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Author for correspondence:

Yihui Zhang

e-mail: yihuizhang@tsinghua.edu.cn

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Postbuckling analyses of frame mesostructures consisting of straight ribbons for mechanically guided three-dimensional assembly

Yuan Liu^{1,2}, Zheng Xu^{1,3}, Keh-Chi Hwang^{1,2}, Yonggang Huang⁴ and Yihui Zhang^{1,2}

¹Applied Mechanics Laboratory, Department of Engineering Mechanics, and ²Center for Flexible Electronics Technology and Center for Mechanics and Materials, Tsinghua University, Beijing 100084, People's Republic of China

³State Key Laboratory for Manufacturing and Systems Engineering, School of Mechanical Engineering, Xi'an Jiaotong University, Xi'an 710049, People's Republic of China

⁴Departments of Civil and Environmental Engineering, Mechanical Engineering and Materials Science and Engineering, Center for Engineering and Health, and Skin Disease Research Center, Northwestern University, Evanston, IL 60208, USA

YH, 0000-0002-0483-8359; YZ, 0000-0003-0885-2067

Mechanically guided assembly through buckling-induced two-dimensional (2D)-to- three-dimensional (3D) transformation represents a versatile approach to the formation of 3D mesostructures, thanks to the demonstrated applicability to a wide range of length scales (from tens of nanometres to centimetres) and material types (from semiconductors, metals to polymers and ceramics). In many demonstrated examples of device applications, the 2D precursor structures are composed of ribbon-type components, and some of them exhibit frame geometries consisting of multiple straight ribbons. The coupling of bending/twisting deformations among various ribbon components of the frame mesostructures makes the analyses more complicated than the case with a single component, which requires the development of a relevant theory to serve as the basis of design optimization in practical applications. Here, an analytic model of compressive buckling in such frame mesostructures is presented in the

framework of energetic approach, taking into account the contributions of spatial bending deformations and twisting deformations. Three different frame geometries are studied, including ‘+’, ‘T’ and ‘H’ shaped designs. As validated by the experiments and finite-element analyses (FEA), the developed model can predict accurately the assembled 3D configurations during the postbuckling of different precursor shapes. Furthermore, the theoretical analyses provide approximate analytic solutions to some key physical quantities (e.g. the maximum out-of-plane displacements and maximum strains), which can be used as design references in practical applications.

1. Introduction

Three-dimensional (3D) mesostructures with different geometrical topologies, feature sizes and material compositions have been intensively investigated in recent years [1–3], showing their promising potentials in various classes of micro/nanotechnologies, such as microelectromechanical systems [4,5], energy storage systems [6,7], biomedical devices [8,9], photonics and optoelectronics [10,11], micro-motors/robotics [12,13], flexible electronics [14–17] and many others. Diverse manufacturing techniques were developed for this purpose, including, for example, additive manufacturing [18–20], microcontact printing [21,22], volumetric optical exposures [23,24], self-rolling/folding induced by residual stresses [25,26] and mechanically guided 3D assembly [27–31]. Of these techniques, the mechanically guided assembly based on compressive buckling represents one of the few approaches that are fully compatible with the well-established planar microsystems technologies, such as those available in the semiconductor industries. In this assembly approach, strategically patterned two-dimensional (2D) precursors were integrated with a prestrained elastomer substrate, during which strong covalent interactions were created selectively at limited locations. Releasing the prestrain then triggers the out-of-plane buckling of the 2D precursors, lifting them off from the substrate surface at the non-bonded weak interfaces, and thereby leading to the formation of 3D mesostructures with programmed configurations. Due to the high level of design flexibility, a variety of complex 3D geometries can be realized [32], including open filamentary frameworks [28,33,34], mesostructures with hybrid geometries of membranes and filaments [11,31], folded constructs (e.g. origami-inspired mesostructures) [27,30] and highly filling nested or entangled networks [29]. Many of the demonstrated 3D mesostructures and functional devices involve ribbon-type components, including single ribbons without any branches/joints, and frame ribbon mesostructures with multiple branches and/or joints. The design of those ribbon-type 3D mesostructures requires the development of relevant theories/models as a theoretical basis. The mechanics of buckling and postbuckling that govern the 2D-to-3D transformation are complex, since the deformation of the ribbon-type mesostructures involves not only planar bending, but also twisting and general spatial bending. Tremendous efforts have been made on the theoretical development of buckling and initial postbuckling for single straight ribbons/beams [35–39], single curved ribbons/beams [33,40–43], and the in-plane postbuckling of frame structures consisting of ribbons [44–49]. The previous theories, however, cannot be used directly to analyse the out-of-plane postbuckling in frame mesostructures consisting of multiple ribbon components.

The current work aims at formulating an analytic model that is capable of predicting the deformed 3D configurations and the maximum strain levels during the out-of-plane postbuckling of the frame mesostructures consisting of multiple straight ribbons at the initial state. Compared with the single ribbons/beams without any joints [33,50,51], the analyses of the frame mesostructures involve additional constraint conditions and continuity conditions at each joint that connects different ribbon components, which greatly complicates the theoretical modelling. Here, three representative types of designs, including ‘+’, ‘T’ and ‘H’ shaped mesostructures [27,28,34], are studied, noting that the presented model can be also extended to designs with other frame shapes. Systematic finite-element analyses (FEA) and experimental

measurement on 3D frame mesostructures with different geometries validate the utility of the model. Approximate analytic solutions are obtained for some key physical quantities, including the maximum out-of-plane displacement, mode ratio and maximum strain, which can facilitate the design optimization in practical applications.

2. An analytic model for the compressive buckling of 3D frame mesostructures

Figure 1 presents a schematic illustration of the mechanically guided assembly for three typical frame mesostructures, including ‘+’, ‘T’ and ‘H’ shaped mesostructures consisting of straight ribbons that are perpendicular to each other at the initial state. The equal-biaxial compression induces the lateral buckling of the 2D precursors, leading to the transformation into 3D configurations, through bending and twisting deformations as well as coordinated translational/rotational motions. A theoretical model is introduced below to analyse such postbuckling processes, and to determine the resulting 3D configurations as well as the strain levels.

(a) Deformation analysis

In the 2D precursor design, each frame mesostructure consists of straight ribbons with two different lengths, $L_0^{(1)}$ and $L_0^{(2)}$, where the superscripts refer to the ribbon (1) and ribbon (2), respectively, as shown in figure 1*a,c,e*. To facilitate the lateral buckling, the frame mesostructures usually adopt an ultrathin geometry, in which the thickness (t) is much smaller than the width (w), corresponding to a large cross-sectional aspect ratio (i.e. $w/t \gg 1$).

