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An experimental method to directly measure $DQE(k)$ at $k = 0$ for 2D x-ray imaging systems

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Abstract

The zero-frequency detective quantum efficiency (DQE), viz., DQE_0 , is defined as the ratio between output and input squared signal-to-noise ratio of an imaging system. In 1963, R. Shaw applied Fourier analysis to generalize DQE_0 to the frequency-dependent DQE, i.e. $DQE(k)$. Under conditions specified by Shaw, $DQE(k)$ is the same as DQE_0 at $k = 0$. The experimental measurement of $DQE(k)$ involves the measurement of system modulation transfer function (MTF) and noise power spectrum (NPS). Although the measurement of MTF is straightforward, the experimental measurements of $NPS(k)$ encountered several challenges. As a result, some experimental methods may yield a nonphysical NPS value at $k = 0$, which makes the measured $DQE(k)|_{k=0}$ deviate from the true zero-frequency DQE. This work presents new results from three aspects: 1) system drift is a significant error source when a large number of independent image acquisitions are involved in measuring NPS and DQE; 2) a cascaded systems analysis shows that the drift induces a global positive offset to the measured autocovariance function, and the offset is quantitatively related to the NPS error at $k = 0$; 3) based on the measured autocovariance data, drift-induced offset can be estimated, so that errors in the measured $NPS(k)|_{k=0}$ and $DQE(k)|_{k=0}$ can be corrected. Both numerical simulations with known ground truth for DQE_0 and experimental studies were performed to validate the proposed measurement method. The results demonstrated that the method mitigates the undesirable influence of system drift in $DQE(k)|_{k=0}$ and DQE_0 , allowing the measured values consistent with the classical definition of zero-frequency DQE.

Keywords

detective quantum efficiency; zero frequency DQE; NPS; x-ray detector; x-ray imaging; photon counting detector

1. Introduction

For an x-ray imaging system, detective quantum efficiency (DQE) ‘describes the efficiency of the statistical sample used to estimate the image, on a scale determined by the maximum

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utilization of the statistics in the incoming radiation stream' (Wagner and Muntz 1979). Zero-frequency DQE, denoted as DQE_0 in this work, quantifies this efficiency of the imaging system 'on a macroscopic scale' (Shaw 1963), namely the efficiency of utilizing all input quanta incident within a large detector area as a whole unit. Since the initial proposal of its concept more than 60 years ago (Fellgett 1958), DQE_0 has found widespread application in the development and routine maintenance of various x-ray imaging technologies and systems such as screen-film mammography (Wagner and Muntz 1979), digital mammography (Jing *et al* 1998, Marshall 2009), active matrix flat-panel imagers (AMFPIs) (Antonuk *et al* 2000, Scheuermann *et al* 2018), mega-voltage (MV) imagers used in image-guided radiotherapy (Star-Lack *et al* 2015), and photon counting detectors (Michel *et al* 2006, Tanguay *et al* 2013, 2015, Xu *et al* 2014, Ji *et al* 2018).

A well-known formula for DQE_0 is given by (Fellgett 1958, Shaw 1963, Wagner and Muntz 1979, Cunningham 2000, Dobbins 2000)

$$DQE_0 = \frac{SNR_{out}^2}{SNR_{in}^2}. \quad (1)$$

Here SNR_{in} and SNR_{out} denote the signal-to-noise ratio (SNR) of an imaging system's input and output quanta, respectively. According to Shaw (1963), the SNR measurement area should be large enough so that each measurement can be considered statistically uncorrelated to similar measurements performed at other nonoverlapping areas. For digital imaging systems, an alternative approach to meet the large area assumption is to bin pixels into larger pixels such that each binned pixel is not correlated with other binned pixels (Burton *et al* 2016).

