Published in final edited form as:

Biomed Phys Eng Express. 2018 July ; 4(4): . doi:10.1088/2057-1976/aaca02.

Computation of Magnetic Field Distortions and Impact on T_2^* -weighted MRI with Application to Magnetic Susceptibility Parameter Estimation

Corey E. Cruttenden¹, Xiao-Hong Zhu², Wei Chen^{2,3}, and Rajesh Rajamani¹

¹Department of Mechanical Engineering, University of Minnesota, 111 Church St. SE, Minneapolis, MN

²Center for Magnetic Resonance Research (CMRR), University of Minnesota, 2021 6th St. SE, Minneapolis, MN

³Department of Biomedical Engineering, University of Minnesota, 312 Church St. SE, Minneapolis, MN

Abstract

A two-step numerical computation of T_2^* signal weighting maps in gradient echo magnetic resonance imaging in the presence of an object with varied susceptibility property is presented. In the first step, the magnetic scalar potential is computed for an arbitrary 2D magnetic susceptibility distribution using an algebraic solver. The corresponding magnetic field disturbance is computed from the magnetic scalar potential. In the second step, nonlinear operations are used to compute T_2^* from the magnetic field disturbance and then to generate a map of T_2^* signal weighting. The linearity of the first step of the solution process is used to implement a superposition of basis solutions approach that increases computational efficiency. Superposition of basis solutions, computed from a system composed of a single node of differing magnetic susceptibility from the surround, herein referred to as the base system, is found to provide an accurate estimation of the scalar potential for arbitrary susceptibility distributions. Afterwards, nonlinear computation of the T_2^* signal weighting maps can be performed. The properties of the algebraic magnetic scalar potential solver are discussed in this work. Finally, the linearity of the magnetic scalar potential solver is used to estimate the magnetic susceptibility of various objects from *in vitro* MR-imaging data acquired at 9.4T.

Keywords

magnetic susceptibility; field distortion; numerical solver

Correspondence Rajesh Rajamani, Address: 111 Church St. SE, Minneapolis, MN 55455, Phone: 612-626-7961, Fax: rajamani@umn.edu.

Conflicts of interest: none

3. Introduction

Magnetic resonance (MR) imaging (MRI) is a clinically important noninvasive imaging modality that can be used to visualize both structural features and functional activity in humans. Such visualizations are used for diagnosis of disease or injury, for targeting treatment e.g. identifying cancerous tissues, and for evaluating the efficacy of treatments. For structural imaging, gradient echo and spin echo MRI techniques are available, and the selection of gradient or spin echo acquisition depends on several factors including the desired image contrast, imaging time, and image proximity to artifact sources. One advantage of gradient echo techniques that rely on T_2^* contrast is the short acquisition time even when scanning the entire brain. However, these gradient echo sequences are more susceptibility artifact prone than spin echo acquisitions. Furthermore, most functional imaging sequences rely on the blood-oxygen- level dependent (BOLD) contrast which detects small changes in the magnetic susceptibility property of tissue based on changes in blood flow, volume, and oxygenation [1–3].

Imaging sequences that rely on T_2^* contrast and/or can respond to the BOLD contrast are highly sensitive to magnetic susceptibility changes in the tissue [4]. A significant change in magnetic susceptibility, such as the presence of a foreign object (e.g. an electrode or implant), causes localized distortions of the static magnetic field in the tissue. These distortions increase proton spin dephasing processes in voxels, affecting the apparent transverse relaxation time (T_2^*) and introducing signal cancellation and imaging artifacts around such magnetic susceptibility changes.

The sensitivity of such popular imaging sequences to changes in magnetic susceptibility makes computation and estimation of the static magnetic field for a known magnetic susceptibility distribution an important topic. The ability to compute the magnetic field given some arbitrary magnetic susceptibility distribution enlightens researchers and clinicians as to what imaging artifacts might be expected using certain imaging sequences or how to minimize the artifacts for obtaining better quality of MRI. Furthermore, T_2^* signal maps can be estimated from the computed static magnetic field distortion maps. Such computations are useful as a design tool. For example, engineers might compute the distortions to the static magnetic field and T_2^* signal around some proposed MR-compatible implantable device to determine if artifacts are expected around a specific device. These computations can be performed on many iterations of the design to evaluate MR-compatibility rather than using precious MR-scanner time to examine artifacts around a physically constructed prototype. After a desirable design is found to have acceptable MR-compatibility through simulation, the designers can construct a physical prototype for testing in an MR-scanner.

Magnetic field solvers that compute static field distortions around changes in magnetic susceptibility have been developed previously. Notably, Bhagwandien and colleagues devised a method based on solving the magnetic scalar potential by reformulating a strategy used in numerical solutions of heat transfer problems [5,6]. Their method introduces a pseudo-time variable to iterate on an initial guess of the magnetic scalar potential until the estimate converges to a solution. Others including Collins et al. [7] and Truong et al. [8] also

implemented an approach based on the results of Bhagwandien and coworkers. Methods based on summation of magnetic dipoles have also been applied to model needle tip artifacts in MRI, which while simple to implement are computationally intensive [9,10]. Yoder and colleagues implemented a solver based on the Bloch equations that is computationally expensive but yields impressive correspondence between simulation and experiment [11].

Additionally, Fourier transform techniques have been used to efficiently compute the magnetic field distortion corresponding to a magnetic susceptibility distribution [12,13], as well as for estimating the magnetic susceptibility distribution that generated a particular magnetic field distortion [14–18]. The forward problem, i.e. computing the field distortion map from the magnetic susceptibility distribution, is carried out by taking the Fourier transform of the z- component of the induced magnetization caused by the susceptibility distribution and then computing the z-component of the B-field distortion in the frequency domain. Afterwards, the inverse Fourier transform is used to generate the map of the B-field distortion in the z-direction [13]. The inverse problem, i.e. estimating the susceptibility distribution from a map of the magnetic field distortion, is referred to as quantitative susceptibility mapping (QSM). In QSM, the magnetic field distortion map is usually obtained by post-processing the phase imaging of a gradient echo acquisition. The inverse problem is ill-conditioned but is solved using approaches such as applying prior spatial constraints on the solution [14,15], combining image information using multiple orientations [18], and filtering in k-space [16,18]. Solution of both the forward and inverse problem using Fourier transform techniques is the state-of-the art approach and can be implemented in 3D while maintaining short computation times.

Here we present a two-step approach using finite difference methods to compute the distortion in an MRI image caused by variations in magnetic susceptibility. Step one involves computation of the magnetic scalar potential for an arbitrary 2D magnetic susceptibility distribution using an algebraic solver. The magnetic field distortions caused by variations in magnetic susceptibility are computed from the solved magnetic scalar potential. Step two includes a nonlinear computation of T_2^* in voxels created by grouping the nodes from step one. Afterwards, T_2^* signal intensity maps are calculated from the computed T_2^* in each simulated voxel. We can take advantage of the linearity of the algebraic solver in step one of this approach by implementing superposition of basis scalar potentials. The magnetic scalar potential can be solved numerically just once for a single node of differing susceptibility from the surrounding medium. Afterwards, the solution to this base system, herein referred to as the basis solution, can be used to compute the scalar potential, and hence the magnetic field, for arbitrary susceptibility distributions at much lower computational expense. This introduces the possibility of solving systems with many nodes rapidly on personal computers, provided that the scalar potential basis solution is known. The basis solution could be computed just once using a supercomputer and then distributed for use to solve arbitrary problems on personal computers. Once the magnetic scalar potential is known, it is possible to compute the magnetic field distortions effectively and quickly. Afterwards, step two can be performed, and T_2^* -weighted signal maps can be simulated for the arbitrary 2D magnetic susceptibility distribution. This approach is also expandable to three dimensions.

