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Modeling of slightly-compressible isentropic flows and its compressibility effects on fluid-structure interactions

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Abstract

In this study, an aeroacoustic fluid model for slightly-compressible isentropic flows is developed and evaluated for its compressibility effects in the context of fluid-structure interactions. This model considers computational feasibility and accuracy by adding compressibility terms directly on the incompressible form of Navier-Stokes equation. Rather than solving for the full compressible form, our slightly-compressible form significantly reduces the complications in establishing stabilization and implementation of its finite element procedure, and yet still captures the fluctuating acoustic waves expected in the compressible form. Using this approach, we demonstrate that generations and propagations of acoustic waves can be accurately captured, without the inclusion of a fully compressible representation of the fluid. Upon the successful verification of its accuracy against analytical and known solutions, we then evaluate the fluid compressibility effect on fluid-structure interactions. Our results show that comparing to an incompressible fluid, a deformable solid generates sound waves while it is driven by the flow and vibrates in the fluid. A periodic volume change in the fluid is also observed, which can be considered as a sound source.

Keywords

slightly-compressible; pseudo compressible; fluid-structure interaction; computational aeroacoustics

1. Introduction

Fluid-structure interactions (FSI) are involved in many engineering and bio-mechanical systems. In many of these FSI problems, air is often the working fluid. Air, compared to water or other liquids, has relatively high compressibility. The compressibility of air changes drastically with Mach number. To study subsonic flows with low Mach numbers ($Ma < 0.3$),

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a straight-forward approach is to treat it as a completely incompressible flow. However, such assumption is not ideal to produce realistic and accurate solutions in capturing acoustic properties. In a FSI problem, the accumulated error could be further amplified in the interacting solid, since part of the fluid force driving the solid is caused by the wave generation and propagation of the flow where the fluid undergoes a compression process.

In the field of acoustics fluid-structure interactions, work has been done toward a submerged elastic body interacting with acoustic fluid. A recent study by Wilkes and Duncan [36] combined Helmholtz and elastodynamic boundary element method to described the acoustic and solid domain physics using a conforming mesh where the two domain meshes are conformed at the interface. The work by Ross and Ross et al. [22, 23] showed a development of an acoustic fluid-structure interaction algorithm using localized Lagrange multipliers that treats the coupled system. Similarly, the fluid used in the study was simplified, which was assumed to be linear acoustic fluid that is compressible, irrotational, inviscid, constant density, and adiabatic. Many of the acoustic fluid-structure interaction analyses made assumptions of fluid being inviscid flow or described as linear wave equations [20] for simplicity. However, fluid motion in the immediate vicinity of a solid surface is usually dominated by viscous stresses, in way of boundary layer, which causes an adjustment in the velocity to comply with the no-slip condition at the fluid solid interface [9]. Particularly in acoustic analysis, the viscous stress cannot be neglected when the Reynolds number is moderate or low. Additionally, the viscous effect is significant in the solid boundary as vortices are generated by sound waves impinging on solid surfaces in the presence of flow. Therefore, a fluid solver including all the non-linear terms would be desirable that resolves both flow and acoustics length scales.

A direct way to resolve the acoustic properties in a flow is using the Direct Numerical Simulation (DNS) employing the compressible Navier-Stokes equations [4]. It is typically considered as an accurate method as it is capable of capturing the dynamics of all physical scales [1]. However, problems still emerge when the Mach number is small and reaches the incompressible limit $M \rightarrow 0$. At low fluid velocity, the compressible equations become very stiff, and the stability can only be maintained when the numerical time step is much smaller than the physical time step [35]. When the time step requirement is satisfied, there remains the accuracy issues [35, 5, 30]. Moreover, because the pressure fluctuation is much smaller than the background pressure, without proper care, it may eventually converge to the thermodynamic pressure in the compressible equations failing to capture small fluctuations [19]. Therefore, the disparity between the fluid velocity u and the sound speed c will restrict the spatial and temporal resolution to an extreme. When proper spatial and temporal resolutions are achieved, it would result in a huge consumption of the computational resources.

The other spectrum of the solution technique in solving pure aeroacoustics is to extrapolate the ideal reference flow to obtain the acoustical field. The difference between the actual flow and the reference flow is identified as a source of sound. In this way, aeroacoustic analogy is a popular solution. It evaluates sound sources based on the solutions of Navier-Stokes equations. Furthermore, Green's function can be applied to integrate over a closed surface to include the acoustic sources inside the closure. Lighthill's equation [15, 16] is considered as

the first aeroacoustic analogy: the acoustic effects from turbulent fluctuation in free flows are calculated and represented as a quadrupole source, which is later used to solve the wave equation, either in differential or integrated form. This method is expanded to Ffowcs Williams-Hawkings analogy (FW-H) [37], which considers not only the flows, but also the monopole and dipole acoustic sources from moving hard surfaces [8]. Using aeroacoustic analogy, one can easily compute sound source terms from the fluid field. However, there are still limits: firstly, the acoustic field is computed over the fluid domain and has no feedback to the fluid; additionally, even with FW-H method, only sound sources generated from hard surfaces can be included. For deformable bodies, which is often the case in FSI problems, it can not be applied anymore. To completely include the effects of nonlinear fluctuation terms, one must be careful on the choice of control surface, where the nonlinear behavior should vanish [17]. This increases the difficulty of applying aeroacoustic analogy to arbitrary-sized problems.

