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Multicolor particle shadow accelerometry

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Abstract

This paper describes the extension of multicolor particle shadow velocimetry (CPSV) to the measurement of local acceleration in an Eulerian frame of reference. A validation experiment was conducted on a pendulous disk undergoing unsteady rigid body rotation. Angular velocity and acceleration profiles by CPSA are presented along with a comparison to recordings by an accelerometer mounted on the pendulum. CPSA is also demonstrated in a fully-developed turbulent pipe flow. Profiles of standard deviation of the local acceleration in the near wall region ($0 < y^+ < 75$) are compared to similar measurements by Christensen and Adrian. A favorable comparison is found between CPSA and particle image accelerometry (PIA). The effect of acceleration time delay, or the time between two velocity estimates, on local acceleration estimates is discussed.

Keywords

particle shadow velocimetry; accelerometry; high-speed imaging; turbulent pipe flow

1. Introduction

This paper describes the extension of multi-Color Particle Shadow Velocimetry (CPSV) [1–3] to measure local acceleration fields ($\mathbf{u}/t$) of a fluid flow, in an Eulerian frame of reference. The resulting technique, multi-Color Particle Shadow Accelerometry (CPSA), builds on both the strengths of CPSV, as well as previous approaches to extend particle image velocimetry (PIV) methods to measure fluid acceleration.

PSV is an optical technique to measure seed particle displacement in a plane, similar to PIV, using images of particle shadows cast by a flashing LED, instead of high-intensity laser light scattered by the particles (LED illumination has also been used to generate light sheets, akin to the PIV configuration [4]). The application of multicolor illumination and color imaging to PSV allows recording particle images multiple times within a single exposure. The extension of CPSV to acceleration measurements therefore requires no additional hardware beyond PSV—only one color camera and one multicolor light source are required. LED light sources capable of operating in the kilohertz range are currently available, enabling time resolved measurements of both velocity and acceleration fields, while avoiding the safety concerns and expense associated with laser illumination.

Fluid acceleration measurements have been performed in the past by extension of optical velocity measurement techniques such as particle image velocimetry (PIV) and particle-tracking velocimetry (PTV). In PIV, tracer particles are illuminated by a pulsed laser light sheet, or swept laser beam. Fluid displacements are estimated by cross-correlation of particle image fields acquired successively at a prescribed time delay. Two consecutive velocity field estimates must be obtained to extend PIV to acceleration measurement, thereby requiring four illumination pulses. For many flows of interest, a basic PIV system, operating at less than 15 Hz, lacks the temporal resolution to perform acceleration measurements. High-speed systems, which are becoming more widely available, may provide the required time resolution, but still entail high cost.

A variety of hardware configurations have been developed to obtain the necessary temporal resolution for acceleration measurements with basic PIV. Christensen and Adrian presented a system using two pairs of pulsed Nd:YAG lasers and two cameras [5, 6]. A single velocity field was measured by each camera and laser pair, such that the two velocity fields were acquired at a fixed time delay. The difference of the sequential velocity fields provided a measurement of the local acceleration. In addition, derivatives in the bulk-flow reference frame were taken by offsetting a camera and laser pair spatially in the streamwise direction. A similar setup was used by Liu and Katz to measure instantaneous acceleration and pressure fields [7]. These PIA systems require the hardware for two separate PIV systems, and setup of these systems requires alignment of four laser sheets and two image planes.

Early efforts to extend basic PIV to acceleration measurement employed continuous-wave laser beams and mirrors to sweep or pulse a laser sheet. Jensen *et al* [8, 9] and Jakobsen *et al* [10] developed systems using two and four cameras, respectively, with a single laser. These systems require careful alignment of multiple image planes. Perhaps the simplest hardware realization of a DPIA system is that of Dong *et al* [11]. The authors developed a system using a single camera and laser, where the laser was swept through each exposure twice. Velocity fields were calculated from autocorrelation of each doubly exposed image. The time derivative of velocity was then calculated by differencing two sequential velocity fields. The directional ambiguity that arises in autocorrelation displacement estimates was resolved through cross-correlation of successive doubly exposed images. A similar system for measurements in a two-phase flow, with a dual pulse laser, was presented by Borowski and Wei [12]. Recent studies have made use of high-speed PIV systems to take time resolved measurements of fluid velocity and acceleration [13–15]. These systems are even more costly than basic PIV systems, as a high-speed camera and laser are required. Perhaps the most advanced system to date is that of Lynch and Scarano, who used twelve cameras and two lasers for tomographic PIV measurements to measure of the full material derivative of velocity in an Eulerian frame [16].

Accelerations in a Lagrangian frame of reference have been measured with particle tracking velocimetry (PTV). Virant and Dracos [17] used a four-camera PTV setup and Halogen arc lamp illumination to measure Lagrangian-frame velocities and accelerations. Silicon strip detectors, more commonly seen in high-energy physics research, have been used to track tracer particles in flows at very high acquisition rates (~ 70 kHz) [18–20]. The full 3D velocity gradient tensor was estimated with a 3D-PTV system by Luthi *et al* [21]. An

adaptive PTV approach was developed by Rossi *et al* [22] to improve the acceleration estimation process from particle tracking data. This algorithm was also used in conjunction with PIV (Eulerian) data, where the motion of virtual tracer particles were integrated in time and Lagrangian acceleration computed from their motion [23]. Gerashchenko *et al* [24] presented a PTV system where a sled holding the camera and illumination optics was traversed down the length of a wind tunnel at the bulk flow velocity.

