{01} = 0$, $\beta_{11} = 0.7$, $\beta_{02} = 0$ and $\beta_{12} = 0.2$).

In truth the two classes were generated to be evenly split and similarly for this dataset the class sizes of the 2-class model fit are around 49.5% for class 1 and 50.5% for class 2. Based on the posterior draw residuals for this one dataset, looking at the diagnostic plots in Figure 2 there is an unexpected deviation in the Q-Q plot for the first class which is a result of being unable to perfectly associate subjects to a group; in the region where class models cross a random allocation occurs leading to an under representation in this region for the smaller group, resulting in the variability of the residuals not being exactly as expected under a normal distribution. However, the histograms and the plots of residuals against X do not highlight any violations to the model assumptions.

3.2. Violating variance homogeneity assumption

In the second evaluation, we explore the effect of violating the assumption of homogeneous variance by simulating a set of data with two classes, one of which has a variance which increases with the value of X while the variability is constant across covariate level for the other class. We consider the case where 6000 observations are split equally between two classes. The values of the covariate are generated as uniform random variables on -3 to 3 while the responses are generated using the base model given in Subsection 3.1, though now the variance of ε_{i1} becomes $[0.4(X_i + 1.5 - \min(X_1, \dots, X_{3000}))]^2$, which creates an increase in variance as X increases, giving a ‘trumpet’ shape to class 1 in the data. Despite the true

underlying model only comprising of two classes, the BIC prefers 3- (or more) class solutions in this case (for 100 simulations all select a 3- (or more) class solution).

Figure 3 provides the scatter plot for one such dataset and a simple visual inspection does not reveal any obvious violations of the assumptions of regression mixture models. For this dataset, the 2-class model gives parameter estimates (standard errors) as $\hat{\beta}_{01} = 1.765(0.076)$, $\hat{\beta}_{11} = 1.322(0.038)$, $\hat{\sigma}_1^2 = 1.683(0.130)$, $\hat{\beta}_{02} = -0.302(0.020)$, $\hat{\beta}_{12} = 0.302(0.012)$ and $\hat{\sigma}_2^2 = 1.648(0.050)$, quite different to the values used to generate the data ($\beta_{01} = 0$, $\beta_{11} = 0.7$, $\beta_{02} = 0$ and $\beta_{12} = 0.2$). Additionally, although the two classes were evenly split, the class sizes of the 2-class model are around 12.8% for class 1 and 87.2% for class 2 illustrating that the estimated model does not describe the underlying data generation very well and hence would likely lead to misinterpretation.

The diagnostic plots based on the posterior draw residuals (Figure 4), highlight potential violations to the model assumptions. In this case, the scatter plot of the residuals versus X in both classes, but especially in class 2, suggest an increasing variance in the residuals, which could allow for the identification of a non-constant variance in one of the classes. This is consistent with how the data was simulated, that is, with one of the classes having a variance which increases with the value of X .

3.3. Model misspecification

The third illustration considers the case where a 2-class regression mixture model with a linear relationship with the covariate X is fit to a dataset with a non-linear relationship. In particular, this example simulates two classes, one of which has an additional quadratic term in X . Once more we consider the case where 6000 observations are split equally between the two classes and values of the covariate are generated as uniform random variables on -3 to 3 while the responses are generated using the base model given in Subsection 3.1, though now $Y_{i1} = 0.7X_i + 0.2X_i^2 + \varepsilon_{i1}$, containing an extra quadratic term to give a non-linear relationship in the first class. As before, the BIC prefers a 3- (or more) class solution in this case (88 of 100 simulated datasets chose a 3- (or more) class model).

Looking at one dataset which would have chosen the 3-class model, inspection of the scatter plot and the best line fit (Figure 5) once again shows no obvious violation of the model assumption. In this case, the 2-class fit parameter estimates are $\hat{\beta}_{01} = 0.573(0.030)$, $\hat{\beta}_{11} = 0.885(0.027)$, $\hat{\sigma}_1^2 = 0.621(0.040)$, $\hat{\beta}_{02} = -0.002(0.024)$, $\hat{\beta}_{12} = 0.146(0.014)$ and $\hat{\sigma}_2^2 = 0.732(0.026)$, not too far from the true values of $\beta_{01} = 0$, $\beta_{11} = 0.7$, $\beta_{02} = 0$ and $\beta_{12} = 0.2$ apart from the $\hat{\beta}_{01}$ intercept term. The class split is quite different to the truth of equal class sizes, with estimated class sizes of around 39.5% for class 1 and 60.5% for class 2, which is likely a product of the model misspecification.

