A novel total variation based ring artifact suppression method for CBCT imaging with two-dimensional antiscatter grids

Timur Alexeev,^{a)} Brian Kavanagh, Moyed Miften, and Cem Altunbas^{a)}

Department of Radiation Oncology, University of Colorado School of Medicine, 1665 Aurora Court, Suite 1032, Mail stop F-706, Aurora, CO 80045, USA

(Received 13 July 2018; revised 11 February 2019; accepted for publication 19 February 2019; published 20 March 2019)

Purpose: Two-dimensional antiscatter grids (2DASG) for cone beam computed tomography (CBCT) is a new area of research to reduce scatter intensity, and consequently improve CBCT image quality. One of the challenges in implementation of 2DASGs is their septa shadows that are impinging on the projections. If these artifacts are not corrected, they may lead to ring artifacts in CBCT images. In this work, we present a novel method to suppress ring artifacts in FPD-based CBCT images.

Methods: Briefly, our method first detects the locations of 2DASG's septa shadows in projections and then, reduces projection pixel values in septa shadows iteratively until a residual-based convergence criterion is met. To suppress the 2DASG's septa shadows, we developed a total variation minimization (TVM) formulation, referred to as adaptive-diffusive total variation minimization (adTVM), where the diffusivity regularization parameter was adaptively adjusted during each iteration based on the magnitude of the local pixel gradients. To test our method, we have acquired CBCT scans of phantoms using three 2DASG prototypes with different grid geometries. Projections were acquired with a linac mounted CBCT system, operated in offset detector geometry. These projections were then corrected in the following steps: first, projections were corrected using a gantry angle-specific gain correction map; next, projections were corrected by applying our adTVM method. CBCT images were reconstructed using FDK filtered backprojection algorithm. To evaluate adTVM's performance, pixel value statistics and contrast-to-noise ratio (CNR) were compared in CBCT images corrected with and without our adTVM method.

Results: Without our adTVM method, all three 2DASG prototypes introduced ring artifacts with varying intensities in CBCT images. With our method, significant reduction in ring artifacts was observed in all test cases. Standard deviation of CT numbers was reduced by 7–74% in uniform density phantom CBCT images, CNR was increased by 8–67%, and CT number accuracy of contrast objects embedded in the phantom was preserved.

Conclusion: We propose a new method to suppress ring artifacts caused by the 2DASG's septa shadows in CBCT images. Our initial investigations indicated that adTVM method could substantially reduce such ring artifacts while preserving CT number accuracy and maintaining good spatial resolution. Therefore, our method may potentially play an important role in enabling the implementation of 2DASGs in flat panel detector based CBCT systems. © 2019 American Association of Physicists in Medicine [https://doi.org/10.1002/mp.13456]

Key words: antiscatter grids, cone beam computed tomography, image enhancement/restoration (noise and artifact reduction), x-ray imaging and computed tomography

1. INTRODUCTION

Cone beam computed tomography (CBCT) irradiates a large volume of interest, producing a large amount of x-ray scatter which reaches the flat panel detector (FPD) and which is one of the main drawbacks of CBCT.¹ Scattered x-rays can significantly degrade CBCT image quality, lead to reduced contrast resolution, corrupted CT number accuracy, and scatter induced image artifacts. One of the common methods for scatter management in CBCT is through utilization of antiscatter grids (ASG). Conventional ASGs were originally developed for radiography and fluoroscopy applications, but are now commonly implemented in CBCT systems, and consist of a one-dimensional array of lead lamellae separated by fiber interspacers. However, scatter rejection performance of

conventional ASGs is often insufficient for CBCT, and improvement in image quality remains limited.

To address this problem, two-dimensional antiscatter grids (2DASG),² a 2D array of through-holes separated by tungsten septa, have been investigated recently. With respect to conventional ASGs, we have shown that 2DASG provides substantially better scatter rejection and primary x-ray transmission performance, and therefore, has the potential to further improve CBCT image quality.³

One of the main challenges in implementation of 2DASG are the artifacts caused by its septa shadows. Since the 2DASG is placed on the FPD surface, its footprint is impinging in the projection images — that is, detector pixels underneath the 2DASG's footprint receive lower x-ray fluence and produce artifacts. Such fixed-pattern artifacts in

projections are known as grid line artifacts (GLA). The GLA in projections may lead to ring artifacts in CBCT images that may impair the visualization of objects, and may impede image analysis techniques applied on CBCT images.⁴

Ring artifacts are also caused by conventional ASGs, and are addressed by a combination of data correction strategies, such as gain correction methods and ring artifact suppression techniques. Although gain correction methods, which are common methods to reduce pixel-to-pixel gain variation in x-ray detectors, can suppress a large fraction of GLA in projections, gain correction alone is not sufficient to achieve the desired reduction in the GLA or ring artifacts in CBCT images. One of the reasons for this is due to the CBCT gantry sag — as the x-ray source and the detector arms flex during the rotation of the CBCT gantry, the x-ray focal spot projects onto a different location in FPD, thus changing the appearance of septa shadows in projections. As a result, gain calibration performed at a single fixed gantry angle may not sufficiently correct the septa shadows at the other gantry angles. This problem can be minimized by gantry angle-specific gain correction approach. Furthermore, additional factors, such as the presence of residual scatter and temporal variation in pixel gain, may lead to the suboptimal correction of GLA and ring artifacts.

Because the ASGs' septa introduce periodic line artifacts in projections, numerous methods^{4–12} based on frequency filtering have been developed to suppress a predetermined artifact frequency in the Fourier domain. Since the grid pitch of conventional ASGs is 0.1–0.2 mm, grid artifacts appear at relatively high spatial frequencies with respect to the frequency bandwidth of the anatomical features in the image. In such cases, the suppression of grid artifact frequency may be achieved with minimal impact on the frequency content of the anatomical features. However, the 2DASGs have a much larger grid pitch (in the order of 1–3 mm), and the spatial frequency of grid artifact may have a large overlap with the imaged object's Fourier spectrum. Thus, the suppression of a 2DASG's frequency signature may also lead to the loss of the imaged object's information, which manifests itself as blurring of the images.

Another potential solution to eliminate the GLA and ring artifacts is to utilize a grid reciprocity scheme, as in the case of radiography and mammography where ASG is moved at high frequency to blur the septa shadows during x-ray exposure.¹³ However, the implementation of the ASG reciprocity schemes can be challenging in the context of CBCT imaging; 2DASG has to be reciprocated for tens of seconds, and the frequency of grid reciprocity may fluctuate due to a change in gravitational forces during gantry rotation, leading to nonuniform blurring of septa shadows.

