Influence of vocal fold cover layer thickness on its vibratory dynamics during voice production

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The influence of vocal fold cover layer thickness on the flow-induced vibration and voice production was studied by using a continuum-mechanics based computational model. The cover-body thickness ratio of a three-layer vocal fold was systematically varied. The effect on the vocal fold stiffness, eigenfrequencies and eigenmodes, fundamental frequencies, glottal flow rate, vocal fold vibratory dynamics, and synchronization of the eigenmodes were analyzed by using the structure eigen analysis and flow-structure interaction simulations. It was found that the cover-body layer thickness ratio significantly affected the strength and synchronization of the eigenmodes during flow-structure interactions, and ultimately affected the fundamental frequency and vibration pattern. With the increasing cover-body thickness ratio, the strength of the wave-type higher-eigenfrequency modes increased, and that resulted in a nonlinear bifurcation of the system in which the system evolved from a regular periodic vibration to a periodic doubling vibration and then back to a regular periodic vibration with increased fundamental frequencies. During the transition, the system vibrated chaotically. Because of the increased strength of the wave-type modes, the maximum divergent angle of the glottis was also increased with the increasing cover-body thickness ratio. © 2019 Acoustical Society of America.

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I. INTRODUCTION

The human vocal fold is a layered structure consisting of the epithelium, the lamina propria and the thyroarytenoid muscle.^{1,2} As shown in Fig. 1(a), the epithelium is a very thin layer (0.05–0.08 mm) that is made up of stratified squamous cells. The lamina propria is non-muscular tissue that is usually divided into three layers according to the density of fibrous components.¹ The superficial layer of the lamina propria consists of mainly loosely organized elastin fibers surrounded by interstitial fluids. In the intermediate layer, the elastin fibers are uniformly oriented in the anterior-posterior direction. The deep layer of the lamina propria mainly consists of collagen fibers aligned in the anterior-posterior direction. Below the deep layer of the lamina propria is the thyroarytenoid muscle which is organized in bundles making it very stiff. These five histological layers are usually grouped into two or three mechanical layers based on their elastic properties.^{1,2} In the two-layer model,^{3–8} the cover layer consists of the epithelium, the superficial and intermediate layers of the lamina propria, and the body layer consists of the deep layer of the lamina propria and the muscle. In the three-layer model,^{9–13} the cover layer consists of the epithelium and the superficial layer of the lamina propria, the ligament layer consists of the intermediate and deep layers of the lamina propria, and the body layer consists of the muscle. Vocal fold vibration is the direct result of the interaction between glottal flow and elasticity of vocal fold layers. Therefore, the two-layer model^{3–8} and three-layer model^{9–13} have been widely used to study the effect of the structure and elasticity of inner layers on vocal fold vibrations.

During phonation, the vocal fold is displaced in a wave-like fashion, creating the so-called mucosal wave. The mucosal wave modulates the airflow through the glottis, generating air pulses which are the primary sound source. The characteristics of the mucosal wave have a direct relationship with sound quality. In clinical practice, the mucosal wave assessment is essential for diagnosing and treating voice disorders. Due to the structure property relationship between vocal fold layers (larger inextensibility into deep layers²), the mucosal wave travels along the superficial layer of the vocal folds.¹⁴ In the three-layer model, the cover layer groups the epithelium and the superficial layer of the lamina propria. Therefore, the mucosal wave travelling is closely related to the cover layer. The present study focuses on the effect of the cover layer thickness (referred to as the depth in the medial-lateral direction) on vocal fold vibration. The cover layer thickness varies with vocal fold conditions. For instance, it increases in aging vocal folds of females.¹⁵ It is affected by voice warm-up and hydration.¹⁶ It can also be altered by vocal pathologies, such as polyps and nodules, and surgical treatments, such as dissection for treating cancers. Understanding how the cover layer thickness affects the vocal fold vibration and voice production will provide valuable insights into the diagnosis and treatment of related voice disorders.

Due to the difficulties of the measurement and control of vocal fold inner layers, past studies on the effect of vocal fold inner layer properties on voice production mostly relied on the theoretical, computational, and physical vocal fold models.^{17–21} A number of studies have reported the effect of cover layer properties, including the thickness, damping, geometry, and elastic modulus, on the threshold pressure, onset frequency, and glottal closure.^{3,22,23} For the cover

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FIG. 1. (Color online) (a) Schematic of the vocal fold tissue in a coronal section. E: epithelium; SLP: superficial layer of the lamina propria; ILP: intermediate layer of the lamina propria; DLP: deep layer of the lamina propria; M: muscle layer. (b) The computational domain and geometry of the vocal folds, larynx, and vocal tract. (c) The three-layer structure of the vocal fold and the boundary conditions. (d) A mid-coronal plane profile of the layered vocal fold showing the definition of the layer thicknesses (T_c , T_l , T_b) in the current study. They add up to 0.875 cm in all the cases. (e) The inner layer structures of the vocal fold in the selective cases in the parametric study.

thickness, it is generally believed, based on the string formula, that a thicker cover layer results in a lower fundamental frequency.^{17,24} By using a theoretical surface wave model and a physical model of the vocal fold, Titze and his colleagues^{25,26} found a positive dependence of the phonation threshold pressure on the thickness of the cover layer. With a lumped mass model, Gunter¹⁶ showed that the mechanical stress during vocal fold collision was negatively correlated with the cover layer thickness. Erath *et al.*²⁷ obtained the same conclusion by using a quasi, 1-D viscoelastic contact model. Using eigen analysis of a continuum vocal fold model, Cook *et al.*²⁸ found that the eigenfrequencies were more influenced by layer elastic modulus rather than thickness. In another study on the eigenmodes of vocal fold, Xue *et al.*¹³ showed that the eigenmodes were not sensitive to the inner layer thickness either.