Lasers are often the most expensive component of a PIV/A or PTV/A system, and require safety training. By using pulsed LEDs for illumination, and a single color camera, CPSV/A is a less expensive, safer alternative to the previously described PIV/A systems. The use of LED illumination of particle shadows is not without challenges, however. Higher magnification must be used with CPSV/A as compared to PIV/A to isolate a narrow depth of field, and to properly resolve particle shadows. Scattered laser light in PIV results in useful particle images even when the tracer particles are smaller than the image resolution. CPSV requires larger tracer particles and/or higher magnification to resolve the particle shadows. The high magnification requirement for PSV is compounded for color imaging, due to the Bayer filter over the camera's sensor. The impact of the Bayer filter on spatial resolution has been addressed by Gogineni *et al* [25]. While this paper focuses on color imaging with a single camera sensor and a Bayer filter, cameras with separate CCD chips for each color channel (R, G, B) are also available. These cameras have been used, for example, to perform simultaneous measurement of the velocity field on several color-coded planes [26].

In this paper, an extension of CPSV to the measurement of local acceleration fields is presented. The CPSA system consisted of a single color camera, with a Bayer filter over the sensor, and a pulsed multicolor LED light source for illumination. Unsteady acceleration fields, in a fixed frame, were estimated from difference of subsequent velocity vector fields. The first experiment measured the local acceleration of unsteady rigid body rotation, similar in spirit to Dong *et al* [11]. The second measured the acceleration root-mean-square in fully-developed turbulent pipe flow, previously characterized with CPSV [3].

2. Multicolor particle shadow velocimetry and accelerometry

2.1. Velocity measurement

This section presents a brief overview of the elements of CPSV necessary to perform local acceleration measurements. For a detailed description of the CPSV technique and an approach to compensate for the issues that arise with multicolor illumination, the reader is directed to [1–3].

Figure 1 shows a schematic of a CPSV system used in this study. A color camera and a multicolor LED lamp are placed on opposite sides of the flow, on the same optical axis, which is perpendicular to the measurement plane. The camera images particle shadows produced by the lamp. The camera lens is used to isolate the field of view, with high magnification to ensure a narrow depth of field. A diffuser is best placed between the lamp and flow field to improve uniformity of the illumination. The field of view is set by focusing the camera on a calibration target placed in the flow field, prior to measurement. This

calibration process also provides means of correcting for any optical distortions, as occur between illuminations by different colors (see, e.g. [3]).

This technique can be used with single color LEDs and a gray-scale camera however; the temporal dynamic range is increased with the use of multi-color LEDs and a color camera. Multicolor PSV (CPSV) was introduced by Goss *et al* [1, 2]. CPSV uses the different color components of a color camera sensor as separate recording channels.

In CPSV, the flash of each lamp color is individually triggered during a single camera exposure. Flashing each lamp color at different times during a camera exposure allows multiple particle shadow images, each corresponding to a different color flash, in a single image. Prior to displacement estimation, the color image is converted into three monochrome images by separating and interpolating the intensity distributions corresponding to each illumination color, using the Bayer filter configuration on the camera's sensor. The holes in the single color intensity maps are filled using bilinear interpolation. The single color intensity maps are then inverted by subtracting the intensity maps from an average of a subset of these intensity maps. For example, an average 'blue' intensity map is computed for subsets of 50 such intensity maps. Each individual blue monochrome image is inverted by subtracting its intensity distribution from the computed average image. These inverted images, with low-intensity backgrounds and high-intensity particle images, are suitable for processing with PIV software to estimate displacements.

The use of multicolor illumination can give rise to a bias error in displacement estimates due to chromatic aberration. This bias error is the result of variations in refraction of the different illumination colors of light along the optical path. Other errors can be introduced by crosstalk between the various illumination colors. More information on chromatic aberration and color crosstalk and practical corrections for each are presented in [3].

2.2. Local acceleration measurement

Extending a CPSV system to acceleration measurements requires no additional hardware. Multicolor illumination provides three possible velocity estimates within a single frame. More combinations are possible for frame-straddled illumination. Even for standard frame rate cameras, very small time delays are possible, given that overdriven LED lamps can operate well into the kilohertz range. Using a high-speed camera results in improved time resolution between velocity and acceleration field estimates.

In this work, a simple, first-order differencing of two successive velocity fields is used to estimate the local acceleration field. The choice of this acceleration time delay naturally depends upon the expected time scales of the measurement, as well as the measurement noise levels. Too large an acceleration time delay results in uncorrelated velocity samples, while and too small a time delay results in an acceleration estimate dominated by noise. As will be demonstrated, these two competing effects bound the selection of acceleration time delay.

Most of the PIA techniques described here employ a first order finite-difference scheme, and, the present work has adopted this same approach. The particular form used here has the

convenient property of alleviating bias errors due to chromatic aberration. Velocity bias error is eliminated in the acceleration estimate when the velocities are differenced. This notion is discussed further in section 3.2.

It should be noted that CPSA offers a variety of possible acceleration estimation schemes, which leverage the improved temporal resolution and multiple velocity estimates obtainable from multicolor illumination/imaging. For example, with double-shutter image acquisition, up to three separate illumination pulses per frame are available. Five sequential velocity fields with similar illumination time delays are thus attainable from a double image pair. More velocity estimates are also available from other combinations of image pairs. These velocity estimates could be used in a second-order finite-difference estimation of local acceleration. CPSV is also capable of acquiring multiple coincident velocity field estimates by simultaneously firing all illumination colors in each frame of a double-image pair. Each color channel of the camera then provides an independent velocity estimate for averaging to ameliorate the effects noise. All the above schemes, which exploit the multicolor capability of CPSV, require chromatic aberration corrections to avoid bias errors in displacement estimation. These schemes are not discussed here, and will be the subject of future work.