In this work, we present a novel method that aims to suppress the GLA caused by the 2DASG's septa shadows in projections, and thus reduce ring artifacts in CBCT images. Our method, referred to as the adaptive-diffusive total variation minimization (adTVM) method, is applied iteratively in the projection domain to suppress GLA, and the convergence is determined based on local gradients of the artifacts. adTVM was evaluated in a variety of phantom CBCT images acquired

with three different 2DASG prototypes that produced different ring artifact patterns and intensities due to the differences in the 2DASGs' physical characteristics.

2. ADAPTIVE-DIFFUSIVE TOTAL VARIATION MINIMIZATION METHOD

2.A. Diffusive TVM formulation

Total variation minimization (TVM) methods are based on the concept of minimizing function deviations from the mean value while preserving discontinuities and edges, and have been successfully applied in image denoising. Strong and Chan¹⁴ present an analytical proof that TV regularization causes piecewise features to remain exactly piecewise with exact preservation of edges, and illustrate the superiority of TV regularization over other denoising methods such as FFT, H1, Haar, and Daubechies4.

Originally proposed by Rudin, Osher, and Fatemi,¹⁵ the image denoising model in bounded variation (BV) has the following form:

$$\min_{u \in BV} \rho \|u(x, y, \alpha)\|_{TV(\Omega)} + \frac{1}{2} \int_{\Omega} (u(x, y, \alpha) - f(x, y, \alpha))^2 d\Omega \quad (1)$$

where f is the measured image containing noise, u is the target image without noise, ρ is a positive diffusivity parameter, Ω is the optimization domain, x and y denote the two orthogonal dimensions in Ω , and α is the CBCT gantry rotation angle. The total variation of the image seminorm is given by Eq. (2):

$$\|u\|_{TV(\Omega)} = \int_{\Omega} |\nabla u| d\Omega \quad (2)$$

Thus, Eq. (1) takes on the following form for smooth u :

$$\min_{u \in BV} \rho \int_{\Omega} |\nabla u(x, y, \alpha)| d\Omega + \frac{1}{2} \int_{\Omega} (u(x, y, \alpha) - f(x, y, \alpha))^2 d\Omega \quad (3)$$

where the first integral in Eq. (3) is the regularization term and the second integral in Eq. (3) is the image fidelity term.

Lysaker et al.¹⁶ showed that fourth-order diffusion-based filters dampen the high frequency spatial components, such as noise, steps, and edges, much faster than the second-order diffusion based filters. For this reason, we formulate Eq. (3) in the form of a Laplacian of the pixel values.

$$\min_{u \in BV} \rho \int_{\Omega} |\Delta u(x, y, \alpha)| d\Omega + \frac{1}{2} \int_{\Omega} (u(x, y, \alpha) - f(x, y, \alpha))^2 d\Omega \quad (4)$$

The l_2 norm tends to have relatively few large residuals as compared to the l_1 norm, and the l_1 norm puts relatively larger emphasis on the small residuals compared to the l_2 norm.¹⁷ Thus, the l_1 TV norm appears to be better at preserving fine object features and makes a more attractive candidate for GLA suppression methods.

Total variation minimization methods have been favorably used in the past to enhance images, and are known for their ability to suppress noise while preserving object edges. However, the problem of suppressing the GLA in projections is fundamentally different, as we are now interested in suppressing the high gradient features produced by GLA, while preserving features that are inherent to the imaged object in the projections. For this reason, we propose an adaptive TVM-based method that detects and diffuses the GLA in the projection domain, and subsequently suppresses the ring artifacts in CBCT images while preserving the imaged object's features.

Our adTVM method diffuses the energy of the GLA, and is applied iteratively until the gradient-based convergence criteria reaches a user specified tolerance, which in our case is 10^{-6} .

Applying the Euler–Lagrange transformation to Eq. (4), we produce the following differential relation:

$$\rho \nabla^2 |\nabla^2 u(x, y, \alpha)| + (u(x, y, \alpha) - f(x, y, \alpha)) = 0 \quad (5)$$

In this work, we are interested in developing the adaptive regularization parameter, ρ . Thus, we consider a reduced order problem for simplicity, decouple the “x” and “y” directions in the image, and hence reduce the biharmonic in Eq. (5) to two decoupled fourth-order ordinary differential equations in “x” and in “y” that are mutually independent. Decoupling Eq. (5), eliminates the cross derivatives to produce the following:

$$\rho \frac{d^4 u(x, y, \alpha)}{dx^4} + (u(x, y, \alpha) - f(x, y, \alpha)) = 0 \quad (6)$$

$$\rho \frac{d^4 u(x, y, \alpha)}{dy^4} + (u(x, y, \alpha) - f(x, y, \alpha)) = 0 \quad (7)$$

Equations (6) and (7) are solved line by line to formulate a system of equations of the following form for each line:

$$\begin{bmatrix} \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ & A_x & B_x & C_x & D_x & E_x & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix}_{y,\alpha} \begin{bmatrix} \vdots \\ u_x \\ \vdots \end{bmatrix}_{y,\alpha} = \begin{bmatrix} \vdots \\ f_x \\ \vdots \end{bmatrix}_{y,\alpha} \quad (8)$$

where subscript “y, α ” for a matrix indicates that the “y and α ” dimensions are held constant. Coefficients on the diagonals are defined as the following:

$$A_x = \frac{\rho}{h_x^4} \quad B_x = -\frac{4\rho}{h_x^4} \quad C_x = \left(1 + \frac{6\rho}{h_x^4}\right) D_x = -\frac{4\rho}{h_x^4} \quad E_x = \frac{\rho}{h_x^4} \quad (9)$$

where “h” is the nondimensional step size corresponding to the pixel edgewise dimensions. After Eq. (8) is solved in the

“x” direction in the projection, a similar system is constructed and solved in the “y” direction.

Although for simplicity, we present the special case solution in this work by decoupling the longitudinal and lateral directions in the projections, the general proposed formulation should be understood as the biharmonic form in Eq. (5).