In consideration of both needs of numerical accuracy and computational feasibility in between the two ends of the spectrum, some more general models have been developed to combine the merits of incompressibility treatment and capturing acoustic perturbation in computing aeroacoustic properties. One popular solution is the splitting methods proposed by Hardin and Pope [6], which is called expansion about incompressible flow (EIF). This method uses two meshes where one is for incompressible flow field and the other is superimposed for acoustic perturbation. Generally, the mesh for acoustic perturbation is much coarser than that for incompressible flow field. It splits the whole computation into two parts, namely incompressible hydrodynamic part and perturbed compressible part. Quantities calculated from incompressible Navier-Stokes equations are used in the perturbation equations for the fluctuation in velocity and pressure fields, and the final quantities are the sum of the incompressible and the perturbed compressible components. After Hardin and Pope, the equations used for splitting method have been developed and extended in various forms that include: linearized perturbed compressible equations (LPCE) [25, 26], acoustic perturbation equations (APE) [3] and linearized acoustic perturbation equations [19], which are several representative methods adopting the splitting approach. Despite its effectiveness, there are still limitations in a splitting method. In particular, the way of superposition neglects non-linear terms; additionally, same as aeroacoustic analogy, the compressible part is calculated based on the incompressible result but it does not provide feedback to the incompressible part before proceeding to the next time step. These factors impact the accuracy and stability of simulations. There is also a family of unified solvers, including CIP-Combined Unified Procedure (CCUP) [39, 14] and Thermo-CCUP (TCUP) [7]. This kind of methods separates one time step into advection, diffusion and acoustic stages. In this family, finite difference method (FDM) and cubic interpolated pseudo-particle method (CIP) [27] are combined to solve different stages. They show good results in both incompressible and compressible limits[39]. However, they rely on certain numerical schemes, which causes difficulties in implementation.

In this work, we develop an alternative aeroacoustic solution technique for such slightly-compressible isentropic flow, without computing the solutions in two parts as is in the splitting methods discussed above. There is no decomposition of variables, i.e., flow and perturbation fields, instead, all the variables are integrated in one set of fluid equations. As a

result, only one set of mesh is necessary to resolve the physical scales in the acoustics flow, other than two sets of mesh for fluid and perturbations, respectively. The fluctuations of pressure is hence reflected in the movement of the flow. The full fluid governing equations include all the nonlinear effects, which are neglected in most of the linearized perturbation methods. The proposed model allows fluid to have slight compression, since the compressibility of air at low Mach number is very small. The slight compressibility, nevertheless, is sufficient in representing air as it does capture the wave generation and propagation. This slightly-compressible or “pseudo-compressible” fluid model is then incorporated into the immersed finite element method (IFEM)[32], a fully-coupled fluid-structure interaction algorithm, to demonstrate how the acoustic properties could have an effect in the solid behavior in FSI problems. In particular, a solid deformation is closely observed in cases where it interacts with a fluid that has slight and no compressibility. It also provides a straightforward way in including the aeroacoustic solutions using an existing incompressible fluid solver.

The paper is organized as follows: in Section 2, we first present the proposed fluid equation for slightly-compressible isentropic flows, then we briefly review the concept of the equation in the context of the FSI in the IFEM context. To verify the accuracy and capability of our model, three cases are studied in Section 3 using OpenIFEM [12], an open-source software developed in our research group. The first two cases are the validations of the slightly-compressible isentropic flow model and the third one is an example of a fluid-structure interaction case that demonstrates the compressibility effects. Finally, the conclusions are drawn in Section 4.

2. Slightly-compressible fluid model for isentropic air

In this section, the slightly-compressible fluid, or pseudo-compressible fluid, with isentropic process is presented along with the assumptions made. All the variables are pertinent to the fluid, unless specifically noted.