3. Demonstration of CPSA using pendulum

A CPSA validation experiment was conducted on a pendulous disk undergoing unsteady rigid body rotation. The pendulum apparatus provided a repeatable local acceleration field to calibrate the CPSA measurement system, similar in spirit to the PIV/A study by Dong *et al* [11]. Here CPSV/A is used to measure velocity and local acceleration fields. These measurements are converted to angular velocity and acceleration and compared to simultaneous recordings by an accelerometer mounted on the disk.

3.1. Hardware setup

Figure 2 shows the pendulum apparatus, consisting of an unbalanced disk, driven by gravity, and undergoing rigid body rotation. The disk, constructed from optically clear acrylic, pivoted on an acetal plastic bearing placed at the disk center. A mass was placed 3.8 mm off the center of rotation of the disk to enhance the pendulous motion. A random arrangement of 0.5 mm dots printed on a transparency and fixed to the disk served as seed particles. An ADXL 212 integrated accelerometer (Analog Devices, Norwood, Massachusetts) was fixed to the disk, along the line from the center of rotation to the added mass, to record the tangential acceleration. The recording axis of the accelerometer was aligned in the plane of rotation, as well as tangent to rotation. An Arduino Duemilanove microcontroller, which contains a 10-bit Atmel (San Jose, California) A/D converter, was used to acquire the accelerometer signal at 100 Hz. Calibration of the accelerometer was conducted following the procedure outlined by Godfrey *et al* [27]. The disk was positioned such that the accelerometer axis was pointing upwards or downwards with respect to vertical, and recording +1 *g* or -1 *g*, respectively. The sensitivity of the accelerometer was then determined from these recordings. The net tangential acceleration recorded by the accelerometer contains both gravitational and inertial contributions [27]:

$$(a_r)_{\text{REC}} = \alpha r + g \sin \theta \quad (1)$$

The angle between the pendulum arm and vertical is given by θ and r is the distance of the accelerometer from the center of rotation. CPSA measurements are converted to angular acceleration α by projecting the $\mathbf{u}/t$ vector onto the unit vector tangential to disk motion, and then dividing by the radial position r . Angular acceleration estimates must then be reformulated using equation (1) to be compared directly with the accelerometer recordings. The first term on the right hand side is given by the angular acceleration measured with CPSA multiplied by the radial distance from the accelerometer. The second term is the gravitational acceleration measured by the accelerometer, which varies with the angular position of the disk, θ . The velocity results were integrated using trapezoidal integration to compute θ . This enabled estimation of the second term, allowing comparison between accelerometer and CPSA measurements.

A Basler (Ahrensburg, Germany) 504kc color camera fitted with a 60 mm Nikon (Tokyo, Japan) lens was used for imaging. The camera was set to record single frame images at 50 Hz with a spatial resolution of 1024×1024 pixels. Illumination was provided by a multicolor LED lamp manufactured by Innovative Scientific Solutions Incorporated (Dayton, Ohio). A Berkeley Nucleonics (San Rafael, California) Model 555 delay generator was triggered by a TTL pulse indicating the opening of the camera shutter and used to control the LED lamp. The accelerometer and CPSA system were temporally aligned by a synchronization pulse sent from the delay generator and recorded by the microcontroller.

Figure 3 shows a synchronization of camera shutter and the red and blue illumination. In each camera exposure, images illuminated by one red and one blue pulse each were recorded. Velocity was estimated using the two images from each camera frame, and the acceleration was computed using the velocity difference between subsequent frames. The camera exposure was 5 ms in duration, with blue and red color lamp flashes occurring at 0.5 ms and 4.5 ms, respectively, relative to the beginning of each camera exposure. Individual color lamp pulses were 5 μ s in duration. The time delay used for velocity calculation was then $t_v = 4$ ms and the time delay for acceleration calculations was $t_a = 20$ ms (the time between subsequent exposures). Image calibration was conducted by imaging a ruler whose end was placed at the center of rotation of the disk. This allowed for identification of the center of rotation in the images, necessary because the center of rotation was located outside the image field of view. A magnification scale factor was also obtained from the portion of the ruler in the image field of view.

3.2. Image processing and displacement estimation

Color images were first color-separated, interpolated, and inverted as discussed in section 2. Velocity vector fields were estimated from the cross correlation of blue- and red-illuminated intensity maps extracted from the same color image. Each color image therefore provided a single velocity vector field. Displacement estimation was conducted in Davis 8.0 by LaVision (Goettingen, Germany). A standard FFT based correlation was used with a multi-

pass scheme starting at a window size of 128×128 pixels, with a second pass of 64×64 pixels. A nearest neighbor filter was used to remove vectors that were three standard deviations from the median of neighboring vectors. A post-processing step of a single pass of a 3×3 averaging mask was passed over the field to smooth the vector fields.