2.B. Adaptive diffusivity parameter

When developing TVM for image denoising, diffusivity term ρ must satisfy the conditions presented by Strong et al.¹⁴:

1. ρ is sufficiently small that all jumps in $f(x, y, \alpha)$ are preserved in the processed image, $u(x, y, \alpha)$
2. ρ is sufficiently large that the noise (and in our case GLA) is completely removed

In order to preserve phantom features and only suppress the GLA, we allow the magnitude of ρ to vary in Ω such that the magnitude of ρ increases exponentially over the grid septa and approaches zero elsewhere, thus developing an adaptive TVM diffusivity scheme. In other words, we develop a regularization parameter $\rho_{x,y,\alpha}$ that changes adaptively to increases in magnitude over the GLA edges:

$$\rho_{x,y,\alpha} = \left| \frac{1}{u_{x,y,\alpha}} \frac{\partial u_{x,y,\alpha}}{\partial h_{x,y}} \right| \frac{1}{u_{x,y,\alpha}} \frac{\partial^2 u_{x,y,\alpha}}{\partial h_{x,y}^2} \quad (10)$$

$$* (GM(x, y, \alpha) - 1) * ResMap(x, y, \alpha)$$

where $GM(x, y, \alpha)$ is the gain map calculated at gantry angle α using Eq. (12) and is the ratio of flood field projections without and with an antiscatter grid, and has nonbinary values between 0 and 1. $ResMap(x, y, \alpha)$ is the residual map of each projection and has local binary values of 0 for pixels that meet the convergence criteria and 1 for pixels that do not. Furthermore, $ResMap$ is evaluated only at GLA locations and is updated at each iteration. Although various residual and correction formulations may be used for $ResMap$, the formulation that was used in this study is a blend of residual and correction expressions and is described by the following:

$$ResMap(x, y, \alpha) = \max \left[\left(\frac{\partial u_{x,y,\alpha}}{\partial h_x} + O(h_x^2) \right) - \left(\frac{\partial u_{x,y,\alpha}}{\partial h_x} + O(h_x) \right)^+, \right. \\ \left| \left(\frac{\partial u_{x,y,\alpha}}{\partial h_x} + O(h_x^2) \right) - \left(\frac{\partial u_{x,y,\alpha}}{\partial h_x} + O(h_x) \right)^- \right|, \quad (11) \\ \left| \left(\frac{\partial u_{x,y,\alpha}}{\partial h_y} + O(h_y^2) \right) - \left(\frac{\partial u_{x,y,\alpha}}{\partial h_y} + O(h_y) \right)^+ \right|, \\ \left| \left(\frac{\partial u_{x,y,\alpha}}{\partial h_y} + O(h_y^2) \right) - \left(\frac{\partial u_{x,y,\alpha}}{\partial h_y} + O(h_y) \right)^- \right| \right]$$

where $O(h^2)$ indicates the second-order approximation of the derivative, $O(h)$ indicates the first-order approximation of the

derivative, superscript (+) indicates the forward difference scheme, and superscript (−) indicates the backward difference scheme. As before, subscripts x and y indicate the orthogonal directions in the projection. Convergence is reached when the change in the $ResMap(x, y, \alpha)$ is sufficiently small.

The adaptive expression in $\rho_{x,y,\alpha}$ is designed via first derivative terms to increase exponentially when encountering high gradient regions in the image and to decrease in low gradient regions. The second derivative acts as a limiter on diffusivity parameter to preserve the natural texture and anatomical features in the image.

The implementation of an angle-specific gain map in our method has a twofold purpose: (a) a multiplicative gain map reduces the severity of GLA and accelerates the convergence of adTVM; and (b) an angle-specific gain map helps adTVM in locating the GLA location by reducing the adTVM regularization parameter for pixels away from the GLA. In our application, gain map is implemented with adTVM, and the $(GM(x,y,\alpha)-1)$ term acts to localize the effect of GLA. The gain map is calculated using Eq. (12):

$$GM(x, y, \alpha) = \frac{FloodProjection_{noasg}(x, y, \alpha)}{FloodProjection_{asg}(x, y, \alpha)} \quad (12)$$

The full adTVM scheme is produced by substituting Eq. (10) into Eq. (5), producing the following nonlinear fourth-order PDE:

$$\begin{aligned} & \left| \frac{1}{u_{x,y,\alpha}} \frac{\partial u_{x,y,\alpha}}{\partial h_{x,y}} \right| \frac{1}{u_{x,y,\alpha}} \frac{\partial u_{x,y,\alpha}}{\partial h_{x,y}} \left| \frac{1}{u_{x,y,\alpha}} \frac{\partial^2 u_{x,y,\alpha}}{\partial h_{x,y}^2} \right| * (GM(x, y, \alpha) - 1) \\ & * ResMap(x, y, \alpha) * \\ & \nabla^2 |\nabla^2 u(x, y, \alpha)| + (u(x, y, \alpha) - f(x, y, \alpha)) = 0 \end{aligned} \quad (13)$$

Nonlinearity in Eq. 13 is lagged and the resulting system of linear equations described by Eq. (8) is solved iteratively in pseudo time.

To accelerate the convergence and improve robustness of adTVM, derivatives in Eq. (10) may be computed using higher order flux approximation.^{18–20} Higher order schemes may be particularly attractive to preserve object gradients and hence high spatial resolution in the image. Furthermore, in order to ensure convergence in the case that the image is not sufficiently smooth, flux limiters may be used in the numerical scheme to ensure that the diffusivity terms are kept finite.^{21,22} adTVM is applied iteratively at GLA locations in the projections, and the scheme is constructed so that $f_{x,y,\alpha}$ in Eq. (13) becomes $u_{x,y,\alpha}$ from the preceding iteration. Since a “clean” image without GLA is not known, $f_{x,y,\alpha}$ is assigned an arbitrary value of a flood field projection in the first iteration. To accelerate convergence, a successive over relaxation method²³ was used to perform the iterations and propagate the solution:

$$u_{x,y,\alpha}^{k+1} = u_{x,y,\alpha}^k + \omega(u_{x,y,\alpha}^{k+1} - u_{x,y,\alpha}^k) \quad (14)$$

where k indicates the iteration level and ω is the relaxation parameter with values ranging between 1 and 2 for equations with low numerical stiffness, and 0–1 for stiff equations. We found experimentally that our system of equations was moderately stiff and observed a good convergence acceleration with ω between 1 and 1.3. Successful convergence was validated with ω equal to 1.

Convergence of adTVM is determined once the residual in Eq. 13 reaches the specified tolerance level, which in our experiments was 10^{-6} .

2.C. adTVM workflow

The adTVM method suppresses GLA in each projection independently. Before processing projections with adTVM, projections are first normalized to minimize the effect of x-ray tube output fluctuations. To monitor x-ray tube output variations, readout values of an exposure-meter embedded in the x-ray tube housing, referred to as KV norm chamber, was utilized. First, a flood projection and KV norm chamber ratio is developed as the ratio of the mean flood projection without an antiscatter grid and the corresponding KV norm chamber output:

$$IOratio(x, y) = \frac{\sum_{n=1}^{N_{proj}} KVnormChamber_{flood}(n)}{\sum_{n=1}^{N_{proj}} FloodProjection_{noasg}(x, y, n)} \quad (15)$$

where $KVnormChamber_{flood}(n)$ are the KV norm chamber outputs, $FloodProjection_{noasg}(x, y, n)$ are the flood projections acquired without an antiscatter grid, and N_{proj} is the total number of projections acquired.

All acquired projections are then divided by the corresponding KV norm chamber reading and multiplied by the flood projection KV norm chamber ratio:

$$R(x, y, \alpha) = \frac{R(x, y, \alpha)}{KVnormChamber(\alpha)} * IOratio(x, y) \quad (16)$$

where $R(x, y, \alpha)$ are the imaged object's projections.

