New insights into the physics of inertial microfluidics in curved microchannels.

II. Adding an additive rule to understand complex cross-sections

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ABSTRACT

Curved microchannels allow controllable microparticle focusing, but a full understanding of particle behavior has been limited—even for simple rectangular and trapezoidal shapes. At present, most microfluidic particle separation literature is dedicated to adding “internal” complexity (via sheath flow or obstructions) to relatively simple cross-sectional channel shapes. We propose that, with sufficient understanding of particle behavior, an equally viable pathway for microparticle focusing could utilize complex “external” cross-sectional shapes. By investigating three novel, complex spiral microchannels, we have found that it is possible to passively focus (6, 10, and 13 μm) microparticles in the middle of a convex channel. Also, we found that in concave and jagged channel designs, it is possible to create multiple, tight focusing bands. In addition to these performance benefits, we report an “additive rule” herein, which states that complex channels can be considered as multiple, independent, simple cross-sectional shapes. We show with experimental and numerical analysis that this new additive rule can accurately predict particle behavior in complex cross-sectional shaped channels and that it can help to extract general inertial focusing tendencies for suspended particles in curved channels. Overall, this work provides simple, yet reliable, guidelines for the design of advanced curved microchannel cross sections.

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I. INTRODUCTION

As we approach three decades of research into “microfluidic” technology,¹ much more is understood about the fundamental physics of these devices. As our understanding has grown, several particle separation and enrichment techniques have been developed,² which, writ large, likely represent the most impactful applications of microfluidics to date.³ In inertial microfluidics applications,^{4,5} it is possible to focus particles at equilibrium positions in microchannels where internal forces are balanced. The main factors that influence

these forces are particle size and the fluid’s shear gradient. Although those parameters should be easy enough to track, a full understanding of the underlying physics of these devices has remained just outside our reach since: (a) experimental results do not align perfectly with theory⁶ and (b) it is prohibitively computationally expensive to simulate the full particle-laden flow.⁷

In straight microchannels, a full understanding is possible since there are only three inertial lift forces competing to determine the final focusing position:^{8–10} (i) F_S , a shear gradient lift force, that

pushes particles down the gradient of the shear rate, $\dot{\gamma}$, (ii) F_W , a wall-induced lift force, which always pushes particles away from the wall, and, (iii) F_Q , a rotation-induced lift force, that drags particles to the centerline of the channel walls. The resultant of F_S , F_W , and F_Q is the net inertial lift force F_{Net} . At locations where these forces are in balance (i.e., $F_{Net} = 0$), particle focusing occurs.

This relatively straightforward force balance becomes much more complex when additional secondary forces are intentionally introduced in the microchannel. By adding curves, obstructions, or external forces to disturb an otherwise straight, smooth microchannel flow profile, it is possible to gain design freedom and superior performance/tunability (at the cost of complexity).

As one notable category of advanced geometries, symmetric or asymmetric contraction/expansion arrays (CEA)¹¹ have been proposed as a means to induce secondary flows. Park *et al.*¹² utilized multifurcated microfluidic channels to focus microparticles at different Reynolds numbers Re . In 2013, Lee *et al.*¹³ separated cancer cells from whole blood using a modified CEA structure. In 2015, malaria parasites were enriched from blood using a rectangular microchannel patterned with CEA structures.¹⁴ In 2016, Wu *et al.*¹⁵ presented a CEA-based microdevice that successfully separated white blood cells from red blood cells. A commonality of microfluidic devices with CEA structures is that larger particles/cells are dragged toward the

expanded part of the channel while smaller particles remain in the middle of the channel.