Local acceleration fields were estimated by differencing subsequent velocity fields, shown in equation (2):

$$\frac{\partial \mathbf{u}}{\partial t}(\mathbf{x}, t) \approx \frac{\mathbf{u}\left(\mathbf{x}, t + \frac{\Delta t_a}{2}\right) - \mathbf{u}\left(\mathbf{x}, t - \frac{\Delta t_a}{2}\right)}{\Delta t_a} \quad (2)$$

Due to the manner in which the acceleration is discretized, the resulting local acceleration field is assigned a time index exactly between the times at which the velocity fields are recorded. Because the two velocity fields that are differenced are similar, i.e. the same illumination color combination and time delays were used; any bias error introduced by chromatic aberration [3] affects each similarly. Therefore, differencing of these two velocity fields removes the chromatic aberration bias, to the extent that these effects are roughly stationary in time. For other acceleration calculation schemes, the chromatic aberration should be corrected for at the velocity field level. Corrections for chromatic aberration are described in detail in [3].

Finally, the tangential components of the velocity and local acceleration of the disk estimated with CPSA were computed. The tangential direction of disk rotation was determined from the center of rotation of the pendulous disk and the vector location. Vectors measured with CPSV/A were then projected onto the tangential direction, and divided by their radial position on the disk, to estimate angular velocities and accelerations.

3.3. Uncertainty analysis

Measurement uncertainty was estimated following the approach of Petrie *et al* [28]. Uncertainties were estimated by uniformly shifting a CPSV image a specified pixel amount. Non-integer shifts were conducted using bilinear interpolation. An image pair composed of the original image and the shifted image was then processed using PIV software and the estimated displacement was compared to the imposed shift. This approach gives an estimate of systematic uncertainty of a displacement estimate (such as pixel locking), and estimates of single sample random uncertainties, with all estimated error sources lumped together. This uncertainty estimation process characterizes the cumulative contributions of PIV/PSV error sources such as illumination, particle size, and pixel resolution, in a practical manner.

3.4. Results and discussion

3.4.1. Vector fields—Figure 4 shows 2D vector field results from the pendulum experiment. Consecutive velocity vector fields are shown in figures 4(a) and (b), and the local acceleration field computed from their difference is shown in (c). The coordinate system origin was located at the pendulum center of rotation. The velocity fields show the expected pattern of rigid body rotation, where speed increases with distance from the center

of rotation, and that the disk was rotating counter-clockwise at this portion of the motion. The vector direction of the local acceleration field in figure 4(c) is opposite that of the velocity fields, showing that the disk was decelerating.

3.4.2. Velocity and acceleration profiles—Figure 5 shows time traces of angular velocity and acceleration from the pendulum experiment. Figures 5(a) and (b) show time traces of angular velocity and acceleration, respectively, at three radial locations on the disk. These results provide an internal comparison of CPSA measurement capabilities. Angular velocity waveforms at different radial locations collapse to a single curve and are all within CPSV uncertainty. A mean squared difference of $3 \times 10^{-4} \text{ rad s}^{-1}$ is found between angular velocity waveforms, 0.01% of the maximum angular velocity value. The angular acceleration waveforms exhibit noise at the crests and troughs, due to reduced signal-to-noise in the angular velocity measurements at low speeds. The mean squared difference between waveforms is 0.19 rad s^{-2} , 1.9% of the maximum angular acceleration value.

Figure 6 compares CPSA results to accelerometer readings. CPSA results are recast using equation (1) for comparison with accelerometer recordings. The mean squared difference between acceleration waveforms measured by CPSA and by the accelerometer is 0.17 m s^{-2} , 2.6% of the maximum accelerometer recording.

4. Demonstration of CPSA in fully-developed turbulent pipe flow

Here CPSA is demonstrated in the near wall region of a turbulent pipe flow ($y^+ < 75$). A previous experiment, in the same facility, compared velocity statistics measured by CPSV and LDV [3]. CPSA results are compared to PIA measurements by Christensen and Adrian [5, 6] taken in a turbulent channel flow. Table 1 shows a comparison of relevant flow and measurement parameters between the two experiments. The Reynolds numbers, given by the tunnel diameter, or channel half width, and the friction velocity, for the two flows are close, as is the absolute size of the measurement field of view. However, the present experiment, using glycerin as the working fluid, has better spatial resolution, focused on the near wall region ($y^+ < 75$). The temporal resolution, relative to the Kolmogorov time scale, is comparable for the two experiments.

4.1. Methods

4.1.1. Glycerin tunnel facility—Figures 7 and 8 show sketches of the glycerin tunnel facility of the Applied Research Laboratory at the Pennsylvania State University. The working fluid is ~99% glycerin and tunnel conditions (glycerin density, glycerin viscosity, flow velocity, pressure drop, and Reynolds number) are controlled through glycerin temperature. A heat exchanger is used to remove friction-generated heat to maintain a desired temperature in the working fluid. A Gould (Seneca Falls, New York) 3450 centrifugal pump and 100HP Westinghouse (Round Rock, Texas) Life-Line electric motor drive the flow. General description of the test facility can be found in [29–31] and an example of CPSV in the glycerin tunnel in [3].

The 1.1 m long acrylic test section has a circular interior cross-section with an inner diameter of 285 mm and square exterior (figure 8). The test section is preceded by 6.6 m of

285 mm diameter stainless steel pipe. A 10 mm serrated trip ring at the entrance of the stainless steel pipe ensures fully developed turbulent flow roughly nine pipe diameters upstream of the test section.

Image calibration was conducted by placing a clear NT57-981 Edmund Optics (Barrington, New Jersey) glass target with printed dots in the field of view. The target dots are 0.25 mm in diameter with centers 0.5 mm apart, in a rectangular array. A scale factor was derived from the known spacing of the dots.