4. Materials and Methods

Simulation Strategy - Part 1: Algebraic Solver Computes B-field for χ Distribution

The magnetic susceptibility is a material property that is defined as the magnetization of a material M [Am^{-1}] per applied magnetic field strength H [Am^{-1}] as in Eqn. 1. As such, the magnetic susceptibility is a unitless property, which can create some confusion regarding its value. For a detailed discussion of magnetic susceptibility, and cgs and SI(mks) units, see Schenck [19].

$$\chi = \frac{M}{H} [\text{unitless}] \quad \text{Equation 1}$$

The magnetic susceptibility is related to the commonly known magnetic permeability through the relationship in Eqn. 2. The magnetic susceptibility is more useful in describing diamagnetic and paramagnetic objects, i.e. “non-magnetic objects”, whereas permeability is used to describe the magnetization property for ferromagnetic objects. This is because for the wide range of non-magnetic objects, the value of μ_r is extremely close to 1. Using the magnetic susceptibility and reporting values in parts per million (ppm) allows us to more easily distinguish the degree of diamagnetism or paramagnetism of a non-magnetic material.

$$\chi = \mu_r - 1 \quad \text{Equation 2}$$

For non-magnetic materials that are potentially compatible with MRI, the relationship between the magnetic induction, or B-field, and the magnetic field strength, or H-field, is linear as in Eqn. 3.

$$B = \mu_0 \mu_r H \quad \text{Equation 3}$$

Furthermore, we can define the magnetic scalar potential Φ according to Eqn. 4, which is analogous to the relationship between the electric field and the electric potential. For a linear system consisting of diamagnetic or paramagnetic materials, we can then write the constraint on the magnetic scalar potential in Eqn. 5 [5].

$$H = -\nabla\Phi \quad \text{Equation 4}$$

$$\nabla \cdot (\mu_r \nabla \Phi) = 0 \quad \text{Equation 5}$$

Eqn. 5 is expanded to Eqn. 6 and applied on a uniform grid using finite difference approximations to get to the form of Eqn. 7. A sample 4×4 grid is illustrated in Fig. 1 for clarity.

$$\frac{\partial \mu_r}{\partial x} \frac{\partial \Phi}{\partial x} + \mu_r \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial \mu_r}{\partial z} \frac{\partial \Phi}{\partial z} + \mu_r \frac{\partial^2 \Phi}{\partial z^2} = 0 \quad \text{Equation 6}$$

$$\begin{aligned} & \frac{\partial \mu_r}{\partial x} \left[\frac{\Phi_{0+\Delta x} - \Phi_{0-\Delta x}}{2\Delta x} \right] + \mu_r \left[\frac{\Phi_{0+\Delta x} + \Phi_{0-\Delta x} - 2\Phi_0}{\Delta x^2} \right] + \frac{\partial \mu_r}{\partial z} \left[\frac{\Phi_{0+\Delta z} - \Phi_{0-\Delta z}}{2\Delta z} \right] + \\ & \dots \mu_r \left[\frac{\Phi_{0+\Delta z} + \Phi_{0-\Delta z} - 2\Phi_0}{\Delta z^2} \right] \approx 0 \end{aligned}$$

Equation 7

Finally, Eqn. 7 is rearranged into the form of Eqn. 8, which explicitly includes scalar constant values weighting a central node Φ_0 and its four adjacent nodes.

$$\begin{aligned} & \Phi_0 \left[\frac{-2\mu_r}{\Delta x^2} + \frac{-2\mu_r}{\Delta z^2} \right] + \Phi_{0-\Delta z} \left[\frac{\mu_r}{\Delta z^2} - \frac{\partial \mu_r / \partial z}{2\Delta z} \right] + \Phi_{0+\Delta z} \left[\frac{\mu_r}{\Delta z^2} + \frac{\partial \mu_r / \partial z}{2\Delta z} \right] + \\ & \dots \Phi_{0-\Delta x} \left[\frac{\mu_r}{\Delta x^2} - \frac{\partial \mu_r / \partial x}{2\Delta x} \right] + \Phi_{0+\Delta x} \left[\frac{\mu_r}{\Delta x^2} + \frac{\partial \mu_r / \partial x}{2\Delta x} \right] \approx 0 \end{aligned} \quad \text{Equation 8}$$

Since Eqn. 8, which constraints a central node and 4 adjacent nodes, can be written for every interior node in a discretized grid, we can form a linear algebraic system in the form of Eqn. 9.

$$\mathbf{A}\vec{\Phi} + \mathbf{B}\vec{u} = \vec{0} \quad \text{Equation 9}$$

Again, consider the example grid illustrated in Fig. 1. In this case, there are $n = 16$ total nodes with $p = 12$ boundary nodes and $m = 4$ interior nodes. In the system of Eqn. 9, $\mathbf{A}$ is an $m \times m$ matrix of weights based on Eqn. 8 corresponding to the vector of magnetic scalar potentials of interior nodes. $\mathbf{B}$ is an $m \times p$ matrix of weights of Eqn. 8 corresponding to boundary magnetic scalar potentials. $\vec{\Phi}$ is an $m \times 1$ vector containing the unknown values of the magnetic scalar potential at each interior node. Finally, $\vec{u}$ is a $p \times 1$ vector containing prescribed boundary values of the magnetic scalar potential. The combination $\mathbf{A}\vec{\Phi} + \mathbf{B}\vec{u}$ then is an algebraic system where each row expresses Eqn. 8 for some interior node in the system.

Previously, others used a pseudo-time approach to solve the magnetic scalar potential Eqn. (6) for an arbitrary magnetic susceptibility distribution [5–8]. The pseudo-time approach requires setting an effective gain parameter that determines the rate of convergence from an initial guess at the solution to the final steady state solution $\vec{\Phi}$. Rather than using this convergent pseudo-time method, we present a direct method to compute the static solution $\vec{\Phi}$.