In Shaw (1963), Shaw generalized the concept of DQE from DQE_0 to frequency-dependent DQE by applying Fourier analysis to the output signal and noise of an analog film-based imaging system. Specifically, the power spectra of system output signal and noise were combined using the DQE_0 framework in equation (1) to derive the following frequency-dependent DQE formula for film-based systems:

$$DQE(\vec{k}) = \frac{(\gamma \log_{10} e)^2 MTF^2(\vec{k})}{q_0 NPS(\vec{k})}, \quad (2)$$

where $\vec{k}$ denotes the spatial frequency vector, and NPS and MTF denote the noise power spectrum and modulation transfer function, respectively, of the imaging system. The quantity q_0 denotes the expected number of input photons per unit area, γ denotes the slope of the characteristic optical density versus log-exposure curve of the film system.

In the modern era of x-ray imaging, the concept of the frequency-dependent DQE has been further extended from analog film systems to digital systems with either nonlinear or linear response. According to the International Commission on Radiation Units and Measurements

(ICRU) Report 54, the following generalized version of equation (2) has been recommended (ICRU 1995):

$$\text{DQE}(\vec{k}) = \frac{(\bar{d})^2 \text{MTF}^2(\vec{k})}{q_0 \text{NPS}(\vec{k})}, \quad (3)$$

where $\bar{d}$ denotes the average pixel value in a large area in the image receptor that receives uniform exposure. As suggested by its name and based on the proof for film-based system in Shaw (1963), the concept of $\text{DQE}(k)$ is applicable to every frequency level within the bandwidth of the imaging system, with the zero frequency being no exception; at $k = 0$, $\text{DQE}(k)$ is the same as DQE_0 in equation (1).

Over time, however, a growing number of $\text{DQE}(k)$ results reported in literature do not include the data point at $k = 0$. As an example, a literature search in the NIH PubMed database with the key words of ‘DQE, NPS, x-ray detector’ identified 30 papers published in the last five years. Among these papers, 18 reported $\text{DQE}(k)$ results, but only six out of these 18 papers provided $\text{DQE}(k)$ value at $k = 0$ (Jain *et al* 2013, Star-Lack *et al* 2014, Xu *et al* 2014, Kuttig *et al* 2015, Larsson *et al* 2016, Myronakis *et al* 2017). Among these six papers, there is one paper (Monte-Carlo simulation only; no experimental measurement) that explicitly reported the $\text{DQE}(k)$ measurement method (Star-Lack *et al* 2014), which is consistent with the definition of $\text{DQE}(k)$ in equation (3) based on the analysis in section 2.

The reason for the obsolete reporting of $\text{DQE}(k)$ at $k = 0$ may be associated with certain technical challenges in accurate experimental NPS measurements at $k = 0$. These challenges will be described later in section 3. If $\text{NPS}(k)|_{k=0}$ is underestimated (e.g. $\text{NPS}(k)|_{k=0} = 0$), according to equation (3), $\text{DQE}(k)|_{k=0}$ will be overestimated, and vice versa. Consequently, the measurement error may make the value of $\text{DQE}(k)|_{k=0}$ deviate from the DQE_0 value given in equation (1), despite the fact that by definition, $\text{DQE}(k)|_{k=0}$ and DQE_0 are the same (Shaw 1963).

The purpose of this work was to (1) identify an error source in experimental NPS and DQE measurements for two-dimensional (2D) x-ray imaging systems, and (2) develop a corresponding correction method to enable accurate and direct measurement of NPS(k) and $\text{DQE}(k)$ at $k = 0$. The paper will use digital systems as examples, although in principle, the method can be generalized to analog systems.

