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Theoretical assessment of unsteady aerodynamic effects in phonation

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Abstract

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NOMENCLATURE

f = frequency

$\bar{F}_f$ = friction force on air in glottis

G = net force on glottal flow (transglottal pressure difference minus friction force)

$h(x, t)$ = glottal half-width

h_A, h_S = glottal half-width at x_A and x_S , respectively

L = axial length of vocal folds

L_d = boundary layer development length scale

L_{eff} = effective axial length of glottis

LG = vocal fold length

n_i = i th component of unit normal vector on vocal fold wall

p = air static pressure

Re = Reynolds number, Uh_S/ν , based on glottal width h_S

S_A, S_S = physical cross sectional areas at x_A and x_S , respectively

S'_S = aerodynamic cross sectional area at x_S , corrected for viscous blockage

S_{wall} = area of vocal fold walls

St = Strouhal number $U/(LT)$

t = time

T = Period of vocal fold vibration

$T_O, T_A, T_{US}, T_D, T_C$ = time scales describing: interval glottis is open, glottal flow acceleration/deceleration, interval time air displacement by vocal fold wall motion is important, viscous diffusion across boundary layer, and motion of separation point, respectively

u = axial component of velocity on centerline

u_i = component of air velocity vector in i th coordinate direction

U_g = glottal volume flow rate

v_W = velocity of vocal fold walls

V_{AB} = glottal volume between axial points x_A and x_B

$\dot{V}_{AB}$ = Time rate of change of V_{AB}

Wo = Womersley number $[h^2/(T\nu)]^{1/2}$

x_i = i th component of spatial coordinate $i=1$: axial, $i=2$, transverse

p_{AD} = pressure difference between x_A and x_D (transglottal pressure difference)

h_S = change in h_S during vocal fold vibration

δ_S = boundary layer thickness in glottis, at separation point

μ = air dynamic viscosity

ν = air kinematic viscosity, μ/ρ

ρ = air density

This paper ranks the importance of unsteady aerodynamic mechanisms in glottal flow. Particular emphasis is given to separation point motion, acceleration of glottal airflow by vocal fold motion, and viscous blockage. How nondimensional parameters such as the Reynolds, Strouhal, and Womersley numbers help in this ranking is also addressed. An equation of motion is derived which includes terms explicitly describing the effects of interest, assuming (1) a symmetrical glottis, (2) zero pressure recovery downstream of the vocal folds, and (3) a quasisteady glottal jet. Estimating the order of magnitude of the terms in this equation, it is shown that the flow is characterized by two temporal regimes: (1) a flow initiation/shutoff regime where local unsteady acceleration and wall motion dominate, and (2) a “quasisteady” regime where the flow is dominated by convective acceleration. In the latter case, separation point motion and viscous blockage are shown to be out of phase with motion of the vocal folds, thereby impacting the shape of the glottal volume flow waveform. The analysis suggests that glottal flow may be considered quasisteady only insofar as traditional assumptions concerning glottal jet behavior can be confirmed.

I. INTRODUCTION

This paper presents a theoretical approach to determine the dynamic relevance of unsteady aerodynamic effects during phonation. The particular focus is on how time variations in (1) inertia, (2) viscous blockage, (3) vocal fold wall motion, and (4) separation point motion influence the shape of the glottal volume flow waveform, the physical determinant of voice power and quality. In doing so, how these effects might cause departures from the commonly assumed quasisteady model of glottal airflow is also addressed. While these effects have previously received attention in the literature, their relative importance and influence on the glottal volume flow waveform have never been assessed directly using the equations of motion.

Consider the flow through the vocal fold during a single cycle of vibration. A schematic of the flow is shown in Fig. 1. Upon opening, air is forced through the glottis by the transglottal pressure gradient. Near the glottal exit, the flow separates, forming the glottal jet. The diffusion of this jet causes the pressure loss which the transglottal pressure gradient must overcome. This airflow continues until the vocal folds close at the end of the cycle. The simplest description of this flow is that the glottal volume flow has a simple dependence on only the transglottal pressure and glottal area. This description, which has been used widely in glottal aerodynamic modeling (see, e.g., Pelorson *et al.*, 1994; Story and Titze, 1995; Lous *et al.*, 1998), implicitly assumes a quasisteady glottal jet with minimal pressure recovery (Hirschberg, 1992). In this model, because the glottal volume flow changes instantaneously in response to changes in either transglottal pressure or glottal area, airflow in the glottis is quasisteady. Several recent studies (Mongeau *et al.*, 1997; Zhang *et al.*, 2002; Vilain *et al.*, 2004) seem to support this approximation.

Several other factors also mediate the relationship between the driving pressure gradient, glottal area, and volume flow. Due to its inertia, a finite length of time is needed for the air in the glottis to accelerate from rest [this was accounted for in the early glottal flow models of Ishizaka and Flanagan (1972)]. Similarly, at the end of the cycle, when the folds close, the flow is shut off over a finite duration during which the flow decelerates to rest. Along the

vocal fold walls, in the direction of the flow, there exist regions, the boundary layers, in which friction is important (see Fig. 1). These regions grow in thickness through diffusion and in response to time variations of glottal flow. Through the displacement effect (see, e.g., Rosenhead, 1963) the boundary layers can decrease the effective aerodynamic area of the glottis. This *blockage effect*, if large enough, increases the transglottal pressure difference needed to push air through the glottis at a given rate [see, e.g., discussion of entrance flow in a straight pipe in Bejan (1984)]. Near the glottal exit, the boundary layers separate from the vocal fold walls, forming the glottal jet. Separation is affected by the glottis shape, volume flow, and boundary layer thickness, as well as the rate at which these factors change in time (Hirschberg, 1992; McGowan, 1993). Finally, the motion of the vocal fold walls displaces air in the glottis, exchanging momentum with it (Flanagan and Ishizaka, 1976; Deverge *et al.*, 2003). Previous consideration of these phenomena has made use of nondimensional parameters such as the Reynolds number or the Strouhal number. Bejan (1984) has pointed out that the interpretation of these parameters is highly dependent on the flow being studied. A careful consideration of the meaning of nondimensional parameters in phonatory airflows has not been done.

In this article, a theoretical approach is taken to address these issues. An equation of motion is derived which explicitly includes the phenomena in question. Using the scales of motion observed in phonation, the order of magnitude of each of the terms in the equation is estimated. Assessing the order of magnitude of each term relative to transglottal pressure provides a clear basis for ranking each effect in terms of its dynamic importance. Using these estimates, the relevant time scales governing each effect are also revealed. The analysis also allows interpretation of nondimensional numbers such as the Reynolds, Strouhal, and Womersley numbers. Neglecting terms two orders of magnitude smaller than transglottal pressure leads to a simplified equation of motion. The solution of this equation is used to express the glottal volume flow in terms of the leading order effects.

II. APPROXIMATE DESCRIPTION OF GLOTTAL AERODYNAMICS

A. Problem definition

The geometry of the airflow through the vocal folds is shown in Fig. 1. The airflow proceeds from left to right through the constriction leading from the cylindrical trachea through the glottis, a rectangular slit, and into the cylindrical pharynx as shown. The coordinate axes show x_1 as the axial coordinate and x_2 the transverse coordinate. The vocal folds lie between $x_A < x_1 < x_C$, and their surfaces are defined by the function $x_2 = h(x_1, t)$ and the components $n_i(x_1, t)$ of the wall unit normal vector $\mathbf{n}$.