Looking at Figure 6, the plot of the posterior draw residuals against X clearly shows non-linearity in class 1, highlighting the violation of the model assumptions, consistent with the data generated where one class has a non-linear relationship with X .

3.4. Smaller sample size and violating variance homogeneity assumption

In Subsection 3.2, we explored the effect of violating the assumption of homogeneous variance by simulating a set of data with two classes, one of which had a variance which increases with the value of X , for a sample of 6000 observations. We now repeat this procedure for smaller sample sizes.

We consider the cases where 3000, 1500 and 500 observations are split equally between two classes. The values of the covariate and responses are generated as in Subsection 3.2. Despite the true underlying model only comprising of two classes, the BIC prefers solutions with more than two classes in all simulations for sample sizes of 3000 and 1500 while in 70% of the simulations more than 2 classes are preferred with 500 observations. Figure 7 provides the scatter plots for one dataset from each of the sample size cases. A simple visual inspection of these scatter plots does not reveal any obvious violations of the assumptions of regression mixture models.

Table 1 presents the parameter estimates $\hat{\beta}_{01}$, $\hat{\beta}_{11}$, $\hat{\beta}_{02}$ and $\hat{\beta}_{12}$ for one simulated dataset for the cases of 500, 1500 and 3000 observations; these are all quite different to the values used to generate the data ($\beta_{01} = 0$, $\beta_{11} = 0.7$, $\beta_{02} = 0$ and $\beta_{12} = 0.2$).

Additionally, although the two classes in the simulated data were evenly split, the class sizes of the 2-class model fitted to the 3000, 1500 and 500 observations datasets are around 14%, 13% and 11% for class 1, respectively, and 86%, 87% and 89% for class 2, respectively, illustrating that the estimated model does not describe the underlying data generation very well and hence would likely lead to misinterpretation.

The diagnostic plots of the residuals against X shown in Figure 8 highlight potential violations to the model assumptions. In these cases, the scatter plots of the residuals versus X in both classes, but especially in class 2, suggest an increasing variance in the residuals, which could allow for the identification of a non-constant variance in one of the classes. This is consistent with how the data was simulated, that is, with one of the classes having a variance which increases with the value of X . Obviously, it is more challenging to see this pattern in the case of the lowest sample size, though it can still be observed. The other diagnostic plots (Q-Q plots and histograms) are essentially the same as for the previous example.

3.5. Multiple covariates with more classes and model misspecification

In Subsection 3.3, we explored the case where a 2-class regression mixture model with a linear relationship with the covariate X is fit to a dataset where one of the two classes has an additional quadratic term in X . We now explore a similar scenario where we have two covariates, X_1 and X_2 , both generated as uniform random variables on -3 to 3 and data with four classes generated using the following models:

$$Y_{i1} = 0.7X_{i1} + 0.5X_{i2} + 0.2X_{i1}^2 + \varepsilon_{i1}, \quad i = 1, \dots, 1500,$$

$$\varepsilon_{i1} \sim \text{Normal}(0, 1 - 0.7^2),$$

$$Y_{i2} = 0.7X_{i1} - 0.3X_{i2} + \varepsilon_{i2}, \quad i = 1501, \dots, 3000,$$

$$\varepsilon_{i2} \sim \text{Normal}(0, 1 - 0.7^2),$$

$$Y_{i3} = 0.2X_{i1} + 0.5X_{i2} + \varepsilon_{i3}, \quad i = 3001, \dots, 4500,$$

$$\varepsilon_{i3} \sim \text{Normal}(0, 1 - (3 \times 0.2^2)),$$

$$Y_{i4} = 0.2X_{i1} - 0.3X_{i2} + \varepsilon_{i4}, \quad i = 4501, \dots, 6000,$$

$$\varepsilon_{i4} \sim \text{Normal}(0, 1 - (3 \times 0.2^2)).$$

Note that in the first class there is an additional quadratic term in X_1 ; plots visualising this can be seen in Figure 9. We consider the case where 6000 observations are split equally between four classes, generating 100 separate simulated datasets.