Another category of advanced design incorporates structures and/or obstacles in the microchannel's walls to controllably deform the fluid streams (i.e., secondary flow pattern). Following this approach, researchers have investigated grooves^{16,17} or herringbone structures^{18,19} to improve performance. Channels containing obstacles^{20,21} or micropillars^{22,23} have also been used to control the shape of streams. Depending on the pattern of the pillar, these channels are capable of focusing particles at an arbitrary width of the channel.²⁴ The downside of adding micropillars, however, is the risk of clogging the device.²⁵

As perhaps the most successful advanced microchannel design, curved microfluidic channels create a relatively strong and stable secondary flow. This type of microchannel design can be categorized as either spiral^{10,26–30} or asymmetric curving channels^{31,32} (sigmoidal channels with alternating curvature and serpentine). Due to the centrifugal forces present in curved channels, fluid tends to flow from the inner wall toward the outer wall, returning through the regions near the channel's top and bottom walls. Therefore, two primary, counter-rotating vortices develop in the cross section of the channel, which are known as Dean vortices.³³ Following Stokes' drag law, Dean flow provides a Dean drag

FIG. 1. Schematic of microchannels. (a) Top view of the spiral microchannels used in this study (i.e., the x-y plane). R and L indicate the radius of channel curvature and channel length, respectively. Also, F-F and B-B denote sections of interest (near the outlet) in forward and backward flows, respectively. (b) Cross-sectional view of the complex shaped channels in the x-z plane. The dashed "M-m-M" and "M-m-m-M" lines designate the centerline of the cross section. These lines are simply referred as M-M centerline. (c) Schematic of the microchip for (i) top-view observation in forward (blue) and backward (red) flows, and (ii) side-view observation in backward flow. (d) Images of the experimental setup for an inverted microscope (i) top view and (ii) side view.

FIG. 2. Schematic of the focusing regions. Regions I, II, and III in the cross section of curved channels with simple geometrical shapes determine the potential focusing areas highlighted in yellow. The boundary dotted lines between the regions are IPs.

force, F_D , which helps control the focusing behavior of particles in the microchannel.^{10,34} While investigating these in detail, Nivedita *et al.*³⁵ reported the formation of secondary vortices where the Dean number is greater than a critical Dean number (i.e., $De > De_c$).

In general, the focusing position of particles in curved microfluidic devices can be determined from the resultant of F_D and F_{Net} . Since all of these forces are a function of flow rate, Q , and particle size, a , the expected location of the final equilibrium point also varies with these parameters. This relationship, though somewhat tunable based on design choice, imposes a limitation in focusing particles of different sizes in a desired position for a wide range of flow rates.

To overcome these limitations, researchers have introduced another layer of complexity via sheath flow,³⁶ coflow,³⁷ viscoelastic fluids,³⁸ and active techniques which rely upon external forces, such as electrophoresis,^{39,40} magnetophoresis,⁴¹ and acoustophoresis.⁴² While this next layer of complexity has shown potential, we propose that complex geometries containing simple, passive flow—if well understood—could represent an equally viable evolutionary pathway to achieve similar results. As such, this paper investigates spiral microchannels with complex—yet generalizable—cross sections. We hypothesize that by understanding passive flow in complex geometries, it may be possible to (i) achieve the same level of control as has been achieved with layers of complexity (e.g., using auxiliary flows or active systems) and (ii) gain insights into both simple and advanced geometries.

II. EXPERIMENTAL

To design complex cross-sectional shapes, we combined three basic geometries: a rectangle, an inward sloping trapezoid, and an outward sloping trapezoid. These were chosen because our prior work extensively investigated inertial focusing mechanisms in individual channels with these basic elements.⁶ Conceptually, these elements be used as building blocks for creating more complex cross sections for controllable inertial focusing. Each of the devices shown in Fig. 1 was tested over a wide range of flow rates (i.e., $Q = 0.5$ – 9 ml/min, $Re \approx 20$ – 450), with three particle sizes,

FIG. 3. Inflection points of Dean flow. Distribution of IPs at $Q = 0.5$ – 9 ml/min in backward flow (i.e., section B-B) and forward flow (i.e., section F-F) of the channels.

and in two flow directions (to study inertial focusing at different Reynolds numbers, Re , and Dean numbers, De). Note: Sec. I of the [supplementary material](#) document provides more detail about the exact geometric parameters of the channels and the calculation of their hydraulic diameter. To perform the experiments, three fluorescent particle suspensions (of $a=6, 10$, and $13\text{ }\mu\text{m}$ diameter particles) at the same concentration were pumped through these spiral microchannels. As is shown in Fig. 1(d), in forward flow, section F-F was observed using an epifluorescence microscope from the top view only. In backward flow, section B-B was imaged from both the top and side views to observe the focusing behavior of the fluorescent microbeads. It should be noted that orientation (e.g., the gravitational force) has no meaningful effect for the range of flow rates tested here.^{43,44}