The local cross-sectional area $S(x_1)$ is defined as $S(x_1) = 2L_G h(x_1)$, where L_G is the length of the glottis in the direction perpendicular to the page in Fig. 1. The presence of boundary layers along the walls means the flow “sees” a smaller cross-sectional area, $S^*(x_1)$. The local flow cross-sectional area, $S^*(x_1)$, corrected for viscous blockage, is defined as $S^*(x_1) = 2(1 - \delta^*(x_1)/h(x_1))L_G h(x_1)$. Here, δ^* is the boundary layer displacement thickness, which depends on the shape of the local axial velocity profile (see, e.g., Rosenhead, 1963; Bejan, 1984), and which scales on the local boundary layer thickness δ .

The locations shown in the sketch denote the regions of the airflow: For $x_1 < x_A$, the air particle velocity is small enough that the motion is linear and essentially acoustic. The locations x_B and x_C denote the inlet and outlet, respectively, of the glottis. The axial location $x_1 = x_S$, downstream of the point of minimum constriction, is where the airflow separates from the wall and a planar jet is formed. In the present work separation point behavior will not be computed, but its behavior will be assumed to be consistent with that described previously (Hirschberg, 1992; Pelorson *et al.*, 1994; Liljencrants, 1996; Lous *et al.*, 1998; Alipour and Scherer, 2004), in which the separation point moves rapidly during glottal opening and closure. The separation point location is important because it indicates not only the distribution of aerodynamic forces driving vocal fold vibration, but also the volume of air allowed to pass through the glottis. In other words, glottal volume flow $U_g = u_S S'_S$. Thus the movement of the separation point helps determine the temporal behavior of U_g . At some point further downstream, $x_1 = x_D$, the jet has diffused enough, through the action of friction and turbulent mixing, to reattach to the vocal tract walls.

The airflow in the region $x_A < x_1 < x_D$ is analyzed using the following assumptions:

1. *The tracheal and pharyngeal cross-sectional areas (S_A and S_D , respectively) are assumed approximately equal, and much larger than the glottal area. In other words, $S_A \approx S_D \gg S_S$.*
2. *The airflow is incompressible*, equivalent to saying that the airflow fluctuations are slow enough that they are communicated to the rest of the region in question during a time interval much shorter than the period of the fluctuation. In other words, the region of interest is acoustically compact—see, e.g., Hirschberg (1992).
3. Vocal fold shape, its motion, and the airflow are symmetrical about the mid-sagittal plane. Although there are indications (see, e.g., Hofmans *et al.*, 2003) that the glottal jet becomes asymmetrical, for purposes of this work, where orders of magnitude are sought, symmetry is assumed. Note that this previous work does not indicate asymmetric flow upstream of the separation point for a symmetrical glottis.
4. No pressure recovery downstream of the separation point (*in the region $x_S < x_1 < x_D$*). *In addition, this process is quasisteady.* This assumption has been widely used in phonation modeling (Ishizaka and Flanagan, 1972; Pelorson *et al.*, 1994; Story and Titze, 1995). This assumption approximates the air pressure downstream of the point of separation to be uniform and equal to ambient pressure p_D . In other words, the pressure $p_S = p_C = p_D$, so that the transglottal pressure difference $p_A - p_D \approx p_A - p_S$. This approximation implicitly assumes that the jet flow is established very rapidly compared to the other time scales of the flow and of vocal fold motion. Note that if this assumption is correct, any asymmetric jet motions that may occur (see, e.g., Hofmans *et al.*, 2003) are not dynamically relevant.
5. The flow in the glottal flow boundary layers is laminar. While this assumption finds support from the experiments of Hofmans *et al.* (2003), Deverge *et al.*

(2003), and Vilain *et al.* (2004), it appears to be unresolved at present. For present purposes, this assumption is sufficient—a turbulent boundary layer will exhibit a higher rate of transverse diffusion and a different separation point location, but qualitatively the boundary layer behaves in the same manner (Bejan, 1984). Note that the boundary layers being laminar does not preclude the glottal jet from transitioning to a fully turbulent flow.

6. Separation point motion occurs during brief intervals (compared to the time the glottis is open), and moves a distance of order of the axial length of the glottis. This assumption has been used previously (Hirschberg, 1992; Liljencrants, 1996; Lous *et al.*, 1998). Although the work of Pelorson *et al.* (1994) and Alipour and Scherer (2004) conflict in terms of the direction of separation point motion, both support this assumption. In the present analysis the effect of separation point motion in both directions is discussed.
7. Convective acceleration in the converging glottis, between the trachea and the separation point, is the dominant mechanism by which transglottal pressure gradient accelerates the airflow. This is equivalent to saying that the quasisteady Bernoulli equation, in which the axial acceleration of the flow in the glottis inlet is balanced by transglottal pressure, is the lowest-order model of phonatory airflow, as described in the Introduction. As a result, all other phenomena are expected to be corrections to this model. This assumption is consistent with the success of past efforts to model vocal fold vibration.

B. Derivation of equation describing glottal flow

The glottal flow equation is derived in a manner similar to the derivation of Bernoulli's equation, but for a more general situation. First, the axial momentum equation is integrated along the axial direction in the region $x_A < x < x_S$ extending from the glottis entrance to the moving separation point. Then, mass conservation is used to express all appearances of velocity in terms of the velocity at the separation point.

The momentum equation along the symmetry streamline ($x_2=0$) is given by

$$\rho \left(\frac{\partial u}{\partial t} + \frac{1}{2} \frac{\partial u^2}{\partial x_1} \right) + \frac{\partial p}{\partial x_1} = 0, \quad (1)$$

where ρ is the fluid density, u is the axial component of velocity, p is the static pressure, and x_1 is the coordinate in the axial direction. Integrating this equation along the symmetry streamline from $x_1=x_A$ to $x_1=x_S(t)$, the following equation is obtained:

$$\rho \frac{d}{dt} \int_{x_A}^{x_S(t)} u(x_1) dx_1 - \rho u_S \dot{x}_S + p_S - p_A + \frac{\rho}{2} (u_S^2 - u_A^2) = \frac{\int_{x_A}^{x_S(t)} \tau_w(x_1) dx_1}{S_{\text{wall}}} \quad (2)$$

where Liebnitz's rule (see Panton, 1994) was used to move the time derivative outside the integral in the first term, resulting in the appearance of a new term which explicitly includes the speed of motion of the separation point. An additional term containing τ_w , the wall shear stress, also appears on the right-hand side, representing the head loss due to friction on the glottis walls. The mass conservation equation (see the Appendix) yields

$$u(x) = \frac{\dot{V}_{xS}}{S'(x)} + (u_S - \dot{x}_S) \frac{S'_S}{S'(x)}. \quad (3)$$

Here $\dot{V}_{xS}$ is the time rate of change of glottal volume from $x_1=x$ to x_S , and $S'(x)$ is the cross-sectional area of the glottis at x_1 (corrected for viscous blockage), to rewrite the integrand. Using this result, the unsteady acceleration term becomes

$$\rho \frac{d}{dt} \int_{x_A}^{x_{S(t)}} u(x) dx = \rho \frac{d}{dt} \int_{x_A}^{x_{S(t)}} \frac{\dot{V}_{xS}}{S(x)} dx + \rho \frac{d}{dt} \left((u_S - \dot{x}_S) \int_{x_A}^{x_{S(t)}} \frac{S'_S}{S(x)} dx \right). \quad (4)$$