2. Brief review of classical definitions of DQE_0 and $\text{DQE}(k)$

2.1 The concept of DQE_0

As described in Shaw (1963), SNR_{out} in the DQE_0 formula in equation (1) can be measured using an external measurement device (such as film densitometer) with a large aperture. For digital systems, the large area measurement can be achieved by binning pixels into larger pixels such that each binned pixel is not correlated with other binned pixels (Burton *et al* 2016). An equivalent strategy is to sum pixel values with a large area A . The summation of pixel values (denoted as d_s) is given by

$$d_s = \sum_{m=1}^M \sum_{n=1}^N d_{m,n}. \quad (4)$$

The ‘signal’ component in SNR_{out} is the expected value of d_s , namely

$$\langle d_s \rangle = \left\langle \sum_{m=1}^M \sum_{n=1}^N d_{m,n} \right\rangle = \sum_{m=1}^M \sum_{n=1}^N \langle d_{m,n} \rangle. \quad (5)$$

Apparently, the average pixel value within the large area ($\bar{d}$ in equation (3)) is related to the summation of pixel value (d_s) by

$$\langle d_s \rangle = MN\bar{d}. \quad (6)$$

The ‘noise’ component in SNR_{out} is the noise standard deviation of d_s , namely

$$\sigma_{d_s} = \left[\langle (\Delta d_s)^2 \rangle \right]^{\frac{1}{2}} = \left[\left\langle \left(\sum_{m=1}^M \sum_{n=1}^N \Delta d_{m,n} \right)^2 \right\rangle \right]^{\frac{1}{2}}, \quad (7)$$

in which the two variables are defined as

$$\Delta d_s = d_s - \langle d_s \rangle \quad \text{and} \quad \Delta d_{m,n} = d_{m,n} - \langle d_{m,n} \rangle. \quad (8)$$

Note that the ensemble averaging operator in equation (7) is applied after the summation operations. Therefore, $\sigma_{d_s}^2$ is usually not equal to the summation of pixel noise variance ($\langle d_{m,n}^2 \rangle$), unless there is no noise correlation or when binned pixels are used in the noise calculation (Burton *et al* 2016) to make noise correlations between binned pixels negligible.

Based on equations (6) and (7), the square of SNR_{out} defined in area A is given by

$$\text{SNR}_{\text{out}}^2 = \frac{\langle d_s \rangle^2}{\sigma_{d_s}^2} = \frac{(MN\bar{d})^2}{\left\langle \left(\sum_{m=1}^M \sum_{n=1}^N \Delta d_{m,n} \right)^2 \right\rangle}. \quad (9)$$

To work out the corresponding mathematical expression for SNR_{in}^2 , a variable Q_A is introduced to denote the total number of input photons received in area A . Based on the definition of q_0 , the expected value of Q_A can be written as

$$\langle Q_A \rangle = q_0 MN \Delta x \Delta y. \quad (10)$$

For a specified measurement area, Q_A is Poisson-distributed, and its variance $\sigma_{Q_A}^2$ is equal to $\langle Q_A \rangle$. SNR_{in}^2 is thus given by

$$\text{SNR}_{\text{in}}^2 = \frac{\langle Q_A \rangle^2}{\sigma_{Q_A}^2} = \langle Q_A \rangle = q_0 MN \Delta x \Delta y. \quad (11)$$

Using the $\text{SNR}_{\text{out}}^2$ formula in equation (9) and the SNR_{in}^2 formula in equation (11), one can readily work out DQE_0 as follows:

$$\begin{aligned} \text{DQE}_0 &= \frac{(MN\bar{d})^2}{q_0 MN \Delta x \Delta y \left\langle \left(\sum_{m=1}^M \sum_{n=1}^N \Delta d_{m,n} \right)^2 \right\rangle} \quad (12) \\ &= \frac{MN(\bar{d})^2}{q_0 \Delta x \Delta y \left\langle \left(\sum_{m=1}^M \sum_{n=1}^N \Delta d_{m,n} \right)^2 \right\rangle} \end{aligned}$$

$$= \frac{MN(\bar{d})^2}{q_0 \Delta x \Delta y \left[\sum_{m,n} \left\langle (\Delta d_{m,n})^2 \right\rangle + \sum_{m' \neq m, n' \neq n} \left\langle \Delta d_{m,n} \Delta d_{m',n'} \right\rangle \right]}. \quad (13)$$

Note that the denominator in equation (13) contains not only noise variance of each pixel $\langle (\Delta d_{mn})^2 \rangle$ but also the covariance of noise $\langle \Delta d_{m,n} \Delta d_{m',n'} \rangle$. If SNR is measured for an individual pixel instead of over a large area A , it will not include the contribution of noise covariance. In that case, the corresponding DQE_0 result will be inconsistent with the one given by equation (13), unless noise is completely uncorrelated across pixels.