III. THEORETICAL BACKGROUND

To interpret the results from complex cross section channels, one first needs to understand the subtle mechanisms at play in spiral microchannels with simple cross section. The key to understanding this relies upon knowledge of the focusing positions—e.g., locations where the resultant of F_D and F_{Net} is equal to zero. Our theory of “Inflection Point Displacement”⁶ states that the local force balance can be determined by the influence distance of Dean flow, δ —a parameter measured as the distance between the real focusing position of a particle and its nearest inflection point, IP, of the secondary flow in the same section of the channel. From this theory, the locations of IPs divide the cross section of the curved microchannel into three essential regions—regions I, II, and III (see Fig. 2). Contrary to the common perception of fixed IPs, the

FIG. 4. Experimental results of the convex channel. (a) Experimental observations of fluorescent beads of 6, 10, and $13\text{ }\mu\text{m}$ diameter at $Q=0.5$ – 9 ml/min in forward flow (i.e., top view at section F-F) and backward flow (i.e., top and side views at section B-B) of the convex channel. Note: The fluorescent color of 6, 10, and $13\text{ }\mu\text{m}$ beads is red, green, and blue, respectively. (b) Focusing positions of microbeads, regardless of how wide the bands are, at different flow rates. (c) Sharpness factor for different particle sizes at all the flow rates.

height—and shape—of these regions are a function of flow rate because of the displacement of IPs. Thus, particles can only focus in certain “potential focusing areas,” that are the inner half of regions I and III as well as the outer half of region II (i.e., areas highlighted in yellow). Outside these areas, F_D and F_Ω are in the same direction—a situation in which inertial lift forces and Dean drag force cannot achieve a balance.

IV. RESULTS AND DISCUSSION

A. Distribution of IPs

The first step of analyzing the mechanism of inertial focusing in curved channels required us to obtain the distribution of IPs in the section of interest (e.g., typically a section near the outlet). Figure 3 shows how the IPs are distributed in the cross section of the complex curved channels of this study at different flow rates. The distribution of IPs was calculated by our previously developed “Dean Flow

Analyzer” software.⁶ From Fig. 3, it can be seen that at low flow rates in backward flow, the IP curves are close to the top and bottom walls and move away from the walls with increasing flow rate. At low flow rates in forward flow, however, the IP curves are close to the M-M centerline and approach the top and bottom walls as the flow rate increases. This contrast originates from the difference between the patterns of the secondary flow where the sign of dR/dL is different.

B. Inertial focusing in a convex channel

1. Experimental results

Figure 4(a) shows the experimental results of the convex channel in forward and backward flows. The focusing trends and sharpness factor proposed by Rafeie *et al.*⁶ [i.e., $s = 1 - (b - a)/W$, where b and W are the band width and the channel width, respectively], of microbeads are shown in Figs. 4(b) and 4(c), respectively. From these results, it can be seen that except for low flow rates

FIG. 5. Inertial focusing in the convex channel. (a) Experimental results of 6, 10, and 13 μm microbeads ($Q = 0.5\text{--}6\text{ ml/min}$, $Re = 24\text{--}291$) in the convex channel and two trapezoidal microchannels with the same relative cross-sectional dimensions – i.e., side wall heights of 80 and 130 μm . (b) Particle equilibrium positions in the cross section of the convex microchannel. (i)–(vi) Schematics of the proposed force balances, moving from low to high flow rate, respectively.

(i.e., $Q \leq 2$ ml/min, $Re \leq 97$), where $13\text{ }\mu\text{m}$ particles gather in the inner half of the channel dispersedly, particles usually focus near the channel's center. As can be seen from side view results [i.e., backward flow in Fig. 4(a)], the focusing band near the upper wall of this channel is much thicker (i.e., less sharp) than the band near the lower wall. Lastly, Fig. 4(c) confirms that the sharpness factor for 10 and $13\text{ }\mu\text{m}$ particles remains above 0.7 and 0.85, respectively, at a flow rate of $Q = 9$ ml/min ($Re = 437$).