After normalizing for the tube output fluctuations, projections are corrected by an angle-specific gain map from Eq. (12) prior to processing with adTVM. Once the projections have been normalized and corrected with a gain map, GLA are suppressed iteratively with adTVM, where convergence is achieved once the local gradients of GLA reach a desired tolerance level. adTVM is applied iteratively, and iterations are stopped for each pixel independently as the local convergence criteria are met.

The following steps summarize the procedure of our method:

1. Normalize all projections, $R(x, y, \alpha)$, to minimize the effect of x-ray tube output fluctuations. $R(x, y, \alpha) = R(x, y, \alpha) * KVnormChamber(\alpha) * IOratio$.
2. Develop an angle-specific gain map from flood-field projections using Eq. (12). (done once prior to data acquisition)

3. Apply an angle-specific gain map to each CBCT projection to reduce the effect of GLA: $\hat{R}(x, y, \alpha) = R(x, y, \alpha) * GM(x, y, \alpha)$.
4. Apply adTVM and calculate the adaptive regularization parameters, $\rho_{x,y,\alpha}$, for each pixel and at every iteration according to Eq. (10)
5. Evaluate Eq. (8) to obtain the new image and update pixel values using Eq. (14)
6. Calculate the residual for each pixel in each projection. If convergence is reached, proceed to step 7), otherwise loop back to step 4).
7. Upon full convergence, back-project the projections to reconstruct the CBCT images.

The reconstruction procedure is performed as usual, and in our case, we implement the filtered back projection to reconstruct the CBCT image.

3. EXPERIMENTAL SETUP

Our CBCT experiments were performed using a Varian TrueBeam STx linac system with a 40 cm $\times$ 30 cm PaxScan 4030CB FPD and a GS-1542 x-ray tube. The TrueBeam system was operated in half-fan geometry, and with a 0.9-mm titanium beam filter. For each CBCT scan, 900 projections were acquired at 125 kVp, 38 mA, and 13 ms per projection. Projections were 2 $\times$ 2 binned, gain corrected, and reconstructed using the FDK method with 0.5 mm³ voxel size.²⁴

The amount of scatter attenuation by 2DASG depends on the grid ratio, where the grid ratio is defined as the ratio between the septa height and pitch, and the primary transmission is determined by the grid pitch. Three different 2DASG prototypes were fabricated for the experiments: 2DASG-R8 with grid ratio 8 and grid pitch of 3 mm, 2DASG-R12 with grid ratio 12 and grid pitch of 2 mm, and 2DASG-R16 with grid ratio 16 and grid pitch of 2 mm. All prototypes had 0.1 mm septa thickness. The 2DASG-R8 prototype was composed of two 2 cm $\times$ 20 cm modules, where as R12 and R16 prototypes were composed of two 3 cm $\times$ 20 cm modules each. For each grid type, two modules were joined together to create a single 40 cm long grid module. 2DASGs were fabricated from tungsten using a Powder Bed Laser Melting process (Philips, Best, the Netherlands).

Since our initial prototypes were smaller than the full detection area of FPD, the area on FPD that was not covered by the 2DASG prototypes was shielded with a 1.6-mm lead foil. Figure 1 shows the 2DASG assembly, which consists of a mounting platform, 2DASG prototype shown in Fig. 1, and lead sheets that cover the FPD area not covered with a 2DASG. CBCT images were acquired for each 2DASG by removing the default 1DASG, and mounting the 2DASG assembly on the FPD surface as seen in Fig. 1.

Additional scatter correction methods, or exposure modulation hardware, such as a bowtie filter, were not used. Desired alignment of 2DASG was selected prior to the CBCT acquisitions by performing a series of flood projection experiments at different detector positions to minimize 2DASG

septa shadows and maximize the primary transmission. The position with the least significant GLA was selected for all CBCT experiments. CBCT images of a CatPhan 504 phantom with an annular uniformity sleeve, pelvis phantom with added bolus material, and thorax phantom were acquired using R8, R12, and R16 2DASG prototypes.

4. EVALUATION OF ADTVM METHOD

Our numerical scheme was developed and validated using three approaches. (a) Purely synthetic 1D phantom was generated and grid-like artifacts were imposed as sawtooth artifacts with the artifact pitch of 10 pixels and the artifact width of 3 pixels to mimic the effect of a 2DSAG prototype in one dimension. The sawtooth artifacts were multiplicatively applied to the underlying 1D phantom image. (b) 2D quasi-synthetic projections were developed from phantom projections acquired without an antiscatter grid. A synthetic grid-like artifact was applied with a pitch of 8 pixels and septa width of 1 pixel. A blurring kernel with a 6 pixel diameter was used to further approximate a “realistic” look of the GLA, and the artifact was multiplicatively applied to the projections. Three artifact intensities were evaluated: 5%, 20%, and 30%. Convergence of adTVM for a 30% artifact was also evaluated with step sizes of h , $2h$, and $4h$. (c) Finally, CBCT experiments were performed with three 2DASG prototypes: 2DASG-R8 with grid pitch 3 mm, 2DASG-R12 with grid pitch of 2 mm, and 2DASG-R16 with grid pitch of 2 mm. Projections were processed using (a) a median filter (b) the conventional TVM described by Chan^{25–27} (c) our proposed approach (adTVM). After processing of projections, CBCT images were reconstructed using the FDK method.

The conventional TVM described by Chan^{25–27} solves the following least square problem:

$$\min_u \|u(x, y, \alpha)\|_{TV} + \frac{\mu}{2} \|(u(x, y, \alpha) - f(x, y, \alpha))^2\| \quad (17)$$

where μ is the regularization parameter. Equation (4) is similar to Eq. (1) with the difference being that Eq. (17) is formulated with the regularization parameter on the TV norm, and Eq. (1) is formulated with the regularization parameter on the data fidelity term.

To quantify the noise and spatial resolution properties of images, noise power spectrum (NPS) and modulation transfer function (MTF) were calculated using formulations similar to the ones presented by Friedman et al.²⁸ NPS was calculated by first subtracting two consecutive slices of the reconstructed image, and then averaging 50 $\times$ 50 pixel² square regions taken at every 10° intervals in the uniform material sleeve of the large CatPhan phantom. NPS was calculated in the resulting averaged square ROI and the calculated amplitude was divided by 2 to account for the doubling of the measured noise. The final NPS was found by averaging NPS found in 5 subtracted slice pairs. Lastly, NPS profiles were obtained by radially averaging the 2D NPS obtained in the previous step. To calculate MTF, 10 slices were first averaged in the reconstructed image and then the mean values of

FIG. 1. A two-dimensional antiscatter grid prototype for use in CBCT. Left: Placement of 2DASG assembly on PaxScan 4030CB FPD. Top Right: 2DASG assembly CAD model. Bottom Right: Additively manufactured 2DASG module prototype with 100 μm septa thickness and grid ratio of 12. [Color figure can be viewed at wileyonlinelibrary.com]

isoreadial pixels were calculated between 1° and 360° around the air (12-o'clock) insert in the CatPhan 504 phantom.

