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Electrically Small Dipole Antenna Probe for Quasistatic Electric Field Measurements in Transcranial Magnetic Stimulation

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Abstract

The present paper designs, constructs, and tests an electrically small dipole antenna probe for the measurement of electric field distributions with the ultimate purpose to directly measure electric fields induced by a transcranial magnetic stimulation (TMS) coil. Its unique features include applicability to measurements in both air and conducting medium, high spatial resolution, large frequency band from 100 Hz to 300 KHz, efficient feedline isolation via a printed Dyson balun, and accurate mitigation of noise. Prior work in this area is thoroughly reviewed. The proposed probe design is realized in hardware; implementation details and design tradeoffs are described. Test data are presented for the measurement of a constant wave capacitor electric field, demonstrating the probe's ability to properly measure electric fields caused by a charge distribution. Test data are also presented for the measurement of a constant wave solenoidal electric field, demonstrating the probe's ability to measure electric fields caused by Faraday's law of induction. Those are the primary fields for the transcranial magnetic stimulation. Further steps necessary for the application of this probe as an instrument for TMS coil design are discussed.

Keywords

Biomagnetics; Electric Field Probe; Magnetic Instruments; Transcranial Magnetic Stimulation

I. INTRODUCTION AND REVIEW OF THE PRIOR ART

TRANSCRANIAL magnetic stimulation (TMS) is a commonly used tool for the clinical treatment of certain neuropsychiatric disorders [1]. In TMS, a coil is positioned above a patient's head and is driven with a sequence of short current pulses, hereby inducing a sequence of electric field pulses in a targeted region of the patient's brain according to Faraday's law of induction, which subsequently modulate the neural activity [2].

Generally, the total electric field can be expressed in terms of potentials, that is

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \varphi \quad (1)$$

whereby φ is the scalar electric potential and $\vec{A}$ is the magnetic vector potential. In the quasistatic approximation to Maxwell's equations commonly used in TMS, the time derivative in the Lorentz gauge, $\frac{1}{c^2} \frac{\partial \varphi}{\partial t} + \nabla \cdot \vec{A} = 0$ is neglected (which gives us the

Coulomb gauge, $\nabla \cdot \vec{A} = 0$) while it is still kept in Eq. (1). As a result, the $-\nabla \varphi$ term in Eq. (1) becomes a conservative electric field contribution due to charge density alone while the $-\frac{\partial \vec{A}}{\partial t}$ term is a solenoidal electric field contribution due to current density alone. Below, a distinction will be made between probes that can only capture the solenoidal contribution or $-\frac{\partial \vec{A}}{\partial t}$, and probes that can capture the complete electric field as described in (1).

The major types of probes that have been used to measure the electric fields patterns of TMS coils in the past are the following:

- a) electric field probes using bioelectrodes;
- b) $\frac{\partial \vec{B}}{\partial t}$ probes using a small loop;
- c) $\frac{\partial \vec{B}}{\partial t}$ probes using a long rectangular loop;
- d) $\frac{\partial \vec{B}}{\partial t}$ probes using a triangular loop.

Among these only a) is designed to be used in tissue or saline solution-based phantoms, and only a) can measure the complete electric field of (1) under general conditions. The rest of the probes are meant to be used in air. The probes in b) and c) can only measure the curl of the solenoidal part of the electric field. The triangular $\frac{\partial \vec{B}}{\partial t}$ probe of d) can be used to make a phantom measurement of the complete electric field found in a spherically symmetric volume conductor by way of a duality argument.

A. E-field Probes Using Bioelectrodes

A bioelectrode is simply an electrode designed to be compatible for measurements in bodily tissues [3]. The free charge carriers in the metal conductor that makes up the electrode are electrons. The free charge carriers in bodily tissue are ions available in solution. The simplest model for a bioelectrode in an electrolytic solution is a built-in electric potential (a voltage source) in series with a parallel resistor and capacitor [3]. This voltage source is sometimes called the half-cell potential or the reversible potential [3]. The resistance is called the charge transfer resistance, or R_{CT} , and represents the change in the voltage drop across the electrode-electrolyte interface with current flow [3]. The capacitance is called the double-layer capacitance, or C_{DL} , and is reviewed in great detail by Bockris in [4].

Bioelectrode-based electric field probes have been successfully used in the past to measure the electric fields of TMS coils [5]–[9]. A popular approach is to use two bioelectrodes to measure the voltage drop over a short distance in the test medium, be it saline solution or a

real tissue sample [10]. The electric field is the measured voltage divided by the electrode separation distance.