As shown in figure 1, $\hat{E}_i$ ($i=1, 2, 3$) are the unit vectors of Cartesian coordinate system. In the deformation analyses of the frame mesostructures, local coordinate systems $E_i^{(k)}$ ($i=1, 2, 3$; $k=1, 2$) are established to characterize the location of the central axis of ribbon components before deformation, as illustrated in figure 1*a,c,e*. Here, $E_3^{(k)}$ refers to the axial direction of the ribbon (k); $E_1^{(k)}$ and $E_2^{(k)}$ are both in the cross-sectional plane, where $E_1^{(k)}$ and $E_2^{(k)}$ are along the out-of-plane direction and the width direction, respectively. The relation between the unit vectors of $E_i^{(1)}$ and $E_j^{(2)}$ is given by

$$E_i^{(2)} = B_{ij}E_j^{(1)}, \quad (2.1)$$

where B_{ij} is an orthogonal matrix in the form of

$$[B_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta) & -\sin(\beta) \\ 0 & \sin(\beta) & \cos(\beta) \end{bmatrix}, \quad (2.2)$$

in which, β is the rotation angle from $E_3^{(2)}$ to $E_3^{(1)}$ along the anticlockwise direction.

Let $Z^{(k)}$ denote the coordinate component associated with $E_3^{(k)}$. Then a point $\mathbf{r}_0^{(k)} = Z^{(k)}E_3^{(k)}$, ($k=1, 2$) on the central axis of the ribbon (k) moves to $\mathbf{r}^{(k)} = \mathbf{r}_0^{(k)} + \mathbf{U}^{(k)} = (U_1^{(k)}, U_2^{(k)}, Z^{(k)} + U_3^{(k)})$ after deformation, wherein $\mathbf{U}^{(k)} = \sum_{i=1}^3 U_i^{(k)}(Z^{(k)})E_i^{(k)}$ is the displacement. According to [51], the unit vector along the tangential direction of deformed central axis can be defined as $e_3^{(k)}$, as shown in figure 1*b,d,f*. The other two unit vectors, $e_1^{(k)}$ and $e_2^{(k)}$, involve the twisting of the cross section along the central axis, and they remain in the cross-sectional plane during deformation, according to the Kirchhoff assumptions [33,41,51]. Then the unit vectors of ribbon (k), ($k=1, 2$) before and after deformation can be related by the direction cosine $A_{ij}^{(k)}$ by

$$e_i^{(k)} = A_{ij}^{(k)} \cdot E_j^{(k)}, \quad (2.3)$$

Figure 1. Illustration of the buckling-guided formation, along with the coordinate system and the key geometric parameters. Three different 3D ribbon network structures, including (a,b) ‘+’ shaped mesostructure, (c,d) ‘T’ shaped mesostructure and (e,f) ‘H’ shaped mesostructure, are considered. (Online version in colour.)

and, $A_{ij}^{(k)}$ can be given in terms of $U_i^{(k)}$ and $\phi^{(k)}$ by Taylor expansion for Bernoulli–Euler beams [51].

$$[A_{ij}^{(k)}] = \begin{bmatrix} 1 & \phi^{(k)} & -U_1^{(k)'} \\ -\phi^{(k)} & 1 & 0 \\ U_1^{(k)'} & 0 & 1 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2}(\phi^{(k)2} + U_1^{(k)'^2}) & 0 & 0 \\ 0 & -\frac{1}{2}\phi^{(k)2} & -U_2^{(k)'} + \phi^{(k)}U_1^{(k)'} \\ 0 & U_2^{(k)'} & -\frac{1}{2}U_1^{(k)'^2} \end{bmatrix}. \quad (2.4)$$

According to Love’s definition [52], the derivatives of the unit vectors after deformation are related to the curvature vector $\kappa^{(k)}$, i.e. $e_i^{(k)'} / \lambda^{(k)} = \kappa^{(k)} \times e_i^{(k)}$ ($i=1, 2, 3$; $k=1, 2$), where $(\cdot)' = d(\cdot)/dZ^{(k)}$, and $\lambda^{(k)} = dz^{(k)}/dZ^{(k)}$ is the stretch along the central axis of the ribbon (k), with $z^{(k)}$ being the axial coordinate after deformation. The bending curvatures, $\kappa_1^{(k)}$ and $\kappa_2^{(k)}$, denote

the curvatures in the $(e_2^{(k)}, e_3^{(k)})$ and $(e_1^{(k)}, e_3^{(k)})$ surfaces; and the twisting curvature $\kappa_3^{(k)}$ is related to the twist angle $\phi^{(k)}$ of the cross section by $\kappa_3^{(k)} = d\phi^{(k)}/dz^{(k)}$. As shown by Liu *et al.* [33], an approximate scaling between the general displacement components ($U_i^{(k)}$ and $\phi^{(k)}$) and the applied strain $\varepsilon_{\text{appl}}$ can be obtained, as given by

$$\left. \begin{aligned} \phi^{(k)} &\propto \sqrt{\varepsilon_{\text{appl}}}, \\ U_1^{(k)} &\propto \sqrt{\varepsilon_{\text{appl}}}, \\ U_2^{(k)} &\propto \varepsilon_{\text{appl}} \\ U_3^{(k)} &\propto \varepsilon_{\text{appl}} \quad (k=1, 2). \end{aligned} \right\} \quad (2.5)$$

and

According to [51], the work conjugate of bending moment and torque is defined as $\kappa^{(k)} = \lambda^{(k)} \kappa^{(k)}$, whose components can be given in terms of $U_i^{(k)}$ and $\phi^{(k)}$ by

$$[\hat{\kappa}_i^{(k)}] = \begin{bmatrix} 0 \\ U_1^{(k)''} \\ \phi^{(k)'} \end{bmatrix} + \begin{bmatrix} \phi^{(k)} U_1^{(k)''} - U_2^{(k)''} \\ 0 \\ 0 \end{bmatrix} \quad (i=1, 2, 3; k=1, 2). \quad (2.6)$$

(b) Analysis for the displacements of the frame mesostructures

(i) Displacement functions of '+' shaped mesostructures

As shown in figure 1a, the origins of the local coordinate systems are the same for both ribbons, i.e. the joint point. Due to the symmetry of the geometry and loading, the two ribbon components of '+' frame mesostructures both bend without twisting, as shown in figure 1b and electronic supplementary material, figure S1(a). For simplicity, we assume the ribbon (2) is longer than ribbon (1), i.e. $L_0^{(2)} \geq L_0^{(1)}$, and in this condition, the maximum out-of-plane displacement of ribbon (1) is located at the middle point, while, the maximum out-of-plane displacement of ribbon (2) is typically located at the other two points (P_1 and P_2), whose initial coordinates are $(0, 0, -L_1^{(2)}/2)$ and $(0, 0, L_1^{(2)}/2)$, respectively.