Of the 100 simulated datasets, using the BIC, 19 would have chosen a 3-class solution, 78 would have chosen a 4-class solution, and 3 would have chosen a 5-class solution. Focussing on one of the simulated datasets where a 4-class solution would be chosen, the 4-class model gives parameter estimates (standard errors) $\hat{\beta}_{01} = 0.568(0.053)$, $\hat{\beta}_{11} = 0.925(0.058)$,

$$\hat{\beta}_{21} = 0.454(0.031), \hat{\beta}_{02} = 0.019(0.040), \hat{\beta}_{12} = 0.704(0.062), \hat{\beta}_{22} = -0.320(0.024),$$

$$\hat{\beta}_{03} = 0.029(0.046), \hat{\beta}_{13} = 0.149(0.030), \hat{\beta}_{23} = 0.487(0.033), \hat{\beta}_{04} = -0.088(0.048),$$

$$\hat{\beta}_{14} = -0.221(0.074), \text{ and } \hat{\beta}_{24} = -0.295(0.045). \text{ These estimates allow identification of each}$$

class, compared to the models we simulated, but are still noticeably different from the true values. Although the four classes were simulated to be evenly split, the class sizes of the 4-class model fit for classes 1, 2, 3 and 4 were 20.6%, 19.7%, 33.2%, and 26.5%, respectively.

The diagnostic plots of the residuals against X shown in Figure 10 highlight potential violations to the model assumptions. In this case, the first scatter plot of the residuals versus X_1 in class one suggests a slight non-linear shape that could provide evidence to suggest that there are violations to the model assumptions.

4. Discussion

This paper explores a simple diagnostic tool that can help to uncover violations of the underlying assumptions of regression mixture models. This is done using standard residual checking diagnostics on the posterior draw residuals. As demonstrated the approach can be effective in identifying violations of variance homogeneity and departures from a linear model. The method is useful for trying to detect why additional classes are selected over a particular solution. They must be selected because of non-normality in the conditional distribution of Y . It is worth pointing out, however, that regression mixture models often account for deviations of the model assumptions by adding additional classes to capture these deviations (George et al. [4]). The posterior draw residuals described here are only useful to uncover if additional classes were selected from a particular solution because of an assumption being violated. This method will only indirectly indicate the correct number of classes. In order to identify violations of the assumption using posterior draw residuals it is helpful if the tail of the covariate distribution(s), X , is well represented (like the uniform distribution used in our illustrations). This is because violations typically are most pronounced at the extremes. Finally, our description and illustrations focussed on data with 2-classes, however, extensions to models with more classes is simple.

Acknowledgements

This research was supported by the National Institute of Child Health and Human Development under grant number R01HD054736, M. Lee Van Horn (PI); and the UK Medical Research Council under grant number MR/L010658/1, Thomas Jaki (PI).

References

- [1]. Bandeen-Roche K, Miglioretti DL, Zeger SL and Rathouz PJ, Latent variable regression for multiple discrete outcomes, *J. Am. Stat. Assoc* 92 (1997), 1375–1386.
- [2]. Desarbo WS, Jedidi K and Sinha I, Customer value analysis in a heterogeneous market, *Strat. Mgmt. J* 22 (2001), 845–857.
- [3]. Ding C, Using regression mixture analysis in educational research, *Pract. Assess. Res. Eval* 11 (2006), 1–11.
- [4]. George MR, Yang N, Jaki T, Feaster DJ, Lamont AE, Wilson DK and Lee Van Horn M, Finite mixtures for simultaneously modeling differential effects and nonnormal distributions, *Multivariate Behav. Res* 48(6) (2013), 816–844. [PubMed: 25717214]
- [5]. George MR, Yang N, Van Horn ML, Smith J, Jaki T, Feaster D, Masyn K and Howe G, Using regression mixture models with non-normal data: Examining an ordered polytomous approach, *J. Stat. Comput. Simul* 83 (2013), 759–772.
- [6]. Grün B and Leisch F, FlexMix Version 2: Finite mixtures with concomitant variables and varying and constant parameters, *J. Stat. Softw* 28 (2008), 1–35. Retrieved from <http://www.jstatsoft.org/v28/i04> [PubMed: 27774042]
- [7]. Kaplan D, Finite mixture dynamic regression modeling of panel data with implications for dynamic response analysis, *J. Educ. Behav. Stat* 30 (2005), 169–187.
- [8]. McLachlan G and Peel D, *Finite mixture models*, John Wiley & Sons, 2004.
- [9]. Muthén LK, Muthén BO, *Mplus, Version 7.2*, Muthén & Muthén, Los Angeles, CA, 2014.
- [10]. R Core Team, *A language and environment for statistical computing*, R Foundation for Statistical Computing, Vienna, Austria, (2013), Retrieved from <http://www.R-project.org/>.
- [11]. Schmiede SJ, Levin ME and Bryan AD, Regression mixture models of alcohol use and risky sexual behavior among criminally-involved adolescents, *Prev. Sci* 10 (2009), 335–344. [PubMed: 19459047]