2. Inertial focusing mechanism

The intuitive reader will notice that the cross section of the convex channel is comprised of two adjoining trapezoids whose larger side is common. When these two simple shapes are added

together in a single complex cross section, the inertial focusing in this channel appears to be a combination of the particle behavior in the two constituent individual channels. As a comparison, particle focusing in two outward and inward sloping trapezoidal channels with similar dimensions were tested [Fig. 5(a)]. In this test, the maximum difference between Re of the constituent channels and the complex channel at any given flow rate is only $\sim 3\%$.

As can be seen in Fig. 5(b), in state (i), where the flow rate is very low but high enough to allow for inertial focusing (i.e., for microchannels of this study; $Q_{min} > 0.5$ ml/min, $Re > 25$), the distance from the inflection point, δ , has a relatively large positive value. In this state, due to the relatively large magnitude of reversed F_D , particles focus in their closest position to the inner wall. In state (ii), which happens at higher flow rates, δ still has a positive

FIG. 6. Experimental results of the concave channel. (a) Experimental observations of fluorescent beads of 6, 10, and $13\text{ }\mu\text{m}$ diameter at $Q = 0.5\text{--}9$ ml/min in forward flow (i.e., top view at section F-F) and backward flow (i.e., top and side views at section B-B) of a concave channel. (b) Focusing positions of microbeads at different flow rates. (c) The sharpness factor for different particle sizes at all the flow rates.

value, but not as large as (i). Hence, particles experience a weaker secondary flow and focus closer to the channel center where F_Ω and the reversed F_D balance locally. In state (iii), δ takes on infinitesimal values resulting in weak local balances between F_Ω and the reversed F_D . At such flow rates, particles become dispersed at this area of the channel width. If particles pass the IP curves, however, they will enter region II and rapidly migrate to the outer region, i.e., from state (iv₁) to (iv₂) where δ takes on a negative value. Similar to an inward sloping trapezoidal channel [Fig. 5(a), middle image], in the outer half of the convex channel the pattern of the secondary flow drags particles into regions I and III at high flow rates. The farther the particles focus from the channel center, the larger the vertical component of F_D exerts upon them [i.e., states (v) and (vi)]. Therefore, particles cannot focus near the outer wall and a balance between F_Ω and F_D can only be achieved near the channel's center [Fig. 5(a), right image].

C. Inertial focusing in a concave channel

1. Experimental results

Figure 6(a) shows the experimental results of the concave channel in forward and backward flows along with the focusing trends and sharpness factor of microbeads shown in Figs. 6(b) and 6(c), respectively. As can be seen in these figures, 6 μm particles are gathered near both side walls. 10 and 13 μm diameter particles are also inclined to focus near side walls. However, at low flow rates (i.e., $Q < 3 \text{ ml/min}$, $Re < 140$), 10 and 13 μm diameter particles mostly focus near the inner wall and then gradually fade and join the outer focusing band as the flow rate increases, until high flow rates where all of them focus near the outer wall. Finally, Fig. 6(c) shows that the sharpness factor for 10 and 13 μm particles remains about 0.9 and 0.95, respectively, up to a flow rate of $Q = 9 \text{ ml/min}$ ($Re = 420$).

FIG. 7. Inertial focusing in the concave channel. (a) Experimental results of 6, 10, and 13 μm microbeads ($Q = 0.5\text{--}6 \text{ ml/min}$, $Re = 23\text{--}280$) in the concave channel as well as two trapezoidal microchannels whose cross-sectional shapes constitute the cross section of this channel. The heights of the side walls of these additional trapezoidal channels are 80 and 130 μm . (b) Particle equilibrium positions in the cross section of the concave microchannel. (i–vi) Schematics of the proposed force balances, moving from low to high flow rate, respectively.