Furthermore, in order to quantify improvements in the image quality, contrast to noise ratio (CNR) and pixel standard deviation (STD) were calculated in the CatPhan material inserts, as well as the center of the phantom. CNR and STD values were calculated for all seven material inserts in the phantom, starting with the insert at the 12-o'clock position and proceeding clockwise. STD was also calculated in the central region of the phantom with a 30 pixel diameter. CNR was calculated as the ratio between the contrast of the insert and noise in the surrounding background material:

$$\text{CNR} = \frac{|C - B|}{\frac{1}{2} \sqrt{\sigma_C^2 + \sigma_B^2}} \quad (18)$$

where C is the mean pixel value in the material insert, B is the mean pixel value in the surrounding background ROI, σ_C is the standard deviation of pixel values in the material insert, and σ_B is the standard deviation of pixel values in the surrounding background ROI.

5. RESULTS AND ANALYSIS

5.A. Evaluations with synthetic and semi-synthetic data

The artifact suppression adTVM method was first validated using a 1D synthetic image shown in Fig. 2. The object in Fig. 2 was developed to contain various features that would test the performance of adTVM. A 5% synthetic sawtooth artifact was multiplicatively applied to the image to model artifacts such as GLA. The synthetic grid in Fig. 2 has a pitch of 10 pixels and septa width of 3 pixels to imitate our various 2DASG prototypes. Although not displayed, adTVM

performed equally well for 1D synthetic grids with grid pitch varying between 5 and 15 pixels.

Performance of the adTVM artifact suppression method in a 1D synthetic image was compared to the artifact suppression performed with a 5 pixel wide median filter, as well as with TVM with $\mu = 0.1$ and $\mu = 1$. Neither the median filter, nor the TVM methods were successful in sufficiently suppressing the artifacts without inflicting major alterations of the underlying image. Although the median filter appears to have suppressed the artifacts, it also resulted in a loss of spatial resolution as some object features were lost, such as the small features located at $0.72 \leq x \leq 0.76$ and $0.87 \leq x \leq 0.88$. The regularization parameter, μ , for the

FIG. 2. First Row: 1D synthetic image with a sawtooth artifact, artifact suppression with a median filter, TVM, and adTVM. Polluted data is produced by multiplicatively applying a sawtooth artifact to the underlying feature. Second Row: Magnified section of the image in the first row, $0.67 \leq x \leq 0.78$. Third Row: absolute error of the recovered image compared to the same image without artifacts. The error is calculated as the absolute percent difference between the values of the known synthetic image without the artifact and the synthetic image with the artifact processed using various methods displayed above. [Color figure can be viewed at wileyonlinelibrary.com]

TVM method had to be kept large enough to preserve the object features, and as a result did not sufficiently suppress the artifacts as can be seen in the error plot. However, when μ was increased, TVM was not only more successful at suppressing artifacts, but also resulted in severe degradation and “smoothing” of the object features in the image. On the other hand, adTVM method appears to have both, suppressed the artifacts and preserved the important object features in the 1D synthetic image. This is explained by adTVM’s adaptive property, where the regularization parameter is adjusted adaptively at the artifact locations. The over and under estimation produced by adTVM at sharp corners may be resolved when using compact high order stencils for calculating the $\rho_{x,y,z}$ diffusion parameter.

The GLA suppression performance of adTVM was also demonstrated in a projection image of the CatPhan phantom acquired without an antiscatter grid (Fig. 3, first row). The projection was modified by imposing a synthetic GLA to mimic the appearance of the 2DASG induced GLA (Fig. 3, second row). Finally, adTVM was applied to the projection to validate the efficacy of adTVM in removing the grid-like GLA in 2D projections (Fig. 3, third row). The error contour plot and the convergence plots in the fourth row in Fig. 3 demonstrate the efficacy of adTVM, where discrepancy between the original image and the image processed with adTVM can be seen as been limited to predominantly the GLA locations. Furthermore, adTVM was seen to produce the same image with different h_x , h_y , and GLA intensity. The nonlinear scheme is demonstrated to reach convergence to the specified tolerance of 10^{-6} in all synthetic cases in Fig. 3.

5.B. Evaluations with CBCT images

Figure 4 displays images of the material inserts section in the CatPhan phantom which were acquired with 2DASG-R8, 2DASG-R12, and 2DASG-R16. The left column in Fig. 4 displays images where the GLA persist after the application of an angle-specific gain map. The center column in Fig. 4 displays the same images where GLA were suppressed with adTVM. The rightmost column displays the difference of the first and second columns and is mostly composed of rings, implying that object features are well preserved. adTVM was not tailored for each grid, and the same algorithm successfully detected and suppressed the GLA in all images.

Figures 5 and 6 show images of the spatial resolution module in the CatPhan phantom with the annular uniform material sleeve and the pelvis phantom acquired with 2DASG-R12. The pelvis phantom was covered with layers of tissue equivalent material to increase its dimensions to assess the performance of our method at low detector entrance exposure and high scatter conditions. Figures 5(a) and 6(a) demonstrate rings that appear in the CBCT images due to 2DASG septa shadows even after the gantry angle-specific gain correction. Large amount of ring artifacts can be seen in both the central and the peripheral regions of the phantom. The artifacts indicated by the red arrows correspond to the

rings due to the abutment region of the 2DASG modules, septa shadows, and flickering pixels in the projections caused by the detector readout. Figures 5(b) and 6(b) were developed by processing all the projections with a median filter, Figs. 5(c) and 6(c) were developed by processing the projections with the conventional TVM^{25–27} with a regularization parameter of 20, Figs. 5(d) and 6(d) were developed by processing the projections with the conventional TVM with a regularization parameter of 1, Figs. 5(e) and 6(e) were developed by processing the projections with the conventional TVM with a regularization parameter of 0.1, and Figs. 5(f) and 6(f) were developed by processing the projections with our adTVM method.

Although TVM with $\mu = 1$ preserves the number of visible line pairs in Fig. 5(f), rings caused by the GLA, the abutment, and flickering pixels are still visible in the image. When the TVM regularization parameter is reduced to the value of 0.1, the spatial resolution is reduced as is demonstrated by the presence of fewer line pairs in Fig. 5(e). Furthermore, while ring artifacts due to grid septa shadows were reduced by the median filter and the conventional TVM, the ring artifacts due to the grid abutment were not suppressed effectively.