Let $\bar{Z}^{(k)} = Z^{(k)}/L_0^{(k)}$ ($k=1, 2$) denote the non-dimensional coordinates (related to $E_3^{(k)}$) of the two ribbon components. FEA results (electronic supplementary material, figure S2(a)) show that the normalized displacement ($U_1^{(1)}/(L_0^{(1)} \sqrt{\varepsilon_{\text{appl}}})$) is approximately a single-variable function of $\bar{Z}^{(1)}$. Similar to the Euler-Buckling theory of single straight ribbons/beams [42], the out-of-plane displacement of ribbon (1) can be also characterized by

$$U_1^{(1)} = a_1^{(1)} \cos^2(\pi \bar{Z}^{(1)}) L_0^{(1)} \sqrt{\varepsilon_{\text{appl}}}, \quad -\frac{1}{2} \leq \bar{Z}^{(1)} \leq \frac{1}{2}, \quad (2.7)$$

in which, $a_1^{(1)}$ is a dimensionless parameter to be determined, typically independent on the applied strain.

The axial displacement ($U_3^{(1)}$) can be decomposed into a uniform part ($U_{3\text{uniform}}^{(1)} = -\bar{Z}^{(1)} L_0^{(1)} \varepsilon_{\text{appl}}$) associated with the global compression from two ends, and a non-uniform part ($U_{3\text{non-uniform}}^{(1)}$) resulted from the spatially varying bending, i.e. $U_3^{(1)} = U_{3\text{uniform}}^{(1)} + U_{3\text{non-uniform}}^{(1)}$. Together with FEA calculations on a wide range of geometries of '+' shaped frame mesostructures (electronic supplementary material, figure S2(b)), the non-uniform part of $U_3^{(1)}$ can be characterized by a sinusoidal function containing two periods. Then, a trial function of $U_3^{(1)}$ can be given by

$$U_3^{(1)} = (a_2^{(1)} \sin(4\pi \bar{Z}^{(1)}) - \bar{Z}^{(1)}) L_0^{(1)} \varepsilon_{\text{appl}}, \quad -\frac{1}{2} \leq \bar{Z}^{(1)} \leq \frac{1}{2}, \quad (2.8)$$

where $a_2^{(1)}$ can be solved as $1/(4\pi)$ from the clamped boundary condition, i.e. $U_3^{(1)'}|_{\bar{Z}^{(1)}=\frac{1}{2}} = 0$.

As shown in electronic supplementary material, figure S2(c), there are typically two peak points (P_1 and P_2) on the curve of out-of-plane displacement ($U_1^{(2)}$) distribution in ribbon (2). These two peak points then divide the curve into three segments, noting that each segment of the displacement distribution can be well described by a sinusoidal function similar to that of ribbon (1) in equation (2.7). Thereby, the trial function of $U_1^{(2)}$ can be given in the form of a segmented function as

$$U_1^{(2)} = \begin{cases} a_1^{(2)} \cos^2 \left(\pi \frac{\bar{Z}^{(2)} + \bar{L}_1^{(2)}/2}{1 - \bar{L}_1^{(2)}} \right) L_0^{(2)} \sqrt{\varepsilon_{\text{appl}}}, & -\frac{1}{2} \leq \bar{Z}^{(2)} \leq -\frac{\bar{L}_1^{(2)}}{2} \\ \left(a_1^{(2)} - a_2^{(2)} \cos^2 \left(\pi \frac{\bar{Z}^{(2)}}{\bar{L}_1^{(2)}} \right) \right) L_0^{(2)} \sqrt{\varepsilon_{\text{appl}}}, & -\frac{\bar{L}_1^{(2)}}{2} \leq \bar{Z}^{(2)} \leq \frac{\bar{L}_1^{(2)}}{2} \\ a_1^{(2)} \cos^2 \left(\pi \frac{\bar{Z}^{(2)} - \bar{L}_1^{(2)}/2}{1 - \bar{L}_1^{(2)}} \right) L_0^{(2)} \sqrt{\varepsilon_{\text{appl}}}, & \frac{\bar{L}_1^{(2)}}{2} \leq \bar{Z}^{(2)} \leq \frac{1}{2}, \end{cases} \quad (2.9)$$

in which, $\bar{L}_1^{(2)} = L_1^{(2)}/L_0^{(2)}$; $a_1^{(2)}$ and $a_2^{(2)}$ are two-dimensionless parameters to be determined. At the peak points (i.e. $\bar{Z}^{(2)} = \bar{L}_1^{(2)}/2$), the continuity of the ribbon curvature requires that

$$U_1^{(2)''} \Big|_{\bar{Z}^{(2)} = \frac{\bar{L}_1^{(2)-}}{2}} = U_1^{(2)''} \Big|_{\bar{Z}^{(2)} = \frac{\bar{L}_1^{(2)+}}{2}}. \quad (2.10)$$

Substitution of equation (2.9) into equation (2.10) then yields a relation between $a_1^{(2)}$ and $a_2^{(2)}$:

$$\frac{a_2^{(2)}}{a_1^{(2)}} = \left(\frac{\bar{L}_1^{(2)}}{1 - \bar{L}_1^{(2)}} \right)^2. \quad (2.11)$$

At the connection of ribbon (1) and ribbon (2), the out-of-plane displacements of the two ribbons should be the same, i.e.

$$U_1^{(1)}|_{\bar{Z}^{(1)}=0} = U_1^{(2)}|_{\bar{Z}^{(2)}=0}, \quad (2.12)$$

which can be also derived from the continuity conditions (see electronic supplementary material, equation (S1) for details). From equation (2.12), we can obtain an equation correlating the three parameters ($a_1^{(1)}$, $a_1^{(2)}$ and $a_2^{(2)}$):

$$a_1^{(1)} = \frac{L_0^{(2)}}{L_0^{(1)}} (a_1^{(2)} - a_2^{(2)}). \quad (2.13)$$

Combining equations (2.11) and (2.13) gives the following relationship:

$$\bar{L}_1^{(2)} = \frac{a_1^{(2)} L_0^{(2)}}{a_1^{(1)} L_0^{(1)}} \left(\sqrt{1 - \frac{a_1^{(1)} L_0^{(1)}}{a_1^{(2)} L_0^{(2)}}} - \left(1 - \frac{a_1^{(1)} L_0^{(1)}}{a_1^{(2)} L_0^{(2)}} \right) \right), \quad (2.14)$$

in which, $a_1^{(2)} L_0^{(2)}/(a_1^{(1)} L_0^{(1)}) = \max(U_1^{(2)})/\max(U_1^{(1)})$. Equation (2.14) thereby reveals that the positions of the two peak points ($\bar{Z}^{(2)} = \pm \bar{L}_1^{(2)}/2$) can be determined directly by the ratio of maximum out-of-plane displacements of the two ribbon components, and are independent on the applied strain. As $\max(U_1^{(2)})/\max(U_1^{(1)}) \rightarrow \infty$, $\bar{L}_1^{(2)}$ reaches its limit of 0.5, according to equation (2.17). Electronic supplementary material, figure S2(d) shows that equation (2.14) can well capture the dependence obtained from FEA.