2. Inertial focusing mechanism

The cross section of the concave channel is comprised of two adjoining trapezoids whose shorter side is common. The particle behavior in either half of this channel is quite similar to a trapezoidal channel with the same cross-sectional shape. To check this observation, the focusing behavior of the same particle sizes in two inward and outward sloping trapezoidal channels was examined [Fig. 7(a)]. In this examination, the maximum difference between Re of the constituent channels and the complex channel at any given flow rate is less than 1%.

As can be seen in Fig. 7(b), in the inner half, δ has a relatively large positive value in state (i)—a value that decreases gradually with the flow rate until becomes negative [i.e., state (ii) and then state (iv)]. As soon as a particle enters region II, the streamlines of

this region transport it to the realm of the outer distinct Dean flow pattern consisting of two independent counter-rotating vortices. As a result, particles progressively abandon the inner half and join the rest of the focused particles [i.e., state (iv)].

In the outer half, on the other hand, at low flow rates, particles are focused in regions I and III (i.e., $Q \leq 1$ ml/min, $Re \leq 47$). Upon increasing the flow rate, they gradually approach the IP curves until δ approaches zero. At such flow rates (i.e., $1 \leq Q \leq 2$ ml/min, $47 \leq Re \leq 93$), particles dissipate and form local equilibrium positions where F_D and the reversed F_D are oppositely equal [i.e., providing a force balance in state (iii)]. As the flow rate increases even further, particles continue their approach to the M-M centerline and eventually enter region II where δ takes on a negative value. Larger absolute values of δ correspond to stronger F_D values which drag

FIG. 8. Experimental results of the ramp channel. (a) Experimental observations of fluorescent beads of 6, 10, and 13 μm diameter at $Q = 0.5$ –9 ml/min in forward flow (i.e., top view at section F-F) and backward flow (i.e., top and side views at section B-B) of the ramp channel. (b) Trends of focusing positions of microbeads at different flow rates. (c) The sharpness factor for different particle sizes at all the flow rates.

particles farther in opposition to F_D . Ultimately, the closest focusing position to the outer sidewall is determined by the balance of F_W , F_D , and F_D [i.e., state (vi)].

D. Inertial focusing in a ramp channel

1. Experimental results

Figure 8 shows the experimental results of the ramp channel in forward and backward flows, together with the focusing trends and sharpness factor of microbeads shown in Figs. 8(b) and 8(c), respectively. As can be seen from these results, at low flow rates (i.e., $Q \leq 1$ ml/min, $Re \leq 47$), two distinct focusing positions in the inner and outer halves are observed. Upon increasing the flow rate, particles which were focused near the inner side wall swiftly merge into the outer focusing band (i.e., $Q \leq 2$ ml/min, $Re \leq 94$). For larger particles, this lateral migration can be accompanied by a dissipation which results in a considerable drop in the s values. Lastly, as the flow rate increases beyond 2.5 ml/min ($Re > 117$), the distance

between the focusing position of particles and the outer wall becomes smaller.

2. Inertial focusing mechanism

Like the convex and concave designs, the cross section of the ramp channel is comprised of simple subset shapes—in this case, two rectangles connected by a trapezoid [Fig. 9(a)]. For comparison, three individual spiral channels with each of the adjoining geometries of the ramp channel cross section were tested. In this test, the maximum difference between Re of the constituent channels and the complex channel at any given flow rate is only $\sim 4\%$.

As can be seen in Fig. 9(b), at low flow rates particles are focused near the inner side of these three sections [i.e., state (i)]. By increasing the flow rate, particles in the trapezoidal section jump to the outer side⁶ and join the focusing band of particles in the inner side of the outer rectangular section. Hence, two distinct focusing bands are formed in the inner side of the rectangular sections [i.e., state (ii) corresponding to $Q < 2.5$ ml/min, $Re < 117$].

FIG. 9. Inertial focusing in the ramp channel. (a) Experimental results of 6, 10, and 13 μm microbeads ($Q = 0.5\text{--}6$ ml/min, $Re = 23\text{--}281$) in the ramp channel as well as two rectangular and one trapezoidal microchannels whose cross-sectional shapes constitute the cross section of this channel. The heights of the rectangular channels are 80 and 130 μm . Also, the heights of the side walls of the additional trapezoidal channel are 80 and 130 μm . (b) Particle equilibrium positions in the cross section of the ramp microchannel. (i)–(vi) Schematics of the proposed force balances, moving from low to high flow rate, respectively.