2.2. DQE(k) at $k = 0$

Since $\text{DQE}(k)$ is closely related to $\text{NPS}(k)$, let us first review the concept of NPS. As shown in Cunningham (2000) and Dobbins (2000), the 2D NPS of a 2D digital imaging system is given by

$$\text{NPS}(u, v) = \frac{\Delta x \Delta y}{MN} \left\langle \left| \sum_{m=1}^M \sum_{n=1}^N (d_{m,n} - \langle d_{m,n} \rangle) e^{-i2\pi \left(\frac{mu}{M} + \frac{nv}{N} \right)} \right|^2 \right\rangle \quad (14)$$

$$= \frac{\Delta x \Delta y}{MN} \left\langle \left| \sum_{m=1}^M \sum_{n=1}^N \Delta d_{m,n} e^{-i2\pi \left(\frac{mu}{M} + \frac{nv}{N} \right)} \right|^2 \right\rangle, \quad (15)$$

where u and v are two integers representing spatial frequency indices. Note that in equation (14), $\langle d_{mn} \rangle$ is the expected value of d_{mn} , not the sample average over area A.

Based on equation (15), the zero-frequency NPS is simply

$$\begin{aligned} NPS(k)|_{k=0} &= \frac{\Delta x \Delta y}{MN} \left\langle \left| \sum_{m=1}^M \sum_{n=1}^N \Delta d_{m,n} \right|^2 \right\rangle \\ &= \frac{\Delta x \Delta y}{MN} \left\langle \left(\sum_{m=1}^M \sum_{n=1}^N \Delta d_{m,n} \right)^2 \right\rangle. \end{aligned} \quad (16)$$

In moving from the first line to the second line of equation (16), we used the fact that the detector outputs are real numbers. With the $NPS(k)|_{k=0}$ formula in equation (16), $DQE(k)|_{k=0}$ is given by

$$\begin{aligned} DQE(k)|_{k=0} &= \frac{(\bar{d})^2}{q_0 NPS(k)|_{k=0}} \\ &= \frac{MN(\bar{d})^2}{q_0 \Delta x \Delta y \left\langle \left(\sum_{m=1}^M \sum_{n=1}^N \Delta d_{m,n} \right)^2 \right\rangle}, \end{aligned} \quad (17)$$

which is the same as the DQE_0 formula in equation (12). In principle, as long as both SNR and NPS are based on large-area measurements and $\langle d_{m,n} \rangle$ in equation (14) is estimated properly, $DQE(k)|_{k=0}$ is nothing but DQE_0 as one would expect.

3. Challenges in experimental measurements of $DQE(k)$ at $k = 0$

As indicated by the expected value operator $\langle \cdot \rangle$ equation (14), the large area measurement must be performed a large number of times to obtain a reliable estimation of NPS. To meet this requirement, one approach is to extract multiple regions-of-interest (ROIs) from each acquired image. This ROI ensemble method, when used in conjunction with repeated acquisitions (image ensemble), usually enables one to obtain a large number of samples without performing too many repeated image acquisitions. For this approach, the estimation of $\langle d_{m,n} \rangle$ in equation (14) usually does not use direct ensemble averaging, because different ROI locations may have different expected output values due to spatial variations of detector response and input photon intensity. Instead, as recommended by IEC, a two-dimensional polynomial function is fitted to the entire image to estimate the image background (IEC 62220-1 2003, IEC 62220-1-1:2015 2015). It is important to follow the IEC guideline when estimating $\langle d_{m,n} \rangle$, otherwise $NPS(k)$ is subject to error at $k = 0$. For example, if the

polynomial fit is performed for each ROI, or if the average pixel value of each ROI is used as $\langle d_{m,n} \rangle$, it will lead to nonphysical results of $\text{NPS}(k)|_{k=0} = 0$ and $\text{DQE}(k)|_{k=0} = \infty$.