2.1. Isentropicity of air

An isentropic process is a reversible adiabatic process where the entropy S is constant. In normal acoustic situations air compression/expansion is much closer to isentropic than other processes such as isothermal process. When air is compressed by shrinking its volume V , for example, not only the pressure P increases, the temperature T increases as well, according to the ideal gas law $PV = nRT$ or $P = \rho RT$ where P is the absolute pressure. However, in a constant-entropy compression/expansion, temperature changes are not given time to diffuse away to thermal equilibrium. Heat diffusion occurs much slower than acoustic vibrations. Therefore, temperature field does not play a major role in the governing equation. As a result, the energy equation involving T does not need to be coupled with continuity and momentum equations. The pressure and density fluctuations from its equilibrium pressure p_0 and density ρ_0 are $p = P - p_0$ and $\rho' = \rho - \rho_0$. For slightly-compressible flows with low Mach number, we can assume p and ρ' are much smaller than the total pressure and the initial density, i.e. $p/P \ll 1$ and $\rho'/\rho_0 \ll 1$.

Isentropic process further requires that the variation of density as a function of the pressure with constant entropy is related to the speed of sound, i.e. $\frac{\partial \rho}{\partial p}|_S = \frac{1}{c^2}$ where c is the speed of sound, and S refers to a constant entropy. With the nearly incompressibility assumption, it can be deduced to

$$p = c^2 \rho'. \quad (1)$$

The isentropic relationship for air can then be written as:

$$P - p_0 = (\rho - \rho_0)c^2. \quad (2)$$

For dry air, the adiabatic index or isentropic expansion factor is $\gamma = 1.4$ and the specific gas constant is $R = 287.06 \text{ J/(kg}\cdot\text{K)}$. The temperature is chosen as a constant value of 273 K. The speed of sound c or the speed of the pressure wave in a fluid is dependent on the bulk modulus of the fluid, κ , where $c = \sqrt{\kappa/\rho}$. For an ideal gas, the isentropic bulk modulus $\kappa = \gamma P$. The speed of sound in air is then:

$$c = \sqrt{\gamma P / \rho}. \quad (3)$$

Similar approach has been presented in [2] where the slight compressibility is adjusted by controlling the bulk modulus, κ . However, for a realistic aeroacoustic model, the air undergoes an isentropic process, where κ is a function of the total pressure P and the adiabatic index γ is set to be 1.4. The outcome of fixing different parameters to adjust compressibility may be the pressure variations in the resulting waves.

2.2. Slightly-compressible governing equations

The continuity equation of a compressible fluid is written as (all partial derivatives are labeled as ','),

$$\rho_{,t} + \nabla \cdot (\rho \mathbf{v}) = 0. \quad (4)$$

The density can be written as the sum of fluctuation and equilibrium density, namely $\rho = \rho' + \rho_0$. We can expand Eq.(4) as:

$$\rho_{0,t} + \rho'_{,t} + \nabla \cdot ((\rho_0 + \rho')\mathbf{v}) = 0. \quad (5)$$

where ρ_0 at equilibrium is a constant, therefore $\rho_{0,t} = 0$. By utilizing the chain rule we can further expand the equation to:

$$\rho'_{,t} + (\rho_0 + \rho')(\nabla \cdot \mathbf{v}) + \mathbf{v} \cdot \nabla(\rho_0 + \rho') = 0. \quad (6)$$

Since $\nabla \rho_0 = 0$, Eq. (6) becomes:

$$\rho'_{,t} + (\rho_0 + \rho')(\nabla \cdot \mathbf{v}) + \mathbf{v} \cdot \nabla \rho' = 0. \quad (7)$$

By substituting Eq.(1) into Eq.(7) we obtain:

$$\frac{p_{,t}}{c^2} + (\rho_0 + \frac{p}{c^2})(\nabla \cdot \mathbf{v}) + \mathbf{v} \cdot \nabla \frac{p}{c^2} = 0. \quad (8)$$

It is simplified by multiplying both sides by c^2 ,

$$p_{,t} + (\rho_0 c^2 + p)(\nabla \cdot \mathbf{v}) + \mathbf{v} \cdot \nabla p = 0. \quad (9)$$

Based on Eq.(2), the term $\rho_0 c^2$ can be evaluated as $\rho_0 c^2 = \rho c^2 - P + p_0$. By further using Eq. (3) it becomes,

$$\rho_0 c^2 = \gamma P - P + p_0 = \gamma P - p \quad (10)$$

Therefore, the continuity equation (4) can be deduced as:

$$p_{,t} + \gamma(p_0 + p)(\nabla \cdot \mathbf{v}) + \mathbf{v} \cdot \nabla p = 0. \quad (11)$$

To clarify, the fluctuation of pressure, p , in the above equation is the typical pressure defined in the Navier-Stokes equations where pressure p is a relative measure, and p_0 is the reference atmospheric pressure.