4.1.2. Image acquisition—Imaging was performed using a PCO Dimax camera, controlled with Camware 2.19. An ISSI RGB LED lamp was used for illumination. It was observed that in glycerin green-illuminated images suffered from poor contrast. As a result only results for red and blue illumination are reported. The lamp was placed facing the camera, along the optical axis of the camera, on the opposite side of the tunnel. The camera shutter and LED lamp were controlled by TTL pulses from a Berkeley Nucleonics 555 digital waveform generator. A sequence of 1400 images was acquired at 1900 Hz, where each image was exposed for 470 μ s. Each image was exposed to each color only once. Images were treated as pairs for illumination control (denoted A and B in table 2). Lamp flashes were triggered relative to the beginning of the first image of the pair, as indicated in table 2. All lamp flashes had a duration of 5 μ s.

The time delay used for acceleration estimation was chosen such that, when normalized by the Kolmogorov time scale, it matches the normalized delay used by Christensen and Adrian. The Kolmogorov time scale was estimated by:

$$t_K = \left(\frac{\nu}{\varepsilon_B} \right)^{1/2}, \quad \varepsilon_B = \frac{u_{\star}^2 U_{cl}}{4D} \quad (3)$$

where ε_B is the bulk dissipation for a turbulent pipe flow, and U_{cl} is the centerline velocity [5, 29]. This normalization ensured the measurements resolved similar flow time scales [5]. The CPSA acceleration time delay was thus set to $t_a = 863 \mu$ s, giving a normalized time delay of $t_d/t_K = 0.27$, to match $t_d/t_K = 0.28$ used by Christensen and Adrian.

4.1.3. Displacement estimation—Velocity was estimated from cross correlation of intra-frame blue and red intensity maps, i.e. each velocity field was obtained from a single color exposure. The displacement engine was a standard FFT based correlation algorithm, as implemented in LaVision 8.0. A multi-pass scheme was used—a first pass of 64×64 pixels, a second and third pass of 32×32 pixels, with 50% window overlap. A nearest neighbor filter removed vectors that exceeded three standard deviations from the median of neighboring vectors. Finally, a single pass 3×3 averaging mask was convolved with the velocity field for smoothing noise reduction.

4.1.4. Local acceleration computation—Local acceleration fields were estimated by the differencing of two velocity fields, according to equation (2). A post-processing noise correction to the local acceleration fields was implemented, following the process developed

by Christensen and Adrian [5, 6]. A separate zero-time-delay measurement set was recorded, with blue and red lamp pulses fired simultaneously within each image frame. These zero-time-delay measurements allowed estimation of the acceleration measurement noise floor. The difference of two zero-time-delay velocity fields provides an estimate of the CPSA measurement noise, and can be used for correction of the standard deviation of u [5, 6]

$$\langle \Delta u_i(\mathbf{x}, \Delta t_a)^2 \rangle_{\text{corr}}^{1/2} = (\langle \Delta u_i(\mathbf{x}, \Delta t_a)^2 \rangle - \langle \Delta u_i(\mathbf{x}, 0)^2 \rangle)^{1/2} \quad (4)$$

where u_i is the different of the i th component of velocity, $\langle \cdot \rangle$ indicates ensemble average, and subscript corr is the corrected value.

Uncertainty for CPSA results were estimated using the expressions from Benedict and Gould [32], which are limited to random uncertainty contributions. Uncertainties shown are 95% confidence interval bands around CPSA results at $y^+ = 20, 40, 60$.

4.2. Results and discussion

4.2.1. Vector fields—Figure 9 shows velocity and local acceleration vector fields in the glycerin tunnel taken with CPSV/A. The mean centerline velocity of the glycerin tunnel (8.8 ms^{-1}) was subtracted from each velocity vector, so that turbulent flow structures are more visible. The advection of a structure is shown between the velocity fields of figures 9(a) and 9(b) at $y^+ \approx 40$. The difference of these two velocity fields, in figure 9(c), has the characteristic ‘mushroom’ acceleration signature of an advected vortical structure, similar to those observed by Christensen and Adrian [5, 6].

4.2.2. Acceleration statistics—The standard deviation of the u (streamwise) and v (wall-normal) components is shown in figure 10, along with the noise floor estimates from zero-time-delay measurements. The signal to noise ratio (SNR) is greater than two for $y^+ \geq 10$ for both components. The noise contributes substantially to both components for $y^+ < 10$, and is especially apparent in the v profile near the wall. Both profiles in figure 10(b) show a fluctuation at $y^+ \approx 35$ where the edge of a clear plug in the glycerin tunnel adds an imaging artifact. The plug is used to gain access to the tunnel to place instrumentation like calibration targets.

Figure 11 shows local acceleration standard deviation, measured by both CPSA and PIA, normalized with $u_\star$ and $t_\star$. Acceleration standard deviation estimates include the noise correction given in equation (4). The normalization used follows Christensen [5], where local acceleration standard deviation profiles were shown to collapse across Reynolds number, when normalized by the friction velocity and Kolmogorov time scale. These results indicated that, in an Eulerian reference frame, the largest velocity fluctuations occur on short temporal scales.

Normalized acceleration standard deviation from CPSA and PIA measurements show good agreement away from the wall ($y^+ > 40$). The large discrepancies in the streamwise component found for $y^+ < 40$, are likely due to differences in spatial resolution. In addition,

the few points nearest the wall from the PIA data may suffer from an over correction of the near wall noise. The Christensen and Adrian present data in the range of $0 < y^+ < 550$. Only the roughly 15% of their total range which overlaps the current results is shown here. Similar effects of the wall are also evident in the CPSA data set, for the points nearest the wall. The current results have a factor of seven (in wall units) improvement in spatial resolution, so suffer less from spatial filtering than the PIA results. As noted by Christensen [5], the dynamics that contribute the most to the standard deviation of the local acceleration signal take place at short temporal scales. Furthermore, the size of motions contributing most to acceleration are likely to become smaller as the wall is approached, so that spatial filtering affects the acceleration estimate most strongly nearest the wall.