Despite these previous studies, the effect of the cover layer thickness on flow-induced vocal fold vibration has not been systematically studied. As the cover layer is closely associated with the propagation of the mucosal wave, we hypothesize that increasing the cover layer thickness would promote the formation and propagation of the mucosal wave, and so promote the excitation of the wave-type eigenmodes of the vocal fold. Such an effect would

further affect the synchronization of the eigenmodes during flow-structure interactions, and finally change the vocal fold vibration pattern, fundamental frequency, frequency spectrum, and voice outcomes. This study aimed to test this hypothesis by using a three-dimensional, continuum mechanics based flow-structure interaction computer model of voice production. The cover-body thickness ratio was systematically varied in a wide physiologically-possible range. The effect of these variations on vocal fold stiffness, eigenfrequencies and modes, fundamental frequency, glottal flow rate, vocal fold vibratory dynamics, and the synchronization of the eigenmodes were analyzed in detail.

II. COMPUTATIONAL METHODOLOGY AND SIMULATION SETUP

The computer model couples a finite element method based continuum vocal fold model and an immersed-boundary method based Navier-Stokes flow model.²⁹ The model has been extensively used in our previous studies^{12,30–32} and the details of the methodology have been published in Mittal *et al.*³³ and Zheng *et al.*³⁴ In this paper, only the key information is reported. The governing equation of the dynamics of the tissues is the Navier equation

$$\rho_{tiss} \frac{\partial^2 d_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho_{tiss} f_i, \quad (1)$$

where ρ_{tiss} is the tissue density, d is the displacement, σ is the stress tensor, and f is the body force. Equation (1) was discretized using the Newmark scheme with a second-order accuracy.

The Kelvin-Voigt model³⁵ was employed for modeling the viscoelastic material,

$$\sigma = E\varepsilon + \eta\dot{\varepsilon}, \quad (2)$$

where σ is the stress tensor, ε is the strain tensor, $\dot{\varepsilon}$ is the strain rate tensor, and E and η are the material constant tensors corresponding to elasticity and viscosity.

The dynamics of the glottal air flow was governed by the three-dimensional unsteady viscous incompressible Navier-Stokes equations

$$\frac{\partial u_i}{\partial x_i} = 0, \quad (3)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}, \quad (4)$$

where u is the velocity, p is pressure, and ρ and ν are flow density and kinematic viscosity. The equations were discretized with the second-order non-dissipative central difference scheme and integrated in time using the second-order fractional step scheme. A sharp-interface immersed-boundary method was employed to treat the moving boundary conditions.

The coupling between the flow and solid solvers was implemented by tracking the aerodynamic loading on the interface mesh as well as its deformed shape and velocity in a Lagrangian fashion. A simple hard-wall contact model was incorporated to model the contact of the vocal folds.^{30,31}

Figure 1(b) shows the geometric models of the vocal folds and airway. The geometry of the vocal fold was based on the model proposed by Titze and Talkin.¹⁰ Although it is a simplified geometry, it reflects the key physiological features of the human vocal fold, including the size, shape of medial surface, anterior-posterior variation, and subglottal angle. The model has been extensively used in the past.^{36,37} The supra-glottal airway was modeled by a curved circular tube with varying cross-section areas. The variation of the cross-section area was based on a neutral vowel model proposed by Story³⁸ and the center-line of the tube was based on the data reported in Story *et al.*³⁹ which was reconstructed from *in vivo* MRI scans. The length of the supraglottal tract was 17.4 cm. The shape of the ventricle and ventricular folds was reconstructed from the CT scan of the larynx of a 30-year-old male subject, which has been used in our previous studies.⁴⁰ Figure 1(c) shows the inner layer structure of the vocal fold, which followed the three-layer scheme proposed by Hirano *et al.*,⁴¹ including the cover, ligament and muscle from the outermost to the innermost. Figure 1(d) shows the mid-coronal plane profile of the layered vocal fold. The cover, ligament, and body layer thicknesses are denoted as T_c , T_l , and T_b , respectively. They add up to 0.875 cm in all the cases. The thicknesses of the cover and ligament layers were

kept the same from the inferior to superior throughout the vocal fold. In the baseline case, their values were based on the mean values of the measurement of the human vocal fold samples⁴¹ where the cover layer thickness was 0.33 mm (denoted by T_{co}) and the ligament layer thickness was 1.11 mm.