Similarly, electronic supplementary material, figure S2(e,f) shows that the axial displacement of ribbon (2) ($U_3^{(2)}$) can be also characterized by a segmented function as follows, where each part

of $U_3^{(2)}$ can be derived/extended from the basic form in equation (2.8),

$$U_3^{(2)} = \begin{cases} \left(a_3^{(2)} \left(\frac{1}{4\pi} \sin \left(4\pi \frac{\bar{Z}^{(2)} + \bar{L}_1^{(2)}/2}{1 - \bar{L}_1^{(2)}} \right) - \frac{\bar{Z}^{(1)} + \bar{L}_1^{(2)}/2}{1 - \bar{L}_1^{(2)}} \right) + \frac{a_4^{(2)}}{2} \right) L_0^{(2)} \varepsilon_{\text{appl}}, & -\frac{1}{2} \leq \bar{Z}^{(2)} \leq -\frac{\bar{L}_1^{(2)}}{2} \\ a_4^{(2)} \left(\frac{1}{4\pi} \sin \left(4\pi \frac{\bar{Z}^{(2)}}{\bar{L}_1^{(2)}} \right) - \frac{\bar{Z}^{(2)}}{\bar{L}_1^{(2)}} \right) L_0^{(2)} \varepsilon_{\text{appl}}, & -\frac{\bar{L}_1^{(2)}}{2} \leq \bar{Z}^{(2)} \leq \frac{\bar{L}_1^{(2)}}{2} \\ \left(a_3^{(2)} \left(\frac{1}{4\pi} \sin \left(4\pi \frac{\bar{Z}^{(2)} - \bar{L}_1^{(2)}/2}{1 - \bar{L}_1^{(2)}} \right) - \frac{\bar{Z}^{(2)} - \bar{L}_1^{(2)}/2}{1 - \bar{L}_1^{(2)}} \right) - \frac{a_4^{(2)}}{2} \right) L_0^{(2)} \varepsilon_{\text{appl}}, & \frac{\bar{L}_1^{(2)}}{2} \leq \bar{Z}^{(2)} \leq \frac{1}{2} \end{cases} \quad (2.15)$$

The axial displacement at the end of the ribbon (2) is $U_3^{(2)}|_{\bar{Z}^{(2)}=-\frac{1}{2}} = \varepsilon_{\text{appl}} L_0^{(2)}/2$, from which we can obtain a relationship between $a_3^{(1)}$ and $a_4^{(1)}$, i.e.

$$a_3^{(2)} + a_4^{(2)} = 1. \quad (2.16)$$

In the displacement functions given by equations (2.8), (2.9) and (2.15), $a_1^{(1)}$, $a_1^{(2)} - a_4^{(2)}$ and $\bar{L}_1^{(2)}$ are the unknown dimensionless parameters. By considering the boundary conditions and the joint/continuity conditions (equations (2.11), (2.13) and (2.16)), the number of independent dimensionless parameters to be determined is reduced to 3. All the unknown parameters are then determined by the energy minimization, as detailed in §2c.

(ii) Displacement functions of 'T' shaped mesostructures

The origins of the local coordinate systems are fixed at the joint point, as shown in figure 1c. By looking into the FEA results, it can be observed that the out-of-plane displacement $U_1^{(1)}$ and the axial displacement $U_3^{(1)}$ in the ribbon (1) of 'T' shaped mesostructures show similar profiles to the shorter ribbon of '+' shaped mesostructures. Therefore, the associated displacement functions can take similar forms to equations (2.7) and (2.8), as

$$U_1^{(1)} = a_1^{(1)} \cos^2(\pi \bar{Z}^{(1)}) L_0^{(1)} \sqrt{\varepsilon_{\text{appl}}}, \quad -\frac{1}{2} \leq \bar{Z}^{(1)} \leq \frac{1}{2} \quad (2.17)$$

and

$$U_3^{(1)} = \left(\frac{1}{4\pi} \sin(4\pi \bar{Z}^{(1)}) - \bar{Z}^{(1)} \right) L_0^{(1)} \varepsilon_{\text{appl}}, \quad -\frac{1}{2} \leq \bar{Z}^{(1)} \leq \frac{1}{2}. \quad (2.18)$$

Compared with the '+' shaped mesostructures, the lateral deformation of the ribbon-(1) component in the 'T' shaped mesostructures results in the non-negligible in-plane displacement component $U_2^{(1)}$ and twisting angle $\phi^{(1)}$. Considering the scaling on the applied strain [33], the in-plane displacement component $U_2^{(1)}$ can be characterized using the following form,

$$U_2^{(1)} = a_2^{(1)} \cos^2(\pi \bar{Z}^{(1)}) L_0^{(1)} \varepsilon_{\text{appl}} - \frac{L_0^{(2)}}{2} \varepsilon_{\text{appl}}, \quad -\frac{1}{2} \leq \bar{Z}^{(1)} \leq \frac{1}{2}, \quad (2.19)$$

in which, $-L_0^{(2)} \varepsilon_{\text{appl}}/2$ represents the applied displacement.

It is worth noting that the twisting deformation has a stronger nonlinearity than the bending deformations. Due to the symmetric deformation of ribbon (1), only half of the ribbon within $\bar{Z}^{(1)} \in [-1/2, 0]$ is studied herein. According to the FEA calculations on a wide range of loading levels (10%, 20% and 30%) and geometries ($0.7 \leq L_0^{(2)}/L_0^{(1)} \leq 2.0$) as shown in electronic supplementary material, figure S3(a,b), the twisting curvatures ($\hat{\kappa}_3^{(1)}$) always have the minimum values (approx. -0.2 after normalization) at around $\bar{Z}^{(1)} = -1/4$, and take the maximum values (approx. 0.8 after normalization) at the end of the ribbon (1) with $\bar{Z}^{(1)} = -1/2$. The normalized

twisting curvature can be then defined as $\hat{\kappa}_3^{(1)*} = \hat{\kappa}_3^{(1)} / |\hat{\kappa}_3^{(1)}|_{\bar{Z}^{(1)} = -1/2} - \hat{\kappa}_3^{(1)}|_{\bar{Z}^{(1)} = -1/4}|$ and we adopt the following piecewise polynomial functions to characterize the dependence, i.e.