Now the mass conservation equation is used to rewrite the u_A^2 term in Eq. (2) in terms of u_S and the speed of the separation point. After some rearrangement, the following equation results:

$$\begin{aligned} & \underbrace{\rho \frac{d}{dt} (u_S L_{\text{eff}})}_{\text{I}} - \underbrace{\rho \frac{d}{dt} (\dot{x}_S L_{\text{eff}})}_{\text{II}} + \underbrace{\rho \frac{d}{dt} \int_{x_A}^{x_{S(t)}} \frac{\dot{V}_{Ax}}{S(x,t)} dx}_{\text{III}} - \underbrace{\rho u_S \dot{x}_S}_{\text{IV}} + \underbrace{\rho \frac{u_S^2}{2} \left(1 - \left(\frac{S'_S}{S_A} \right)^2 \right)}_{\text{V}} \\ & - \underbrace{\rho u_S \frac{2S'_S}{S_A^2} \dot{V}_{AS}}_{\text{VI}} - \underbrace{\rho u_S \frac{2S'^2_S}{S_A^2} \dot{x}_S}_{\text{VII}} - \underbrace{\rho \frac{\dot{V}_{AS}^2}{S_A^2}}_{\text{VIII}} - \underbrace{2 \frac{\dot{V}_{AS}^2}{S_A^2} S'_S \dot{x}_S}_{\text{IX}} + \underbrace{\frac{S'^2_S}{S_A^2} \dot{x}_S^2}_{\text{X}} + \underbrace{p_S - P_A}_{\text{XI}} = \underbrace{\frac{\int_{x_A}^{x_{S(t)}} \tau_w(x) dx}{S_{\text{wall}}}}_{\text{XII}} \end{aligned} \quad (5)$$

where

$$L_{\text{eff}} = \int_{x_A}^{x_{S(t)}} \frac{S_S}{S(x)} dx \sim L \quad (6)$$

(see, e.g., Pelorson *et al.*, 1994). The first four terms in Eq. (5) represent unsteady acceleration of the air in the glottis. From left to right, these terms represent (I) unsteady acceleration of glottal volume flow, (II) a correction term due to the unsteady motion of the separation point, (III) acceleration due to the motion of the vocal fold walls, and, finally, (IV) a second correction due to the motion of the separation point. The first term (I) is the familiar unsteady acceleration term that appears even when vocal fold wall and separation point motion is neglected. Terms V-X represent convective acceleration of the air in the

glottis, including contributions from movement of the walls (VI, VIII, and IX) and the separation point (VII, IX, and X). The last two terms are the two axial forces acting on the flow: (XI) the transglottal pressure, where assumption 3 has been invoked so that $p_A - p_D = p_A - p_S$, and (XII) the net friction force. Note that vocal fold wall motion, separation point motion, and blockage ratio contribute not only to the unsteady acceleration, but also to the convective acceleration.

If the effects of wall motion and separation point movement are neglected, then Eq. (5) reduces to the familiar unsteady Bernoulli equation, as expressed in Hofmans *et al.* (2003), supplemented by friction:

$$\rho \frac{d}{dt}(u_S L_{\text{eff}}) + \rho \frac{u_S^2}{2} \left[1 - \left(\frac{S'_S}{S_A} \right)^2 \right] + p_S - p_A = \frac{\int_{x_A}^{x_{S(t)}} \tau_W(x) dx}{S_{\text{wall}}}, \quad (7)$$

III. SIMPLIFICATION BY SCALE ANALYSIS

Having derived Eq. (5), which describes the motion of air in a time-varying glottis, the next step is to determine the hierarchy of effects, by ranking each term according to its order of magnitude. By comparing these estimates to those for the dominant terms, three results are obtained: (1) the dynamic relevance of the effect represented by each term, (2) the duration over which a given term might be dynamically relevant, and (3) the nondimensional parameters describing the flow and their interpretation.

In this section, the scales of motion are first identified in Sec. III A. In Sec. III B, the technique just described is applied to Eq. (7) in order to demonstrate the approach. Section III C then studies the behavior of viscous blockage from boundary layers, and under what conditions this effect is unsteady. In Secs. III D and III E, the effects of wall motion and movement of the separation point, respectively, are then determined using the same approach. Section III F then uses the results of Secs. III A-III E to develop a simplified equation of motion, and in Sec. III G the implications for the behavior of the glottal jet speed and the glottal volume flow waveform are discussed.

A. Scales of motion

The length scales describing glottal flow are those which describe the volume of the time-varying glottis itself. The length of the volume under consideration is $L \sim x_C - x_A \sim L_{\text{eff}}$, which is approximately 1 cm for adult humans. The glottal volume is approximately this length times the cross-sectional area, $S'_S(t)$, which is approximately $h_S(t) L_G (1 - \delta_S/h_S)$. The length scale describing the maximum glottal width, h_S , is of order 1 mm.

There are several time scales describing glottal flow. The first two characterize the motion of the vocal fold walls: the period of vocal fold vibration, T , and the length of time, T_o , that the glottis is open. Because the interest is the order of magnitude of terms, rather than their exact value, these two quantities will be used interchangeably: $T_o \sim T$. Here, motions of order 100 Hz (speaking voice for adult humans) are considered, so T is approximately 0.01 s. The

unsteady acceleration of glottal flow during flow initiation and shutoff has its own time scale, T_a . A second time scale of acceleration, T_{us} , relates the time during which the acceleration of the air in the glottis is dominated by motion of the vocal fold walls. Another relevant time scale, T_c , describes separation point motion. Finally, there is a time scale, T_v , governing the rate of diffusion of the boundary layers. This time scale controls the behavior of viscous blockage.

The scales which characterize the velocities involved in glottal flow are the flow speed at the separation point u_s , which is approximately 30 m/s, and the transverse speed of the vocal fold walls $v_W \approx h_s/T \sim O(0.1)$ m/s. Here, and in what follows the notation $f \sim O(x)$ means that f has the same order of magnitude as x (see, e.g., Bejan, 1984).

B. Example: Traditional model—wall motion, separation point neglected

Before considering Eq. (5) in full, the “traditional” case, where acceleration due to wall motion and separation point motion are neglected, will be addressed. This will demonstrate how the current approach applies in this familiar case, in which the only effects considered are the local unsteady flow acceleration, the convective acceleration, the driving transglottal pressure gradient, and friction, which acts to resist the flow.

1. Unsteady acceleration versus convective acceleration—In the traditional case, it is clear that the driving force (transglottal pressure difference) will accelerate the air using two mechanisms, (1) unsteady acceleration, which emphasizes time variations in the flow speed, and (2) convective acceleration, in which the flow is accelerated spatially by squeezing the air through the narrowing glottal inlet. Traditionally, the unsteady acceleration has been neglected. The most concise statement of how this neglect is justified is given by Hirschberg (1992). The magnitude of the two acceleration terms are estimated as follows using scales of motion:

$$\rho \frac{d}{dt}(u_s L_{\text{eff}}) \sim \rho \frac{u_s L}{T}, \quad \rho \frac{u_s^2}{2} \left(1 - \left(\frac{S'_S}{S_A} \right)^2 \right) \sim \rho u_s^2, \quad (8)$$

where T is the period of vibration and $S'_S / S_A \sim (10^{-2})$, by assumption. The ratio of the unsteady to the convective acceleration terms thus has order of magnitude $\text{St} = (L/u_s T)$, where St is the Strouhal number. Because St is the ratio of the unsteady and the convective acceleration, and because $\text{St} \sim O(10^{-2})$, the argument goes, unsteady acceleration may be safely neglected. The soundness of this argument will now be considered.