One important requirement not explicitly stated in prior works is that the passive interface impedance of the bioelectrodes, e.g. R_{CT} in parallel with C_{DL} , should ideally be much lower than the input impedance of the downstream differential amplifier. This is because the passive interface impedance is not stable; both R_{CT} and C_{DL} can change with time, temperature, frequency and electrolyte concentration [11]. Sometimes special processing has to be administered to lower the interface impedance. For example, Yunokuchi [7] needed to plate his electrodes with platinum black in order to lower their impedance.

A second important requirement is that the feedlines must be routed together as closely as possible. The voltage seen by the downstream differential amplifier is the superposition of the voltage drop along the bioelectrodes due to (1) and the contributions of $-\partial\vec{A}/\partial t$ along the feedlines. Exact cancellation of the feedline contributions can only occur if the feedlines are coincident in space as explained and demonstrated experimentally by Glover [10].

B. $\partial\vec{B}/\partial t$ Probes Using a Small Loop

A small loop can be used to measure the magnetic field or flux. If the x, y and z components of the magnetic field over space are sampled properly, the inverse problem for the vector potential $\nabla \times \vec{A} = \vec{B}$ could be solved. These probes can only measure the solenoidal electric field. The advantage of small loop probes is that they are very easy to construct and that the measurements can be made in air. The disadvantage is that only the solenoidal electric field can be derived, and that three very small orthogonal loops and very many measurements are required to accurately restore the electric field.

C. $\partial\vec{B}/\partial t$ Probe Using a Long Rectangular Loop

Epstein [12] pioneered a clever rectangular loop probe to measure the solenoidal electric field without solving the inverse problem. His approach was successfully applied by Salinas [13], [14] where he refers to the rectangular loop as a 3D eddy current probe. The basic construction of this $\partial\vec{B}/\partial t$ probe consists of a long rectangular loop, whose long sides are determined by the amount of distance required for the magnetic vector potential to fall off to negligible levels [12]. The short sides are determined by the desired spatial resolution. One short side is placed at the location and in the direction for the desired primary electric field measurement, while the other short side is connected to some voltage measuring devices like an oscilloscope. If the long sides are centered along an axis of symmetry with respect to the magnetic vector potential, then the $-\partial\vec{A}/\partial t$ contributions along the long sides cancel out and the total *emf* is simply the length of the short side multiplied by its $-\partial\vec{A}/\partial t$; this is the voltage that is ultimately measured.

Tofts [15] criticized this approach for only being able to measure the solenoidal electric field along an axis of symmetry. However, that assessment is only true if a single measurement is permitted. Salinas [14] claims the design can be applied off of the axis of symmetry by using two measurements. The advantage of the rectangular loop is that measurements can be made

in air and that it is relatively easy to construct. Salinas [13] suggested a simple way to create a two-axis probe using two orthogonal rectangular loops on a single support rod. The disadvantages of the probe are that it can only measure the solenoidal electric field and that it may only be able to measure the field along an axis of symmetry as Epstein [12] claimed.

D. $\partial \vec{B} / \partial t$ Probe using a Triangular Loop

The triangular loop-based $\partial \vec{B} / \partial t$ probe was created by Nieminen [16]. His probe, called the TMS calibrator, was inspired by prior work of Ilmoniemi on the inverse and forward problems of magnetoencephalography (MEG) [17], [18]. It can be argued by reciprocity that the *emf* measured by the triangular loop is the same as the complete electric field integrated along the tangential length in a spherically symmetric volume conductor [16]. An elegant graphical proof of this can be found in Nieminen's paper [16]. Nieminen's triangular loop-based probe can only be used for spherical symmetry.

E. Other Electric Field Probes

In a review of standard probes for electromagnetic field measurements, Kanda [19] discusses the idea of an electrically short dipole antenna loaded with a capacitive load. Due to its capacitive self-impedance, the electrically short dipole antenna will yield a frequency independent transfer characteristic when driving a capacitive load. Kanda [19] experimentally demonstrates this feature using a large 15-cm dipole (6-mm radius) connected to a 13 pF FET amplifier to achieve a flat frequency response from 2kHz to 400MHz. The low frequency cutoff of Kanda's [19] E-field probe is determined by the RC time constant made between the input resistance of the FET amplifier and the combined input capacitance of the FET amplifier together with the antenna capacitance of the dipole. The end application of Kanda's capacitively loaded small dipole antenna was in the measurement of EMI and EMC problems and determination of safe levels for electromagnetic radiation [19]. In recent years, other researchers have continued to use electrically small dipoles to create E-field probes for EMC and EMI test purposes [20], [21].