Our goal is to solve the magnetic scalar potential vector $\vec{\Phi}$. We can prescribe boundary conditions to populate the vector $\mathbf{u}$, and $\mathbf{A}$ and $\mathbf{B}$ are known because we defined the magnetic susceptibility distribution. Therefore, such a system could be solved by inversion of Eqn. (9), as in Eqn. 10.

$$\vec{\Phi} = \mathbf{A}^{-1}(-\mathbf{B}\vec{u}) \quad \text{Equation 10}$$

However, $\mathbf{A}$ is a large, sparse matrix. Rather than inverting $\mathbf{A}$, which requires significant computational effort, we can solve by forming the augmented matrix-vector system in Eqn. 11, and computing its reduced row echelon form in Eqn. 12.

$$[\mathbf{A}|\mathbf{B}]\begin{bmatrix} \vec{\Phi} \\ \vec{u} \end{bmatrix} = 0 \quad \text{Equation 11}$$

$$[\mathbf{I}|\mathbf{K}]\begin{bmatrix} \vec{\Phi} \\ \vec{u} \end{bmatrix} = 0 \quad \text{Equation 12}$$

From Eqn. 12 we can see that $\vec{\Phi} + \mathbf{K}\vec{u} = 0$, so we use Eqn. 13 to compute $\vec{\Phi}$ for a known (prescribed) $\vec{u}$.

$$\vec{\Phi} = -\mathbf{K}\vec{u} \quad \text{Equation 13}$$

Then by Eqns. 3 and 4, we can compute the magnetic induction $\mathbf{B}$ from Eqn. 14. Finally, we can express the magnetic induction in Hz (as is common in MRI) by using the gyromagnetic ratio γ , which relates the Larmor (precession) frequency of protons to the magnetic induction $\mathbf{B}$ (Eqn. 15).

$$\mathbf{B} = \mu_0\mu_r(-\nabla\Phi) \quad \text{Equation 14}$$

$$\nu = \gamma B \quad \text{Equation 15}$$

As mentioned in the description of the numerical solution technique, it is necessary to assign boundary conditions to the system in order to compute $\vec{\Phi}$. The boundaries are shown as shaded nodes in Fig. 1. Assuming that at far boundaries the nodes are not influenced by a central object with differing magnetic susceptibility from the background, we can compute the scalar potential by considering Eqn. 4. Because, the transverse field in MRI is extremely small, Eqn. 4 is approximately equivalent to Eqn. 16.

$$H \approx -\frac{\partial \Phi}{\partial z} \quad \text{Equation 16}$$

Rearranging Eqn. 16, integrating, and substituting in Eqn. 3 results in the expression of Eqn. 17.

$$\Phi \approx -\frac{BZ}{\mu_0 \mu_r} \quad \text{Equation 17}$$

Eqn. 17 was used to compute the top and bottom boundaries of the discretized system. Further, if no object is present to distort the field, we have a linear magnetic scalar potential gradient to produce an undisturbed H-field, so the side boundaries were taken by interpolation between the top and bottom boundaries

Because the boundary nodes must be far from the object of interest for Eqn. 17 to be valid, we used an enlargement scheme similar to that used previously in the literature [5]. Specifically, the following scheme was used to evaluate the B-field distortions around an arbitrary susceptibility distribution:

1. Define the susceptibility distribution of interest
2. Resize the susceptibility distribution such that the field of view (FOV) is enlarged by a factor of 64 (8x enlargement in both the x and z directions) but the number of nodes remains the same (resolution is lost)
3. Compute the boundary conditions according to Eqn. 17 for the enlarged system
4. Form **A** and **B** for the enlarged system according to Eqns. 8 and 9
5. Solve the magnetic scalar potential Φ for the enlarged system using Eqn. 13
6. Use Φ that was solved in the enlarged system to get the boundary conditions for the original system of interest; interpolate to get the correct resolution in the original FOV
7. Form **A** and **B** for the original susceptibility distribution of interest according to Eqns. 8 and 9
8. Solve Φ for the susceptibility distribution of interest using Eqn. 13
9. Compute H from Φ according to Eqn. 4
10. Compute B from H according to Eqn. 3

For the computation of H from Φ we used the central difference method. However, we found that the error at the boundaries of susceptibility change could be reduced using a different method. Whereas Collins et al. used a forward or backwards difference method at these boundaries [7], we used a different approach. Specifically, we took the value on either side of the susceptibility boundary and its slope, and extrapolated the adjacent value using the

slope and the node size to estimate the value at the boundary nodes. We found this to reduce the error more than using a forward or backwards finite difference.

Analytical Solutions

In order to evaluate the accuracy of the first part of our solver, we can compare our numerical solution to analytical solutions which exist for simple geometries such as cylinders.

For a cylinder with radius R and susceptibility χ_i in a medium with susceptibility χ_e , the following equations apply [20]:

$$B_i \approx B_0 \left(1 + \frac{\chi_e}{2} - \frac{\chi_i}{6}\right) \quad \text{Equation 18}$$

$$B_e \approx B_0 \left(1 + \frac{\chi_e}{3} + \frac{(\chi_e - \chi_i)R^2(x^2 - z^2)}{2(x^2 + z^2)^2}\right) \quad \text{Equation 19}$$

where B_i is the field inside of the cylinder of radius R and B_e is the field outside of the cylinder. The static magnetic field (B_0) produced by a MR scanner is in the z -direction, and the Lorentz correction has already been applied in the forms of Eqns. 18 and 19.

Superposition Approach Improves Computational Efficiency in Part 1

Recall that we solved the algebraic system of Eqn. 11 to compute the magnetic scalar potential for an arbitrary susceptibility distribution. We then computed the magnetic field from the magnetic scalar potential. For a system with many nodes, this process is computationally intensive. However, because linear systems obey the principle of superposition (Eqn. 20), we can compute the magnetic scalar potential for a system with a single node of differing magnetic susceptibility, and use superposition of magnetic scalar potentials to compute the Φ for an arbitrary system.

$$\alpha f(x) + \beta f(y) = f(\alpha x + \beta y) \quad \text{Equation 20}$$

The advantage of using superposition of magnetic scalar potentials is that we can compute a magnetic scalar potential basis solution for a system with a single node of differing susceptibility just one time. As previously mentioned, this is computationally intensive. Afterwards, we can use this basis solution to compute the magnetic scalar potential for other arbitrary systems very quickly. This becomes significant when evaluating many systems, for example in the design process. To obtain accurate results, the following approach is used:

1. Compute the basis Φ for a system of $n \times n$ nodes where the central node has $\chi = \chi_{\text{base}}$

2. Define new χ distribution with matrix size $n/2 \times n/2$
3. Scale and shift the basis solution Φ and superimpose them to compute Φ for the new χ distribution at significantly reduced computation expense

Simulation Strategy - Part 2: Nonlinear Computation of T_2^* Signal Maps

Once the B-field has been computed by scaling the solutions of the magnetic scalar potentials as described above, artifacts in gradient echo MR magnitude maps can be estimated by computing T_2^* signal maps. To generate the estimates of image artifacts, a linear frequency encoding gradient is superimposed with the computed magnetic field. Generally, this frequency encoding gradient should correspond to an experiment, or at least take on a typical value used in MRI. Afterwards, nodes in the numerical solution are grouped to create voxels corresponding to the simulated MRI map. Then, T_2^* can be computed for each voxel according to Eqn. 21 [21], where B_{in} is the inhomogeneity of the B-field within a voxel computed as the difference between the maximum and minimum B-field value within each voxel.