Note that the estimate of the unsteady acceleration uses the vibration cycle period T as its time scale. But does that time scale really characterize the motion of the air? Measurements of glottal jet speed (see, e.g., Berke *et al.*, 1989; Alipour *et al.*, 1995; Mongeau *et al.*, 1997) suggest that in fact the glottal jet speed u_s changes much more rapidly, especially during flow initiation and shutoff. During initiation, the flow speed is small, but rapidly changing. As a result, the nonlinear convective acceleration terms (proportional to u_s^2) are small

compared to the local unsteady acceleration term (proportional to u_S). Once the flow is initiated, however, the rate of change of flow speed decreases, but the flow is still being accelerated spatially (i.e., convectively) through the glottal inlet. This suggests that two temporal regimes exist for the flow: (1) a regime at the beginning and end of the cycle, of duration T_a , in which the unsteady acceleration dominates, and (2) a regime in the middle of the cycle, of approximate duration $T_o - 2T_a$, in which the convective acceleration may dominate. This model has support from Mongeau *et al.* (1997). What fraction of the cycle do each of these regimes occupy? In other words, what is T_a/T ?

T_a is the time it takes for the flow to initiate, so it is also a measure of the duration over which the flow has accelerated sufficiently that the two acceleration terms are the same order of magnitude:

$$\rho \frac{d}{dt}(u_S L_{\text{eff}}) \sim \rho \frac{u_S^2}{2} \left(1 - \left(\frac{S'_S}{S_A} \right)^2 \right). \quad (9)$$

The order of magnitude of the unsteady acceleration term is then estimated as

$$\rho \frac{d}{dt}(u_S L_{\text{eff}}) \sim \rho \frac{u_S L}{T_a} \sim \rho \frac{u_S L}{T} \frac{T}{T_a}. \quad (10)$$

Combining (8) and (9),

$$\rho \frac{u_S L}{T} \frac{T}{T_a} \sim \rho \frac{u_S^2}{2}. \quad (11)$$

Rearranging these, the following relation is obtained:

$$\frac{L}{u_S T} \frac{T}{T_a} \sim \text{St} \frac{T}{T_a} \sim O(1) \quad (12)$$

From this result, the fraction of the cycle occupied by the unsteady regime is given by

$$\frac{T_a}{T} \sim \text{St}. \quad (13)$$

Thus the Strouhal number can be interpreted as the fraction of the vibration period that it takes for the flow to accelerate from, or decelerate to, rest. This fraction is a few percent of the vocal fold vibration period for adult speech, but may reach 10% for infants, and an even greater percentage for high-frequency sung notes.

If Eq. (7) is sufficient to describe glottal flow, then in the second, middle temporal regime, it reduces to a quasisteady Bernoulli equation, corrected by wall shear stress,

$$\rho \frac{u_S^2}{2} - \Delta P_{AD} = \frac{\int_{x_A}^{x_{S(t)}} \tau_W(x) dx}{S_{\text{wall}}} = \bar{F}_f, \quad (14)$$

which is quadratic in u_S , so that u_S may be expressed as

$$u_S = \sqrt{\frac{2(\Delta P_{AD} - \bar{F}_f)}{\rho}}, \quad (15)$$

Note that friction acts to oppose the effect of the transglottal pressure difference, reducing the jet speed u_S .

2. Skin friction behavior—Friction is often neglected in glottal flow modeling (see, e.g., Story and Titze, 1995; Pelorson *et al.*, 1994; Lous *et al.*, 1998). The accuracy of this approximation may be found by comparing the relative orders of magnitude of the friction and convection terms. The friction term may be rewritten as

$$\begin{aligned} \bar{F}_f &= \frac{\int_{x_A}^{x_{S(t)}} \tau_W(x) L_G dx}{\int_{S_A}^{S_{S(t)}} dS_{\text{wall}}} \\ &= \frac{\int_{x_A}^{x_{S(t)}} \frac{1}{2} \rho u(x)^2 c_L \sqrt{\nu / u(x)} L_G dx}{\int_{S_A}^{S_{S(t)}} dS_{\text{wall}}}, \end{aligned} \quad (16)$$

where (see, e.g., Bejan, 1984) τ_W , for a laminar boundary layer, is proportional to $\rho u(x_1)^2 c_L \text{Re}^{1/2} (x_1/h_S)^{1/2}$. In Eq. (15), ν is the kinematic viscosity, the ratio μ/ρ of the dynamic viscosity and the fluid density. Here, c_L is in general a factor of $\mathcal{O}(1)$ (e.g., 5.0 for a laminar boundary layer on a flat plate in a uniform flow), which depends on the instantaneous glottal shape, and $\text{Re} = u(x_1)h_S/\nu$ is the Reynolds number based on glottal width, h_S . By using the mass conservation equation (neglecting displacement of air by wall motion), the friction term may be further rewritten as

$$\begin{aligned}\bar{F}_f &= \frac{L_G}{S_{\text{wall}}} \int_{x_A}^{x_{S(t)}} \tau_W(x) dx \\ &= \frac{L_G}{S_{\text{wall}}} \frac{1}{2} \rho u_S^2 h_S^2 c_L \sqrt{\frac{\nu}{u_S h_S}} \int_{x_A}^{x_{S(t)}} \frac{dx}{\sqrt{h(x, t)^3 x}}.\end{aligned}\quad (17)$$

The integral is dominated by its behavior where $h(x, t)$ is minimum, which can be approximated as a $h_S \times h_S$ region, where $h(x, t) \approx h_S$. As a result, the order of magnitude of friction force term is

$$\bar{F}_f \sim \frac{L_G h_S}{S_{\text{wall}}} \rho u_S^2 c_L \sqrt{\frac{\nu}{u_S h_S}}. \quad (18)$$

Using $S_{\text{wall}} \sim L_G L$, the order of magnitude of the friction term relative to the convective acceleration is then

$$\frac{\bar{F}_f}{\rho_\infty (u_S^2 / 2)} \sim \frac{h_S}{L} c_L \sqrt{\frac{\nu}{u_S h_S}}, \quad (19)$$

so that the relative contribution of friction to the dynamics of glottal flow is proportional to the ratio of the glottal aspect ratio h_S/L and the square root of the Reynolds number $\text{Re} = u_S h_S / \nu$. The traditional argument states that the Reynolds number, the ratio of inertia to friction, is sufficient to determine whether friction is negligible. Here, it is seen that the glottal aspect ratio also plays a role.

3. Summary of analysis of traditional case—The preceding analysis shows how, for the traditional case, estimation of the order of magnitude of the terms in the equation of motion may be used to determine not only which terms are dominant, but also the duration of each term's importance. It was shown that two temporal regimes exist, one in which the unsteady acceleration dominates, and another in which the convective acceleration dominates. In addition, the interpretation of nondimensional parameters is clarified. First, the Strouhal number is seen to set the fraction of the vibration period that the unsteady acceleration will dominate. Second, the Reynolds number alone is not sufficient to ascertain the importance of friction.

C. Unsteadiness of viscous blockage

As described above, viscous blockage reduces the effective glottal aerodynamic cross-sectional area S'_G , relative to the true cross-sectional area S_G . Mongeau *et al.* (1997) and Zhang *et al.* (2002) have quantified this effect using discharge coefficients, assuming that during phonation, the boundary layers are quasisteady. In this case, S'_G and S_G are in phase.

Deverge *et al.* (2003) and Vilain *et al.* (2004) have also studied the behavior of viscous blockage, intuitively deriving a scaling law for the blockage ratio and using it to interpret measurements from a model glottal flow. Neither study provided direct evidence that the boundary layers are in fact quasisteady. In fact, Deverge *et al.* (2003) showed that friction is likely greater during glottal closure, which suggests that the boundary layers are not quasisteady. In this section, a relation for the viscous blockage ratio is derived from the equation of motion. In addition, the possibility that the boundary layers are unsteady (S'_S and S_S are *not* in phase) is considered.