F. Our Design

After reviewing all available design variations, we suggest to use the capacitively-loaded, dipole antenna described by Kanda [19] as a starting point. The dipole is intentionally operated far below its first resonance in order to take advantage of its capacitive self-impedance to create a frequency-independent voltage transfer characteristic between the dipole and its downstream capacitive load. The dipole length cannot be designed to be arbitrary small as there is a fundamental tradeoff between spatial resolution and signal-to-noise ratio as discussed in this paper and previously by Smith [22]. Feedline isolation is required, and for this we employed a printed Dyson balun [23]–[24]. Electric circuitry that buffers and amplifies the transduced voltage signal over the frequency band from 100Hz-300kHz at a high signal-to-noise ratio is presented, which covers a wide variety of TMS pulse applications.

II. DIPOLE ANTENNA, FEEDLINE, AND CIRCUIT IMPLEMENTATION

A small dipole with a total length of 5 mm (tip to tip) shown in Fig. 1a was printed to satisfy the spatial resolution requirements of the application. A simplified printed version of the Dyson balun [22]–[23] is considered, which is equivalent to a shielded, balanced two-conductor transmission line. This dipole is self-resonant at 30 GHz (in air) meaning that the antenna impedance is purely capacitive around 3 kHz with virtually zero radiation resistance. Instead of attempting to match this antenna we suggest coupling it to the inputs of an instrumentation amplifier (iNA) with high input impedance as shown in Fig. 1c.

Since our dipole is extremely electrically small, being 10^{-7} wavelengths, an attempt to match it would result in negligible bandwidth (assuming both our antenna and the matching network were lossless) as explained by Chu [25] and Wheeler [26]. Given the very short electrical length the problem of designing the receiving circuits is best viewed from the electroquasistatic (EQS) perspective as opposed to the full wave perspective. Instead of thinking in terms of power waves and aiming to maximize power transfer, we are mainly interested in voltages and aiming to achieve a flat voltage gain over our passband. Since the self-impedance of an electrically small dipole is capacitive the only way to achieve a flat voltage gain is to ac-couple the dipole to a capacitive load, which in our case is the input impedance of the balun and the INA.

Fig. 1b shows the dipole, balun and instrumentation assembled together. The dipole and balun were implemented using a 4-layer, FR4-based PCB; the instrumentation was implemented on a 2-layer, FR4-based PCB. The two PCBs were bridged using a SMA connector assembly. The loop made by the SMA connector bridge was a source of unwanted $\vec{\partial B} / \partial t$ injection during solenoid field testing; the parasitic loop was eliminated by cutting the feedline traces at the input and output of the SMA connectors and making the connections with tightly coupled flying wires as shown in Fig. 1b.

Fig. 1c shows a simplified schematic for the complete electric field probe; biasing and other ancillary circuitry are omitted. The dipole-facing and output facing circuits are galvanically isolated via U_4 and separate batteries are used to supply the bias power. The component values used for our implementation are: $R_B = 30 \text{ M}\Omega$, $C_{IN} = 22 \text{ pF}$, $R_F = 2 \text{ k}\Omega$, $R_G = 200 \Omega$, $R_A = 1 \text{ k}\Omega$, $R_F = 15 \text{ k}\Omega$, $R_{VI} = 82 \text{ k}\Omega$, $R_{IV} = 82 \text{ k}\Omega$, $R_E = 50 \Omega$, $C_1 = 10 \text{ pF}$, $C_2 = 5 \text{ pF}$, $Q_1 = \text{ON Semiconductor BC856}$. All resistors excepting R_e and R_B should be 1% tolerant (preferably 0.1%). C_{IN} should match to better than 1pF and utilize a Class 1 dielectric (e.g. preferably NP0). All op-amps were implemented using the Texas Instruments OPA2325; U_{1A} and U_{1B} must be copackaged. The Broadcom HCNR201 was used to implement the matched opto-coupler depicted with U_4 .

VREF was pinned to 1.8V above VLF and was developed using the Texas Instruments TLV431B shunt regulator. The net label VLF represents the return of the antenna-facing floating circuits.

The bias resistance R_B sets the DC bias point for the INA. Without it, the DC bias point would be determined by printed circuit board leakage paths, op-amp input bias currents and

board contamination/cleanliness. With a bias resistance as high as 500 M Ω functionality was confirmed but a sensitivity to solder flux residue was observed in our testing.