$$\frac{1}{T_{2*}} = \frac{1}{T_2} + \gamma \Delta B_{in} \quad \text{Equation 21}$$

From T_2^* , we can generate a map that estimates the signal strength in each voxel using Eqn. 22, where S is the signal intensity. While this simple approach only accounts for the influence of T_2^* and the echo time (T_E) on the magnitude map, reasonable correspondence to the experimental data can be obtained without having to estimate longitudinal relaxation time (T_1), proton density, etc. throughout the sample, assuming that the MRI signal is acquired under fully relaxed condition.

$$S \propto \exp\left(-\frac{T_E}{T_{2*}}\right) \quad \text{Equation 22}$$

Phantom Preparation for In Vitro Experiments

Agar phantoms were prepared to demonstrate the ability to estimate χ from experimental data. Three samples were prepared using: a silver wire with outer diameter of 254 μm , a tungsten wire with outer diameter of 127 μm , and a prototype CNT neural electrode 282 μm wide with approximately 5 μm conductive functionalized CNT on 76 μm polyimide and insulated by polydimethylsiloxane estimated to be of similar thickness to the polyimide layer. The CNT electrode is described in detail elsewhere [22]. The samples were held in plastic tubes such that the cross sections of interest would be oriented perpendicular to the static magnetic field in the MRI experiment. Then, agar 2% by weight in water was prepared by heating and stirring on a hot plate. The agar was poured around the samples in the plastic tubes and allowed to set overnight before imaging.

Magnetic Resonance Imaging

Imaging was performed on a 9.4T-31 cm horizontal bore animal research scanner (Varian/VNMRJ, CA). Phantoms were positioned into a cradle and a proton transmit-receive surface coil was positioned above the sample. Afterwards, the samples were inserted into the magnet near the isocenter and the coil was tuned and matched to the proton Larmor frequency (400 MHz). 3D shimming was performed to improve the homogeneity of the magnetic field. Gradient echo imaging was used to verify the position of the sample near the isocenter, and the sample was moved where necessary. Gradient echo imaging (RF pulse flip-angle 30 degrees, repetition time (TR) = 100 ms, $T_E = 4.71$ ms) was performed on the samples with field of view (FOV) 40×40 mm and matrix size 512×512 (in plane resolution 78.1 μm), slice thickness was 1 mm and signal averaging was used to improve signal-to-noise ratio (SNR).

Images were loaded into MATLAB using the open-source AEDES (<http://aedes.uef.fi>) toolbox and code was written to compare the *in vitro* MRI data with simulated images.

Technique for Estimation of Magnetic Susceptibility from Experimental MRI Images

The linearity of the first step in the solution can also be leveraged to estimate the magnetic susceptibility of an object from *in vitro* MR imaging. An object of interest is placed into an agar gel phantom and gradient echo imaging is performed. The voxel dimensions, echo time, and frequency encoding gradient used in the experiment should be noted, as they are used in the estimation of χ from the imaging data.

Because the magnetic scalar potential is linear, we can compute Φ for an object with dimensions corresponding to the object of interest but with an arbitrary value of χ , for example $\chi = 1$ ppm. Afterwards, it is easy to scale the solution for any value of χ , so long as the value remains in the range such that a linear relationship between B and H exists, when the object is diamagnetic or paramagnetic. Specifically, we scaled the solution from 1 ppm to the range of values from -100 ppm to 100 ppm in 0.5 ppm steps (401 values). This was done for three systems: a cylinder of diameter $254 \mu\text{m}$, a cylinder with diameter of $127 \mu\text{m}$, and a composite system corresponding to the CNT electrode prototype. The composite system was $282 \mu\text{m}$ wide and contained a $4.88 \mu\text{m}$ layer of scalable magnetic susceptibility between a $78 \mu\text{m}$ layer of polydimethylsiloxane and a $78 \mu\text{m}$ layer of polyimide. These dimensions approximate the physical samples that were imaged. The magnetic scalar potential basis solutions were computed at 512×512 nodes in a FOV of $2500 \mu\text{m} \times 2500 \mu\text{m}$. With this approach, the B-field distortion can be rapidly computed for many possible values of χ for each object of interest.

Once the B-field has been computed by scaling the solutions of the magnetic scalar potentials as described above, artifacts in gradient echo MR magnitude maps can be estimated using part two of our solver. To generate the estimates of image artifacts, a linear frequency encoding gradient corresponding to the *in vitro* experiment is superimposed with the computed magnetic field (herein, 14.678 G cm^{-1}). Afterwards, nodes in the numerical solution are grouped to create voxels with dimensions also corresponding to the *in vitro* experiment. In this case, the experimental voxel size was $78.125 \mu\text{m}$, and our node size was

$2500\text{ }\mu\text{m} / 512 = 4.88\text{ }\mu\text{m}$. Therefore, each simulated voxel consisted of 16×16 nodes. Then, T_2^* was computed for each voxel according to Eqn. 21. T_2 was estimated for water as 40 ms at 9.4T [23] and set to 500 μs for polyimide and 10 ms for polydimethylsiloxane. We then computed the signal strength map from the map of T_2^* using Eqn 22. T_E was 4.71 ms, corresponding to the *in vitro* experiment. The phantom images and simulated signal strength maps were then interpolated by a factor of 64 to improve the visualization for comparison.

After computing S for a wide range of potential values of χ , we compare the signal strength map with experimental data. The *in vitro* image is cropped and aligned to the simulated images. The experimental data and simulated images are normalized such that data values are between 0 and 1. Then, a threshold is applied so that regions of signal loss can be compared. Specifically, values above 0.25 are set equal to 1, and values below the 0.25 threshold remain unaltered. Then, two comparisons are performed on a zoomed view that just encompasses the experimental image signal void. The absolute difference (AD) between the *in vitro* data and each simulated image is computed and then normalized across all the comparisons. The minimum AD was found and the corresponding χ value for the simulated image with the minimum AD is an estimate of the value of χ for the experimental sample. Similarly, the 2D correlation coefficient is computed between the *in vitro* data and each simulated image. This value is subtracted from 1 such that the minimum of (1 - correlation coefficient) indicates the best match between the simulation and experiment and the corresponding value of χ is an estimate of the value of χ for the sample.

5. Results

Comparison Between Numerical and Analytical Solutions

The accuracy of the numerical solution was evaluated by comparing a cylindrical system with a known analytical solution to the numerical solution for the same system. A cylinder of radius 16 μm was centered in a field of view of $128\text{ }\mu\text{m} \times 128\text{ }\mu\text{m}$ with $128\text{ nodes} \times 128\text{ nodes}$. The cylinder was assigned χ_i of 10 ppm and the background had χ_e of 0 ppm. Eqns. 18 and 19 were used to analytically compute the B-field, and the background field strength was then subtracted to compute the B-field distortion. The distortions were converted to a frequency shift map using Eqn. 15. The analytical result is presented in Fig. 3. We observe a uniform distortion within the cylinder, and symmetric positive shifts along the cylinder's z-direction, and negative shifts along the x-direction.