For the parametric study, seven different values of the cover thickness were considered, including $T_c = 0.5 T_{co}$, T_{co} , $1.5 T_{co}$, $2 T_{co}$, $2.5 T_{co}$, $3 T_{co}$, and $5 T_{co}$. In these cases, the ligament layer thickness was unchanged. The body layer thickness [T_b in Fig. 1(d)] was adjusted accordingly to keep the total volume of the vocal fold unchanged. Hence these seven cases correspond to the cover-body thickness ratio (T_c/T_b) of 0.02, 0.05 (baseline), 0.07, 0.09, 0.12, 0.15, and 0.28, respectively. In the rest of the paper, the T_c/T_b value is used to represent the cases. Figure 1(e) shows the inner layer profiles at the mid-coronal plane of the selective models. It should be pointed out that $T_c/T_b = 0.28$ ($5 T_{co}$) might be an exaggerated value from a physiological point of view. However, the fact that a periodic doubling bifurcation occurred when $T_c/T_b = 0.15$ ($3 T_{co}$), which would be discussed later, indicated that the system was during a transition between two steady states and one of the steady states occurred at $T_c/T_b = 0.28$. That is the reason we included this case in the study. Although this case may be exaggerated physiologically, it provided a complete picture of the underlying mechanism driving the nonlinear dynamic transition of the system. Such knowledge would be very useful to understand the nonlinear dynamics observed in real vocal systems, such as those reported in Berry *et al.*,⁴² Tokuda *et al.*,⁴³ and Zañartu *et al.*⁴⁴

The three layers of the vocal fold tissues were modeled as transversely isotropic viscoelastic materials. The transverse Young's moduli (E_p) of the cover, ligament, and body layers were 2.01, 3.31, and 3.99 kPa, respectively. The transverse and longitudinal Poisson's ratio for all the layers are 0.9 and 0.0, respectively.⁴⁵ These values were adopted based on the model in Alipour *et al.*⁴⁵ in which a summary of the measurement of the elastic prosperities of vocal fold tissues was done for a proper choice of model parameters. The longitudinal Young's moduli of the cover, ligament, and body layers were 40, 66, and 80 kPa, which were 20 times the values of the transverse plane. The high values were chosen to constrain the displacement in the longitudinal direction according to Cook *et al.*⁶ The values of the viscosity of the three layers were 5, 7.5, and 12.7 Poise, respectively.

The boundary conditions on the vocal fold are shown in Fig. 1(c). A zero-displacement boundary condition was applied at the anterior, posterior and lateral surfaces while the rest were free to move. The aerodynamic loading was applied on the vocal fold surfaces in contact with the airflow, as shown by the shaded parts in Fig. 1(c).

The density and viscosity of the air were 1.1455 kg/m³ and 1.65×10^{-5} m²/s, respectively, corresponding to the human body temperature. Gauge pressures of 0 and 1.0 kPa were applied at the exit and inlet, respectively. Non-slip and non-penetration boundary conditions were applied at the airway walls.

The airway and vocal fold models were immersed into a $2.4 \times 10.8 \times 14.5$ cm rectangular computational domain [Fig. 1(b)]. The domain was discretized by a $64 \times 256 \times 192$

nonuniform Cartesian mesh with the highest mesh density around the vocal folds. Such grid density was based on our previous simulation experience with similar configurations^{12,31,32} which would generate grid-independent solution. Each vocal fold was discretized by 28 997 tetrahedral elements. A small time step of $1\ \mu\text{s}$ was employed for both the flow and the solid solvers.

III. RESULTS AND DISCUSSIONS

A. Effect on vocal fold stiffness

The cover layer has the lowest transverse Young's modulus among the three layers.^{4,10,46} A thickened cover layer will result in a reduced stiffness of the vocal fold, and vice versa. It should be noted that, in this study, a distinction was made between the elastic property and stiffness. The elastic property is the material property just related with the material itself while the stiffness measures the local resistance under deformations. The stiffness is influenced by the elastic property as well as the geometric dimensions, loading direction, and type of constraint. A numerical indentation technique was applied to measure the stiffness on the vocal fold surface. It was performed by applying a uniform pressure on a small area on the vocal fold surface and then numerically solving the static deformation using the finite element method. Let P be the pressure load, A be the loading area, and Δx be the maximum deformation, the stiffness k was computed as $k=PA/\Delta x$. The details of this method are referred to Geng *et al.*⁴⁷ In the current study, $P=4\text{ kPa}$ and $A=1\text{ mm}^2$. The constant pressure loading was applied to create a loading condition comparable to that during flow-structure interactions. The stiffness was measured at three locations at the mid-coronal plane of the vocal fold, including the superior and inferior side of the medial surface and a location on the subglottal surface, as shown in Fig. 2(a). Ideally, if the tissue surface is significantly larger than the size of the indentation (for example, a half infinite-surface), the stiffness only depends on the material properties, and if the material properties are uniform, it does not change along the surface. However, for the vocal fold model, its size is comparable to the indentation size, therefore, its local stiffness is significantly affected by the geometries. The indentation depth varied with the location and the model. The smallest indentation depth was 0.26 mm and occurred at the

inferior surface of the vocal fold model with the thinnest cover layer ($T_c/T_b=0.02$), while the largest indentation depth was 0.72 mm and occurred at the superior edge of the medial surface of the vocal fold model with the thickest cover layer ($T_c/T_b=0.28$).