$$\hat{\kappa}_3^{(1)*} = \begin{cases} a_3^{(1)}(\bar{Z}^{(1)})^3 + a_4^{(1)}(\bar{Z}^{(1)})^2 + a_5^{(1)}(\bar{Z}^{(1)}) + a_6^{(1)}, & -\frac{1}{2} \leq \bar{Z}^{(1)} \leq -\frac{1}{4} \\ a_7^{(1)}(\bar{Z}^{(1)})^2 + a_8^{(2)}(\bar{Z}^{(1)}) + a_9^{(1)}, & -\frac{1}{4} \leq \bar{Z}^{(1)} \leq 0. \end{cases} \quad (2.20)$$

Considering the continuity conditions (electronic supplementary material, equations (S4)–(S6)), the number of independent dimensionless parameters among $a_3^{(1)}-a_9^{(1)}$ is reduced to only 2.

According to equation (2.6), the twisting angle can be expressed as the integral of twisting curvature, i.e.

$$\phi^{(1)} = \left(a_{10}^{(1)} \int \hat{\kappa}_3^{(1)*} d\bar{Z}^{(1)} + a_{11}^{(1)} \right) \sqrt{\varepsilon_{\text{appl}}}, \quad -\frac{1}{2} \leq \bar{Z}^{(1)} \leq 0, \quad (2.21)$$

in which, the dimensionless parameter $a_{10}^{(1)}$ determines the amplitude of $\phi^{(1)}$, and the integration constant $a_{11}^{(1)}$ can be determined by the following boundary condition $\phi^{(1)}|_{\bar{Z}^{(1)} = -\frac{1}{2}} = 0$.

Similar to the ‘+’ shaped mesostructure, we still let the point P_1 with material coordinates $(0, 0, -L_1^{(2)})$ denote the peak point with the maximum out-of-plane displacement in the ribbon (2). Inspired by equation (2.9), the out-of-plane displacement of ribbon (2) of ‘T’ shaped mesostructures can be characterized by a segmented function as

$$U_1^{(2)} = \begin{cases} a_1^{(2)} \sin^2 \left(\pi \frac{\bar{Z}^{(2)} + 1}{2(1 - \bar{L}_1^{(2)})} \right) L_0^{(2)} \sqrt{\varepsilon_{\text{appl}}}, & -1 \leq \bar{Z}^{(2)} \leq -\bar{L}_1^{(2)} \\ \left(a_2^{(2)} + a_3^{(2)}(\bar{Z}^{(2)} + \bar{L}_1^{(2)}) + a_4^{(2)}(\bar{Z}^{(2)} + \bar{L}_1^{(2)})^2 \right. \\ \quad \left. + a_5^{(2)}(\bar{Z}^{(2)} + \bar{L}_1^{(2)})^3 + a_6^{(2)}(\bar{Z}^{(2)} + \bar{L}_1^{(2)})^4 \right) L_0^{(2)} \sqrt{\varepsilon_{\text{appl}}}, & -\bar{L}_1^{(2)} \leq \bar{Z}^{(2)} \leq 0. \end{cases} \quad (2.22)$$

The continuity at the point of $\bar{Z}^{(2)} = -\bar{L}_1^{(2)}$ (electronic supplementary material, equations (S7)–(S9)) requires that the number of independent dimensionless parameters among $a_1^{(2)}-a_6^{(2)}$ and $\bar{L}_1^{(2)}$ is only 2.

Similarly, the in-plane displacement can be given by a segmented function in the following form:

$$U_3^{(2)} = \begin{cases} \left(a_7^{(2)} \left(\frac{1}{4\pi} \sin \left(4\pi \frac{\bar{Z}^{(2)} + \bar{L}_1^{(2)}}{2(1 - \bar{L}_1^{(2)})} \right) - \frac{\bar{Z}^{(2)} + \bar{L}_1^{(2)}}{2(1 - \bar{L}_1^{(2)})} \right) + a_8^{(2)} \right) L_0^{(2)} \varepsilon_{\text{appl}}, & -1 \leq \bar{Z}^{(2)} \leq -\bar{L}_1^{(2)} \\ \left(a_9^{(2)} \left(\frac{1}{4\pi} \sin \left(2\pi \frac{\bar{Z}^{(2)}}{\bar{L}_1^{(2)}} \right) - \frac{\bar{Z}^{(2)}}{2\bar{L}_1^{(2)}} \right) + a_{10}^{(2)} \right) L_0^{(2)} \varepsilon_{\text{appl}}, & -\bar{L}_1^{(2)} \leq \bar{Z}^{(2)} \leq 0. \end{cases} \quad (2.23)$$

The boundary condition of the in-plane displacement $U_3^{(2)}$ is given by

$$U_3^{(2)}|_{\bar{Z}^{(2)} = -1} = \varepsilon_{\text{appl}} \frac{L_0^{(2)}}{2}. \quad (2.24)$$

By taking into account equation (2.24) and electronic supplementary material, equation (S10), there are just two independent dimensionless parameters among $a_7^{(2)}-a_{10}^{(2)}$.

In the above displacement modes (equations (2.17)–(2.19) and (2.21)–(2.23)), $a_1^{(1)}-a_{11}^{(1)}$, $a_1^{(2)}-a_{10}^{(2)}$ and $\bar{L}_1^{(2)}$ are the unknown dimensionless parameters to be determined. By taking into account the aforementioned boundary conditions and continuity conditions, the total number of the independent dimensionless parameters is only 7.

(iii) Displacement functions of ‘H’ shaped mesostructures

The local coordinate systems of the ribbon components are illustrated in figure 1e, where the origins are located at the midpoint of each ribbon. Considering the similar loading conditions and structural geometries, the displacement mode of the ribbon (1) component in the ‘H’ shaped

mesostructures is very similar to that in the ‘T’ shaped mesostructures, as evidenced by the FEA results (electronic supplementary material, figure S1(b,c)). Therefore, we adopt the same trial functions of the out-of-plane displacement ($U_1^{(1)}$), lateral displacement ($U_2^{(1)}$), axial displacement ($U_3^{(1)}$) and twisting angle ($\phi^{(1)}$) as those in equations (2.17)–(2.19) and (2.21), respectively. The normalized twisting curvature ($\hat{\kappa}_3^{(1)*}$) also has the same form as equation (2.20), as supported by the FEA calculations (electronic supplementary material, figures S3(a,b) and S4(a,b)). The boundary conditions and continuity conditions are similar to those of ‘T’ shaped structures (see the details in electronic supplementary material).