Next, we compute the numerical solution for the same system using the approach outlined in Eqns. 5 – 15. The frequency shift computed with the numerical approach is shown on the left-hand side in Fig. 4, and the difference between the numerical solution and the analytical solution is shown on the right-hand side of the figure. The computation of the magnetic scalar potential using the numerical approach took 30.3 seconds on a personal computer with a 2.3 GHz Intel Core i5 processor and 4 GB of RAM. From Fig. 4, we see that the numerical solution is in good agreement with the analytical one. The largest errors in the solution are at the boundary of magnetic susceptibility change and are observed to oscillate around the boundary. The errors within the cylinder and in the background field are nearly zero. The boundary errors can be easily reduced by applying a smoothing filter at the known boundaries, such as a moving average filter, though this was not performed herein.

Evaluation of Superposition Approach

We also computed the magnetic field distortions for the same system using the superposition approach to demonstrate the decrease in computation time using this approach. First, the magnetic scalar potential basis solution was computed at 256×256 nodes. Admittedly, this computation takes longer than solving a system of 128×128 nodes one time. However, the base system of 256×256 nodes can be computed a single time and used to compute arbitrary systems of 128×128 nodes rapidly, rather than having to solve the scalar potential for each different 128 node system separately. A zoomed view of the magnetic scalar potential basis solution is presented in Fig. 5. The node size used was $1 \mu\text{m}$, but it is easy to scale the solution to different node sizes. The magnetic scalar potential solution has a dipole-like appearance.

The superposition approach was used to compute the magnetic field distortions for the cylindrical system and nearly identical results were obtained when compared to the solution presented in Fig. 4. The superposition solution is presented in Fig. 6 on the left-hand side, and the difference between the superposition approach and the analytical solution is shown on the right-hand side of the figure. Again, the errors are concentrated at the susceptibility boundary and are nearly zero within the cylinder and outside of the cylinder in the background. The computation time for the magnetic scalar potential using the superposition approach (not including the single computation performed to obtain the basis) was just 1.5 seconds. Therefore, a 20x improvement in computation time was obtained using the superposition approach.

The difference between the superposition of scalar potentials approach and the full numerical solution approach is illustrated in Fig. 7. As illustrated by the figure, the differences between the two solutions are minor. Interestingly, there is a difference between the two solutions at the boundary. Recall that in the full numerical simulation, a system enlarged by a factor of 8 in each direction with lower spatial resolution is solved initially. Then, the boundaries for the final FOV are extracted from the initial solution with lower spatial resolution and interpolated by a factor of 8 to generate the boundary conditions for the final FOV grid. Conversely, in the superposition approach, no boundary conditions are imposed on the final FOV. Rather they are computed in the procedure of scaling, shifting, and superposition of solutions. The differences observed at the boundary of the two simulation approaches, which can be seen as a periodic oscillation, are caused by imposing the boundary conditions in one solution compared to having free boundaries in the other solution. Counting the periods of oscillation in the difference map reveals that there are 16 periods over the 128 nodes. This equates to one period for every 8 nodes, where 8 is the factor of interpolation from the enlarged FOV solution to generate the final FOV boundary conditions in the full numerical simulation approach.

Susceptibility Parameter Estimation from Experimental MRI Images

Experimental *in vitro* gradient echo imaging was performed on a silver wire with diameter $254 \mu\text{m}$, a tungsten wire with diameter $127 \mu\text{m}$, and a prototype CNT electrode [22]. We then utilized the scalability of Φ with respect to χ for estimating the magnetic scalar potential value of these three samples. In each case, the magnetic scalar potential for a

geometrically matched cross section and $\chi = 1$ ppm was computed. Then, the solution was scaled for χ from -100 ppm to 100 ppm in 0.5 ppm steps. That is, the magnetic scalar potential was computed for 401 possible values of χ . Computation of the scalar potential basis solution was performed on a supercomputer and took approximately 90 minutes. Therefore, without the scaling approach, computation of the 401 possible values would have taken an unreasonably long time. Afterwards the B-field was computed and signal strength was estimated around the simulated objects. The MRI signal intensity map for each value of χ was compared to the *in vitro* experimental imaging data (after thresholding as described in *Methods*) and the minimum absolute difference and strongest 2D correlation was found. The simulated signal intensities (pre-thresholding) are shown in Fig. 8. For the silver wire, the minimum χ value was found as -13.5 ppm using absolute difference and -14 ppm using the 2D correlation coefficient. For the tungsten wire, minimum χ value was found as 86 ppm using the absolute difference and 85.5 ppm using the 2D correlation coefficient. For the CNT electrode sample, the minimum χ value was found as -3 ppm using absolute difference and -3.5 ppm using the 2D correlation coefficient. The results for all three samples are shown in Fig. 9. The estimated values for χ (with respect to water) are in close agreement with the expected values based on the literature [19]. For the silver wire, we would expect the minimum A% value to be -15 ppm, and for the tungsten wire we expect to find the minimum χ value as 86 ppm. For CNT, the expected value of χ is between -1 and -21 ppm when bulk-scale density of CNT is used to convert mass magnetic susceptibility reported in literature to volume magnetic susceptibility [22]. These estimates are quite close considering the simplicity of implementing this approach.

6. Discussion

A 2D magnetic field simulator has been developed using an algebraic solver for the magnetic scalar potential. The solver can estimate the magnetic field distortions around an object with arbitrary shape and susceptibility distribution. The solution obtained with the numerical approach was in good agreement with the analytical solution for a cylinder, as shown in Fig. 4.

Additionally, the linearity of the solver in step 1 of the 2-step simulator was used to implement a superposition of basis magnetic scalar potentials approach. This approach was shown to give nearly identical results as the full numerical simulation approach, but dramatically reduced the computation time. The advantage of this approach compared with previous finite difference method solvers is that the magnetic scalar potential basis solution, which only need be computed once, could be distributed to many users. With the basis solution, the superposition approach enables rapid computation of the magnetic field distortion for many systems (so long as the matrix size is $n/2 \times n/2$ if the basis solution is $n \times n$). This is particularly advantageous if users are simulating field distortions around a proposed MRI-compatible device design, for instance implanted electrode(s) for neural recording or brain stimulation, since many embodiments might need to be simulated prior to building prototypes for *in vitro* experiments. We have shown that the computations can even be performed on a personal computer quickly. While the simulations herein were limited to 2D and only 128×128 nodes, such an approach could be expanded to larger 3D systems with similar merits and benefits. The basis solution could be computed using a

supercomputer and distributed to users that could then compute the solutions to arbitrary problems using the superposition method.

The superposition of scalar potentials approach in step 1 of our solver allows a single solution of the magnetic scalar potential to be applied to a variety of problems. The linearity of the solution allows for the scaling and superposition of solutions in several ways including:

1. Addition of basis solutions to compute Φ for arbitrary geometries
2. Scaling the basis solution with χ to compute Φ for different materials
3. Superposition of basis solutions to compute Φ for arbitrary geometries and materials (combination of 1 and 2)
4. Scaling basis solutions with node size to alter the object size and field of view
5. Scaling basis solutions with node size and χ allows for different object sizes and materials (combination of 2 and 4)

The implementation of these cases is discussed in detail in the supplementary information for the interested reader.