Figure 2(b) shows the stiffness values at the three locations and their variations with the cover-body layer thickness ratio. When the cover-body layer thickness ratio increased from 0.02 to 0.28, the stiffness decreased 31.2% at the superior side of the medial surface, 52.1% at the inferior side of the medial surface, and 46.1% at the inferior surface. The decreasing rate of the stiffness decreased with the increasing cover-body thickness ratio, indicating a reduced sensitivity of the stiffness to an increased cover-body thickness ratio. It was also observed that the stiffness increased from the superior to the inferior. It was caused by two factors. The first is related to the asymmetric geometry of the vocal fold from inferior to superior and that the extra subglottal part provided more resistance to the loading according to Geng *et al.*⁴⁷ The second is related to the fact that some parts of the superior measurement area only had the resistance from the cover layer below it while all the inferior measurement area had the resistance from the cover, ligament, and body layers.

B. Effect on eigenmodes and eigenfrequencies

Since the ultimate vibration pattern of the vocal fold is determined by the shape of the excited eigenmodes, especially the low order modes, the effect of the cover-body layer thickness ratio on the eigenmodes and eigenfrequencies was evaluated. Figure 3(a) displays the profile of the mid-coronal plane of the vocal fold at the two extreme positions of the five lowest-eigenfrequency modes of the baseline model. The dashed line represents the original shape and the solid lines represent the deformed shapes. The modes are ordered in a way that the eigenfrequency increases with the mode number. Modes 1 and 2 primarily represented a vertical motion and a medial-lateral motion of the vocal fold, respectively. Starting from mode 3, the wave motions were observed on the vocal fold surface. From mode 3 to mode 5, the wavelength gradually decreased with the mode order. Moreover, in mode 3, the wave was primarily on the superior surface of the vocal fold, while, in modes 4 and 5, the waves were on the entire vocal

FIG. 2. (Color online) (a) The three indentation locations in the superior and inferior sides of the medial surface and the inferior surface of the vocal fold. All of them were in the mid-coronal plane with the loading $P=4\text{ kPa}$ and loading area $A=1\text{ mm}^2$. (b) The stiffness variation with cover-body layer thickness ratio in the three indentation locations.

FIG. 3. The profile of the mid-coronal plane of the vocal fold of eigenmodes 1–5 in (a) the baseline case; (b) $T_c/T_b = 0.28$ case. The solid lines represent the two extreme deformed shapes and the dashed line represents the original shape.

fold surface. A close look at the wave patterns in modes 4 and 5 revealed that these two modes generated an out-of-phase motion between the superior and inferior edges of the medial surface of the vocal fold, which was essential for the energy transfer from the airflow to the vocal fold.

For the cases where the cover-body thickness ratio was between 0.02 and 0.15, the eigenmodes were very similar to those in the baseline case. In the case $T_c/T_b = 0.28$, the eigenmodes were different. Figure 3(b) shows the shape of the five lowest-eigenfrequency modes for the $T_c/T_b = 0.28$ case. A close comparison of these modes to those in the baseline case revealed that modes 1, 2, and 5 were highly similar to the corresponding modes in the baseline case, while modes 3 and 4 switched the modal order. Specifically, modes 3 and 4 in this case were highly similar to modes 4 and 3 in the baseline case, respectively. To precisely quantify the similarity between the modes, the dot-product between the eigenmodes of the two cases were computed.¹³ The method is described below. The j th mode was written into a single vector $\bar{q}_j = (u_{j1}, v_{j1}, w_{j1} \dots u_{jN}, v_{jN}, w_{jN})$, where u_{jm} , v_{jm} , and w_{jm} represent the displacements of the m th grid point in the x , y , and z directions, respectively. This vector is normalized with its own magnitude leading to a normalized vector

$\bar{Q}_j = \bar{q}_j / |\bar{q}_j|$, where $|\bar{q}_j| = \sqrt{\sum_{i=1}^N (u_{ji}^2 + v_{ji}^2 + w_{ji}^2)}$ and N is the total number of grid points. The dot-product of any two normalized modes is indicative of the similarity between the two modes with the value of one corresponding to an exact match, and zero indicating orthogonality. Figure 4 shows the dot-products between each mode in the $T_c/T_b = 0.28$ case and each mode in the baseline case. It is clear that modes 1, 2, and 5 in the $T_c/T_b = 0.28$ case were highly similar to the corresponding modes in the baseline case with the dot-products ranging between 0.79 and 0.94. Modes 3 and 4 in the $T_c/T_b = 0.28$ case were highly similar to modes 4 and 3 in the baseline case with the dot-product value of 0.72 and 0.78, respectively. Since mode 4 was the lowest-eigenfrequency mode in the baseline case presenting the elastic wave on the entire vocal fold surface, its moving-up in the modal order in the $T_c/T_b = 0.28$ case implied that a very thick cover would promote the formation of the elastic waves on the vocal fold surface.