Similar to the ‘+’ and ‘T’ shaped mesostructures, the normalized out-of-plane displacement ($U_1^{(2)}/(L_0^{(2)}/\sqrt{\varepsilon_{\text{appl}}})$) and the normalized axial displacement ($U_3^{(2)}/(L_0^{(2)}\varepsilon_{\text{appl}})$) of ‘H’ shaped mesostructures are also approximately independent on the applied strain (electronic supplementary material, figure S4(c,d)). FEA results further show that the displacements of ribbon (2) can be characterized by

$$U_1^{(2)} = a_1^{(2)} \cos(\pi \bar{Z}^{(2)}) (1 + a_2^{(2)} \bar{Z}^{(2)^2}) L_0^{(2)} \sqrt{\varepsilon_{\text{appl}}} + U_1^{(1)}|_{\bar{Z}^{(1)}=0}, \quad (2.25)$$

wherein, the $U_1^{(1)}|_{\bar{Z}^{(1)}=0}$ is the out-of-plane displacement of ribbon (1) at the midpoint, and

$$U_3^{(2)} = a_3^{(2)} \left(\frac{1}{4\pi} \sin(4\pi a_4^{(2)} \bar{Z}^{(2)}) - a_4^{(2)} \bar{Z}^{(2)} \right) L_0^{(2)} \varepsilon_{\text{appl}}. \quad (2.26)$$

In this case, the total number of independent dimensionless parameters is 6.

(c) Energy approach

Due to the ultrathin geometry of the frame mesostructures, the membrane energy can be neglected. As such, the total strain energy (Π_{tot}) of a buckled frame mesostructure mainly consists of the in-plane bending energy $\Pi_{\text{bend(in)}}$, out-of-plane bending energy $\Pi_{\text{bend(out)}}$ and twisting energy Π_{twist} of the ribbons, i.e.

$$\Pi_{\text{tot}} = \Pi_{\text{bend(in)}} + \Pi_{\text{bend(out)}} + \Pi_{\text{twist}}. \quad (2.27)$$

For any prescribed geometries (e.g. $L_0^{(2)}/L_0^{(1)}$) and external strain ($\varepsilon_{\text{appl}}$), the minimization of the total energy by searching over a reasonable range of value for each dimensionless parameter then gives solutions for the unknowns. The process can be implemented numerically (e.g. using the commercial software MAPLE or Matlab). After determining the displacement components, the coordinates of every material point in the frame mesostructures during the postbuckling can be obtained.

Specifically, the total strain energy of ‘+’ shaped mesostructures is given by

$$\Pi_{\text{tot}} = EI_2 \left(\int_0^{1/2} \hat{\kappa}_2^{(1)} d\bar{Z}^{(1)} + \int_0^{1/2} \hat{\kappa}_2^{(2)} d\bar{Z}^{(2)} \right), \quad (2.28)$$

because of the zero in-plane bending energy and twisting energy. The negligible membrane energy also suggests that the total arc lengths of the two ribbons components keep almost unchanged during the postbuckling. Such constraint conditions can be written as

$$2 \int_0^{1/2} (U_1^{(k)'} + (1 + U_3^{(k)'})^2) d\bar{Z}^{(k)} - L_0^{(k)} = 0, \quad (k = 1, 2). \quad (2.29)$$

With consideration of these constraint conditions, the number of independent dimensionless parameters can be reduced to 1, which simplifies the minimization process of the total energy.

Figure 2. Experimental, numerical and theoretical studies of the ‘+’ shaped mesostructures. (a) Configurations of 3D frame mesostructures with different length ratios (1.0, 2.0 and 3.0) under different levels of compressive strains (0%, 10%, 20% and 30%). The colour of the FEA results represents the magnitude of maximum principal strain. All of the scale bars in the experiments are 5 mm. (b–e) Analytic and FEA results on the distributions of dimensionless out-of-plane displacements and in-plane displacements for the two ribbon components in the frame mesostructures (with $L_0^{(2)}/L_0^{(1)} = 2.0$), under different levels of compressive strains. The geometric parameters include ($w/L_0^{(1)} = 0.04$, $t = 75 \mu\text{m}$ and $w = 600 \mu\text{m}$). (Online version in colour.)

For ‘T’ shaped mesostructures, the total strain energy is given by

$$\Pi_{\text{tot}} = 2 \left(EI_1 \int_{-1/2}^0 \hat{\kappa}_1^{(1)} d\bar{Z}^{(1)} + EI_2 \int_{-1/2}^0 \hat{\kappa}_2^{(1)} d\bar{Z}^{(1)} + GI_p \int_{-1/2}^0 \hat{\kappa}_3^{(1)} d\bar{Z}^{(1)} \right) + EI_2 \int_{-1}^0 \hat{\kappa}_2^{(2)} d\bar{Z}^{(2)}. \quad (2.30)$$

As FEA results show that the peak point of ribbon (2) keeps almost at the same material point during the postbuckling, the constraint conditions resulted from the unchanged ribbon lengths can be written as

$$2 \int_{-1/2}^0 (U_1^{(1)2} + U_2^{(1)2} + (1 + U_3^{(1)})^2) d\bar{Z}^{(1)} - L_0^{(1)} = 0, \quad (2.31)$$

$$\int_{-\bar{L}_1^{(2)}}^0 (U_1^{(2)2} + (1 + U_3^{(2)})^2) d\bar{Z}^{(2)} - L_1^{(2)} = 0 \quad (2.32)$$

and

$$\int_{-1}^{-\bar{L}_1^{(2)}} (U_1^{(2)2} + (1 + U_3^{(2)})^2) d\bar{Z}^{(2)} - (L_0^{(2)} - L_1^{(2)}) = 0. \quad (2.33)$$

Figure 3. Experimental, numerical and theoretical studies of the 'T' shaped mesostructures. (a) Configurations of 3D frame mesostructures with different length ratios (0.7, 1.0, 1.5 and 2.0) under different levels of compressive strains (0%, 10%, 20% and 30%). The colour of the FEA results represents the magnitude of maximum principal strain. All of the scale bars in the experiments are 5 mm. (b–e) Analytic and FEA results on the distribution of dimensionless out-of-plane displacement, twist angle and in-plane displacements of ribbon-(1) component in the frame mesostructures (with $L_0^{(2)}/L_0^{(1)} = 1.5$), under different levels of compressive strains. (f,g) Similar results on the distribution of dimensionless out-of-plane displacement, in-plane displacement of the ribbon-(2) component. The geometric parameters include ($w/L_0^{(1)} = 0.04$, $t = 75 \mu\text{m}$ and $w = 600 \mu\text{m}$). (Online version in colour.)