Being a small dipole, the antenna capacitively couples into the inputs of the INA. The low frequency cutoff, f_{LOW} , is equal to $(2\pi R_B C_{TOT})^{-1}$, where C_{TOT} is made up of double the antenna's impedance, double the differential line-to-line capacitance of the two-conductor transmission line, the whole line-to-shield capacitance of the two-conductor transmission line, the common-mode input capacitance of INA op-amp, the parasitic shunt capacitance of R_B , and any additional external input capacitance C_{IN} . If no additional C_{IN} is inserted, C_{TOT} will be approximately 30 pF for our design with the dominant contribution coming from the capacitances associated with the two-conductor transmission line. The additional input capacitance C_{IN} is a degree of freedom for lowering f_{LOW} . At the same time, C_{IN} also attenuates the input signal resulting in a degradation of the SNR.

The SNR of the system is determined by the INA input stage alone. Downstream stages do not contribute into the noise calculation due to the high gain of the INA, which should be set as large as possible given bandwidth and accuracy requirements; in our implementation it was set to 20 V/V, which reduces the closed loop bandwidth of our selected op-amp to 500 kHz. The noise contributors are the equivalent noise voltage and equivalent noise current of the INA op-amps, e_n and i_n , respectively, and the thermal noise from R_B shown in Fig 2a. It is assumed that INA feedback resistors, R_G and R_F shown in Fig. 1 are chosen such that their noise contributions are negligible.

The total input referred noise is approximately given by $\sqrt{(\pi/2)f_{BW}e_n^2 + k_B T / C_{TOT}}$, where f_{BW} is the bandwidth of the INA op-amp, k_B is Boltzmann's constant and T is the absolute temperature in Kelvin. Even with $R_B = 30\text{M}\Omega$ the noise contribution from i_n turns out to be negligible compared with that of e_n for the OPA2325, and thus it was omitted. The input signal is determined by the full-scale input field E_{FS} , the dipole length l_A , and the capacitive divider made between the half-circuit antenna capacitance C_A and C_{TOT} . The expression for the resultant SNR is obtained in the form:

$$SNR = 20 \log_{10} \left(\frac{E_{FS} l_A C_A}{C_{TOT} \sqrt{(\pi/2)f_{BW}e_n^2 + k_B T / C_{TOT}}} \right), \quad (2)$$

A plot of Eq. 2 versus $(C_{TOT} - C_A)$ is shown in Fig. 2b. We plot the SNR of our system using the OPA2325 with $e_n = 10\text{nV}/\sqrt{\text{Hz}}$ in blue and the SNR of a system using a noiseless amplifier in red; the difference between the curves is the noise figure of our circuit. In order to minimize this noise figure (i.e. keep it less than 5dB) we should keep C_{TOT} less than 100 pF. We also see that the SNR of our system falls off at two different slopes. When C_{TOT} is less than 20 pF but greater than 0.1pF the SNR degrades at a rate of 10 dB/decade; whereas, when C_{TOT} is greater than 20pF the SNR degradation slope becomes 20 dB/decade. This difference in slopes is due to the fact that the noise power saturates to a constant level determined by e_n alone when C_{TOT} is greater than 20 pF, whereas the noise power decreases at a rate of 10 dB/decade when C_{TOT} is less than 20 pF. The plateau seen at minimal $(C_{TOT} - C_A)$

– C_A) can be understood analytically by taking the limit of Eq. (2) as ($C_{TOT} \rightarrow C_A$). Since the self-capacitance of an electrically small dipole is proportional to its length l_A [19] we can replace C_A in our expression with κ/l_A , where constant κ has units of F/m and is the first coefficient of the power series expansion, and neglect the contribution from e_n altogether, which yields

$$\lim_{C_{TOT} \rightarrow C_A} SNR = 20 \log_{10} \left(\frac{E_{FS}}{\sqrt{k_b T}} \sqrt{\kappa} (l_A)^{3/2} \right), \quad (3)$$

Eq. (3) illustrates a tradeoff between the spatial resolution and the maximum achievable SNR. For a given dipole length, l_A , the only means to improve the SNR is to increase the full-scale electric field or to decrease temperature. The derivation presented here is simplified; a more detailed derivation of a similar system can be found in [22].

The design of the rest of the system is less critical and we will present it expeditiously. The instrumentation amplifier gives us a differential output. We will ultimately need a single ended output and more gain. To accomplish this, a simple difference amplifier was used to do the differential to single-ended conversion.

The last part of the design is galvanic isolation via the Broadcom HCNR201 matched opto-coupler shown in Fig. 1c. The purpose of the galvanic isolation is to suppress any common-mode current. The optical isolation amplifier of Fig. 1c depends on tight matching of photodiodes PD1 and PD2 and equal illumination by the LED, which is guaranteed by the design of the HCNR201. The first op-amp drives the PNP and ultimately the LED to make the current in PD1 and PD2 equal to V_{IN}/R_{VI} . Hence the output voltage, V_{OUT} , will be equal to V_{IN} to within the bandwidth of the cascade of amplifiers, which is set by C_1 and C_2 .