While our solver has demonstrated an improvement in computation speed compared to previous finite difference method-based solvers through use of the superposition of basis solutions approach, admittedly it is slower than the state-of-the-art techniques that utilize Fourier transforms to solve the problem in the frequency domain. However, because our solution uses a finite difference method-based approach, it is possible to adapt the solution strategy to solve a system with non-uniform node spacing. Conversely, techniques using the Fourier transform require uniform node spacing across the problem space since the effective sampling frequency is constant in evaluating the Fourier transform. The ability to use non-uniform node spacing might be useful in certain applications where extremely high resolution is required in only a small portion of a large FOV.

Estimation of the Magnetic Susceptibility of an Object from In Vitro MRI Data Acquired at 9.4T

We also demonstrated the ability to apply the linearity of the magnetic scalar potential in estimation of the χ value from in vitro images. The base magnetic scalar potentials were solved according to the geometry of two wire samples and a CNT electrode prototype using a base χ value of 1 ppm. The basis solutions were performed at high resolution (512×512 nodes) and took approximately 90 minutes to compute with a supercomputer. Afterwards, the solutions were scaled for 401 different values of χ and the closest match between the simulated signal map and the *in vitro* image was found. In each case the process took less than 5 minutes, and the values for tungsten and silver were within 1–2 ppm of the expected value based on literature. For the CNT sample, the estimate of χ (approximately –3 ppm) is within the range of values (–1 ppm to –21 ppm) provided in the literature when conversion from reported mass susceptibility to volume susceptibility is performed with the CNT bulk-scale density. Previously, we estimated the susceptibility value of the CNT sample to be near –30 ppm (χ of –21 ppm) based on a limited number of manual visual

comparisons to simulated images and only a few potential values of χ for CNT (−11, −31, −51, −71, and −91 ppm) [22]. However, using the improved strategy presented herein, a more appropriate estimate for the magnetic susceptibility of the sample is −12 ppm ($\chi = -3$ ppm). Considering that only the magnitude image maps were used to estimate the value of χ , and that only the T_E / T_2^* term was computed in our estimates of signal magnitude, these estimates are impressive. Using phase map data from the gradient echo imaging to additionally compare the B-field distortions could further improve the results but would also require additional processing steps used in QSM such as background field removal, phase unwrapping, and noise filtering. We limited ourselves to the magnitude imaging comparisons alone for simplicity and because we were still able to achieve reasonable estimates of the magnetic susceptibility parameters. Nonetheless, we have demonstrated a case where using the linearity of the solution through scaling provided massive time savings compared to the case where the full numerical solution had been computed for each value of χ . Such computation would have been wasteful and likely not feasible.

Compared to the field of QSM, the approach used here to estimate the magnetic susceptibility parameter is quite limited. However, the implementation of our approach is much simpler and requires fewer processing steps than QSM. We iteratively compute the forward solution and compare the result to the experimental data. This avoids the issues of inverting the experimental phase imaging map as done in QSM, specifically regarding noise propagation, artifacts, and the issue of singularities at the so-called magic angle in k-space [15,18].

Considering the Validity of the Boundary Condition Assumption

Finally, we would like to discuss the validity of the boundary condition assumption that is critical in finding a solution to the magnetic scalar potential for a system with an arbitrary magnetic susceptibility distribution. We can use the analytical solution for a cylinder to estimate the distance at which the presence of the cylinder becomes negligible. Recall Eqn. 19 for the cylinder, which is reproduced here for convenience:

$$B_e \approx B_0 \left(1 + \frac{\chi_e}{3} + \frac{(\chi_e - \chi_i) R^2 (x^2 - z^2)}{2(x^2 + z^2)^2} \right) \quad \text{Equation 19}$$

We want to find some distance where there is no distortion to the magnetic field from the cylinder. Note that this equation already includes the Lorentz correction. Therefore, there is no distortion due to the cylinder if the external field B_e is equal to the expression in Eqn. 23, which is the Lorentz-corrected (microscopic) field without distortion.

$$B_e \approx B_0 \left(1 + \frac{\chi_e}{3} \right) \quad \text{Equation 23}$$

Therefore, we can define δ such that it represents the distortion to the ideal homogeneous magnetic field caused by the presence of the cylinder. Assume that B_0 is 1T for simplicity but with no loss of generality. In that case, we can write Eqn. 24.

$$\delta = \frac{(\chi_e - \chi_i)R^2(x^2 - z^2)}{2(x^2 + z^2)^2} \quad \text{Equation 24}$$

Suppose that we would like to find where δ is less than some specified value, δ_{spec} , where the distortion δ could be positive or negative. Therefore we consider the magnitude of δ .

$$\text{abs}\left(\frac{(\chi_e - \chi_i)R^2(x^2 - z^2)}{2(x^2 + z^2)^2}\right) \leq \delta_{spec} \quad \text{Equation 25}$$

We can switch to polar coordinates with $x = r\cos(\theta)$ and $z = r\sin(\theta)$. Then Eqn. 25 is rewritten (after cancellation of r^2 in both the numerator and denominator) in the form of Eqn. 26.

$$\text{abs}\left(\frac{(\chi_e - \chi_i)R^2\cos(2\theta)}{2r^2}\right) \leq \delta_{spec} \quad \text{Equation 26}$$

We can then write the following inequality:

$$\text{abs}\left(\frac{(\chi_e - \chi_i)R^2\cos(2\theta)}{2r^2}\right) \leq \text{abs}\left(\frac{(\chi_e - \chi_i)R^2}{2r^2}\right) \quad \text{Equation 27}$$

The right-hand side of Eqn. 27 can be rewritten in an equivalent form shown in Eqn. 28.

$$\text{abs}\left(\frac{(\chi_e - \chi_i)R^2}{2r^2}\right) = \text{abs}(\chi_e - \chi_i)\left(\frac{R^2}{2r^2}\right) \quad \text{Equation 28}$$

Therefore, we can conservatively write Eqn. 29. That is, if Eqn. 29 holds, then Eqn. 25 also holds.

$$\text{abs}(\chi_e - \chi_i)\left(\frac{R^2}{2r^2}\right) \leq \delta_{spec} \quad \text{Equation 29}$$

We can rearrange Eqn. 29 to the form of Eqn. 30, which gives a bound on the distance r that a node must be from a cylinder of radius R with susceptibility difference χ from the surround such that the distortion is less than δ_{spec} .

$$r \geq R \sqrt{\frac{\text{abs}(\Delta\chi)}{2\delta_{\text{spec}}}} \quad \text{Equation 30}$$

The expression of Eqn. 30 was rearranged to plot the distortion in ppm to the static magnetic field at various distances r/R for different values of χ . The plot is presented in Fig. 10. We observe in the figure that even for moderate values of χ , we should place the boundaries quite far away from the object of interest if the assumption of no distortion of the boundary nodes is to hold. Certainly, for the system used to compute the basis solution presented herein, where a single node of 1 ppm difference was located in the center of a matrix of 256×256 nodes, $r/R = 256$ so we are far below 10^{-3} ppm distortion at the boundaries which is essentially negligible. However, care must be taken when scaling the value of the scalar potential basis solution, as this effectively shifts between the curves of Fig. 10.