Considering equations ((2.31)–(2.33)), there are just four independent dimensionless parameters to be determined from the minimization of the total energy.

For 'H' shaped mesostructures, the total strain energy is written as

$$\Pi_{\text{tot}} = 2 \left(EI_1 \int_{-1/2}^0 \hat{\kappa}_1^{(1)} d\bar{Z}^{(1)} + EI_2 \int_{-1/2}^0 \hat{\kappa}_2^{(1)} d\bar{Z}^{(1)} + GI_p \int_{-1/2}^0 \hat{\kappa}_3^{(1)} d\bar{Z}^{(1)} \right) + EI_2 \int_0^{1/2} \hat{\kappa}_2^{(2)} d\bar{Z}^{(2)}, \quad (2.34)$$

and the constraint conditions are given by

$$2 \int_{-1/2}^0 (U_1^{(1)2} + U_2^{(1)2} + (1 + U_3^{(1)'})^2) d\bar{Z}^{(1)} - L_0^{(1)} = 0 \quad (2.35)$$

Figure 4. Experimental, numerical and theoretical studies of the ‘H’ shaped mesostructures. (a) Configurations of 3D frame mesostructures with different length ratios (0.5, 1.0, 1.5 and 2.0) under different levels of compressive strains (0%, 10%, 20% and 30%). The colour of the FEA results represents the magnitude of maximum principal strain. All of the scale bars in the experiments are 5 mm. (b–e) Analytic and FEA results on the distribution of dimensionless out-of-plane displacement, twist angle and in-plane displacements of ribbon-(1) component in the frame mesostructures (with $L_0^{(2)}/L_0^{(1)} = 1.5$), under different levels of compressive strains. (f,g) Similar results on the distribution of dimensionless out-of-plane displacement, in-plane displacement of the ribbon-(2) component. The geometric parameters include ($w/L_0^{(1)} = 0.04$, $t = 75 \mu\text{m}$ and $w = 600 \mu\text{m}$). (Online version in colour.)

and

$$2 \int_0^{1/2} (U_1^{(2)/2} + (1 + U_3^{(2)})^2) d\bar{Z}^{(2)} - L_0^{(2)} = 0. \quad (2.36)$$

Considering equations (2.35) and (2.36), there are also four independent dimensionless parameters to be determined from the minimization of the total energy.

3. Results and discussion

We carried out experiment measurements and full 3D FEA to validate the above theoretical model for the three different shaped mesostructures. In the experiments, the polyimide (PI) film ($t = 75 \mu\text{m}$) was used to fabricate the patterned 2D precursor. The geometric parameters of the

Table 1. Approximate solutions of $a_1^{(1)}$ and $a_1^{(2)}$.

	$a_1^{(1)}$	$a_1^{(2)}$
‘+’ shaped structures	0.59	$0.59(0.48 + 0.52/(L_0^{(2)}/L_0^{(1)}))$
‘T’ shaped structures	0.59	$0.59(1.07 + 0.31/(L_0^{(2)}/L_0^{(1)}))$
‘H’ shaped structures	0.59	$0.59(1.19 - 0.41/(L_0^{(2)}/L_0^{(1)}))$

2D precursor mesostructure include $w/L_0^{(1)} = 0.04$, and $w = 600 \mu\text{m}$. In the FEA, two-node beam elements were used to model the 2D precursors. Refined meshes were adopted to ensure the computational accuracy. The critical buckling strains and corresponding buckling modes resulted from linear buckling analyses served as initial imperfections in the postbuckling simulations, which then give the deformed configurations and strain distributions at different levels of applied strain. The elastic modulus (E) and Poisson’s ratio (ν) of PI film are $E_{\text{PI}} = 2.8 \text{ GPa}$ and $\nu_{\text{PI}} = 0.32$.

In the postbuckling of slender ribbons ($L_0^{(1)}$ and $L_0^{(2)} \gg w$), material parameters (E and ν) and cross-sectional geometric parameters (e.g. w/t , $w/L_0^{(1)}$ and $w/L_0^{(2)}$) play negligible roles [33,51], while the length ratio ($L_0^{(2)}/L_0^{(1)}$) and the applied strain ($\varepsilon_{\text{appl}}$) represent the dominant parameters. Figures 2*a*, 3*a* and 4*a* present the theoretical, FEA and experimental results on the deformed configurations under three different levels of applied strain for a wide range of frame geometries (e.g. length ratios ($L_0^{(2)}/L_0^{(1)}$) from 1.0 to 3.0 for ‘+’ shaped mesostructures, from 0.7 to 2.0 for ‘T’ shaped mesostructures, and from 0.5 to 2.0 for ‘H’ shaped mesostructures). Good agreements can be observed for all of the different cases studied herein. Figures 2*b–e*, 3*b–g* and 4*b–g* provide the distribution of dimensionless displacement components for the representative geometries of the three frame mesostructures, with $L_0^{(2)}/L_0^{(1)} = 2.0$, 1.5 and 1.5, for ‘+’, ‘T’ and ‘H’ shaped mesostructures. These quantitative results show further evidence on the accuracy of the analytic model.

After validating the theoretical model, we further exploit this model to analyse the effects of length ratio ($L_0^{(2)}/L_0^{(1)}$) and applied strain ($\varepsilon_{\text{appl}}$) on the configurations of assembled 3D frame mesostructures, aiming to provide explicit, approximate solutions that can facilitate the design optimization. The maximum out-of-plane displacement of the ribbon components represents a key quantity that measures the buckled 3D configuration. According to the trial functions shown in equations (2.7), (2.9), (2.17), (2.22) and (2.25), the maximum out-of-plane displacements of the three frame mesostructures are determined, once the two parameters, $a_1^{(1)}$ and $a_1^{(2)}$, are known. By fitting the analytic results for a wide range of length ratio ($L_0^{(2)}/L_0^{(1)}$), approximate solutions to these two parameters ($a_1^{(1)}$ and $a_1^{(2)}$) can be obtained, as shown in table 1.

Figure 5*a,b,d,e* displays the dimensionless maximum out-of-plane displacements ($\max(U_1^{(k)})/L_0^{(k)}$ ($k = 1, 2$)) of the ribbon components versus the length ratio ($L_0^{(2)}/L_0^{(1)}$), for ‘+’ and ‘T’ shaped mesostructures. The ratio of maximum out-of-plane displacements of the two ribbon components increases almost in a linear manner with increasing the length ratio (figure 5*c,f*). During the out-of-plane postbuckling, the ‘H’ shaped mesostructures are transformed into a double-layer 3D configuration. The dimensionless heights of the two layers are denoted by $\max(U_1^{(1)})/L_0^{(1)}$ and $\max(U_1^{(2)} - U_1^{(1)}|_{\bar{z}^{(1)}=0})/L_0^{(2)}$, respectively, and their dependences on the length ratio are described in figure 5*g,h*. The height ratio of the second layer to the first layer increases approximately linearly with the increase of length ratio, as shown in figure 5*i*.