Figure 5 shows the value of eigenfrequencies of modes 1–5 in each case and their variations with the change of the cover-body thickness ratio. The eigenfrequencies decreased with the increasing cover-body thickness ratio. This was due to the reduced vocal fold stiffness.

FIG. 4. The similarity of the eigenmodes 1–5 between the $T_c/T_b = 0.28$ case and baseline case. The i th bar in the category of mode j represents the dot product between the j th mode in the baseline case and the i th mode in the $T_c/T_b = 0.28$ case.

FIG. 5. (Color online) The eigenfrequencies of modes 1–5 and the fundamental frequency (F0) in each case.

C. Effect on glottal flow rate and spectrum

Figure 6 shows the time variation of the volumetric flow rate through the glottis in each case. Interestingly, it was seen that when the cover-body thickness ratio was low ($T_c/T_b = 0.02$ – 0.07), the flow rate reached the periodic, steady cycles very quickly. When the cover-body thickness ratio was increased to $T_c/T_b = 0.09$, the waveform of the flow rate became irregular and non-periodic with large variations in the frequencies and peak values from cycle to cycle. The irregularity continued in the $T_c/T_b = 0.12$ case. When the thickness ratio was increased to $T_c/T_b = 0.15$, the flow rate showed a periodic waveform again, but with alternating low and high peaks, indicating a period-doubling behavior characterized by two alternating peaks in each cycle. Keeping increasing the thickness ratio to $T_c/T_b = 0.28$, the system was back to the regular and periodic vibration again, but

FIG. 6. The time history of the volumetric glottal flow rate in each case.

with a much longer transition time to reach the steady cycles compared to the baseline case.

The changes of the flow waveforms clearly indicated a nonlinear bifurcation of the system associated with the changes of the cover-body thickness ratio. Period-doubling is often associated with the appearance of a subharmonic in the frequency spectrum. To examine this behavior, Fig. 7 shows the spectrum of the flow rate in each case. The spectrum was obtained through performing fast Fourier transform in all the cases with the time period of about 0.05 s, indicating a frequency resolution of about 19 Hz. The values of the fundamental frequency and signal-to-noise ratio (SNR) are also shown in each sub-figure. The spectrum and SNR were calculated based on the data of the steady cycles or late cycles if there was no steady state. In the cases where the cover-body thickness ratio was between 0.02 and 0.07, the spectrum showed a typical pattern of limit-cycle oscillations, dominated by the fundamental frequency and its superharmonics. The SNR decreased with the increasing cover-body thickness ratio. In the $T_c/T_b = 0.09$ case, the spectrum became noisy. The SNR decreased by about 53% from the baseline case. The spectrum showed two nearly equal peaks at 165 and 201 Hz with the peak at 201 Hz slightly higher than the peak at 165 Hz. The superharmonics of the peaks could no longer be distinguished. This case indicated the onset of the bifurcation. In the $T_c/T_b = 0.12$ case, the spectrum was still noisy. The SNR further decreased to just about 12% of that in the baseline case. In the $T_c/T_b = 0.15$ case, the spectrum clearly showed a $\frac{1}{2}$ subharmonic frequency (frequency below the fundamental frequency in a ratio of 1/2) and it was dominated by the fundamental frequency, the subharmonic and the linear combination of the two. This case indicated the complete of the period-doubling bifurcation. The system was stable again at this new state and the SNR increased to 42% of that in the baseline case. Further increasing the cover-body thickness ratio to 0.28 created another bifurcation that the system returned to the periodic, limit-cycle oscillation with the spectrum dominated by the fundamental frequency and its superharmonics again and the SNR as high as 96% of that in the baseline case.

D. Effect on the fundamental frequency and mode synchronization

The fundamental frequency of each case is superimposed in Fig. 5 using the diamond symbols. It was interesting that, while the fundamental frequency decreased when T_c/T_b was increased from 0.05 to 0.07 and from 0.09 to 0.28, which was as expected since the stiffness of the vocal fold was reduced, it increased when the thickness ratio was increased from 0.07 to 0.09. This singular behavior violates the general understanding that the fundamental frequency decreases with the decreasing stiffness of the vocal fold based on the string formula. By noticing that the case $T_c/T_b = 0.09$ was the onset of the period-doubling bifurcation as observed in Fig. 7, we hypothesized that the period-doubling bifurcation and the jump of the fundamental frequency were related and their occurrence were due to the increased strength of the higher-frequency modes in the modal

FIG. 7. The spectrum of the flow rate waveform in each case. The fundamental frequency (F_0) and signal-to-noise ratio (SNR) of each case are denoted in the subfigures. The emergence of the subharmonic is highlighted in the $T_c/T_b = 0.15$ case.

synchronization process that broke the initial modal synchronization and attracted the modes to synchronize at higher frequencies.