Although extracting multiple ROIs from each image helps to reduce total data collection time, NPS and DQE measured using this approach represent system performance averaged over these ROI locations. For certain detector systems, especially those composed of tiled modules, there is often a need to characterize NPS and DQE separately for each module. If that is the case, using ROI ensemble is no longer appropriate. Similarly, for systems with relatively small area detectors or with long range noise covariance, the number of uncorrelated ROIs available from each image may be very limited. In these cases, it is more desirable to only use image ensemble generated from a large number of independent acquisitions.

When the number of acquisitions increases, the influence of system drift becomes more pronounced. To analyze the impact of drift, a term f_t is introduced to denote the contribution of drift to the t th image frame. Since in the image ensemble method, the measurement is performed at a fixed ROI location within a given detector module, drift can be considered to be dominated by variation in tube output and is independent of pixel position within the ROI. Correspondingly, detector output measured at a time frame t can be modeled as

$$d'_{m,n}(t) = d_{m,n} + f_t, \quad (18)$$

where f_t denotes the offset imposed by drift to the t th frame. As shown in the appendix, the offset will propagate to the measured autocovariance function as follows:

$$K_{d'}(m', n') = K_d(m', n') + \sigma_f^2, \quad (19)$$

where m' and n' denotes the relative distance (units: pixels) between image pixels, and $K_d(m', n')$ denotes the true autocovariance function. The term σ_f^2 in equation (19) is related to f_t by (appendix)

$$\sigma_f^2 = \frac{1}{T-1} \sum_{t=1}^T \left(f_t - \frac{1}{T} \sum_{t=1}^T f_t \right)^2, \quad (20)$$

where T denotes the total number of time frames.

Since the zero-frequency NPS is given by the spatial integration of the autocovariance function (Cunningham 2000, Dobbins 2000), σ_f^2 will propagate to the zero-frequency NPS and get amplified as follows:

$$\begin{aligned}
\text{NPS}_{d'}(k)|_{k=0} &= \Delta x \Delta y \sum_{|m'| \leq \frac{M}{2}} \sum_{|n'| \leq \frac{N}{2}} K_{d'}(m', n') \\
&= \Delta x \Delta y \sum_{|m'| \leq \frac{M}{2}} \sum_{|n'| \leq \frac{N}{2}} K_{d'}(m', n') + \Delta x \Delta y MN \sigma_f^2 \\
&= \text{NPS}_{d'}(k)|_{k=0} + A \sigma_f^2.
\end{aligned} \tag{21}$$

This result dictates that even a small σ_f^2 can result in a significant error in the measured zero-frequency NPS due to the amplification factor of A . Based on equation (3), this amplified positive error in the measured zero-frequency NPS will lead to an underestimation of $\text{DQE}(k)|_{k=0}$.

Using similar analysis method, it can be shown that drift adds an error in term $\sigma_{d_s}^2$ in equation (7) as follows:

$$\sigma_{d_s}^2 = MN \sum_{|m'| \leq \frac{M}{2}} \sum_{|n'| \leq \frac{N}{2}} K_{d'}(m', n') = \sigma_{d_s}^2 + (MN)^2 \sigma_f^2. \tag{22}$$

In other words, the term σ_f^2 gets amplified by a factor of $(MN)^2$ in the measured large area noise variance, which results in an underestimation of SNR_{out} and DQE_0 .