Despite the fact that the median filter and TVM with $\mu = 20$ and $\mu = 1$ suppress the ring artifacts to some degree, these artifacts can still be seen upon closer examination of the images. Furthermore, artifacts can also be seen in images processed with TVM with $\mu = 0.1$, which reduces the image noise more than the grid line artifacts, and consecutively enhances the appearance of ring artifacts in Figs. 5(e) and 6(e). On the other hand, most ring artifacts appear to have been suppressed and the spatial resolution well preserved in the image processed with adTVM. Figure 7 shows the spatial resolution section of the CatPhan phantom with a uniform material sleeve in a large field of view. The conventional TVM with regularization parameters of 1.0 and 0.1 was unable to suppress all the rings caused by the flickering pixels in the periphery of the phantom and the grid abutment zone.

Figure 8 shows the regions of interest (ROIs), as well as the improvement in CNR and STD, when comparing the CatPhan 2DASG-R8/2DASG-R12/2DASG-R16 images processed with and without the adTVM method. An improvement of 8–67% in CNR and 7–74% in STD is seen across all seven material inserts and the central location of the phantom.

Figure 9 shows the CT number profiles and subtracted difference of the pelvis phantom image processed with adTVM [Fig. 6(f)] and without adTVM [Fig. 6(a)]. All images are displayed at window level 0 and window level 1000. The difference image indicates that object features and CT number accuracy were well preserved with the adTVM method.

Figures 10 and 11 show the MTF and NPS calculated for the adTVM CatPhan images. The 1D NPS plots in the left column of Fig. 11 display the radially averaged profiles of the 2D NPS images in Fig. 10. As seen in Figs. 10 and 11, the

FIG. 3. First row: A CatPhan projection acquired without an antiscatter grid. Second row: The same projection with added synthetic GLA. Third row: GLA suppression via adTVM method. All projections were displayed at the same window/level. Fourth row, Left column: Percent difference between projections in first and third rows. Fourth row, Middle column: adTVM convergence to 10^{-6} with various h . Fourth row, right column: adTVM convergence to 10^{-6} with various magnitudes of artifact intensity. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 4. CBCT images of material inserts section in the CatPhan 504 phantom with an annular uniform material sleeve acquired with 2DASG-R8, 2DASG-R12, and 2DASG-R16. Columns on the left and center correspond to images processed without and with adTVM, respectively. Column on the right, displays the subtracted difference of images without and with adTVM. The first row displays images acquired with 2DASG-R8, the second row displays images acquired with 2DASG-R12, and the third row displays images acquired with 2DASG-R16. All images are displayed at 0 HU window level and 1000 HU window width. [Color figure can be viewed at wileyonlinelibrary.com]

NPS of images processed with and without adTVM have similar overall trends; however, the magnitude of the NPS was also significantly reduced in the images processed with

the conventional TVM and adTVM. Furthermore, adTVM also appears to suppress the higher frequency content of the image. The MTF calculated in the 12-o'clock air cavity in the

FIG. 5. CBCT images of the high spatial resolution section in the CatPhan 504 phantom with an annular uniform material sleeve acquired with 2dASG-R12 and reconstructed with the FDK algorithm. Following processing methods were applied to projections. (a) Unprocessed projections. (b) Median filter with 3×3 kernel. (c) Conventional TVM with $\mu = 20$. (d) Conventional TVM with $\mu = 1$. (e) Conventional TVM with $\mu = 0.1$. (f) Our adTVM method. All images displayed with window width of 1000 HU and window level 200 HU. [Color figure can be viewed at wileyonlinelibrary.com]

CatPhan material inserts section and displayed in the right column of Fig. 11 are consistent with the observed spatial resolution in Fig. 5.

A maximum reduction in MTF of 14% can be seen between the uncorrected image and the image corrected with adTVM. Maximum reduction in MTF of 29% can be seen between the uncorrected image and the image corrected with the most aggressive TVM with $\mu = 0.1$. At 10% MTF, the spatial frequency decreased by approximately 0.1/mm for images processed with adTVM and approximately 0.2/mm for images processed with TVM with an aggressive regularization parameter of $\mu = 0.1$. Both, the NPS and MTF confirm the observation, made in Fig. 5, that while some spatial resolution is lost, adTVM seems to suppress line and streak artifacts in CBCT images without significant alterations of the underlying image.

Figure 12 displays a CBCT image of a thorax phantom acquired with 2DASG-R12 where ring artifacts are

FIG. 6. CBCT images of the large pelvis phantom acquired with 2DASG-R12 and reconstructed with FDK algorithm. Following processing methods were applied to projections. (a) Unprocessed projections. (b) Median filter with 3×3 kernel. (c) Conventional TVM with $\mu = 20$. (d) Conventional TVM with $\mu = 1$. (e) Conventional TVM with $\mu = 0.1$. (f) Our adTVM method. Red arrows indicate various artifacts due to grid septa and suspected detector gain switching. All images displayed with window width of 1000 HU and window level 0 HU. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 7. Large CatPhan phantom, spatial resolution. Projections were processed with conventional TVM using $\mu = 1$ in (a), $\mu = 0.1$ in (b), and adTVM in (c). Red arrows indicate ring artifacts due to suspected detector gain switching. All images are displayed with window width of 1000 HU and window level 0 HU. [Color figure can be viewed at wileyonlinelibrary.com]

corrected with gain map only (first column), where artifacts are corrected using adTVM (second column), and the difference between the first two columns (third column). Figure 12 demonstrates that adTVM can suppress the ring artifacts caused by GLA, the abutment, and the flickering pixels, all while preserving the very fine anatomical features. The difference image in the third column further demonstrates that, for the most part, adTVM only suppresses various ring artifacts while preserving anatomical features.

FIG. 8. CNR and STD improvement in a CBCT image of the CatPhan phantom material inserts section acquired with a 2DASG-R8/2DASG-R12/2DASG-R16 and processed with adTVM ring suppression method. ROI 1–7 correspond to material inserts numbered in a clockwise direction, starting with the insert in the 12-o'clock position. ROI 8 correspond to a circular central area in the CatPhan phantom, with a diameter of 30 pixels. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 9. Left: Profile cuts in pelvis images processed with and without adTVM, made along the horizontal dashed lines in Fig. 6. Right: Artifacts suppressed by adTVM in CBCT images of the pelvis phantom. Computed as the difference of pelvis images in first row and third row in Fig. 6(a) and 6(f), and displayed at 0 HU window level with window width of 1000 HU. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 10. 2D NPS of the CatPhan 504 outer uniform section as shown in Fig. 4. Left: Uncorrected image, window width 5000; Center: Image processed with TVM ($\mu = 0.1$), window width 500 (tightened due to low NPS values); Right: Image processed with adTVM, window width 5000.