Using

$$\frac{\delta_S}{L} \sim \frac{\delta_S}{h_S} \frac{h_S}{L} \sim c_L \sqrt{\frac{\nu}{u_S h_S}} \sqrt{\frac{h_S}{L}} \quad (20)$$

(see, e.g., Bejan, 1984), where δ_S is the boundary layer thickness, Eq. (19) can be rewritten as:

$$\frac{\bar{F}_f}{\rho(u_S^2/2)} \sim \frac{\delta_S}{L} \sqrt{\frac{h_S}{L}} \sim \frac{\delta_S}{h_S} \left(\frac{h_S}{L}\right)^{3/2}, \quad (21)$$

which shows that the contribution of friction is directly proportional to the blockage ratio. Equating the rightmost term of (21) to the right-hand side of relation (19), the blockage ratio δ_S/h_S is shown to be

$$\frac{\delta_S}{h_S} \sim \frac{c_L}{\sqrt{Re_{h_S}}} \sqrt{\frac{L}{h_S}}, \quad (22)$$

supporting the arguments of Deverge *et al.* (2003) and Vilain *et al.* (2004) that the controlling parameter for the importance of blockage is Re_{h_S}/L , where friction and viscous blockage become most important as $Re_{h_S}/L \rightarrow 0$.

Under what conditions is a boundary layer and the resulting viscous blockage unsteady? The temporal response of a boundary to a change in the flow is governed by the time it takes for the change in the core flow to diffuse through the boundary layer. Thus, for a laminar boundary layer, where diffusion is governed by molecular processes, the viscous diffusion time $T_{v \sim \delta_S^2/\nu}$. The character of the time response of the boundary layer is governed by the ratio of T_v to the period of vibration. If $T_v/T \ll 1$, then changes are communicated through the boundary layer instantaneously, the boundary layer grows very rapidly, and the flow is quasisteady. If $T_v/T \gg 1$, then the boundary layer growth rate is so slow that the boundary layers are thin for the whole vibration cycle and the blockage ratio is negligible. If, however, $T_v/T \sim O(1)$, then the boundary layers grow appreciably during the cycle and may thus be

thicker during glottal closure than during opening. In this case, viscous blockage depends not only on glottal area and Reynolds number, but also on the time since the glottis opened. The result is that S'_S experiences a phase lag relative to S_S , a lag of order T_ν .

The ratio of diffusion time to glottal period is given by

$$\frac{T_\nu}{T} \sim \frac{\delta_S^2}{\nu T} \sim \frac{h_S^2}{\nu T} \frac{\delta_S^2}{h_S^2} \sim \text{Wo}^2 \frac{\delta_S^2}{h_S^2}, \quad (23)$$

where the Womersley number, Wo (based upon glottal width), is given by

$\text{Wo} = (h_S^2 / (\nu T))^{1/2}$. In this relation, Wo^2 appears as a proportionality between T_ν/T and the blockage ratio (squared). In other words, (1) the higher the blockage ratio, the more likely the boundary layer will be unsteady, and (2) the Womersley number gives the intensity of this effect. Wo in glottal flow is 2.6, using the scales from Sec. III A. Thus, for full blockage ($\delta_S/h_S=0.5$), $T_\nu/T=1.69$, so it appears that the boundary layers will be unsteady when the blockage ratio is high. When the blockage is small, e.g., $\delta_S/h_S=0.01$, then $T_\nu/T \sim O(10^{-4})$. The crossover, where $T_\nu/T=1$, occurs for a blockage ratio of 0.43. During the closure of the glottis, the boundary layers are thicker than during opening, and decreasing glottal area will likely result in an increase in the instantaneous blockage ratio during that interval. As a result, viscous blockage depends explicitly on time, so that S'_S and S_S are out of phase, especially during closure.

D. Acceleration by wall motion

Now the importance of the terms of Eq. (5), which describe the effect of wall motion, is determined, for the interval $T_a < t < T_o - T_a$, in which the convective acceleration dominates relative to the local unsteady acceleration, as described in Sec. III B above. In this section, the terms relating to separation point motion are neglected for simplicity. The equation of motion for glottal flow in this case is given by

$$\rho_\infty \frac{u_S^2}{2} + p_A - p_D + \bar{F}_f = \rho_\infty \frac{\dot{V}_{AS}^2}{S_A^2} + \rho_\infty u_S \frac{2S'_S}{S_A^2} \dot{V}_{AS} - \rho_\infty \frac{d}{dt} \int_{x_A}^{x_S(t)} \frac{\dot{V}_{xS}}{S(x,t)} dx \quad (24)$$

The order of magnitude of the left-hand terms has already been discussed in the previous section. In order to estimate the right-hand side terms, the proper scaling for the rate of air volume displacement by wall motion, dV_{AS}/dt , and the integral term need to be determined. The volume V_{xS} , between $x_1=x > x_A$ and $x_1=x_S$, is given by

$$V_{xS} = \int_x^{x_S} S(x_1, t) dx_1.$$

The time rate of change of this volume is

$$\dot{V}_{xS} = \int_x^{x_S} \dot{S}(x_1, t) dx_1 = 2 \int_x^{x_S} v(x_1, t) L_G dx_1.$$

For $x_1 = x_A$, this quantity has the following order of magnitude:

$$\dot{V}_{AS} \sim v_w L_G L. \quad (25)$$

For the first term on the right-hand side of Eq. (24), its magnitude relative to convection is

$$\frac{\rho(\dot{V}_{AS}^2 / S_A^2)}{\rho(u_S^2 / 2)} \sim \frac{\rho \dot{V}_{AS}^2}{\rho(u_S^2 S_S'^2 / 2)} \left(\frac{S_S'}{S_A} \right)^2 \sim \left(\frac{v_w S_{\text{wall}}}{u_S S_S'} \right)^2 \left(\frac{S_S'}{S_A} \right)^2. \quad (26)$$

Before estimating the magnitude of the other two wall motion terms, some attention will be given to interpretation of (26). It shows that the importance of wall motion is governed by the ratio of two volume fluxes, the air volume flux due to wall displacement, $v_w S_{\text{wall}}$, and the glottal throughflow volume flux $U_S S_S'$. Because $v_w \sim h_S / T_o$ and $S_S' \sim h_S L_G$, the volume flux ratio may also be written as

$$\frac{v_w S_{\text{wall}}}{u_S S_S'} \sim \frac{v_w L_G L}{u_S h_S L_G} \sim \frac{v_w L}{u_S h_S} \sim \frac{L}{u_S T} \sim St, \quad (27)$$

supporting an interpretation of Deverge *et al.* (2003) that the Strouhal number indicates the relative strength of volume flux due to wall displacement and the transglottal volume flux. Deverge *et al.* (2003) also noted that the volume flux ratio is likely small when the glottis is wide open, but it should become larger as the glottis closes and the air velocity decreases, resulting in an essentially unsteady flow. In this case,

$$\frac{v_w L_G L}{u_S S_S'} \sim \frac{v_w L}{u_S h_S} \sim O(1), \quad (28)$$

which means that the wall motion is important when

$$\frac{v_w}{u_S} \sim \frac{h_S}{L} \sim \frac{S_S'}{S_{\text{wall}}}. \quad (29)$$

When does this occur? The wall speed is never an appreciable fraction of the flow speed, except for the instants just upon opening and closure. During these intervals the flow has either just been initiated, or is being extinguished, but the walls are moving. However, if the glottis is long and narrow, the area over which displacement by wall motion takes place is

larger than the glottal area, so that the volume fluxes may then be commensurate. However, this is rarely the case. Clearly, the shape of the glottis is relevant in determining the importance of wall displacement, as shown by the experiments of Deverge *et al.* (2003).