The previous discussion raises another question: is it valid to use the analytical solution for a cylinder in deriving Eqn. 30 for a Cartesian grid such as the one used in this work? To answer this question, we computed the analytical solution for a cylinder of $\chi = 5$ ppm and diameter of 2 nodes, as well as the numerical solution for a square rod with $\chi = 5$ ppm and side length of 2 nodes. The matrix size was 256×256 nodes, and the resultant maps are scaled from -10^{-3} ppm to 10^{-3} ppm to illustrate field distortions near the boundaries. The results are presented in Fig. 11. The difference between field distortions at the boundaries is negligible (the mean difference of all boundary nodes is 2.31×10^{-6} ppm), suggesting that it is reasonable to use the cylinder equation in the derivation leading up to Eqn. 30, despite the use of a Cartesian grid in our work. Note that there are differences between the solutions in the center of the field maps, since one is the solution for a cylinder and the other a square rod. However, these differences cannot be seen with the scaling that was used to compare the behavior of the *solutions at the boundaries*. The purpose of the comparison was to show that as we get far away from a square node, the solution appears the same as if the node were cylindrical. Therefore, we can use the chart of Fig. 10 that demonstrates how far away a node must be before the influence from the node of different susceptibility is negligible. Note that in these figures, $\chi = 5$ ppm, and $r/R = 128$, so we expect less than 10^{-3} ppm distortion at the boundaries, which is what we found.

Summary and Conclusions

In summary, a two-part numerical solver was presented that enables rapid computation of magnetic field distortions and T_2^* signal maps around an arbitrary 2D object of defined magnetic susceptibility. In part one of the solver, the static magnetic scalar potential was computed by finding the reduced row echelon form of an augmented matrix, rather than using a pseudo-time approach. Afterwards, the magnetic field distortions were computed from the magnetic scalar potential solution. This approach was shown to have advantages stemming from the ability to use the superposition of basis scalar potentials. In this method,

the magnetic scalar potential for a base system consisting of a single node of differing magnetic susceptibility is solved a single time. This initial computation of the magnetic scalar potential basis solution is computationally intensive. However, the basis solution can be scaled, shifted, and added together to compute the magnetic scalar potential for arbitrary distributions without the computational expense that is involved in solving each system from scratch, that is without the basis solution superposition approach. This is a major advantage if many computations are to be performed. We showed for 128×128 nodes a 20-fold decrease in computation time using the superposition approach. Despite the 2D grid and limited field of view, the approach could be applied in 3D in theory to create a fast solver with more geometric complexity.

The second part of the solver involves nonlinear computation of T_2^* signal maps from magnetic field distortions computed in part one. The signal maps are generated by grouping nodes from part one into voxels and calculating the inhomogeneity within each voxel. Afterwards, Eqns. 21 and 22 were applied.

Finally, we demonstrated the ability to estimate χ from *in vitro* images by using the linear scaling property of the magnetic scalar potential. We compared 401 different values of χ in simulation to the *in vitro* images to obtain the closest reasonable estimates of the magnetic susceptibility of our experimental samples. These comparisons were performed in just a few minutes because the scaling approach was used, whereas the single solution of the base magnetic scalar potentials for the systems took approximately 90 minutes each. This demonstrates the enormous time savings that can be had by using the linearity of the magnetic scalar potential.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

Acknowledgements

This work was supported in part by an Institute for Engineering in Medicine (IEM) Group Grant at the University of Minnesota, the University of Minnesota MnDrive RSAM Initiative Grant, the Minnesota Supercomputing Institute (MSI) at the University of Minnesota, NIH grants R01 MH111413, R01 NS070839, R24 MH106049, P41 EB015894, P30 NS076408, the W.M. Keck Foundation, and NSF-IGERT DGE-1069104.

8. References

- [1]. Ogawa S, Tank DW, Menon R, Ellermann JM, Kim SG, Merkle H, et al. Intrinsic signal changes accompanying sensory stimulation: functional brain mapping with magnetic resonance imaging. *Proc Natl Acad Sci* 1992;89:5951–5. [PubMed: 1631079]
- [2]. Bandettini PA, Wong EC, Hinks RS, Tikofsky RS, Hyde JS. Time course EPI of human brain function during task activation. *Magn Reson Med* 1992;25:390–7. [PubMed: 1614324]
- [3]. Kwong KK, Belliveau JW, Chesler DA, Goldberg IE, Weisskoff RM, Poncelet BP, et al. Dynamic magnetic resonance imaging of human brain activity during primary sensory stimulation. *Proc Natl Acad Sci* 1992;89:5675–9. [PubMed: 1608978]
- [4]. Ogawa S, Menon RS, Tank DW, Kim SG, Merkle H, Ellermann JM, et al. Functional brain mapping by blood oxygenation level-dependent contrast magnetic resonance imaging. A comparison of signal characteristics with a biophysical model. *Biophys J* 1993;64:803. [PubMed: 8386018]