6. DISCUSSION AND CONCLUSION

In this work, we developed a novel method to reduce ring artifacts in CBCT images acquired with 2DASG prototypes.^{2,3} Although the general formulation is presented as the full biharmonic form, in this work, we considered a special case solution by decoupling the “ x ” and “ y ” directions in the projections. Artifacts caused by 2DASG septa shadows

appear as GLA in projections, and may spread to more than one adjacent pixel row or column. For this reason, interpolation is challenging to implement in suppressing artifacts as it may lead to significant loss of information. In addition to artifacts caused by 2DASG, we also observed occasional flickering pixels in projections, and which manifested as ring artifacts in CBCT images. These flickering pixels appear in septa shadows only, and flicker between two pixels values when the values are in the range of approximately 9,000 and 10,000. While we did not understand why only the pixels underneath the septa shadows flicker, we speculate that this is related to the PaxScan 4030CB’s dual gain readout.²⁹

Our adTVM method automatically detects GLA in projections and selectively reduces the amount of 2DASG septa shadows to minimize the effect of ring artifact suppression on spatial resolution. This allows adTVM to successfully suppress artifacts in CBCT images acquired with all three 2DASG prototypes. Although the median filter and conventional TVM were able to suppress GLA and ring artifacts to some degree, these algorithms do not differentiate between

FIG. 11. Image analysis of the CatPhan 504 material inserts section original image, image processed with TVM ($\mu = 0.1$), and image processed with adTVM. Left: Radially averaged NPS profiles in Fig. 10; Right: MTF at 12-o'clock air insert. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 12. Left: Thorax phantom processed with Gain Map only. (1) Ring due to abutment, (2) Artifacts due to flickering pixels, (3) Rings due to grid septa. Center: Large thorax phantom with adTVM correction. Right: Subtracted difference between images in first and second columns. All images displayed with window width of 1000 HU and window level -500 HU. [Color figure can be viewed at wileyonlinelibrary.com]

the GLA and the anatomical features, and may adversely impact important image characteristics such as the spatial resolution.

By using adTVM, the CNR values improved by up to 67% and STD values decreased by up to 74% in images where grid line artifacts were suppressed with adTVM. The actual STD and CNR improvements varied from grid to grid due to the fact that each grid prototype introduced GLA with different intensity, where the variation in GLA intensity is explained by the primary and scatter transmission characteristics of each grid and their alignment with respect to the x-ray divergence. Our method suppressed ring artifacts without substantial reduction in the image spatial resolution as was demonstrated by the images of the high spatial resolution bar pattern section and the MTF of the 12-o'clock air insert in the large CatPhan phantom. The difference in CBCT images processed with and without adTVM shows the ring artifacts only, whereas the contrast features embedded in the phantom are not visible, which implies that the shape of anatomical features are largely preserved with our method. This observation was valid in both, the relatively uniform phantoms such as the pelvis and CatPhan phantoms, as well as in the phantom with fine features such as the thorax phantom.

We observed that the appearance of 2DASG induced ring artifacts depends on numerous factors, such as the physical properties of 2DASGs (e.g., grid pitch, grid ratio, and septa thickness), the alignment of 2DASG with respect to x-ray source, imaging dose, and CBCT image resolution. For this reason, we evaluated our adTVM method with three different 2DASGs with different grid pitch and grid heights. Furthermore, alignment conditions for each 2DASG prototype were

slightly different due to the way the prototypes were mounted on the FPD, thus contributing to the variation of ring artifacts in the final images. CBCT images with smaller voxel size, or smaller pixel size in projections, caused more pronounced rings artifacts as expected, due to the appearance of artifacts in relatively higher spatial frequencies. Due to the diversity of parameters that affect 2DASG induced ring artifacts, we performed our evaluations in imaging conditions for CBCT guided radiation therapy.

Each 2D grid introduces unique grid artifacts in projections and ring artifacts in reconstructions due to their physical characteristics and the imaging conditions. For example, the R8 grid appears to introduce more intense ring artifacts compared to the R12 grid, which we believe is due to slightly sub-optimal alignment of the R8 grid. On the other hand, the quantity of rings appear to be much less in images acquired with the R8 grid. We believe this is due to the larger grid pitch of the R8 grid, which leads to sparser ring artifacts in CBCT images. When adTVM method is applied on projections acquired with the R8 grid, smoothing is performed on relatively lower percentage of pixels due to the relatively large grid pitch. However, the magnitude of smoothing applied by our adTVM method is larger in such pixels due to a larger intensity of grid artifacts. Compared to the R8 grid, smoothing was applied to more pixels with R12 and R16 grids, due to their smaller grid pitch. Therefore, the degree of CNR improvement provided by our method (Fig. 8) largely depends on the quantity and intensity of ring artifacts.

During the radiation treatment sessions, kV radiographs are often acquired to check the positioning of the bony anatomy or the implanted fiducial markers. Therefore, in a

clinical setting, a 2DASG would be in place during both CBCT and 2D kV imaging sessions. Since our method reduces both, ring artifacts in CBCT images and grid artifacts in projections, it may have a utility in the applications of 2DASGs for both 2D and 3D kV imaging in radiation therapy. Although not optimized for speed and clinical implementation at this point, we believe that our adTVM method can be effectively parallelized and optimized for clinical implementation. Since our method operates only in projection domain, computationally expensive back projection/forward projection iterations are not needed for artifact suppression. In this work, CBCT reconstruction was done with the FDK algorithm and implemented on a Tesla K40 GPU, whereas the adTVM algorithm was not implemented on GPU and the computational performance was not benchmarked at this point. Furthermore, we limited the maximum allowable iterations to 300 in the interest of time. As the convergence is assessed on a pixel-by-pixel basis in each projection, we validated that vast majority of pixels reach convergence in less than 300 iterations. It is important to note that assessment of convergence on a pixel-by-pixel basis is an important element of adTVM, and the method would not perform as intended in the absence of a residual-based convergence criteria. For instance, presetting the iteration limit to 300 without a convergence criteria will lead to the over-smoothing of projections and a loss of the spatial resolution.

While our method provided large reduction in ring artifacts, we believe that there are additional areas that we can improve upon in our future work. First, there was a tendency of adTVM to enhance sharp edges such as the faint outer boundary of the phantom visible in Fig. 9 and the object edges shown in Fig. 2. Although it was not demonstrated in this work, we believe that this effect may be lessened with the implementation of the full biharmonic formulation, as well as higher order stencils for gradient evaluations during each adTVM iteration.