Having estimated the order of magnitude of the rate of wall volume flux, the remaining wall motion terms in Eq. (24) may be likewise estimated. The magnitude of the righthand side terms, relative to convection, are given by

$$\frac{\rho(\dot{V}_{AS}^2/S_A^2)}{\rho(u_S^2/2)} \sim \text{St}^2 \left(\frac{S'_S}{S_A}\right)^2 \sim O(10^{-6}), \quad (30)$$

$$\frac{\rho u_S(2S'_S/S_A^2)\dot{V}_{AS}}{\rho(u_S^2/2)} \sim \text{St} \left(\frac{S'_S}{S_A}\right)^2 \sim O(10^4), \quad (31)$$

$$\frac{\rho \frac{d}{dt} \int_{x_A}^{x_S(t)} [\dot{V}_{xS}/S(x,t)] dx}{\rho u_S^2/2} \sim \frac{L^2}{u_S^2 T^2} \sim \text{St}^2 \sim O(10^{-4}). \quad (32)$$

Over what time scale are these terms important? The time interval T_{us} during which the wall volume displacement and glottal volume displacement are commensurate may be estimated by using $v_w \sim h_S/T_{us}$, where h_S is the change in glottal width during the time T_{us} during which wall motion is relevant. Then,

$$\frac{v_w L}{u_S h_S} \sim O(1) \sim \frac{\Delta h_S L}{u_S h_S T_{us}} \sim \frac{\Delta h_S L}{u_S h_S T} \frac{T}{T_{us}} \sim \text{St} \frac{T}{T_{us}} \frac{\Delta h_S}{h_S}. \quad (33)$$

T_{us} has the fraction of a vibration period:

$$\frac{T_{us}}{T} \sim \text{St} \frac{\Delta h_S}{h_S} \sim \frac{T_a}{T} \frac{\Delta h_S}{h_S}, \quad (34)$$

where (13) has been used. Thus the time interval during which wall displacement is important is of the same magnitude as the interval T_a (see Sec. III B 1, above) the flow takes to accelerate from, or decelerate to, zero. This suggests that glottal airflow acceleration by wall motion is negligible except during the initial and final few percent of the period the glottis is open, and is thus confined to that temporal regime.

E. Importance of separation point motion

Equation (5), after neglecting wall motion terms, may be rewritten in the following manner:

$$\rho \frac{u_S^2}{2} - \Delta p_{AD} + \bar{F}_f = -\rho \frac{d}{dt}(\dot{x}_S L_{\text{eff}}) + \rho_\infty u_S \dot{x}_S + \rho_\infty u_S \frac{2S_S'^2}{S_A^2} \dot{x}_S + 2 \frac{\dot{V}_{AS}}{S_A^2} S_S' \dot{x}_S - \frac{S_S'^2}{S_A^2} \dot{x}_S^2, \quad (35)$$

where the terms under consideration in this section have been moved to the right-hand side. If the separation point is assumed to move the length of the glottis in a time T_c , the separation point speed $dx_S/dt \sim L/T_c \sim (L/T)(T/T_c)$.

The magnitude of these separation point motion terms, relative to the convective acceleration term, which is of order ρu_S^2 , are thus given as follows:

$$\frac{\rho \frac{d}{dt}(\dot{x}_S L_{\text{eff}})}{\rho (u_S^2/2)} \sim \rho \frac{L^2}{u_S^2 T^2} \frac{T^2}{T_c^2} \sim \text{St}^2 \frac{T^2}{T_c^2} \sim O(10^{-4}) \frac{T^2}{T_c^2}, \quad (36)$$

$$\frac{\rho u_S \dot{x}_S}{\rho (u_S^2/2)} \sim \frac{L}{u_S T} \frac{T}{T_c} \sim \text{St} \frac{T}{T_c} \sim O(10^{-2}) \frac{T}{T_c}, \quad (37)$$

$$\frac{\rho u_S (2S_S'^2/S_A^2) \dot{x}_S}{\rho (u_S^2/2)} \sim \frac{L}{u_S T} \frac{T}{T_c} \frac{S_S'^2}{S_A^2} \sim \text{St} \frac{T}{T_c} \frac{S_S'^2}{S_A^2} \sim O(10^{-6}) \frac{T}{T_c}, \quad (38)$$

$$\frac{2\rho (\dot{V}_{AS}/S_A^2) S_S' \dot{x}_S}{\rho_\infty (u_S^2/2)} \sim \frac{v_W L_G L}{u_S S_S'} \frac{L}{u_S T} \frac{T}{T_c} \frac{S_S'^2}{S_A^2} \sim \text{St}^2 \frac{T}{T_c} \frac{S_S'^2}{S_A^2} \sim O(10^{-6}) \frac{T}{T_c}, \quad (39)$$

$$\frac{\rho (S_S'^2/S_A^2) \dot{x}_S^2}{\rho (u_S^2/2)} \sim \frac{L^2}{u_S^2 T^2} \frac{T^2}{T_c^2} \frac{S_S'^2}{S_A^2} \sim \text{St}^2 \frac{T^2}{T_c^2} \frac{S_S'^2}{S_A^2} \sim O(10^{-6}) \frac{T^2}{T_c^2}. \quad (40)$$

The first two terms are the largest relative to the convection term, the rest being of at least four orders of magnitude smaller. The importance of these two largest terms depends critically on the ratio T/T_c . If $T/T_c \sim O(10^2)$, for example, then these two terms have the same importance as convection. On the other hand, if $T/T_c \sim O(10)$, as suggested by the computations of Alipour and Scherer (2004), and the modeling of Pelorson *et al.* (1994),

then only the second term is relevant and represents a 10% correction to convection, when the separation point is in motion.

Thus, under the assumption that $T/T_c \sim O(10)$, all the separation point terms but the second are negligible, and Eq. (32) reduces to

$$\rho \frac{u_S^2}{2} - \rho \dot{x}_S u_S - \Delta p_{AD} + \bar{F}_f = 0. \quad (41)$$

Note that the surviving separation point motion term comes directly from the unsteady acceleration term [see Eq. (2)]. This result strongly supports the arguments of Sobey (1983) and McGowan (1993) that a moving separation point results in an inherently unsteady flow, as an approximately 10% correction to the convective acceleration.

F. Behavior of jet speed and glottal volume flow

The analysis of Secs. III B-III E has identified two temporal regimes, one in which unsteady effects such as local unsteady acceleration and wall motion are relevant, and one in which the only relevant unsteady mechanism is due to separation point motion. This article focuses on this second regime, in which Eq. (5) reduces to

$$\rho \frac{u_S^2}{2} - \rho \dot{x}_S u_S - G = 0, \quad (42)$$

where

$$G = \Delta p_{AD} - \bar{F}_f. \quad (43)$$

This equation is a quadratic in u_S , so

$$u_S = \dot{x}_S + \sqrt{\dot{x}_S^2 + \frac{2G}{\rho}}. \quad (44)$$

Given that the first term under the square root sign is $O(L^2 / T_c^2)$ and the second term is $O(u_S^2)$, their ratio is proportional to $Sr^2 T^2 / T_c^2 \sim O(10^{-2})$ and the approximate expression for the air speed at the separation point is

$$u_S = \dot{x}_S + \sqrt{\frac{2G}{\rho}} \quad (45)$$

From this equation the dominant effects determining flow through the glottis can be determined:

1. The jet speed u_S is determined primarily by the in-stantaneous net force $G = p_{AD} - F_f$. The two terms in G vary differently with time, but the time variations in u_S are in phase with the time variations in G .
2. Friction retards the flow, particularly toward the end of the cycle, as the flow is “squeezed” through the growing boundary layers by the closing vocal fold walls. As a result, friction increases with time during glottal closure, reducing the net driving force G at the end of the cycle. This behavior is also indicated by the experiments of Deverge *et al.* (2003).
3. The speed of separation point motion appears directly as a correction to the speed of glottal air flow. When the separation point moves downstream (is positive), the jet speed is enhanced, while upstream separation point motion retards jet speed. If the separation point moves upstream during closure, the jet speed will decrease faster than a Bernoulli prediction.