The approach of particles to the M-M centerline at higher flow rates brings about their exit from regions I and III of the inner rectangular section [i.e., unstable state (iii₁)]. Therefore, particles which were focused near the inner wall migrate to the outer half and integrate with the focusing band in the inner side of the outer rectangular section [i.e., state (iii₂)]. Upon increasing the flow rate further (i.e., $Q > 2.5$ ml/min, $Re > 117$), the absolute value of δ increases and so particles experience stronger F_D (i.e., $|\delta_{iii2}| < |\delta_{iv}| < |\delta_v| < |\delta_{vi}|$). As a result, the local balance between F_{Ω} and F_D is achieved in closer locations to the outer wall.

According to the underlying physics of outward trapezoidal curved channels,⁶ the steeper the ramp section of the channel is, the smoother transition from the inner rectangular section to the outer rectangular section happens. Conversely, if the slope of the ramp section reduces, the inner section of the channel will tend to keep particles focused until relatively higher flow rates. Ultimately, particles in a ramp channel with a very gentle slope will gradually migrate from the inner half to the outer half, much like a rectangular curved channel while steeper slopes result in a more abrupt shift.

E. Generalizing the trends with an additive rule

The numerical and experimental results of the three complex microchannels presented above suggest that there is a clear relationship, termed the additive rule, which can explain the behavior of particles in curved channels with complex cross

sections. As the term implies, the behavior of particles in a curved microchannel—whose cross section is comprised of multiple simple, adjoining shapes—can be extrapolated from the particle behavior in each individual adjoining cross section. In addition, each of the adjoining sections has a focusing capability which enters a competition between the sections. Thus, at low flow rates, particles tend to focus in each distinct region, but at higher flow rates particles find their way to the dominant cross section. This generalized perspective allows for a straightforward but accurate prediction of inertial focusing in complex-shaped microchannels.

To take this a step further, we believe the additive rule will hold for even more complex geometries. To verify this, we also designed a complex spiral microchannel comprised of three outward trapezoidal component shapes [Fig. 10(a)]. The experimental results of 6, 10, and 13 μ m particles in this three-component jagged channel at $Q = 0.5$ –9 ml/min ($Re = 10$ –183) are shown in Fig. 10(b). As can be seen, there are two distinct particle bands near the sidewalls as well as another one in the middle of the jagged channel. The focusing trends of particles in this channel appear as if three outward sloping trapezoidal channels are competing to focus particles (see Sec. III of the [supplementary material](#)). These trends are very similar to the two-component designs shown above. That is, at low flow rates, particles tend to focus near the inner side of each trapezoidal component, which is similar to the tendency of particles in individual outward sloping trapezoidal curved channels.⁶ By increasing the flow rate, focused particles migrate toward the outer side of each component. This migration is followed by a gradual aggregation of particles

FIG. 10. Inertial focusing in a jagged channel. (a) The cross section of a curved jagged microchannel with three teeth. (b) Experimental results of the jagged channel using 6, 10, and 13 μ m particles at $Q = 0.5$ –9 ml/min ($Re = 10$ –183). (c) Schematics of Dean vortices formed in a concave and a jagged channel. In comparison, the cross section of the jagged channel shown in (A) has been filleted due to micromilling limitations. (d) Contours and vectors of the Dean velocity in section B-B of the jagged channel at $Q = 9$ ml/min ($Re = 183$).

FIG. 11. Examples of advanced designs of curved microchannels. (a) A proposed curved channel for focusing suspended particles near the inner wall at relatively high flow rates. (i)–(iii) The design steps, respectively. (b) A proposed curved channel for focusing particles at an arbitrary lateral location.

in the middle component, which is the dominant section of this complex channel. Since the proposed additive rule holds true for complex channels comprised of two components (i.e., the convex and concave channels) and three components (i.e., the ramp and jagged channels) with arbitrary shapes, by induction it should continue to apply as a general rule for all complex channels consisting of simple, adjoining cross sections.