To test the hypothesis, we computed the normalized modal coefficient of each eigenmode in each case. The coefficient reflected the contribution of each mode to the unit length of the displacement vector of the vocal fold vibration. The approach is described below. The vocal fold vibration was assumed to be a linear superposition of the eigenmodes as shown in Eq. (5),

$$V(t) = c_1(t)V_1 + c_2(t)V_2 + \cdots + c_n(t)V_n, \quad (5)$$

where $V(t)$ is the vocal fold displacement vector at the time instant of t , V_i ($i = 1 \sim n$) is the i th eigenmode, and $c_i(t)$ is the corresponding eigen coefficient that is a function of time as it is different at different time steps. Based on this assumption, the vocal fold displacement vector was transferred from the general three-dimensional physical coordinates system to a modal coordinates system. The modal coefficients were the amplitudes of each modal component. It needs to be pointed out the equilibrium position of the

vocal fold during vibration was shifted from the initial rest position due to the subglottal pressure. Therefore, the displacement vector, V , was computed based on the equilibrium position which was the average of the coordinates of the vocal fold nodes throughout the steady cycles. Based on the orthogonality of the eigenmodes ($V_j^T[M]V_i = \delta_{ij}$), the modal coefficients were computed as

$$c_i(t) = V(t)^T[M]V_i, \quad (6)$$

where $[M]$ is the global mass matrix. The modal coefficients were further normalized by the magnitude of the displacement vector as $c'_i(t) = c_i(t) / \sqrt{V(t)^T \cdot V(t)}$. The modal coefficients represented the amplitudes of the vibration of each mode per unit length of the displacement vector.

Figure 8 shows the root-mean-square (RMS) values of $c'_i(t)$ of the five lowest-eigenfrequency modes in each case. It was found that when T_c/T_b was between 0.02 and 0.07, the amplitudes of modes 1, 2, and 3 were much higher than those of modes 4, 5. According to Fig. 5, the fundamental frequencies of these cases were close to the eigen frequencies of modes 2 and 3. It was further found that in these cases, the

FIG. 8. (Color online) The root-mean-square (RMS) values of c'_i of eigenmodes 1–5 in each case. c'_i stands for the amplitude of the vibration of each mode per unit length of the displacement vector.

amplitudes of modes 2 and 4 decreased with the increase of the cover-body thickness ratio while that of mode 3 increased. However, when the thickness ratio was increased from 0.02 to 0.15, the amplitudes of modes 4 and 5 quickly increased with the amplitude of mode 3 dropping quickly and the amplitudes of modes 1 and 2 remaining nearly unchanged. This behavior confirmed our hypothesis that the jump of the fundamental frequency and the chaotic vibration at $T_c/T_b = 0.09$ were associated with the increase of the strength of the higher-frequency modes. With the continuous increase of the cover-body thickness ratio, the amplitudes of modes 4 and 5 continued to increase and the amplitude of mode 3 continued to drop. At $T_c/T_b = 0.15$, they had nearly the same amplitudes, indicating a comparable strength among them, which may be the reason for the periodic doubling vibration which has two energy peaks (fundamental and its subharmonics). With the further increase of the cover-body thickness ratio, at $T_c/T_b = 0.28$, although the amplitudes of mode 4 only increased slightly, the amplitude of mode 3 dropped significantly; therefore, the relative strength of modes 4 and 5 was still increased. This has led the system back to a new regular periodic vibration state but with the fundamental frequency near the eigenfrequency of the higher-frequency modes. As mode 2 represents a medial-lateral motion involving all the layers and modes 4 and 5 represent wave motions on the vocal fold surface, the energy decrease of mode 2 when T_c/T_b increased from 0.02 to 0.07 and the energy increase of modes 4 and 5 when T_c/T_b increased from 0.07 to 0.28 suggest that increasing the cover-body thickness ratio would make the vibration more concentrated on the superficial layer of the vocal fold and generate higher frequencies. It may have implications to modal-falsetto transition as high-frequency falsetto is mostly with cover and ligament layer vibrating and low-frequency modal register also involves the body layer. The results imply that the vocal fold vibration in falsetto might be dominated by the higher-order wave-type of eigen modes and increasing the cover-body thickness ratio would be easier to generate falsetto.

Figure 9 shows the frequency spectrum of the modal coefficients of the five lowest-eigenfrequency modes in each case. The frequency resolution of all the cases is about 19 Hz which is the same with the ones in Fig. 7. It will allow us to examine the vibration frequency of each eigenmode so that the modal synchronization can be analyzed. The five lowest eigenfrequencies were also indicated by the vertical lines in each subfigure to show the relationship between the vibration frequencies and the eigenfrequencies. The spectra clearly show that, for the cases where the cover-body thickness ratio was between 0.02 and 0.07, all the modes were vibrating at the same frequencies. When T_c/T_b was increased to 0.09, the modes started to show different spectrum patterns, indicating that the modal synchronization was broken. It was noticed that, for all the modes, there were two dominant peaks, one close to the eigenfrequency of mode 3 and the other close to the eigenfrequency of mode 5, indicating that the higher-frequency modes (modes 4 and 5) started to act as another basin of attraction for the system. This was due to the increased strength of the two modes, as seen in Fig. 8. At $T_c/T_b = 0.12$, the system showed the similar two-peak behavior except that the two peaks were close to the eigenfrequency of modes 1 and 5, respectively. When $T_c/T_b = 0.15$, the two-peak behavior vanished and all the modes only showed one peak which was close to the eigenfrequency of mode 5. Moreover, modes 2–5 all have energy concentration at the subharmonic frequency (98 Hz). With the further increase of the energy portion of modes 4 and 5, at $T_c/T_b = 0.28$, all the modes were entrained to the same frequency close to the eigenfrequency of mode 4.