The glottal volume velocity $U_g = u_S S'_S$ is given by the following expression:

$$U_g \approx S'_S(t) \dot{x}_S + S'_S(t) \sqrt{2 \frac{G}{\rho_\infty}}. \quad (46)$$

From this expression, it can be seen that glottal volume flow waveform is affected by temporal fluctuations in three mechanisms: (1) G , (2) S'_S , through variations in both glottal geometry and the blockage ratio δ_S/h_S , and (3) movement of the separation point x_S , where the effect of separation point movement is seen to be a correction of $O(10^{-1})$. For example, the effect of the time variation of p_{AD} (due to vocal tract inertance) is that the volume velocity waveform is skewed such that the flow shutoff occurs more rapidly than the flow initiation. This effect is well known (see, e.g., Titze, 1994) to strongly affect the sound power and quality of the voice. From Eq. (46), it is clear that other mechanisms can further enhance the rapidity of the volume flow shutoff. First, an upstream movement of the separation point causes a deceleration of U_g , both because dx_S/dt is negative and because S'_S decreases with time in a diverging glottis. (A converse effect occurs if the separation point moves downstream.) Furthermore, the effective flow area S'_S is a strong function of the blockage ratio δ_S/h_S , which is expected to increase during the vibration cycle because h_S decreases much faster than δ_S during closure. This results in a faster decrease of the effective flow area S'_S than that of the true cross-sectional area S_S . Again, the validity of these statements does not necessarily apply to the final deceleration of the flow during the final few percent of the interval where the glottis is open, during which time the wall motion terms become important.

G. Discussion

The present analysis explains why *in vitro* experiments in which the vocal folds are mimicked by a time-varying diaphragm or shutter (see, e.g., Coker *et al.*, 1996; Mongeau *et al.*, 1997; Barney *et al.*, 1999; Zhang *et al.*, 2002) produce convincing replications of glottal

volume flow waveforms even though the glottis in these experiments does not change shape (from a converging to a diverging shape) during a glottal cycle. Because the volume flow is determined largely by the transglottal pressure gradient and the glottal flow area $S'_S(t)$, the vibration pattern is almost irrelevant, as long as $S'_S(t)$ has similar behavior to the real flow. Because separation point motion is due more to changing glottal shape than other factors, the behavior of the separation point [and hence $S'_S(t)$] in these experimental approximations *will* differ from that found in real vocal fold vibration. In the experiments mentioned, the flow separation most likely (1) occurs near the “glottal” exit and (2) does not move very much.

The results presented here rely on the validity of the underlying assumptions laid out in Sec. I, which are now discussed. The first assumption is that the glottal jet is essentially quasisteady, resulting in a uniform pressure downstream of the separation point (i.e., that $p_S \approx p_D$). Experimental evidence (Hofmans *et al.*, 2003) strongly suggests that this assumption is not correct. In fact, it seems likely that, because (1) the volume occupied by the jet and (2) the momentum distribution inside that volume both exhibit strong time variations (including turbulent fluctuations), $p_S - p_D$ is a nonzero function of time. Thus, the appearance of a “quasisteady” regime, identified using the theoretical approach used here, rests on the traditional assumption that the jet development is itself quasisteady. Recent studies (Mongeau *et al.*, 1997; Vilain *et al.*, 2004), which apparently confirm this approximation for glottal flow as a whole, did so using transglottal measurements across a model glottis in which the supraglottal pressure tap was either vented to an open room (Mongeau *et al.*, 1997; Deverge *et al.*, 2003; Vilain *et al.*, 2004) or placed far downstream of the glottal exit. This experimental arrangement essentially *enforces* the *assumed* condition that the pressure downstream of separation is uniform or, equivalently, that the jet is quasisteady. The degree to which the glottal jet is unsteady, and how it might thereby contribute to glottal flow unsteadiness, is currently under study by the authors.

Another assumption used is that the vocal fold motion and the flow are symmetric about the centerline. In a symmetrical glottis, the flow will not exhibit asymmetrical behavior upstream of the separation point. In other words, flow in a symmetrical glottis can only be asymmetrical in the jet region. The experiments of Hofmans *et al.* (2003) show that the glottal jet may become asymmetrical, but not the flow upstream of the separation point. The assumption of uniform pressure in the jet region is equivalent to saying that no matter what form glottal jet motion takes, its dynamic importance is limited to an instantaneous and total diffusion of jet momentum. In other words, the dynamic effect of flow asymmetry is negligible, under the assumption that the jet is quasisteady. For cases of mild vocal fold asymmetry, at least while the motion of the folds is synchronized, the same is likely true. Once vocal fold motion becomes desynchronized, vocal fold and flow asymmetry become severe, so the validity of the current analysis is likely compromised.

It is clear, then, that most unsteady effects can be neglected only if the glottal jet is quasisteady and the glottis is symmetrical. This strongly motivates a thorough investigation of the behavior of the glottal jet and the effect of vocal fold asymmetry on the unsteadiness of glottal flow.

IV. SUMMARY

This paper has presented a theoretical evaluation of the common assumption that glottal flow essentially involves a balance between transglottal pressure gradient and convective acceleration, i.e., that all other effects (separation point motion, wall motion, viscous effects such as friction and viscous blockage) are negligible. An equation of motion was derived in a form which explicitly includes the effects of vocal fold wall motion, viscous blockage, and motion of the separation point. Using the scales of motion describing phonation, the order of magnitude of the terms describing each effect was estimated and compared to that of the dominant convective acceleration term.

Two temporal regimes were identified. The first is dominated by unsteady effects, the local acceleration and acceleration by the vocal fold walls. The Strouhal number, which is typically $O(10^{-2})$, indicates the magnitude of this interval relative to the vibration period. The second temporal regime was the major focus of the paper: the interval in which the flow appears to be essentially quasisteady, with second-order corrections due to the motion of the separation point and unsteady diffusion of the boundary layers.

Expressions for the jet speed and volume flow rate were obtained from the simplified equation of motion. These show that the glottal volume flow waveform is determined primarily by the transglottal pressure gradient and the cross-sectional area at the separation point, but is also influenced by temporal fluctuations in three other factors: (1) the movement of the separation point, (2) the viscous blockage, and (3) friction, all of which are not likely to vary in phase with either the transglottal pressure or the glottal area. Thus these effects also contribute to the asymmetry of the volume flow waveform.

The importance of friction, relative to convection, was shown to be controlled by the quotient of the glottis aspect ratio and the square root of the Reynolds number, a result equivalent to the assertion of Deverge *et al.* (2003) that the blockage ratio is determined by the reciprocal of the square root of the product of Reynolds number and the aspect ratio. The Womersley number was also shown to reflect how strongly the blockage ratio affects the transverse diffusion time and the degree to which the true glottal area S_G and the effective glottal area S'_G are in phase. For typical normal phonation, an increase of the blockage ratio and a phase shift between S_G and S'_G are expected during glottal closure.