1. Focusing competition

To understand the apparent “focusing competition” between the components of complex channels, the Dean flow patterns in the concave and jagged channels were compared. As can be seen in Fig. 10(c), two pairs of Dean vortices are formed in a concave channel while three pairs are created in a three-component jagged channel. Our simulation results indicate that the outer pair of vortices in the concave channel is stronger than the inner pair [Fig. 7(b)]. Also, Fig. 10(d) shows that in the jagged channel the middle pair is the strongest one. The experimental results of these channels elucidate that the component with the strongest flow pattern wins the focusing competition and, given a sufficient channel length, should eventually accumulate almost all of particles.

The observed focusing competition along with the additive rule motivates one to refine the confinement ratio criterion for complex geometries. We suggest that “local CR” values—where each component’s hydraulic diameter is considered—provide better metrics for particle focusing in each subsection of a complex shaped channel (particularly the dominant section(s) of the cross section). For more information on the confinement ratio criterion for complex curved channels, see Sec. IV of the [supplementary material](#).

2. General focusing tendencies in curved channels

Having compared and contrasted particle behavior in more than twenty spiral microchannels with different simple or complex cross sections,^{6,10,26–29,43,45–48} we have found some general inertial focusing tendencies, φ_i , that appear to hold for suspended particles in curved microchannels:

- φ_1 : Particles tend to focus near the inner wall at low flow rates—larger particles have more tendency (i.e., $\varphi_1 \propto a$).
- φ_2 : Particles tend to focus near the outer wall at high flow rates—this tendency is more in channels with larger heights (i.e., $\varphi_2 \propto H$).
- φ_3 : Particles tend to focus in positions where the channel height is larger—this tendency itself is proportional to the channel height (i.e., $\varphi_3 \propto H$), and, smaller particles have more of this tendency (i.e., $\varphi_3 \propto a^{-1}$).

F. Implications

The general tendencies extracted from the analysis above can help outline a simple, but practical, set of guidelines toward the design of curved microchannels. Along with the overarching additive rule, it can be deduced that there is a competition between these tendencies in curved channels (i.e., $\Phi = \sum \varphi$) which determines the final focusing position at each flow rate and for each particle size. Here are some characteristic examples of how this competition can be put to beneficial use:

Example 1—Particle focusing near the inner wall at high flow rates:

In this example, φ_2 counteracts and needs to be dampened with the aid of φ_3 . Therefore, instead of a simple rectangular channel [Fig. 11(a-i)], an inward trapezoidal channel can meet this requirement [Fig. 11(a-ii)]; however, since $\varphi_3 \propto H$, it is advisable to reduce the channel height more dramatically at locations closer to the outer wall [see Fig. 11(a-iii)].

Example 2—Particle focusing in an arbitrary lateral location for a wide range of flow rates:

To achieve such a requirement, a convex channel with a modified middle-vertex location, x , can be employed [Fig. 11(b)]. The inner geometric element of such a cross section quickly drives particles toward the x location because $\varphi_3 \propto H$. Sharper slopes help φ_3 beat φ_1 easier. The outer element also pushes particles toward the x location. However, regarding φ_2 , it is better to design the x parameter a bit shorter than the required focusing location, especially if high flow rates are required.

V. CONCLUSIONS

In this work, we contributed both practical and theoretical knowledge to the field of inertial microfluidics via a detailed investigation of complex shaped spiral microchannels. The experimental and numerical results of this study show that the pattern of the secondary flow can be controllably broken into multiple pairs of Dean vortices in the cross section of a curved channel which may enable focusing bands to be engineered at arbitrary lateral locations of the channel. We also proposed an additive rule that can be applied to curved channels whose cross-sectional shapes are composed of adjoining simple components. As a simple, yet consequential concept, this additive rule leads to new insights into the focusing tendencies of suspended particles in curved microchannels (e.g., simple or complex cross-sectional shapes). We believe these insights should enable designers to easily estimate how suspended particles will behave in the next generation designs of curved

microchannels. We also believe that the findings of this paper offer an alternative control mechanism for inertial microfluids (e.g., an alternative to approaches with simple external geometries but complex inner workings induced by sheath flow, obstructions, or external forces). We hope that this will open the door for more complex external geometries with bespoke passive internal flow.