- [12]. Van Horn ML, Jaki T, Masyn K, Ramey SL, Smith JA and Antaramian S, Assessing differential effects: Applying regression mixture models to identify variations in the influence of family resources on academic achievement, *Dev Psychol* 45(5) (2009), 1298. [PubMed: 19702393]
- [13]. Van Horn ML, Smith J, Fagan AA, Jaki T, Feaster DJ, Masyn K, Hawkins JD and Howe G, Not quite normal: consequences of violating the assumption of normality in regression mixture models, *Struct. Equ. Modeling* 19(2) (2012), 227–249. [PubMed: 22754273]
- [14]. Vermunt JK and Magidson J, *Technical Guide for Latent GOLD 5.0: Basic, Advanced, and Syntax*, Statistical Innovations Inc, Belmont, MA, 2013.
- [15]. Wang CP, Hendricks Brown C and Bandeen-Roche K, Residual diagnostics for growth mixture models: Examining the impact of a preventive intervention on multiple trajectories of aggressive behavior, *J. Am. Stat. Assoc* 100 (2005), 1054–1076.
- [16]. Wedel M and Desarbo WS, A review of recent developments in latent class regression models, Bagozzi RP, ed., *Advanced Methods of Marketing Research* Blackwell, Cambridge, MA, 1994, pp. 352–388.

Figure 1.
Scatterplot of 2-class dataset where no assumptions have been violated, with the best fit lines
for a 2-class model.

Figure 2. Diagnostic plots based on posterior draw residuals. Left panel showing class 1 and right panel class 2. The first row displays Q-Q plots, the second histograms of the residuals and the final row plots the residuals versus X .

Figure 3.
Scatterplot of 2-class dataset with one class violating the assumption of variance homogeneity together with the best fit lines for a 2-class model.

Figure 4. Diagnostic plots based on posterior draw residuals. Left panel showing class 1 and right panel class 2. The first row displays Q-Q plots, the second histograms of the residuals and the final row plots the residuals versus X .

Figure 5.
Scatterplot of 2-class dataset with one class violating the assumption of linearity in X together with the best fit lines for a 2-class model.

Figure 6. Diagnostic plots based on posterior draw residuals. Left panel showing class 1 and right panel class 2. The first row displays Q-Q plots, the second histograms of the residuals and the final row plots the residuals versus X .

Figure 7. Scatterplots of 2-class datasets for the cases of 3000, 1500, 500 observations, with one class violating the assumption of linearity in X together with the best fit lines for a 2-class model.

Figure 8. Plots the posterior draw residuals versus X . Top row is the case with 3000 observations, middle row is the case with 1500 observations, bottom row is the case with 500 observations. Left panel showing class 1 and right panel class 2.

Figure 9.

Scatterplots showing the response, Y , against the covariate X_1 of a 4-class dataset with one class including a quadratic term in X_1 . Left scatterplot shows only the data generated in the class with the quadratic term. Right scatterplot shows all classes.

Figure 10. Plots of the residuals against X_1 for the case of a 4-class dataset, with one class including a quadratic term in X_1 .

Table 1.

Parameter estimates $\hat{\beta}_{01}$, $\hat{\beta}_{11}$, $\hat{\beta}_{02}$, and $\hat{\beta}_{12}$ (standard errors) for one simulated dataset for the cases of 500, 1500 and 3000 observations

Number of observations	$\hat{\beta}_{01}$ (SE)	$\hat{\beta}_{11}$ (SE)	$\hat{\beta}_{02}$ (SE)	$\hat{\beta}_{12}$ (SE)
500	1.612 (0.107)	1.212 (0.064)	-0.234 (0.032)	0.300 (0.017)
1500	1.624 (0.124)	1.318 (0.069)	-0.281 (0.040)	0.305 (0.022)
3000	1.662 (0.387)	1.548 (0.234)	-0.145 (0.074)	0.325 (0.050)