Finally, the above conclusions are only as valid as the assumptions upon which they are based. The assumptions that (1) glottal jet behavior is quasisteady and (2) that the pressure downstream of the separation point is uniform are currently under study.

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APPENDIX: MASS CONSERVATION EQUATION IN TIME-VARYING VOLUME

In this appendix the conservation of mass equation for the incompressible flow through a control volume V_{AB} bounded by a control surface S consisting of a time-varying inlet and

outlet, as well as impermeable, moving walls, is presented. From this equation, the expression for the flow speed at any location x_1 inside V_{AB} is derived. This relation is used to derive Eq. (5).

The volume is shown in Fig. 2. The inlet (area S_A) is located at $x_1=x_A(t)$, the outlet (area S_B) at $x_2=x_B(t)$. The wall orientation is denoted by the unit outward normal vector, n , whose components in the x_1 , x_2 , and x_3 directions are n_1 , n_2 , and n_3 , respectively. The conservation of mass equation for this case is given by (Panton, 1994)

$$\int_{S(t)} (v_i - u_i) n_i dS = - \frac{dV_{AB}}{dt}, \quad (A1)$$

where v_i is the i th component of the velocity of the wall. Because of the no-slip and no-flux boundary conditions, $v_i - u_i = 0$ on the solid walls, the only contributions from the surface integral come from the inlet and outlet, so that

$$(u_B - \dot{x}_B) S'_B - (u_A - \dot{x}_A) S'_A = - \frac{dV_{AB}}{dt}, \quad (A2)$$

where $u_A S'_A$ and $u_B S'_B$ are the inlet and outlet volume fluxes (volume velocities), respectively, averaged over their respective cross-sectional areas. In addition, S'_A and S'_B are the physical cross-sectional areas, corrected for viscous blockage. The other terms represent the change in V_{AB} by the motion of the inlet and outlet surfaces, at speeds dx_A/dt and dx_B/dt , respectively.

In the derivation of Eq. (6), an expression is needed for $u(x_1)$, i.e., u at a x_1 location inside V_{AS} . For a control volume bounded by $x_1=x$ on the right, and $x_1=x_B(t)$ on the left, then

$$u(x) = (u_B - \dot{x}_B) \frac{S_B}{S(x)} + \frac{1}{S(x)} \frac{dV_{AB}}{dt}. \quad (A3)$$

References

- Alipour F, Scherer R. Flow separation in a computational oscillating vocal fold model. *J. Acoust. Soc. Am.* 2004; 116:1710–1719. [PubMed: 15478438]
- Alipour F, Scherer R, Patel V. An experimental study of pulsatile flow in canine larynges. *ASME J. Fluids Eng.* 1995; 117:577–581.
- Barney A, Shadle CH, Davis POAL. Fluid flow in a dynamic mechanical model of the vocal folds and tract. I. Measurements and theory. *J. Acoust. Soc. Am.* 1999; 105:444–455.
- Bejan, A. *Convection Heat Transfer*. Wiley-Interseience; New York: 1984.
- Berke G, Moore D, Monkewitz P, Hanson D, Gerratt B. A preliminary study of particle velocity during phonation in an in vivo canine model. *J. Voice.* 1989; 3(4):306–313.
- Coker, CH; Krane, MH; Reis, BY; Kubli, RA. Search for unexplored effects in speech production; *Proceeding of ICSLP '96*; Philadelphia, PA. 1996;

- Deverge M, Pelorson X, Vilain C, Lagrée P-Y, Chentouf F, Willems J, Hirschberg A. Influence of collision on the flow through *in vitro* rigid models of the vocal folds. *J. Acoust. Soc. Am.* 2003; 114:3354–3362. [PubMed: 14714815]
- Flanagan JL, Ishizaka K. Computer model to characterize the air volume displaced by the vibrating vocal cords. *J. Acoust. Soc. Am.* 1976; 63:1559–1565.
- Hirschberg A. Some fluid dynamic aspects of speech. *Bull. Comm. Parl.* 1992; 2:1–30.
- Hofmans G, Groot G, Rancci M, Graziani G, Hirschberg A. Unsteady flow through in-vitro models of the glottis. *J. Acoust. Soc. Am.* 2003; 113(3):1658–1675. [PubMed: 12656399]
- Ishizaka K, Flanagan JL. Synthesis of voiced sounds from a two-mass model of the vocal cords. *Bell Syst. Tech. J.* 1972; 51:1233–1268.
- Liljencrants, J. Experiments with analysis by synthesis of glottal airflow; Proceedings of ICSLP '96; Philadelphia, PA. 1996;
- Lous NJC, Hofmans GCJ, Velduis RNJ, Hirschberg A. A symmetrical two-mass vocal-fold model coupled to vocal tract and trachea, with application to prosthesis design. *Acta Acust. (Beijing)*. 1998; 84(6):1135–1150.
- McGowan RS. The quasi-steady approximation, boundary-layer separation, and vibrations in the vocal tract. *J. Acoust. Soc. Am.* 1993; 93:2295.
- Mongeau L, Franchek N, Coker CH, Kubli RA. Characteristics of a pulsating jet through a small, modulated orifice, with application to voice production. *J. Acoust. Soc. Am.* 1997; 102:1121–1133. [PubMed: 9265759]
- Panton, R. Incompressible Flow. Wiley; New York: 1994.
- Peloroson X, Hirschberg A, van Hassel RR, Wijnands APJ, Auregan Y. Theoretical and experimental study of quasi-steady flow separation within the glottis during phonation. Application to a modified two-mass model. *J. Acoust. Soc. Am.* 1994; 96:3416–3431.
- Rosenhead, L, editor. Laminar Boundary Layers. Dover; New York: 1963.
- Sobey IJ. The occurrence of separation in oscillatory flow. *J. Fluid Mech.* 1983; 134:247–257.
- Story B, Titze I. Voice simulation with a body-cover model of the vocal folds. *J. Acoust. Soc. Am.* 1995; 97:1249–1260. [PubMed: 7876446]
- Titze, IR. Principles of Voice Production. Prentice-Hall; Englewood Cliffs, NJ: 1994.
- Vilain CE, Perorson X, Frayssé C, Deverge M, Hirschberg A, Willems J. Experimental validation of a quasi-steady theory for the flow through the glottis. *J. Sound Vib.* 2004; 276:475–490.
- Zhang Z, Mongeau L, Frankel S. Experimental verification of the quasi-steady approximation for aerodynamic sound generation by pulsating jets in tubes. *J. Acoust. Soc. Am.* 2002; 112:1652–1663. [PubMed: 12398470]

FIG. 1.

Geometry of glottal flow passage. Trachea ends at $x_1 = x_A$. x_B is the location of the glottis entrance, x_C of the glottis exit. The location of the separation point is $x_1 = x_S(t)$, where $x_B < x_S < x_C$. The wall shape is described by the function $x_2 = h(x_1, t)$. The boundary layer thickness is given by $x_2 = \delta(x_1, t)$. The flow fills the passage until $x_1 = x_S$, at which point the flow separates and forms a jet, whose boundary is shown. At the separation point, the boundary layer thickness is δ_S . The location where the jet reattaches to the wall is designated $x_1 = x_D$.

FIG. 2.

Sketch of control volume for flow through a duct of time-varying shape, and whose entrance and exit locations (x_A and x_B , respectively) change with time. The unit outward normal vector is $\mathbf{n}$, and the wall velocity is $\mathbf{v}(x_1, t)$.