Finally, the momentum equation for slightly-compressible fluid becomes:

$$\rho \mathbf{v}_{,t} + \rho(\mathbf{v} \cdot \nabla) \mathbf{v} = \nabla \cdot \boldsymbol{\sigma} + \mathbf{f} \quad (12)$$

where ρ is the fluid density which is evaluated based on the pressure fluctuation based on Eq. (1), $\boldsymbol{\sigma}$ is the fluid stress tensor and $\mathbf{f}$ is the body force, which will be explained more in detail in the following in the context of FSI. The constitutive relationship between the stress and state variables velocity $\mathbf{v}$ and pressure p for compressible fluid is:

$$\boldsymbol{\sigma} = (-p + \frac{2}{3}\mu \nabla \cdot \mathbf{v})\mathbf{I} + \mu(\nabla \mathbf{v} + (\nabla \mathbf{v})^T), \quad (13)$$

where μ is the dynamic viscosity of the fluid.

2.3. Slightly-compressible model in fluid-structure interactions

To study the impact of compressibility of fluid on the solid, the aeroacoustic model is integrated into the immersed finite element method (IFEM) [42, 41, 38]. The IFEM is a volume-based finite element fluid-structure interaction numerical method. Finite elements are not only used as a way of discretization, but the entire IFEM formulation is derived from the weak form, or principle of virtual work, where the total work in the entire fluid-structure coupled system is maintained. In the original development of the IFEM, the fluid is assumed to be incompressible. To further treat large material disparities between the fluid and the solid, such as density and viscosity, and enable solid dynamics to take a dominant effect, a modified IFEM (mIFEM) [32, 34] algorithm is developed. The mIFEM is more modular where the dynamics of the solid and the fluid solvers are independently solved. Therefore the modifications for the slightly-compressible fluid can be easily implemented. The method has been rigorously derived and verified with 2-D and 3-D studies [31, 44].

For completeness, we first briefly introduce the basic concept of the mIFEM. The detailed derivation can be found in [34]. The basic idea of mIFEM is to assume that the fluid exist everywhere in the domain, the overlapped domain where both fluid and solid exist is called “artificial fluid”. Fluid has an Eulerian description and solid is Lagrangian. The solid domain Ω^s is completely immersed in its surrounding fluid Ω^f , depicted in Figure 1.

Since the solid domain is immersed in the fluid domain, the solid boundary Γ^s is also the common interface intersecting with the fluid domain, called the fluid-structure interface $\Gamma^{\text{FSI}} \equiv \Gamma^s$. During the computation, the Lagrangian mesh changes its position due to deformation, while Eulerian mesh does not. Therefore, it is necessary to identify the overlapping domain so that the solid effects are considered. In mIFEM, we used an indicator function $\mathbf{I}(\mathbf{x})$ as a mapping function [32]. It is used to identify the location of the solid in the fluid domain so that proper material properties, *e.g.* density, can be applied appropriately. The indicator $\mathbf{I}$ ranges from 0 to 1, where $\mathbf{I} = 0$ if an element is in fluid region and $\mathbf{I} = 1$ if an element is in solid region. The density is appropriately assigned using the indicator function in the entire computational domain.

$$\bar{\rho} = \rho^f + (\rho^s - \rho^f)\mathbf{I}(\mathbf{x}), \quad (14)$$

In fluid-structure interactions, the external body force applied onto the solid is $\mathbf{f}$ in the fluid governing equation Eq.(12). When interacting with the fluid, the solid experiences an external load from the fluid. This force is labeled as the fluid-structure interaction force $\mathbf{f}^{\text{FSI},f}$. The fluid-structure interaction force is an artificial force defined in the whole solid

domain[34]. This force is first calculated in solid domain and then distributed onto the fluid domain to compensate the difference of internal forces and inertial forces between the fluid and the solid. The derived fluid-structure interaction force is first evaluated in the solid domain as:

$$\mathbf{f}^{\text{FSI},s} = \nabla \cdot \boldsymbol{\sigma}^s - \nabla \cdot \boldsymbol{\sigma}^f - (\rho^s - \rho^f) \left(\frac{D\mathbf{v}^f}{Dt} \right) \quad (15)$$

The force $\mathbf{f}^{\text{FSI},s}$ is then distributed to the fluid domain $\mathbf{f}^{\text{FSI},f}$ as an external force applied to the fluid.

The whole process of mIFEM can be summarized as the following:

1. Solve solid governing equations for solid deformation, given solid boundary conditions;
2. Calculate indicator $\mathbf{I}$ and fluid-solid interaction force $\mathbf{f}^{\text{FSI},f}$;
3. Solve for fluid using the modified fluid governing equation;
4. Interpolate fluid velocity onto the solid-fluid interface to obtain solid boundary conditions and back to step (1).

Using this concept of the mIFEM, the compressibility of air is easily incorporated into the governing equations of the FSI framework. It is a necessary and important step towards the study of the interactions between air and aeroelastic structures.