- [5]. Bhagwandien R, Van Ee R, Beersma R, Bakker CJG, Moerland MA, Lagendijk JJW. Numerical analysis of the magnetic field for arbitrary magnetic susceptibility distributions in 2D. *Magn Reson Imaging* 1992;10:299–313. [PubMed: 1564997]
- [6]. Bhagwandien R, Moerland MA, Bakker CJG, Beersma R, Lagendijk JJW. Numerical Analysis of the Magnetic Field for Arbitrary Magnetic Susceptibility Distributions in 3D. *Magn Reson Imaging* 1994;12:101–7. [PubMed: 8295497]
- [7]. Collins CM, Yang B, Yang QX, Smith MB. Numerical calculations of the static magnetic field in three-dimensional multi-tissue models of the human head. *Magn Reson Imaging* 2002;20:413–24. [PubMed: 12206867]
- [8]. Truong T-K, Clymer BD, Chakeres DW, Schmalbrock P. Three-dimensional numerical simulations of susceptibility-induced magnetic field inhomogeneities in the human head. *Magn Reson Imaging* 2002;20:759–70. [PubMed: 12591571]
- [9]. Muller-Bierl B, Graf H, Lauer U, Steidle G, Schick F. Numerical modeling of needle tip artifacts in MR gradient echo imaging. *Med Phys* 2004;31:579–87. doi: 10.1118/1.1640971. [PubMed: 15070257]
- [10]. Muller-Bierl B, Graf H, Steidle G, Schick F. Compensation of magnetic field distortions from paramagnetic instruments by added diamagnetic material: Measurements and numerical simulations. *Med Phys* 2005;32:76–84. doi:10.1118/1.1828674. [PubMed: 15719957]
- [11]. Yoder DA, Zhao Y, Paschal CB, Fitzpatrick JM. MRI simulator with object-specific field map calculations. *Magn Reson Imaging* 2004;22:315–28. [PubMed: 15062927]
- [12]. Salomir R, De Senneville BD, Moonen CTW. A fast calculation method for magnetic field inhomogeneity due to an arbitrary distribution of bulk susceptibility. *Concepts Magn Reson Part B Magn Reson Eng* 2003;19:26–34. doi:10.1002/cmr.b.10083.
- [13]. Marques JP, Bowtell R. Application of a fourier-based method for rapid calculation of field inhomogeneity due to spatial variation of magnetic susceptibility. *Concepts Magn Reson Part B Magn Reson Eng* 2005;25:65–78. doi:10.1002/cmr.b.20034.
- [14]. De Rochefort L, Brown R, Prince MR, Wang Y. Quantitative MR susceptibility mapping using piece-wise constant regularized inversion of the magnetic field. *Magn Reson Med* 2008;60:1003–9. doi:10.1002/mrm.21710. [PubMed: 18816834]
- [15]. De Rochefort L, Liu T, Kressler B, Liu J, Spincemaille P, Lebon V, et al. Quantitative susceptibility map reconstruction from MR phase data using bayesian regularization: Validation and application to brain imaging. *Magn Reson Med* 2010;63:194–206. doi:10.1002/mrm.22187. [PubMed: 19953507]
- [16]. Haacke EM, Tang J, Neelavalli J, Cheng YCN. Susceptibility mapping as a means to visualize veins and quantify oxygen saturation. *J Magn Reson Imaging* 2010;32:663–76. doi:10.1002/jmri.22276. [PubMed: 20815065]
- [17]. Schweser F, Deistung A, Lehr BW, Reichenbach JR. Differentiation between diamagnetic and paramagnetic cerebral lesions based on magnetic susceptibility mapping. *Med Phys* 2010;37:5165–78. doi: 10.1118/1.3481505. [PubMed: 21089750]
- [18]. Wharton S, Schäfer A, Bowtell R. Susceptibility mapping in the human brain using threshold-based k-space division. *Magn Reson Med* 2010;63:1292–304. doi:10.1002/mrm.22334. [PubMed: 20432300]
- [19]. Schenck JF. The role of magnetic susceptibility in magnetic resonance imaging: MRI magnetic compatibility of the first and second kinds. *Med Phys* 1996;23:815–50. [PubMed: 8798169]
- [20]. Chu K-C, Xu Y, Balschi JA, Springer CS. Bulk magnetic susceptibility shifts in nmr studies of compartmentalized samples: use of paramagnetic reagents. *Magn Reson Med* 1990;13:239–62. doi:10.1002/mrm.1910130207. [PubMed: 2156125]
- [21]. Chavhan GB, Babyn PS, Thomas B, Shroff MM, Haacke EM. Principles, techniques, and applications of T2*-based MR imaging and its special applications. *Radiographics* 2009;29:1433–49. doi: 10.1148/rg.295095034. [PubMed: 19755604]
- [22]. Cruttenden CE, Taylor JM, Hu S, Zhang Y, Zhu X, Chen W. Carbon nano-structured neural probes show promise for magnetic resonance imaging applications. *Biomed Phys Eng Express* 2018;4:15001.

- [23]. de Graaf R a, Brown PB, McIntyre S, Nixon TW, Behar KL, Rothman DL. High magnetic field water and metabolite proton T1 and T2 relaxation in rat brain in vivo. *Magn Reson Med* 2006;56:386–94. doi:10.1002/mrm.20946. [PubMed: 16767752]

Author Manuscript

Author Manuscript

Author Manuscript

Author Manuscript

Figure 1.

Illustration of nodes using a sample 4×4 grid. Consider writing Eqn. 8 at node (2,2); in this case, the adjacent nodes are labeled according to their position relative to the central node at (2,2). Shaded nodes are boundary nodes where boundary conditions must be applied, whereas unshaded nodes are interior nodes where the value of Φ is to be computed.

Figure 2.

Agar phantom containing 254 μm silver wire in a plastic tube. The plastic tube outer diameter is approximately 28 mm, and the tube is filled with 35 mL of agar 2% by weight in water. The silver wire passes through the phantom perpendicular to the plastic tube's primary axis, such that it is perpendicular to the static magnetic field during the MRI experiment.

Figure 3.

Analytical solution of frequency shift in Hz/T for a cylinder with radius $16 \mu\text{m}$ and $\chi = 10$ ppm in a background of $\chi = \chi$ ppm. 128×128 nodes.

Figure 4.

(a) Numerical solution of frequency shift in Hz/T for a cylinder with radius $16 \mu\text{m}$ and $\chi = 10 \text{ ppm}$ in a background of $\% = \chi \text{ ppm}$. 128×128 nodes. Scalar potential took 30.3 seconds to evaluate on a personal computer. (b) Error between numerical and analytical solutions

Figure 5.

A zoomed-in view of the magnetic scalar potential basis solution Φ , for a system consisting of 256×256 nodes where node ($z = 128$, $x = 128$) has $\chi = 1$ ppm and the background $\chi = 0$ ppm. This scalar potential solution can be shifted, scaled, and summed to solve arbitrary systems of 128×128 nodes rapidly.

Figure 6.

(a) Superposition solution of frequency shift in Hz/T for a cylinder with radius $16\ \mu\text{m}$ and $\chi = 10\ \text{ppm}$ in a background of $\chi = 0\ \text{ppm}$. 128×128 nodes. Scalar potential took 1.5 seconds to evaluate on a personal computer. The results are nearly identical to Fig. 4, but computation time is dramatically reduced. (b) Difference between superposition solution and analytical solution

Figure 7.

Difference between the superposition approach and the full numerical solution for a cylinder with radius $16\ \mu\text{m}$ and $\chi = 10\ \text{ppm}$ in a background of $\chi = 0\ \text{ppm}$. 128×128 nodes. The superposition approach was 20x faster to compute than the full numerical solution.

Figure 8.

Zoomed in views of the estimated signal intensity in voxels nearby the object of interest.

Tungsten, silver, and CNT probe are shown from left to right. The signal intensity maps are normalized with respect to the maximum value within each signal map.

Figure 9.

Estimation of the magnetic susceptibility from *in vitro* MR imaging. A) Silver wire with diameter of 254 μm , B) tungsten wire with diameter of 127 μm , C) neural probe prototype, 282 μm wide with approximately 5 μm thick CNT on 76 μm polyimide substrate and insulated with PDMS of similar thickness to polyimide. Phantom and best match simulation images shown for each sample above a plot of the normalized image comparisons for a range of magnetic susceptibility values. The simulated images are T2* weighted with thresholding to set all voxel values above the threshold equal to 1; this produces a clear

delineation of the region of signal drop-out in the imaging. Black solid line is (1 - correlation coefficient) between images and dashed blue line is the normalized absolute difference between images.

Figure 10.
PPM distortion to the magnetic field at the distance r/R from a cylinder of χ .

Figure 11.

Magnetic field distortion (ppm) from a cylinder (left) and square rod (right). The colormaps are set to highlight the distortions near the boundaries of the system, resulting in saturation of the central nodes. We observe a high degree of similarity between the solutions at the boundaries. Note that although the solutions appear identical throughout the whole FOV (due to saturation in the middle), they are not.