Further from the wall, PIA results agree with CPSA measurement to within the CPSA uncertainty. The wall-normal component shows a bump at $y^+ \approx 35$, due to an imaging artifact caused by the glycerin tunnel plug.

4.2.3. Acceleration standard deviation: variation with time delay—Here we explore how local acceleration standard deviation depends on t_a , the time delay used to compute instantaneous acceleration, i.e. the time between the two velocity estimates. Acceleration in a turbulent flow occurs over a broad range of time scales. The choice of t_a affects the acceleration standard deviation estimate magnitude, where acceleration fluctuations faster than t_a may not contribute to the estimate. As such, t_a acts as a filter parameter. Ideally, the time delay is chosen so that the maximum signal energy contributes to acceleration standard deviation, while excluding as much contribution from noise as possible. The contributions of noise, and associated corrections for it, have been addressed in detail for PIA [5, 6] and are not repeated here. Results in this section are shown relative to the noise floor and do not have any noise correction applied. First, for reasons that will become clear, the velocity difference used to compute the acceleration is presented. Second, the acceleration standard deviation estimate using t_a is presented. Both results are used in guidance for the choice of t_a .

Standard deviation of u and v at $y^+ = 10$ are shown in figure 12 as a function of t_a . Also plotted are curves indicating bounds for the standard deviation profiles, set by measurement noise and velocity statistics. As t_a is increased beyond the velocity integral time scale, the measurement is bounded by the statistics of the velocity field. The velocity difference variance is given by:

$$\text{var} [v(t + \Delta t_a) - v(t)] = \text{var} [v(t + \Delta t_a)] + \text{var} [v(t)] - 2\text{cov} [v(t + \Delta t_a) - v(t)] \quad (5)$$

For long enough t_a , the two velocities become uncorrelated, and their variances become essentially identical, for a stationary velocity signal. Thus, u and v standard deviation estimates are bounded at long acceleration time delay by $\sqrt{2}$ times σ_u and σ_v , the respective velocity standard deviations. Figure 12 shows that as the acceleration time delay is increased, u and v standard deviation estimates approach this bound. The relative distance between the bounds is smaller for the wall normal component. While streamwise and wall normal components have similar noise levels at $y^+ = 10$, the velocity standard deviation is

lower for the wall normal component. Note that the points corresponding to the CPSA acceleration standard deviation estimates given in figures 10 and 11 are denoted by filled symbols, for reference.

Figure 13 shows the data from figure 12 as local accelerations. Profiles of u'/t and v'/t standard deviation are shown, normalized by the friction velocity u_* and the Kolmogorov time scale t_K . The curves bounding the measurements are put into proper form by dividing by t_a , and are similarly normalized with the friction velocity and Kolmogorov time scales.

5. Summary

An extension of multicolor particle shadow velocimetry to measure local acceleration was presented. Local acceleration fields were estimated by differencing successive velocity fields, which were obtained in the usual manner.

Two proof of concept studies were performed. In the first, rigid-body motion of a pendulous disk was measured both by CPSA and an accelerometer mounted to the disk. CPSA measurements compared well with accelerometer recordings, within measurement uncertainty. In the second study, measurements of acceleration statistics in a fully-developed turbulent pipe flow were measured. These were compared to PIA measurements performed earlier by Christensen and Adrian [5, 6]. Improvements in hardware since the time of that study, as well as the use of glycerin for the working fluid, resulted in improved spatial and temporal resolution. Comparison of the two measurement techniques at nearly identical temporal resolution compared favorably, within uncertainty. The results diverged near the pipe wall, due to a combination of noise and the improved spatial resolution in the current study. Improved temporal resolution also allowed the acceleration to be estimated for a range sampling times, showing the acceleration standard deviation increases with the inverse of sampling time.

The CPSA technique provides velocity and local acceleration measurement capabilities using a single camera and a single illumination source. The simplicity of the setup, and its low cost, make it an attractive alternative compared to PIA, when the distance between illumination and imaging is short, and small fields of view are acceptable. The results presented here are limited by temporal resolution such that the acceleration estimates do not seem to converge for the range of differencing time delays used. Further work is necessary, therefore, to quantify how temporal resolution and velocity measurement noise combine to filter the acceleration estimate, and whether this approach can yield acceleration estimates independent of t_a for such turbulent flow cases. The use of multiple velocity estimates enabled by the multicolor illumination and imaging may provide a way forward.