2.4. Finite element weak form

With the governing equations (11) and (12), let us assume there exist defined finite-dimensional trial solution and test function spaces for velocity and pressure: $\mathcal{V}_v$, $\mathcal{V}_p$, $\mathcal{S}_v$ and $\mathcal{S}_p$. The goal is to find $\mathbf{v} \in \mathcal{V}_v$ and $p \in \mathcal{V}_p$ such that for all test functions of velocity and pressure satisfy $\forall \delta \mathbf{v} \in \mathcal{S}_v$ and $\delta p \in \mathcal{S}_p$.

This work follows the stabilized equal-order finite element formulation developed in Refs. [28, 29]. The stabilization technique was designed for incompressible flows as detailed in [28]. Although compressibility is included in this governing equations, the formulation derived for isentropic air is still regarded as slightly-compressible fluid where the energy equation is not solved simultaneously with continuity and momentum equations. Stabilized test functions $\delta \tilde{v}_i$ and $\delta \tilde{p}$ along with the stabilization parameters τ^m and τ^c defined in [28] are used:

$$\delta \tilde{v}_i = \delta v_i + \tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i} \quad (16a)$$

$$\delta\tilde{p} = \delta p + \tau^c \delta v_{i,i} \quad (16b)$$

where τ^m and τ^c are dependent of the computational grid, time step size, and flow variables.

The weak form of the continuity equation (4) is obtained by multiplying the pressure test function $\delta\tilde{p}$ and integrating over the computational domain Ω ,

$$\int_{\Omega} (\delta p + \tau^c \delta v_{i,i}) [p_{,t} + \gamma(p_0 + p)(\nabla \cdot \mathbf{v}) + \nabla p \cdot \mathbf{v}] d\Omega = 0. \quad (17)$$

Similarly, the semi-discrete weak form of the momentum equation (12) is achieved by multiplying the velocity test function $\delta\tilde{v}_i$,

$$\begin{aligned} \int_{\Omega} (\delta v_i + \tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}) [\bar{p}(v_{i,t} + v_j v_{i,j}) - f_i^{\text{FSI},f}] d\Omega - \int_{\Omega} \delta v_i \sigma_{ij,j}^f d\Omega \\ - \sum_e \int_{\Omega_e} (\tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}) \sigma_{ij,j} d\Omega = 0. \end{aligned} \quad (18)$$

Applying integration by parts and the divergence theorem, we can rewrite Eq. (18) as

$$\begin{aligned} \int_{\Omega} (\delta v_i + \tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}) [\bar{p}(v_{i,t} + v_j v_{i,j}) - f_i^{\text{FSI},f}] d\Omega - \int_{\Gamma_{h_i}} \delta v_i h_i d\Gamma \\ + \int_{\Omega} \delta v_{i,j} \sigma_{ij} d\Omega - \sum_e \int_{\Omega_e} (\tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}) \sigma_{ij,j} d\Omega = 0, \end{aligned} \quad (19)$$

where individual finite elements are denoted with the subscript e and the stabilization supplementary terms are evaluated at the element interiors.

Let us assume that there is no traction applied on the fluid boundary, *i.e.* $\int_{\Gamma_{h_i}} \delta v_i h_i d\Gamma = 0$, the

final weak form yields

$$\begin{aligned}
& \int_{\Omega} (\delta v_i + \tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}) [\bar{p}(v_{i,t} + v_j v_{i,j}) - f_i^{\text{FSL,f}}] d\Omega + \int_{\Omega} \delta v_{i,j} \sigma_{ij} d\Omega \\
& - \sum_e \int_{\Omega_e} (\tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}) \sigma_{ij,j} d\Omega + \int_{\Omega} (\delta p + \tau^c \delta v_{i,i}) [p_{,t} + \gamma(p_0 + p) v_{j,j} + v_j p_{,j}] d\Omega \\
& = 0.
\end{aligned}$$

(20)

The full finite element discretization is then performed by discretizing the test and trial functional spaces into finite elements. The nonlinear fluid solver is solved with the Newton-Raphson method. GMRES iterative algorithm is used to minimize the equation residuals [24, 43]. With this stabilization technique, we did not observe any deterioration in solution convergence in the fluid. The interface conditions such as no-slip and traction free conditions, are almost automatically satisfied based on the way the coupled solutions are derived. The residual at the interface is still evaluated to guarantee convergence of the coupled system. It typically takes no more than a couple of iterations within the time loop to reach a set tolerance. A set of mesh convergence studies for the IFEM coupled solutions can be found in our previous work [33, 40].

3. Numerical examples

To verify the compressible Navier-Stokes implementation with an isentropic flow, we first examine a simple case against known analytical solution. It is then extended into more complicated cases with fluid structure interactions. The success of these simulations establishes the basis for many future acoustics studies involving fluid-structure interaction simulations. In all cases, both solid and fluid parts use GMRES iterative scheme to solve the linear system, and the tolerance for relative residual is set to $tol = 1e-6$.