Both sets of circuits, on either side of the isolation barrier, are battery powered to eliminate any potential common-mode noise issues.

III. FREQUENCY RESPONSE OF THE PROBE

A. Experimental Setup

The differential-mode and common-mode frequency responses of the probe were measured. The differential-mode frequency response is defined as the magnitude and phase responses relating the probe output voltage to the input electric field strength at the dipole feed. The common-mode frequency response is defined as the magnitude and phase responses relating the probe output voltage to the floating circuit return voltage, i.e. VLF as shown in Fig 1c.

The differential mode frequency response was measured using the parallel plate capacitor setup described in section V together with the AP instruments AP300 network analyzer. The dipole feed of the electric field probe was positioned in the center of the capacitor test setup with the dipole wings aligned along the axis of symmetry. The top plate of the capacitor test setup was connected to both the injection source and the input channel (i.e. A) of the network analyzer. The output port, V_{OUT} , of the electric field probe was connected to the

output channel (i.e. B) of the network analyzer. The bottom plate of the capacitor test setup and the outer conductors of all coaxial cables used (for the injection source, input channel A and output channel B) were connected to the return of the output port, V_{OUT} , of the electric field probe. The output power of the injection source was set to 7dBm, corresponding to a voltage amplitude of 0.7V across the plates of the capacitor test setup. The frequency of the injection source was swept from 10 Hz to 1 MHz, and both the magnitude and phase responses of chosen network (i.e. B/A) were measured. The capacitor setup was chosen to evaluate the differential mode frequency response due to the relative ease-of-use of the setup. The test capacitor has very low capacitance making it very easy to drive with the network analyzer's injection source.

The common-mode frequency response was evaluated using the instrumentation PCB assembly alone with the Agilent 4395A network analyzer. The SMA connector inputs of the instrumentation PCB assembly were shorted together. Both the injection source and input channel (i.e. A) of the network analyzer were connected to the floating circuit return, VLF. The output channel (e.g. B) of the network analyzer was connected to the output port, V_{OUT} . The outer conductors of all coaxial cables used (for the injection source, input channel A and output channel B) were connected to the return of the output port, V_{OUT} , of the electric field probe. The output power of the injection source was set to 15dBm, corresponding to a voltage amplitude of 1.78V across the optical isolation barrier. The frequency of the injection source was swept from 100Hz to 1MHz, and both the magnitude and phase responses of chosen network (i.e. B/A) were measured.

B. Results

The differential mode and common-mode frequency responses of our electric field probe are shown in Fig. 3a, b. The measured differential magnitude response results were normalized around the differential mode gain at 3 kHz. The absolute differential-mode conversion factor of the electric-field probe at 3kHz is 0.227 V output variation per 1 V/cm of electric field variation at the dipole feed. The differential-mode ac measurements confirm our claimed 100Hz to 300kHz bandwidth.

Ideally, the common-mode magnitude response of the electric field probe would be extremely small. At 3kHz the measured common-mode magnitude response is approximately -50dB as shown in Fig. 3b.

IV. TRANSIENT RESPONSE OF THE PROBE

A. Experimental Setup

The transient response was measured using the parallel plate capacitor setup described in Section V together with an Agilent 33250A function generator. The dipole feed of the electric field probe was positioned in the center of the capacitor volume with the dipole wings aligned along the axis of symmetry. The top plate of the capacitor test setup was connected to function generator output. The bottom plate of the capacitor test setup and function generator return were connected to the return of the output port, V_{OUT} , of the electric field probe. The load impedance setting of the function generator was set to high

impedance. The square wave function was used to drive function generator output at an amplitude of 5V. The output port of the electric field probe was connected to a Tektronix MSO4000B oscilloscope. Transient response waveforms given a function generator frequency of 100 Hz, 10 kHz and 100 kHz were captured.

B. Results

Given the frequency response plots of Section III, the transient response is expected to have an AC-coupled characteristic at frequencies below 1 kHz as demonstrated in Fig. 4a.

At frequencies greater than 1 kHz, the step response of the probe follows the electric field between the plates with a high fidelity as shown in Fig. 4b. Even at 100 kHz the electric field probe is able to follow the external electric field as shown in Fig. 4c, d. However, there seems to be an artifact in the output at the onset of the step at 100 kHz. This is due to the common-mode gain of the probe. If the probe orientation is flipped, the artifact appears in the opposite direction. If the floating circuit return, VLF, is connected to the output voltage return, effectively shorting out the optical isolation, this artifact completely disappears as shown in Fig. 4d proving that indeed common-mode injection caused the artifact.