Another area of improvement will be in identification of septa shadows in projections. Currently, we acquire flood field projections with 2DASG in place and create a continuous gain map, where large values in the gain map correspond to grid septa shadows. As the gain map is a continuous map, as opposed to a binary map, grid septa locations cannot be precisely determined. Because adTVM only targets GLA locations, important image features may be suppressed if grid shadow locations are overestimated via the gain map. On the other hand, GLA may not be fully suppressed if grid shadow locations are underestimated. We believe that this problem may be addressed by developing a more robust septa shadow detection method.

In conclusion, the scattered radiation is one of the major roadblocks in improving the quality of CBCT images. The use of 2DASGs in FPD based CBCT is a promising approach to reduce the effects of scatter on image quality. However, the implementation of 2DASG with FPDs requires new methods to minimize the artifacts caused by septa shadows. While conventional gain correction techniques help to reduce the septa

shadows, they are not sufficient to achieve an acceptable level of artifact reduction. In this study, we demonstrated that such artifacts could be suppressed by alternative approaches, such as our adTVM method. With its effectiveness in reducing artifacts caused by the 2DASG's septa shadows, we believe that our adTVM method may play an important role in utilizing 2DASGs in CBCT imaging in the future.

ACKNOWLEDGMENT

This project was supported in part by NIH/NCI Award Number R21CA198462. Tesla K40 GPU used in this project was provided by NVIDIA Corporation.

^{a)} Author to whom correspondence should be addressed. Electronic mails: timur.a.alexeev@gmail.com; cem.altunbas@ucdenver.edu.

REFERENCES

1. Siewerdsen JH, Jaffray DA. Cone-beam computed tomography with a flat-panel imager: magnitude and effects of x-ray scatter. *Med Phys.* 2001;28:220.
2. Altunbas C, Kavanagh B, Alexeev T, Miften M. Transmission characteristics of a two dimensional antiscatter grid prototype for CBCT. *Med Phys.* 2017;44:3952.
3. Alexeev T, Kavanagh B, Miften M, Altunbas C. Two-dimensional antiscatter grid: a novel scatter rejection device for Cone-beam computed tomography. *Med Phys.* 2018;45:529.
4. Sadi F, Lee SY, Hasan MK. Removal of ring artifacts in computed tomographic imaging using iterative center weighted median filter. *Comput Biol Med.* 2010;40:109.
5. Raven C. Numerical removal of ring artifacts in microtomography. *Rev Sci Instrum.* 1998;69:2978.
6. Sijbers J, Postnov A. Reduction of ring artefacts in high resolution micro-CT reconstructions. *Phys Med Biol.* 2004;49:N247.
7. Boin M, Haibel A. Compensation of ring artefacts in synchrotron tomographic images. *Optics Express.* 2006;14:12071.
8. Tang X, Ning R, Yu R, Conover DL. In: Boone JM, Dobbins JT, eds. *Medical Imaging 1999: Physics of Medical Imaging*, Vol. 3659. Bellingham, WA: International Society for Optics and Photonics; 1999:806–817.
9. Tang X, Ning R, Yu R, Conover D. Cone beam volume CT image artifacts caused by defective cells in x-ray flat panel imagers and the artifact removal using a wavelet-analysis-based algorithm. *Med Phys.* 2001;28:812.
10. Ketcham RA. In: Bonse U, eds. *Developments in X-ray tomography V*, Vol. 6318. Bellingham, WA: International Society for Optics and Photonics; 2006:631800.
11. Hudhud G, Turner MJ. Digital removal of power frequency artifacts using a Fourier space median filter. *IEEE Signal Process Lett.* 2005;12:573.
12. Sasada R, Yamada M, Hara S, Takeo H, Shimura K. In: Galloway RL, eds. *Medical Imaging 2003: Visualization, Image-Guided Procedures, and Display*, Vol. 5029. Bellingham, WA: International Society for Optics and Photonics; 2003:688–698.
13. Patel T, Sporkin H, Peppard H, Williams MB. In: Gimi B, Krol A, eds. *Medical Imaging 2016: Biomedical Applications in Molecular, Structural, and Functional Imaging*, Vol. 9788. Bellingham, WA: International Society for Optics and Photonics. 2016:978805.
14. Strong DM, Chan TF. *Exact solutions to total variation regularization problems*. Department of Mathematics University of California: Los Angeles, CA; 1996.
15. Rudin LI, Osher S, Fatemi E. Nonlinear total variation based noise removal algorithms. *Physica D.* 1992;60:259.
16. Lysaker M, Lundervold A, Tai X-C. Noise removal using fourth-order partial differential equation with applications to medical magnetic

- resonance images in space and time. *IEEE Trans Image Process.* 2003;12:1579.
17. Boyd S, Vandenberghe L. *Convex optimization*. Cambridge, UK: Cambridge university press; 2004.
 18. Shu C-W. In: Quarteroni A, eds. *Advanced numerical approximation of nonlinear hyperbolic equations*. Berlin, Germany: Springer. 1998:325–432.
 19. Lele SK. Compact finite difference schemes with spectral-like resolution. *J Comput Phys.* 1992;103:16.
 20. Morinishi Y, Lund TS, Vasilyev OV, Moin P. Fully conservative higher order finite difference schemes for incompressible flow. *J Comput Phys.* 1998;143:90.
 21. Sweby PK. High resolution schemes using flux limiters for hyperbolic conservation laws. *SIAM J Numer Anal.* 1984;21:995.
 22. Tamamidis P, Assanis DN. Evaluation of various high-order-accuracy schemes with and without flux limiters. *Int J Numer Methods Fluids.* 1993;16:931.
 23. Carre B. The determination of the optimum accelerating factor for successive over-relaxation. *Comput J.* 1961;4:73.
 24. Feldkamp L, Davis L, Kress J. Practical cone-beam algorithm. *J Opt Soc Am A.* 1984;1:612.
 25. Chan SH, Khoshabeh R, Gibson KB, Gill PE, Nguyen TQ. An augmented lagrangian method for total variation video restoration. *IEEE Trans Image Process.* 2011;20:3097.
 26. Wang Y, Yang J, Yin W, Zhang Y. A new alternating minimization algorithm for total variation image reconstruction. *SIAM J Imaging Sci.* 2008;1:248.
 27. Wahlberg B, Boyd S, Annergren M, Wang Y. An ADMM algorithm for a class of total variation regularized estimation problems. *IFAC Proceedings Volumes*, 45(16):83–88, 2012.
 28. Friedman SN, Fung GS, Siewerdsen JH, Tsui BM. A simple approach to measure computed tomography (CT) modulation transfer function (MTF) and noise-power spectrum (NPS) using the American College of Radiology (ACR) accreditation phantom. *Med Phys.* 2013;40:051907.
 29. Roos PG, Colbeth RE, Molloy I, et al. In: Yaffe MJ, Flynn MJ, eds. *Medical Imaging 2004: Physics of Medical Imaging*, Vol. 5368. Bellingham, WA: International Society for Optics and Photonics; 2004:139–150.