3.1. Wave propagation in a duct with closed ends

To verify the compressible Navier-Stokes implementation with an isentropic flow, we examine wave propagation in a rigid duct with both of its ends closed after an initial Gaussian pulse applied at the inlet. Wave propagation speed and reflective wave speed are examined and compared to analytical solutions. The units used in these studies are the cm-gram-s (CGS) units. The rectangular duct has a size of 5.0 cm in length and 0.6 cm in width. The model setup is shown in Figure 2.

At the beginning of the simulation a Gaussian planar wave is introduced at the left end, and hereafter both ends are closed. The top and bottom boundaries of the duct have free-slip boundary condition. The Gaussian pulse applied on the left inlet boundary is

$$U_0 = 6 \cdot \exp\left(-\frac{(t - 5 \times 10^{-5})^2}{2 \cdot (1.5 \times 10^{-5})^2}\right), \text{ where the inlet velocity } U_0 \text{ has a unit of cm/s and time } t \text{ in s.}$$

The fluid is isentropic air with an equilibrium density of $\rho_0=1.293 \text{ kg/m}^3$ at 273 K, a dynamic viscosity of $1.8 \times 10^{-5} \text{ Pa}\cdot\text{s}$, and an initial pressure set as the atmospherical pressure of $p_0 = 101.3 \text{ kPa}$. The theoretical speed of acoustic wave propagation can be estimated as $c = \sqrt{\gamma p_0 / \rho_0} \approx 331.18 \text{ m/s}$. A uniform mesh with 1875 quadrilateral elements (125 elements in x -direction and 15 in y -direction) is used in this case; the time step size is set to be $2.5 \times 10^{-8} \text{ s}$, sufficiently small to capture the wave propagation without observable decaying in the amplitude of the pressure. The simulation was run for 50000 time steps, which corresponds to 0.00125 s in simulated time.

The snapshots from the simulation of the pressure field as the wave travels in the duct are shown in Figure 3. The wave travels in the duct and reflects off the closed ends. The reflective wave form is identical to the initial wave.

To further quantitatively verify our results, we examine the acoustic wave speed by using the concept of acoustic impedance. The characteristic specific acoustic impedance of a medium, say the air, is $\rho_0 c$, the acoustic pressure applied to the system is the pressure variation p (not the absolute pressure), the acoustic ‘current’ is the fluid velocity u . Therefore, the relationship $\rho_0 c = p/u$ is obtained [18, 21, 13]. Here, we plot the pressure variation normalized by density p/ρ_0 against velocity u at various axial locations across the duct ($x=0, 1, 2, 3 \text{ cm}$) in Figure 4. All the data collected at each of these axial locations collapsed onto a single line with the slope of 33138.7 cm/s. The computational results reproduce the linear relationship indicated above, which can be expressed as $\frac{p/\rho_0}{u} = c$. It is also shown that the acoustic wave speed is successfully captured from the relationship found in the collected data, recovering the theoretical speed of sound.

A comparison between the numerical solution and theoretical prediction is shown in Table 1. In comparison, the numerical results reproduce the hard wall reflection phenomenon. As is also shown in Figure 4, the linear relationship between acoustic pressure and particle velocity, denoting the specific acoustic impedance, is also in agreement with theoretical prediction. The error of sound speed is less than 0.1%.

3.2. Wave propagation in an open ended duct as a resonator

The second verification case is to examine the acoustic wave propagation in an one-end open rigid duct where the left end is closed after the initial entering of the wave while the right end remains open. To truly simulate an open end without having an infinitely large computational domain, we attach a far-field region on the side of the open duct. The far-field is significantly larger comparing to the size of the duct. It is designed so that it takes a long period of time for the radiated sound wave to reach the numerical boundary and an equally longer period for any reflected wave to travel back to the duct. A semi-circular shape is defined for the far-field as a resonator in order to accurately capture the cylindrical wave that radiates out of the duct. It is discretized with gradually increasing grid spacing in order to dissipate any numerical reflection at the boundary to bounce back into the duct. These setups are to ensure that during the lifetime of the simulation the waves in the duct will not be disturbed or contaminated by numerical wave reflections. The schematic model setup and applied boundary conditions are shown in Figure 5. The dimensions are not drawn in scale

as the duct is too small to show comparing to the far-field. The duct has dimensions of $17.3 \text{ cm} \times 2.8 \text{ cm}$, where the inlet is on the left. The semi-circular far-field has a radius of 1000 cm. The duct is discretized with a uniform mesh comprised of 433×70 quadrilateral elements (with each element size of $0.399 \text{ cm} \times 0.04 \text{ cm}$), while the far-field domain has 11520 elements with the mesh coarsening as it goes from the duct opening to the far end, as shown in Figure 6.