V. MEASURING ELECTRIC FIELD DISTRIBUTION OF A PARALLEL PLATE CYLINDRICAL CAPACITOR

A. Experimental Setup

A parallel-plate, air-filled capacitor with plate separation distance of 4.128 cm, circular plate diameter of 10 cm, and plate thickness of 0.25 mm was constructed. The plates of the test capacitor were driven by a bipolar function generator created using an Agilent 33250A function generator together with a 1:1 transformer to create a complementary output so that the potentials of the top plate and bottom plate were equal in magnitude but perfectly output of phase; the differential voltage amplitude across the plates was 7.6V; the frequency of excitation was 20 kHz; and the wave shape was sinusoidal. The test frequency was chosen to accommodate the frequency response of the 1:1 transformer. The output port of the electric field probe was connected to a Tektronix TDS2000C oscilloscope. The resultant constant wave (CW) electric field was measured at different points in space by measuring the peak-to-peak amplitude on the scope with a single shot acquisition.

B. Results

The measured quasistatic electric field pattern was qualitatively evaluated. The fringing fields near the perimeter of the plates and the outside-facing plate centers were evaluated for correct relative field polarities. The polarization of the dipole probe was evaluated. The electric field strength between the plates was measured to be nearly constant as shown in Fig. 5. The only peculiarity noted was an apparent increase in the measured electric field strength as the proximity of the dipole to either plate increased, which is not entirely understood. The increase of the electrical field strength close to capacitor plates could possibly be explained by the effect of the dipole charge/current on the charge distribution on the nearby plate which may deviate from a uniform distribution. This effect should be investigated in a future work. Outside of this peculiarity the performance of the probe in the

presence of a quasistatic electric field within the capacitor was found satisfactory and met our expectations.

The measurement accuracy of this test setup is limited by DC vertical accuracy specification of the Tektronix TDS2000C scope and by apparent signal noise. The vertical sensitivity of the scope was adjusted as measurements were made in order to always maximize the dynamic range utilized of the oscilloscope's digitizer. Even when half of the oscilloscope screen is filled the quantization accounts for less than 1% of measurement error. The limiting factor for measurement accuracy for electric field intensities greater than 0.67V/cm in magnitude is the DC vertical accuracy of the TDS2000C, which is $\pm 3\%$. For electric field intensities less than 0.67V/cm the limiting factor for measurement accuracy is the amplitude of the apparent signal noise, which was 20mV/cm peak-to-peak.

VI. MEASURING INDUCED ELECTRIC FIELD OF A SOLENOIDAL COIL

A. Experimental Setup

The schematic for the solenoidal test system is shown in Fig 6. This circuit applies a single-shot sinusoidal current pulse to the coil via a series resonant RLC circuit made by the coil inductance L_{FLD} , the resonant capacitance C_{RES} , and the total series resistance from the MOSFET Q_1 and the equivalent series resistances of reactive components and interconnecting wires. The pulse current amplitude, neglecting damping, is equal to the initial voltage of C_{RES} divided by the tank impedance, e.g. $\sqrt{L_{FLD} / C_{RES}}$; and the pulse frequency is equal to $(2\pi\sqrt{L_{FLD}C_{RES}})^{-1}$. The resonant frequency of the test circuit was designed to be 3.846kHz; this was chosen as it is representative of the excitation for a TMS pulse. A larger resonant frequency would have resulted in a larger induced electric field, which would have been easier to measure. We chose to stay close to 3kHz in order to evaluate our probe's suitability for the desired end application.

To operate the test setup, a bias voltage is applied to the V_{BIAS} port. The current pulse is initiated by pulsing the V_{FG} with a manually triggered 1 μ s, 5V single-shot pulse using a pulse generator. The peak current pulse amplitude is given by

$$i_{coil} \approx \frac{R_{F1}}{(R_{F1} + R_B)} \frac{V_{BIAS}}{\sqrt{L_{FLD} / C_{RES}}} \quad (4)$$

The largest bias voltage utilized in our testing was 60 V, resulting in peak coil current of 36 A. Since significant reactive power is processed in the operation of this circuit it is critical to reliably turn off the switch at the end of the bi-stable pulse. For this reason, our test setup implements a zero-crossing detector to automatically soft-switch Q_1 off.