In summary, the increased cover-body thickness ratio has promoted the excitation of the wave-type eigenmodes of the vocal fold (modes 4 and 5), which gradually became a stronger basin of attraction for the system, driving the system to a new periodic doubling vibration state and then back to the regular periodic vibration with higher fundamental frequencies. During the transition between the two steady states, the modes were vibrating at different vibration frequencies, resulting in complex chaotic and period-doubling vibrations.

E. Effect on vocal fold vibration pattern

Figure 10 shows the phase-plane plot of the lateral displacements of the points at the superior and inferior sides of the medial surface in the mid-coronal plane of the vocal fold in each case. The positions of the two points are denoted in Fig. 2(a). It clearly shows that when T_c/T_b was between 0.02 and 0.07, the systems underwent the limit-cycle vibrations. At $T_c/T_b = 0.09$ and 0.12, the systems underwent the chaotic vibrations with both the vibration amplitude and pattern varying from cycle to cycle. At $T_c/T_b = 0.15$, the system showed the period-doubling vibration with the alternative larger and smaller vibration amplitudes. At $T_c/T_b = 0.28$, the limit-cycle vibration resumed. Throughout all the cases, the vibration amplitude at both points increased with the increasing cover-body thickness ratio in accordance with the decreased stiffness on the vocal fold surface. Moreover, a 45° dash line was superimposed in the figures to examine the

FIG. 9. (Color online) The spectrum of each modal coefficient (c_i) of eigenmodes 1–5 in each case. The vertical lines in each subfigure indicate the eigenfrequencies of eigenmodes 1–5 in the corresponding case. The frequency of the subharmonic (98 Hz) is denoted in the $T_c/T_b = 0.15$ case.

FIG. 10. The phase-plane plot of the lateral displacement of the points at the superior and inferior sides of the mid-coronal medial surface of the vocal fold in each case. The positions of the two points are denoted in Fig. 2(a). The 45° dashed line indicates a straight glottal channel shape. The plot goes clockwise direction chronologically.

glottal shape change. The points on the 45° line indicate a straight glottal channel, the points below the line indicate a divergent channel and the points above the line indicate a convergent channel. Moreover, the distance from the point to the 45° line reflected the glottal angle. When the point was more away from the line, the glottal angle was larger. Figure 10 reflected that, with the increasing cover-body thickness ratio, the phase plot gradually shifted from upper left to lower right. This change indicated a gradually decreased convergent angle and increased divergent angle during vocal fold vibration. By carefully examining the vibrations of the vocal folds, it was noticed that this effect was due to the larger opening of the superior part of the vocal folds with the increased cover-body thickness ratio. Briefly, when the cover-body thickness ratio was low (between 0.02 and 0.07), the inferior and superior parts of the vocal fold reached their maximum openings at almost the same time when the glottis was straight. After that, they both started to close. When the cover-body thickness ratio was increased, when the inferior part had reached the maximum and was about to close, the superior was still in the opening motion. This has resulted in a longer time of a divergent glottis as well as a larger maximum divergent angle. Therefore, the phase plot was shifted to the right.

Figure 11 plots the maximum divergent angle in each case. The angle was calculated by tracing the lateral displacement of the inferior and superior side points. The plot clearly shows that the maximum divergent angle increased with the increasing cover-body thickness ratio. When the cover-body thickness ratio was between 0.02 and 0.07, the angle was below or around 8° . At $T_c/T_b = 0.09$, the angle was quickly increased to 13° . When $T_c/T_b = 0.12$ and 0.15 , it was further increased to 16° . At $T_c/T_b = 0.28$, the angle had another quick increase to a value as high as 31° . The two quick increases of the maximum divergent angle occurred at the onset of the two bifurcations. As the larger divergent angle indicated larger amplitude of the mucosal wave propagation, such effect was in accordance with our previous

FIG. 11. (Color online) The maximum divergent angle between the two vocal folds during vocal fold vibration in each case.

analysis that the increase of the cover-body thickness ratio excited the wave-type eigenmodes, which had caused the bifurcations of the dynamics of vocal fold vibrations.

IV. CONCLUSION

The influence of the vocal fold cover-body layer thickness ratio on vocal fold vibration and voice production was studied by using a continuum-mechanics based computational model of the flow-induced vibration of vocal fold. The vocal fold was modeled as a three-layer structure with different material properties in each layer. The cover-body layer thickness ratio was systematically varied in a wide range. The effect of the cover-body layer thickness ratio on the vocal fold stiffness, eigenfrequencies and eigenmodes, fundamental frequencies, glottal flow rate, vocal fold vibratory dynamics, and synchronization of the eigenmodes were analyzed in detail. The main conclusions are summarized below.