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References

1. Goss, L; Estevadeordal, J. Parametric characterization for particle-shadow velocimetry (PSV). 25th AIAA Aerodynamics Measurement Technology and Ground Testing Conf; San Francisco, CA. 5–8 June 2006; 2006.
2. Goss, L; Estevadeordal, J; Crafton, J. Kilohertz color particle shadow velocimetry (PSV). 37th AIAA Fluid Dynamics Conf. and Exhibit; 25–28 June 2007; Miami, FL. Reston, VA: American Institute of Aeronautics and Astronautics; 2007.
3. McPhail MJ, Fontaine AA, Krane MH, Goss L, Crafton J. 2015; Correcting for color crosstalk and chromatic aberration in multicolor particle shadow velocimetry. *Meas Sci Technol.* 26:025302.
4. Willert C, Stasicki B, Klinner J, Moessner S. 2010; Pulsed operation of high-power light emitting diodes for imaging flow velocimetry. *Meas Sci Technol.* 21:075402.
5. Christensen, KT. PhD Thesis. University of Illinois Urbana-Champaign; 2001. Experimental investigation of acceleration and velocity fields in turbulent channel flow.
6. Christensen K, Adrian RJ. 2002; Measurement of instantaneous Eulerian acceleration fields by particle image accelerometry: method and accuracy. *Exp Fluids.* 33:759–69.
7. Liu X, Katz J. 2006; Instantaneous pressure and material acceleration measurements using a four-exposure PIV system. *Exp Fluids.* 41:227–40.
8. Jensen A, Sveen JK, Grue J, Richon JB, Gray C. 2001; Accelerations in water waves by extended particle image velocimetry. *Exp Fluids.* 30:500–10.
9. Jensen A, Pedersen GK. 2004; Optimization of acceleration measurements using PIV. *Meas Sci Technol.* 15:2275.
10. Jakobsen ML, Dewhurst TP, Greated CA. 1997; Particle image velocimetry for predictions of acceleration fields and force within fluid flows. *Meas Sci Technol.* 8:1502.
11. Dong P, Hsu TY, Atsavapranee P, Wei T. 2001; Digital particle image accelerometry. *Exp Fluids.* 30:626–32.
12. Borowsky J, Wei T. 2006; Simultaneous velocimetry/accelerometry measurements in a turbulent two-phase pipe flow. *Exp Fluids.* 41:13–20.
13. Kurtulus DF, Scarano F, David L. 2007; Unsteady aerodynamic forces estimation on a square cylinder by TR-PIV. *Exp Fluids.* 42:185–96.
14. Foucaut JM, Coudert S, Stanislas M. 2009; Unsteady characteristics of near-wall turbulence using high repetition stereoscopic particle image velocimetry (PIV). *Meas Sci Technol.* 20:074004.
15. David L, Jardin T, Farcy A. 2009; On the nonintrusive evaluation of fluid forces with the momentum equation approach. *Meas Sci Technol.* 20:095401.
16. Lynch KP, Scarano F. 2014; Material acceleration estimation by four-pulse tomo-PIV. *Meas Sci Technol.* 25:084005.
17. Virant M, Dracos T. 1997; 3D PTV and its application on Lagrangian motion. *Meas Sci Technol.* 8:1539.
18. La Porta A, Voth GA, Crawford AM, Alexander J, Bodenschatz E. 2001; Fluid particle accelerations in fully developed turbulence. *Nature.* 409:1017–9. [PubMed: 11234005]
19. Voth GA, La Porta A, Crawford AM, Bodenschatz E, Ward C, Alexander J. 2001; A silicon strip detector system for high resolution particle tracking in turbulence. *Rev Sci Instrum.* 72:4348–53.
20. Voth GA, la Porta A, Crawford AM, Alexander J, Bodenschatz E. 2002; Measurement of particle accelerations in fully developed turbulence. *J Fluid Mech.* 469:121–60.
21. Lüthi B, Tsinober A, Kinzelbach W. 2005; Lagrangian measurement of vorticity dynamics in turbulent flow. *J Fluid Mech.* 528:87–118.
22. Ferrari S, Rossi L. 2008; Particle tracking velocimetry and accelerometry (PTVA) measurements applied to quasi-2D multi-scale flows. *Exp Fluids.* 44:873–86.
23. Rossi L, Bocquet S, Ferrari S, Garcia de la Cruz JM, Lardeau S. 2009; Control of flow geometry using electromagnetic body forcing. *Int J Heat Fluid Flow.* 30:505–13.
24. Gerashchenko S, Sharp NS, Neuscamman S, Warhaft Z. 2008; Lagrangian measurements of inertial particle accelerations in a turbulent boundary layer. *J Fluid Mech.* 617:255–81.

25. Gogineni S, Goss L, Pestian D, Rivir R. 1998; Two-color digital PIV employing a single CCD camera. *Exp Fluids*. 25:320–8.
26. Pick S, Lehmann FO. 2009; Stereoscopic PIV on multiple color-coded light sheets and Its application to axial flow in flapping robotic insect wings. *Exp Fluids*. 47:1009–23.
27. Godfrey, A; Hourigan, T; Laighin, GMO. Pendulum analysis of an integrated accelerometer to assess Its suitability to measure dynamic acceleration for gait applications. 2007 Annual Int. Conf. of the IEEE Engineering in Medicine and Biology Society; Lyon, France. 22–26 August 2007; Piscataway, NJ: IEEE; 2007. 4891–4.
28. Petrie, H; Fontaine, A; Krane, M; Weldon, M; Elbing, B; Buchanan, J; Lorenz, D. PIV Uncertainty Workshop; Las Vegas, NV. 11–13 May 2011; 2011.
29. Bakewell HP Jr, Lumley JL. 1967; Viscous sublayer and adjacent wall region in turbulent pipe flow. *Phys Fluids*. 10:1880.
30. Chevrin, PA. PhD Thesis. The Pennsylvania State University; 1988. The structure of Reynolds stress in the near wall region of a turbulent pipe flow.
31. Fontaine, AA. PhD Thesis. The Pennsylvania State University; 1993. Suppression of the near wall burst process of a fully developed turbulent pipe flow.
32. Benedict LH, Gould RD. 1996; Towards better uncertainty estimates for turbulence statistics. *Exp Fluids*. 22:129–36.