The component values used in our implementation are the following: $R_{FG}=50 \Omega$, U_1 =Texas Instruments CD40106B, U_2 =Texas Instruments CD4001UB, U_3 = Texas Instruments CD4013B, U_4 =Advanced Linear Devices ALD2302, U_5 =Texas Instruments UCC27322, Q_1 =International Rectifier IRL2910PBF, D_1 = 1N4148, $R_{IN}=10 \Omega$, F_1 =250V/500mA fuse,

$C_B = 33\mu\text{F}$, 100V, aluminum electrolytic with $\sim 1\text{ohm}$ ESR, $R_B = 200\ \Omega$ resistor network, $R_{F1} = 2\ \text{k}\Omega$ resistor network, $R_{F2} = 1\ \text{k}\Omega$ resistor network, $C_{RES} = 30\mu\text{F}$ made using $3\times$ parallel connected Panasonic ECQ-E2106KF $10\mu\text{F}$ film capacitors, $C_F = 470\text{pF}$, NP0, ceramic capacitor, $C_{HYS} = 10\text{pF}$, NP0, ceramic capacitor. The R_{F1} network needs to be sized for DC power dissipation. The R_B network can be sized according to its single pulse power curves. The MOSFET driver U_5 needs to be placed close to Q_1 . The interconnecting loop made by L_{FLD} , C_{RES} , Q_1 should have negligible area. The maximum attainable coil current is limited by power dissipation in Q_1 ; therefore, adequate heat sinking of Q_1 is critical.

The measured impedance of the solenoid and a description of its winding is given in Table 1.

B. Results

The magnetic vector potential, $\vec{A}$, of a solenoid (assuming the Coulomb gauge) forms a circulating vector field pattern centered at the coil axis. The induced electric field pattern, or $\vec{E} = -\partial\vec{A}/\partial t$, should follow this pattern. For a sinusoidal current pulse, the expected induced field waveform would therefore be a cosine pulse. Our initial attempts to measure this field pattern were unsuccessful. Both the waveform shape and field pattern were different from the expected ones. We found two issues that prevented us from correctly measuring the induced electric field. Once these issues were resolved we were able to measure the field $\vec{E} = -\partial\vec{A}/\partial t$ rather accurately.

First, we needed to eliminate solely quasistatic electric fields generated in the process of pulsing the coil. These fields are due to electric charges from the resonant capacitor, the interconnecting conductors, and the charges generated at the coil terminals themselves (see Fig. 7a). They are unimportant for in-tissue/phantom measurements since the corresponding quasistatic electric fields will be blocked at a conductivity boundary, between the air and sample, identical to classical electrostatics.

To quantify this effect, we oriented the dipole along the length of the coil, which is perpendicular to the magnetic vector potential. After pulsing the coil, we observed a very large signal at the output of the probe, which cannot be $-\partial\vec{A}/\partial t$ due to the perpendicular polarization. This signal was all but eliminated when a shield in the form of an aluminum plate was inserted in between the coil and the dipole as in Fig. 7a.

Second, the SMA connector bridge between the dipole/balun and the instrumentation PCB forms a perfect capacitive closed loop for exciting a parasitic *emf* signal proportional to $\partial\vec{B}/\partial t$. The loop area was large enough so that the induced *emf* voltage was greater than the induced voltage at the dipole feed. This problem was solved by cutting the feedlines at the SMA connector interfaces and instead completing the electrical connections between the feedlines and the inputs to the instrumentation by way of a closely-spaced differential pair made of flying magnet wire; the function of the SMA connector assembly was reduced to structural support and shield biasing.

A plot of the measured $\vec{E} = -\partial\vec{A}/\partial t$ versus a theoretical prediction is shown in Fig. 7c. The theoretical electric field was computed by numerically evaluating the magnetic vector

potential of a finite solenoid. We hypothesize that the remaining difference between the measured and computed data is due to stray magnetic field linking a small but finite gap between the feedlines. Finite element simulations could be performed on the present dipole (including its two-conductor transmission line) to confirm this hypothesis or to verify that the probe possesses no higher-order modes other than the electrical dipole mode.

However, such simulations are out of the scope of the present study but should be investigated in a separate article.

The measurement accuracy of this test setup is limited by both the measurement accuracy of the scope and the output noise of the probe at the small signal levels measured. The oscilloscope used for these measurements was the Tektronix MSO4034B. All measurements were made at a vertical sensitivity of 5mV/div with 64 samples for averaging to reduce the output random noise voltage by a factor of 8. The DC vertical accuracy of the scope at this vertical sensitivity is $\pm 1.5\%$. The peak-to-peak noise envelope of the measured signal after averaging over 64 trials was 4.4mV/cm. The quantization of the oscilloscope's digitizer was 0.86mV/cm for all measurements.