Two views are shown in Figure 6(b): the overall view of the mesh in Figure 6(a) and the zoomed-in view of the duct and a small portion of the far-field. An initial Gaussian pulse wave $U_0 = 6 \cdot \exp\left(-\frac{(t - 5 \times 10^{-5})^2}{2 \cdot (1.5 \times 10^{-5})^2}\right)$, is applied. The time step size is set to be $1 \times 10^{-7} \text{ s}$, and the simulation lasts 0.0012 s in physical time. The top and bottom boundaries of the duct are set to free-slip; so that the input wave remains planar as it first travels through the duct and there is no reflections off from these boundaries. The far-field outlet boundary is stress-free, thus an open boundary. Waves traveling in the far-field will gradually dissipate with the increased grid spacing so that the waves are almost entirely disappeared when reaching the outlet boundary.

The theoretical prediction of an acoustic wave propagation in an open ended duct is that the wave initially remains planar in the duct, as soon as it exits the duct, it radiates out of the duct and becomes cylindrical. The cylindrical wave is to conform with the open space. As the cylindrical wave continues to travel outward, some negative reflections start traveling back to the duct. This acoustic wave radiation and reflection at a duct opening is also analyzed using numerical simulations [10]. The open-ended duct, which is served as an impedance, measured using p/v , to the wave, is shown in the frequency domain for its amplitude and phase, in Figure 7.

The impedance amplitude and phase in the frequency domain, shown in Figure 7, have nearly identical shape and magnitude as presented in [10], where the peaks and troughs of the input impedance become less and less pronounced as it exits the duct, and finally vanishes as the frequency increases to 5000Hz. The decreasing impedance amplitude for increasing frequency shows that components with higher frequencies have lower power [11]. The reflection is shown to have antiphase with negative impedance phase values. Its theoretical prediction for a pulse wave traveling in an open duct can also be found in [11], where the impedance amplitude diminishes as frequency increases and both radiated waves and reflected waves are represented in positive and negative impedance phases, respectively.

A series of snapshots at consecutive time instants of the wave propagation and radiation out of the duct are shown in Figure 8. At $t = 0.00055 \text{ s}$ the planar wave travels rightward in the straight duct. At $t = 0.00065 \text{ s}$ the wave exits the duct and starts to radiate into the open space. At $t = 0.00075 \text{ s}$, the radiated wave travels farther into the open space showing a roughly cylindrical contour, while a negative reflected wave starts to form at the duct opening. At $t = 0.00085 \text{ s}$, the radiated wave propagates even farther in the open space, while the negative reflection travels leftward back into the duct.

The wave, initially planar while traveling in the duct, becomes cylindrical once it radiates from the open-end into the far-field. Meanwhile, a negative reflection is observed traveling back into the duct.

3.3. 2-D leaflet in a channel

The third example is to examine the effects of the solid when considering slight compressibility in waves in the context of fluid-structure interactions. In this problem, a deformable leaflet of 0.5 cm in width and 1 cm in height is fixed at the bottom 3 cm from the inlet flow boundary in a 2-D rectangular fluid channel of size 8cm $\times$ 2cm. The leaflet has density of $\rho^s = 1.001 \text{ g/cm}^3$ and is made of linear elastic material with Young's modulus of 1000 Pa and Poisson's ratio of 0.3. Plane-strain assumption is made for the 2-D solid. The left side of the channel has a steady inflow velocity of $U_0 = 20 \text{ cm/s}$. The bottom and top boundaries are no-slip, and the outlet on the right has an outflow boundary condition. The equilibrium density of the slightly-compressible air is $\rho_o^f = 1.3 \times 10^{-3} \text{ g / cm}^3$. The problem statement is illustrated in Figure 9.

The mesh used for fluid domain consists of 114×29 uniform quadrilateral elements with each element size being $0.07 \text{ cm} \times 0.069 \text{ cm}$, as shown in Figure 10, and the solid mesh has 50 uniform quadrilateral elements. A time step size of $1 \times 10^{-4} \text{ s}$ is employed for a total 2000 steps. Two cases are performed with the same setup but with and without compressibility for comparison. The case with incompressible flow was thoroughly examined in our previous study [32].

To compare the two cases, the observable difference is the movement of the leaflet. Here, we trace the displacement of the top left node of the leaflet, as labeled in Figure 9. From tracking the displacement history in time and frequency domain in Figure 11, we observe that while the oscillation of the leaflet in the two cases have the same fundamental frequencies, the amplitudes at the two dominant frequencies (15 Hz and 55 Hz) are different. The natural frequency of the solid is still the governing vibrational frequency and it is not impacted by the aeroacoustic frequency. Therefore, the fundamental frequency of the solid is the same with or without compressibility. The slightly-compressible flow model yields slightly lower amplitude at 15 Hz and higher amplitude at 55Hz. Allowing slight compressibility provides small oscillatory impact on the solid. This indicates that a slightly-compressible model is more sensitive to higher frequency than the full-incompressible model.