VI. METHODS

A. Design and microfabrication

Each of the three devices was first designed in SolidWorks®, then micromilled (Whits Technologies, Singapore, and BioMEMS Supply, Sydney, Australia), and then cast in polydimethylsiloxane polymer (PDMS, Sylgard 184 Silicone Elastomer Kit, Dow Corning) using soft lithography. To ensure consistency—and transparency for observation—the PDMS devices were all mixed at a ratio of 10:1 (base:curing agent), degassed, and then prebaked in their aluminum molds for 2 h inside an oven at 65 °C. The designs were then peeled from the molds, 0.5 mm diameter fluidic access holes were punched into them using a Uni-Core™ puncher (Sigma-Aldrich Co. LLC, SG), and the devices were capped via a chemical bond with a thick, flat PDMS slab using an oxygen plasma unit (PDC-002, Harrick Plasma, Ossining, NY). Last, the sealed devices were baked again for 2 h at 65 °C to improve the bonding quality.

B. Bead suspensions

Three working fluids at 0.01% volume fraction of fluorescent microbeads (from Fluoresbrite® Microspheres, Polysciences Inc., Singapore) of 6 μm, 10 μm, and 13 μm were created. This resulted in test solutions with the concentration of 2.10×10^4 particles/ml. Each suspension was stabilized with phosphate buffered saline (PBS), 2 mM of EDTA supplemented, and 0.5% bovine serum albumin (BSA) (MACS Buffer, Miltenyi Biotec, Germany). These agents were used because they prevent particle agglomeration and adhesion with the walls and tubing.

C. Experimental setup

To record the particle focusing behavior, a programmable syringe pump (Chemyx Fusion 200, Chemyx Inc., USA) was used to produce various flow rates (0.5–9 ml/min, at 0.5 ml/min increments) inside the devices, which were viewed by an inverted phase contrast/epifluorescence microscope (Olympus IX73 microscope, Olympus Inc., USA) equipped with a DP80 dual chip CCD camera (Olympus Inc., USA). The camera was set at a constant 50 ms exposure time and an ISO setting in the range of 800–1600, which ensures a relatively fast shutter speed. The syringe pump was programmed to keep the initial flow rate (i.e., $Q = 0.5$ ml/min) for 25 s and then automatically increase the flow rate every 5 s with the step size of 0.5 ml/min. Also, TechSmith® Camtasia Studio® was used to record the screen. We used our previously developed “Experiment Info Displayer” software to synchronize the screen recording with the syringe pump.⁶ Lastly, screen recordings were rendered as uncompressed Audio Video Interleave (AVI) files at 30 frames per second. For more information on experimental data acquisition, see Sec. II of the [supplementary material](#).

D. Numerical simulations

Numerical simulations of these devices—for all conditions mentioned in the experimental setup section—were carried out using ANSYS-FLUENT® 17.0 in the high-performance computing facilities at UNSW.⁴⁹ The spiral microchannels were assumed to have incompressible laminar flow with constant (albeit variable) inlet velocities and a constant outlet pressure (i.e., 1 atm). The no-slip boundary condition was imposed at all walls of the device and constant thermophysical properties were assumed (e.g., 20 °C liquid water with a density of 998.2 kg/m³ and a viscosity of 1.003×10^{-3} kg/m s). The SIMPLE scheme was used as the pressure-based segregated algorithm for steady-state calculations with the pressure-velocity coupling method. A mesh independence study was conducted, which revealed that fully hexagonal grids with the minimum height of 1.21 μm and maximum height of 3.95 μm were sufficient to assure accurate numerical results. We used our previously developed “Post-Processing Assistant” software to create Tecplot® macro files dynamically and generate proper graphs automatically.⁶

SUPPLEMENTARY MATERIAL

See the [supplementary material](#) for information about geometric parameters of the microchannels, experimental data acquisition, the comparison of inertial focusing in a jagged channel and an outward sloping trapezoidal channel, and the confinement ratio criterion for complex curved channels.

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