Figure 1. Schematic of a color particle shadow velocimetry (PSV) system. The color camera and color LED lamp are shown on either side of a particle laden flow field. The field of view is noted, and is set by the camera's lens. The color camera images tracer particle shadows cast by the LED lamp.

Figure 2.

Schematic of the pendulous disk experiment, and CPSA system. LED lamp casts shadows of 'seed particles' on the rigid disk. Particle shadows are imaged with the Basler high-speed color camera. Pendulum acceleration is measured using accelerometer attached to disk. Synchronization of the camera shutter, illumination pulses, and accelerometer digitization is accomplished with digital waveform generator, which is triggered by sync pulse from camera shutter. Waveform generator then sends control TTL pulses to accelerometer A/D and LED lamps (see also figure 3).

Figure 3.

Timing diagram for CPSA, illustrating camera and LED lamp activity as a function of time.

Time delays t_a used to estimate velocity, and t_a to estimate acceleration, are shown.

Figure 4.

Subsequent instantaneous velocity (*a*, *b*) and local acceleration (*c*) fields of the pendulum measured with CPSA. Every other vector is removed for clarity. Velocity fields are a result of cross-correlation of blue and red monochrome images. The local acceleration field is computed as the difference of the two shown velocity fields. Reference vectors are shown in each image.

Figure 5.

Time traces of angular velocity (*a*) and angular acceleration (*b*) of the pendulum measured with CPSA. +: $r = 6.6$ cm, ○: $r = 7.8$ cm, △: $r = 10.1$ cm, —: Accelerometer.

Figure 6.

Time traces of angular acceleration of the pendulum measured with CPSA. +: $r = 6.6$ cm, O: $r = 7.8$ cm, Δ: $r = 10.1$ cm, —: Accelerometer.

Figure 7.

Schematic of the glycerin tunnel facility (not to scale), reproduced from [3]. Measurements were conducted in transparent acrylic test section (see figure 8). Flow in upper leg reaches fully-developed condition approximately 1.7 m upstream of acrylic test section. Working fluid temperature is controlled using a heat exchanger (not shown). Because working fluid viscosity is a strong function of temperature, the heat exchanger is used to control flow speed.

Figure 8. Schematic of test section, showing dimensions and location of measurement field of view (FOV), reproduced from [3]. (a) Side view, (b) end view. Nominal measurement FOV dimensions were 21.2 mm (streamwise) $\times$ 42.3 mm (radial).

Figure 9.

Example velocity and acceleration fields in fully developed turbulent pipe flow. (a, b) Successive instantaneous velocity fields, differenced to estimate (c) local acceleration. Bulk flow is from left to right. A uniform streamwise velocity (8.8 m s^{-1}) has been subtracted from the velocity fields. Spatial resolution has been reduced by a factor of three for clarity.

Figure 10.

Standard deviation profiles of u (streamwise) (a) and v (wall-normal) (b) components from CPSA ($\square$) with the noise floor from zero-time-delay measurements (---). Uncertainty is shown to a 95% confidence interval at $y^+ = 20, 40, 60$.

Figure 11.

Standard deviation of the time derivative of velocity for streamwise (a) and wall-normal (b) components normalized with friction velocity $u_\star$ and the Kolmogorov time scale t_κ . Profiles from CPSA ($\square$) with ($t_d/t_\kappa = 0.27$) and Christensen and Adrian [7] ($\circ$) with ($t_d/t_\kappa = 0.28$) are shown.

Figure 12.

Standard deviation of u and v at $y^+ = 10$. (a) Streamwise and (b) wall-normal components are shown. Standard deviation estimates are plotted as a function of the acceleration time delay Δt_a . Velocity difference bounds are given by the noise floor (---) and $\sqrt{2}$ times the velocity standard deviation (—). The measurement corresponding to the time delay used in figures 10 and 11 is denoted by a filled square.

Figure 13.

Local acceleration standard deviation at $y^+ = 10$, normalized by friction velocity $u_\star$ and the Kolmogorov time scale t_κ . Standard deviation estimates are plotted as a function of the acceleration time delay t_a normalized by the Kolmogorov time scale. (a) Streamwise and (b) wall-normal components. The noise floor (---) and $\sqrt{2}$ times the velocity standard deviation (—) are shown, divided by t_a , and similarly normalized. The measurement corresponding to the time delay used in figures 12 and 13 denoted by a filled square.

Table 1

Comparison of flow and measurement parameters between glycerin tunnel CPSA and Christensen and Adrian [5, 6] wind tunnel PIA experiments.

	CPSA	Christensen and Adrian PIA
Working fluid	Glycerin	Air
Configuration	Cylindrical tunnel	Rectangular channel
Image size (pixels)	1008×2016	1000×1016
Measurement domain (mm)	21.2×42.3	25.4×25.4
Wall-normal range (wall units)	$0 < y^+ < 75$	$0 < y^+ < 550$
Spatial resolution (wall units)	0.65	4.74
# of velocity pairs	1400	875
Friction velocity u_* (m s ⁻¹)	0.42	0.34
$Re_* = u_* D / \nu$ or $u_* h / \nu$	544	550
Acceleration time delay t_a (μ s)	863	200
Inner time scale t_b (μ s)	1247	137
Kolmogorov time scale t_K (μ s)	3172	723

Table 2

Illumination timings used in glycerin tunnel experiment, for two subsequent images.

Frame	Flash	Time (μ s)
A	Blue	4
	Red	124
B	Blue	866
	Red	986

Note: Times indicated are given relative to the first shutter opening, as depicted in figure 3.