As the cover-body thickness ratio increased, the stiffness of the vocal fold decreased. This resulted in the decrease of the eigenfrequencies and the increase of the vibration amplitude of the vocal fold. A significant increase of the cover-body thickness ratio ($T_c/T_b = 0.28$ in the present study) also affected the order of the eigenmodes in which the wave-type eigenmodes moved up with lower eigenfrequencies.

The cover-body layer thickness ratio significantly affected the strength and synchronization of the eigenmodes during vibration, and ultimately affected the fundamental frequency and vocal fold vibration pattern. It was found that when the cover-body thickness ratio was low, the vocal fold vibration was dominated by the eigenmodes 1–3, which synchronized all the eigenmodes to a fundamental frequency near their eigenfrequencies. Because these three modes were not wave-type modes in the medial surface, they did not create a strong mucosal wave on the vocal fold surface; therefore, the divergent angle of the vocal fold vibration was small. In these cases, the fundamental frequency decreased with the increasing cover-body thickness ratio. When the cover-body thickness ratio was increased beyond a critical value (0.09 in this work), the strength of the wave-type higher-eigenfrequency modes was significantly increased. These modes became a stronger basin of attraction, breaking the initial synchronization of the modes and attracting other modes to vibrate at higher frequencies. As a result, the modes started to vibrate at different frequencies, resulting in chaotic vibrations. Also, because more energy was shifted to higher frequencies, the fundamental frequency was increased. Further increasing the cover-body thickness ratio continued to increase the strength of the higher-frequency modes and created a new steady state where the system underwent a period-doubling vibration. At this state, all the modes were synchronized at the fundamental frequency and its $\frac{1}{2}$ subharmonics. The presence of the subharmonic was likely due to the comparable strength of eigenmodes 1–3 and higher-eigenfrequency modes. At last, further increasing the cover-body thickness ratio continuously increased the strength of the higher-eigenfrequency modes, resulting in a new steady regular periodic vibration where all the modes were synchronized at the fundamental frequency near the

higher eigenfrequencies. During this process, because of the continuous increase of the strength of the wave-type modes, the mucosal wave on the vocal fold surface became stronger, and so the maximum divergent angle of the glottis was increased.

In summary, our simulation results showed that the cover-body thickness ratio had a complex nonlinear effect on the vocal fold vibration and voice production. Increasing the cover-body thickness ratio would promote the excitation of the wave-type modes of the vocal fold, which were also higher-eigenfrequency modes. These modes became a stronger basin of attraction for the system, driving all the modes to vibrate at higher frequencies, creating a period-doubling bifurcation state and eventually a new steady regular vibration at a fundamental frequency near the higher eigenfrequencies. Because of the increased strength of the higher-eigenfrequency modes, the fundamental frequency could have a sudden increase with the increasing cover-body thickness ratio although the stiffness of the vocal fold was decreased. During the transition, the modes vibrated at different vibration frequencies, resulting in chaotic vibrations. The increased cover-body thickness ratio also resulted in the increase of the maximum divergent angles during the vibration.

In the end, it is useful to point out the limitations of the study. First, the study was based on one set of model parameters including the geometry of the vocal fold and inner structures, the lung pressure and the elastic properties. Different choices of these parameters may result in different system behaviors. For example, a higher or lower value of the lung pressure might result in the transition happening at different values of the cover-body thickness ratio. Moreover, changing the Young's moduli might change the eigenmode shape and order, onset of the nonlinear bifurcation, bifurcation type, vocal fold vibration pattern, etc. However, the main finding of the study was that a larger cover-body layer thickness ratio would reduce the stiffness on the vocal fold surface and promote the excitation of the wave-type eigenmodes during vocal fold vibration, which could cause complex nonlinear behaviors. This effect was due to the fact that the elastic modulus of the cover layer is smaller than that of the other two layers. We think the conclusion has general validity in normal phonation condition because the elastic modulus of the cover layer is normally lower than the other two layers. Second, the effect of intrinsic laryngeal muscle activation was not considered due to the complexity of the model. The stiffness of the vocal fold would be significantly affected by the muscle activations and it might change the eigenmode shape and order, onset of the nonlinear bifurcation, bifurcation type, vocal fold vibration pattern, etc. Last, the vocal fold model was based on the three-layer assumption. Ideally, a more complicated vocal fold model which consists of the five histological layers with their accurate mechanical properties would provide more physiologically realistic predictions on vocal fold vibration patterns. For the cover layer, it was reported that the epithelium has a much larger elastic modulus than the superficial layer of the lamina propria.¹ The experiments on synthetic vocal folds^{18,19} found that adding a stiff epithelium layer with an elastic modulus higher than the other layers would improve the closure and mucosal wave of vocal fold vibrations. However, there exists

very little measurement of the mechanical properties of vocal fold tissues based on the five layers. Most of the measurement were based on either the two-layer or three-layer assumption.^{48,49} Furthermore, the focus of the current study was on the effect of cover layer thickness whose variation is mostly due to the changes in the superficial layer of the lamina propria. Based on these considerations, we think the three-layer assumption is reasonable for the current study. However, it is important to study the effect of the epithelium in the future.

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