Figure 6*a,b* presents results of the dimensionless length ($L_1^{(2)}/L_0^{(2)}$) that determines the peak points (with the maximum out-of-plane displacement) of ribbon (2) for ‘+’ and ‘T’ shaped mesostructures. These results show that $L_1^{(2)}/L_0^{(2)}$ increases with increasing the length ratio ($L_0^{(2)}/L_0^{(1)}$), and is almost independent of the applied strain. In the ‘T’ and ‘H’ shaped mesostructures, the interaction between the ribbon components results in both bending and

Figure 5. Results of model calculations, approximate solution and FEA on the dimensionless maximum out-of-plane displacements of the two ribbons components in frame mesostructures and the ratio of maximum out-of-plane displacements at three different levels of compressive strains, for the three different frame mesostructures: (a–c) ‘+’ shaped mesostructure, (d–e) ‘T’ shaped mesostructure and (g–i) ‘H’ shaped mesostructure. (Online version in colour.)

twisting deformations in the ribbon (1) component. A physical quantity, namely, the mode ratio, which is defined as the ratio of the average twisting curvature to the average out-of-plane bending curvature ($\rho = |\hat{\kappa}_3|_{\text{avg}}/|\hat{\kappa}_2|_{\text{avg}}$) [28], can be used to characterize the deformation mode of ribbon (1) in the ‘T’ and ‘H’ shaped mesostructures. Figure 6c,d shows that such a mode ratio ($\rho^{(1)}$) decreases gradually with increasing the length ratio ($L_0^{(2)}/L_0^{(1)}$), suggesting an intensified bending-twisting coupling at relatively small length ratios.

The maximum principal strain of a certain cross section is determined by the out-of-plane bending strain and twisting strain, i.e.

$$\varepsilon_{\text{principal}} = \frac{\varepsilon_{\text{bending}}}{2} + \sqrt{\left(\frac{\varepsilon_{\text{bending}}}{2}\right)^2 + \left(\frac{\gamma_{\text{torsion}}}{2}\right)^2}, \quad (3.1)$$

in which, $\varepsilon_{\text{bending}} = t|\hat{\kappa}_2|/2$, $\gamma_{\text{torsion}} = t|\hat{\kappa}_3|$, and the in-plane bending strain is neglected. Since the bending deformations usually play a dominant role (figure 6c,d), the maximum principal strain typically occurs at the midpoints ($\bar{Z}^{(1)} = 0$) of the ribbon-(1) component, for the cases studied herein. According to equations (2.6), the curvatures at the midpoints ($\bar{Z}^{(1)} = 0$) can be obtained as

$$\hat{\kappa}_2^{(1)}|_{\bar{Z}^{(1)}=0} = U_1^{(1)''}|_{\bar{Z}^{(1)}=0} = -2\pi^2 a_1^{(1)} \frac{1}{L_0^{(1)}} \sqrt{\varepsilon_{\text{appl}}} \quad (3.2)$$

Figure 6. (a,b) Dimensionless material coordinates of the positions of maximum out-of-plane displacement in ribbon-(1) component for the ‘+’ and ‘T’ shaped mesostructures, respectively. (c,d) The mode ratios of the ribbon-(2) component for the ‘T’ and ‘H’ shaped mesostructures, respectively. (Online version in colour.)

Table 2. Approximate solutions of $a_9^{(1)}$ and $a_{10}^{(1)}$.

	$a_9^{(1)}$	$a_{10}^{(1)}$
‘T’ shaped structures	$0.46(L_0^{(2)}/L_0^{(1)})^{0.5}$	$6.19/(L_0^{(2)}/L_0^{(1)})$
‘H’ shaped structures	$0.43(L_0^{(2)}/L_0^{(1)})^{0.5}$	$5.32/(L_0^{(2)}/L_0^{(1)})$

and

$$\hat{\kappa}_3^{(1)}|_{\bar{z}^{(1)}=0} = \phi^{(1)'}|_{\bar{z}^{(1)}=0} = a_9^{(1)} a_{10}^{(1)} \frac{1}{L_0^{(1)}} \sqrt{\varepsilon_{\text{appl}}}. \quad (3.3)$$

Then, the normalized maximum strains can be derived as

$$\frac{\varepsilon_M L_0^{(1)}}{t \sqrt{\varepsilon_{\text{appl}}}} = \frac{\pi^2 a_1^{(1)}}{2} + \sqrt{\left(\frac{\pi^2 a_1^{(1)}}{2}\right)^2 + \left(\frac{a_9^{(1)} a_{10}^{(1)}}{2}\right)^2}. \quad (3.4)$$

Similar to $a_1^{(1)}$ and $a_1^{(2)}$, we obtained the approximate solutions to the parameters $a_9^{(1)}$ and $a_{10}^{(1)}$ (table 2), by fitting the analytic results for a wide range of length ratio $(L_0^{(2)}/L_0^{(1)})$. Figure 7 presents the normalized maximum strains for the three different shaped mesostructures. Here, the discrepancy between theoretical predictions and FEA can be mainly attributed to the underestimate of the maximum bending curvatures.

Figure 7. Results of model calculations, approximate solution and FEA for the maximum principal strain versus the length ratios for the three frame mesostructures, including (a) ‘+’ shaped mesostructure, (b) ‘T’ shaped mesostructure and (c) ‘H’ shaped mesostructure. (Online version in colour.)

4. Conclusion

This work presented systematic postbuckling analyses of three representative frame mesostructures (‘+’, ‘T’, and ‘H’ shaped), through combined theoretical modelling, FEA calculations and experimental measurement. The energy-based theoretical model provided accurate predictions of key physical quantities related to the buckling-guided 3D assembly, including the maximum out-of-plane displacements, mode ratios and maximum strains of the frame mesostructures. The developed model can be also extended to the postbuckling analyse of other frame geometries (e.g. those consisting of more ribbon components or even arc components). The results presented in this paper can be useful in the design optimization of frame-shaped 3D micro-devices through the buckling-guided 3D assembly.

Data accessibility. All data are contained within the paper.

Authors' contributions. Y.Z. conceived and supervised the project. Y.L. built the analytic model and conducted the numerical simulations. Z.X. and Y.L. conducted the experiments. Y.L., K.C.H., Y.H. and Y.Z. analysed the results. Y.L. and Y.Z. wrote the manuscript.

Competing interests. We declare we have no competing interests.

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