VII. CONCLUSIONS

In this study we have designed, constructed, and tested a basic electrically small dipole antenna probe for the measurement of electric field distributions over the frequency band from 100 Hz to 300 KHz. Its unique features include applicability to measurements in both air and conducting medium, high spatial resolution, efficient feedline isolation via a printed Dyson balun, and accurate mitigation of noise. Given the proper electrostatic insulation, our probe is the only one that can measure the solenoidal electric field in air in the general case.

The wide bandwidth of our probe given its electrically small size seems to violate the Wheeler-Chu limit as described in [25]–[26]. The Wheeler-Chu limit specifies the minimum theoretical Q of a lossless linearly polarized small antenna. The Wheeler-Chu limit represents a bound on the probe's bandwidth only when the antenna is matched to maximize power flow at a particular center frequency. In this case the bandwidth of small antenna would be inversely proportional to the minimum Q (see [27],[28]). Since the receiving network for our dipole antenna is a capacitive load we achieve a frequency-independent, wideband voltage gain characteristic by way of the capacitive divider formed. With this approach our electric field probe's bandwidth is not limited by its minimum Q as dictated by the Wheeler-Chu limit.

Test data have been presented for the measurement of a CW capacitor electric field, demonstrating the probe's ability to properly measure electric fields caused predominantly by charge distributions per the EQS approximation. Test data have also been presented for the measurement of a CW solenoidal electric field, demonstrating the probe's ability to measure electric fields caused predominantly by currents by way of Faraday's law of induction per the magnetoquasistatic (MQS) approximation.

Our next step will be to convert the designed probe to a high-accuracy instrument for TMS coil design that will enable characterizing the fields of the TMS coil/discharge circuit. For

the antenna in air, we will need to develop a means to effectively isolate the solenoidal electric field contribution created by the TMS coil, and be immune to the electric fields not predicted by the MQS approximation. Several ways of doing so are under investigation including directly shielding the outer surface of the supporting PCB.

For the case when the dipole antenna would be embedded within a tissue, its capacitance will be somewhat altered by the tissue permittivity. Hence, the voltage gain of our capacitively coupled dipole antenna will change. We believe that this problem can be resolved using an automatic gain control circuit and a proper calibration.

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Fig. 1.

a) – Zoom-in of the 5mm printed dipole; b) – the dipole, balun and instrumentation assembled together; c) – a simplified schematic for the complete electric field probe; biasing and other ancillary circuitry are omitted.

Fig. 2.

a) – Simplified INA half-circuit noise model; b) – computed signal-to-noise ratio following Eq. (1); note that i_{in} is set to 0 for these calculations.

Fig. 3. Differential mode (a) and common-mode (b) frequency responses of the probe. The differential mode magnitude response is normalized to the data point at 3kHz, where 0.227V was measured on the probe output given 1V/cm of input electric field strength. The common-mode gain is due to the parasitic capacitance bridging the isolation barrier of the HCNR201.

Fig. 4.

Transient response of the probe. CH1 (blue) is voltage of the driven plate of the parallel plate capacitor. CH2 (red) is the output voltage of the electric field probe. a) –AC-coupled probe output response to a 100 Hz square wave; b) – response to a 10 kHz square wave; c) – response to a 100 kHz square wave with an artifact; d) –the same result as in c) but after shorting the isolation barrier.

Fig. 5. Dominant electric-field component inside and outside the parallel-plate capacitor setup described in Section V. The dotted lines represent the capacitor plates.

Fig. 6. Schematic for a bi-stable current pulse generator for induced E-field measurements with a solenoid. Decoupling/bypass capacitors and unused circuits (in multi-circuit parts) are not shown

Fig. 7.

a) – Experimental setup with the blocking shield in order to measure the induced electric field amplitude versus distance; b) –orientation and position of the dipole probe with respect to the solenoid; c) – plot of the measured induced electric field strength versus distance from the coil center. The excitation frequency is 3.846kHz.

TABLE I

IMPEDANCE OF SOLENOID FOR INDUCED ELECTRIC FIELD MEASUREMENTS

Frequency (kHz)	Resistance (ohm)	Reactance (ohm)	Inductance (uH)
0.3	0.1400	0.1064	56.45
1.0	0.1364	0.3538	56.30
3.0	0.1380	1.0564	56.04
10	0.1495	3.5173	55.98
30	0.2168	10.514	55.78
100	0.4881	34.737	55.29
300	1.0015	103.44	54.88
1000	2.1080	343.75	54.71

LFLD coil is wound using AWG 18 copper magnet wire to make a 35-turn single layer solenoid with a diameter of 5.3cm and a length of 3.58cm. The impedance was measured with the HP 4284A Precision LCR meter.