To further examine the effects of compressibility, we re-run the cases with a much stiffer solid to capture the wave produced by the solid. The natural frequency of linear elastic solid is proportional to $\sqrt{E/\rho}$, where E is Young's modulus and ρ refers to the solid density. By increasing Young's modulus to $3.2 \times 10^8 \text{ Pa}$, the natural frequency becomes $\sim 8500 \text{ Hz}$ and the corresponding wave length is around 4 cm. We are then able to capture a whole wavelength in the computational domain. This simulation uses $t = 1 \times 10^{-6} \text{ s}$ for 50000 steps to correctly capture the high-frequency vibration and waves.

To quantify the difference, the deviations of the pressure between slightly-compressible and fully incompressible tests are plotted at $y = 0.4$ cm on a horizontal line where $x = 4$ cm to 8 cm. The plots at different time steps are shown in Figure 12. A clear wave form is observed and varies with time, with the amplitude of around 0.003 Pa. The wave length and the frequency are as expected for the setup, as mentioned before. The wave form represents the sound wave produced from the interaction between solid and fluid during the solid vibration.

The motion of the solid and the drag force from the fluid-solid interaction could cause the change in the fluid volume. The volumetric expansion or contraction could be considered as a source of sound production. We choose $x_A = 2$ cm and $x_D = 4.5$ cm to measure inflow and outflow rates respectively, and evaluate the rate of volume change in Figure 13. The slightly-compressible flow shows a periodic change at the natural frequency of the solid, while the full-incompressible form only contains non-uniform noise which results from the immediate response of the surrounding incompressible fluid to the volume change of the solid through the FSI.

4. Conclusion

In this study, a method to simulate slightly-compressible flows for the simulation of FSI problems is proposed. This method allows flows with low Mach number to have slight compression to capture sound generation and propagation during the FSI. Different from a set of fully employed compressible flow governing equations or decomposition of the flow, our model avoided calculation of energy transportation, and does not require high spatial or temporal resolution, or two different meshes, hence it could be efficient and easily implemented.

The test cases show that this slightly-compressible model is able to capture the sound generation, propagation as well as reflection in flows with low Mach number. In the first case, the speed of sound is correctly demonstrated as the pressure wave travels in the duct. In the second case, the behavior of the reflected wave at the opening of the duct is similar to theoretical solutions. In the third case, compared to a fully incompressible flow model, it shows superiority in capturing oscillations with higher frequency when using the same size of time step. Moreover, the slightly-compressible model successfully captures the periodic expansion and contraction of the flow from the FSI as well as the generated sound wave. Based on the results, the new model is accurate and realistic. With this model, it becomes possible to solve complex aeroelastic fluid-structure interaction problems.

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Highlights

- Developed a slightly-compressible fluid model that captures generations and propagations of acoustic waves without full compressible terms
- Derived compressible terms for isentropic flows (aeroacoustics) that can be directly added onto the incompressible form of Navier-Stoke equation
- Verified accuracy of acoustic behaviors in test cases
- Captured acoustic waves in fluid-structure interactions and found differences in acoustic behaviors comparing to incompressible form

Figure 1:
Computational domain decomposition.

Figure 2:
Geometry and boundary conditions of acoustic wave traveling in a closed rectangular duct.

Figure 3:
Snapshots of the pressure field as the wave propagates and reflects in the duct with two closed ends

Figure 4:

Pressure variation over density versus velocity at several locations along the duct to calculate acoustic wave speed.

Figure 5:
Geometry and boundary conditions for one-end open duct with far-field.

(a) Overall view of the duct with far-field

(b) Zoomed-in view of the duct

Figure 6:
Overall view and zoomed-in-view mesh of the one-end open duct with far-field.

Figure 7:
Acoustic impedance amplitude and phase in the frequency domain for an initial Gaussian pulse wave traveling in an open-ended duct.

Figure 8:
Pressure contours in the one-end open duct with far-field.

Figure 9:
Problem statement of a 2-D leaflet in a channel.

Figure 10:
Fluid mesh for the 2-D leaflet case.

(a) Time domain

(b) Frequency domain

Figure 11:
Displacement of the top left node of leaflet in a wind channel.

Figure 12:

The pressure difference between incompressible and slightly-compressible model along a horizontal line in the downstream on $y = 0.4$ cm.

Figure 13:
The rate of fluid volume change near the solid.

Table 1:

Comparison between theoretical prediction and numerical solution for acoustic wave traveling in a closed duct.

	Theoretical prediction	Numerical solution
Reflection from the boundaries	Yes	Yes
Relationship between p and u	Linear	Linear
Speed of sound	33118 cm/s	33139 cm/s