4. Proposed measurement method to correct drift-induced error

To correct the drift-induced error, the proposed method utilizes *a priori* knowledge that the true autocovariance is usually very close to zero at long spatial distances (i.e. large m' or n' values). Based on equation (19), the apparent elevation of the measured autocovariance at these long distances can be attributed to the global offset term σ_f^2 . Therefore, σ_f^2 can be estimated from the plateaued 'tail' of the measured autocovariance curve as

$$\sigma_f^2 = \frac{1}{MN - (2I)^2} \left[\sum_{|m'| \leq \frac{M}{2}} \sum_{|n'| \leq \frac{N}{2}} K_{d'}(m', n') - \sum_{|m'| \leq I} \sum_{|n'| \leq I} K_{d'}(m', n') \right], \tag{23}$$

Where I is a distance threshold (units: pixels) beyond which the true autocovariance of the imaging system can be considered reasonably close to zero. One approach to determine I is to plot the measured autocovariance curve and subjectively determine the spatial distance beyond which the curve is plateaued; alternatively, I can be determined objectively based on a metric that characterizes the shape of the autocovariance curve. The metric used in this

work was the full width at half maximum (FWHM) of the autocovariance curve; the curve's minimal value was subtracted from it before FWHM measurement to reduce the impact of the global offset on FWHM result. Once FWHM was measured, the distance threshold l was determined by $2 \times \text{FWHM}$, as autocovariance are usually negligible at distances longer than two FWHMs.

Once σ_f^2 is estimated using equation (23), corrections to the measured NPS(0) can be applied based on equation (21) as follows:

$$\text{NPS}_d(k)|_{k=0} = \text{NPS}_{d'}(k)|_{k=0} - A\sigma_f^2. \quad (24)$$

Similarly, correction to the measured large area noise variance is given by

$$\sigma_{d_s}^2 = \sigma_{d'}^2 - (MN)^2 \sigma_f^2. \quad (25)$$

The corrected $\text{NPS}_d(k)|_{k=0}$ and $\sigma_{d_s}^2$ are used to calculate $\text{DQE}_d(k)|_{k=0}$ and DQE_0 , respectively.

5. Validation of the proposed method

To validate the proposed drift correction method, both numerical simulations studies with known ground truth for DQE_0 and experimental studies were performed.

5.1. Simulation methods

The simulation study began with a relatively simple toy-model x-ray imaging system: except the finite x-ray absorption probability (denoted as ϵ), the detector system does not contain any other DQE-degrading mechanism, nor does it contain any mechanism to introduce noise covariance or spatial resolution loss. In other words, each absorbed input photon always contributes to one output count. For this system, the ground truth value for DQE_0 is entirely determined by ϵ , namely

$$\text{DQE}_0^{\text{truth}} = \epsilon. \quad (26)$$

In order to evaluate the robustness of the proposed method in the presence of noise covariance, the simulation also used a detector model that contains noise spatial correlation introduced by apodizing output of the toy model system using a Gaussian filter. The width of the filter (quantified by 2σ , or equivalently, 0.85 of FWHM) was adjusted from four pixels (0.4 mm) to 20 pixels (2 mm) to obtain different autocovariance functions. Based on the properties of convolution, this filtering operation does not change the summation of pixel values in a large area, namely d_s in equation (4). Therefore, $\text{DQE}_0^{\text{truth}}$ was maintained at 0.8 for these simulated systems.

To introduce system drift to the simulated images, the following drift function was used:

$$f_t = \frac{\beta}{1 + \exp(-\alpha[t - T/2])} + \sum_{k=0}^3 c_k t^k. \quad (27)$$

Here a shifted sigmoid function was combined with a polynomial function in order to add complexity to the drift model (figure 1). Values of the parameters in the drift functions were arbitrarily chosen (table 1). Once the simulated images were generated by one of the co-authors (KL), they were sent to another co-author (MF) who was blinded to DQE_0^{truth} , f_b statistical distribution of the image data, and the spatial filter applied. M.F. first measured the autocovariance function from drift-contaminated images, then estimated the drift-induced error term (σ_f^2) using equation (23). Before calculating DQE_0 , drift correction was applied to the measured output noise variance using equation (25). Similarly, the proposed drift correction was applied to the measured $NPS(k)|_{k=0}$ when calculating $DQE(k)|_{k=0}$.

For comparison purpose, $DQE(k)|_{k=0}$ was also measured using the classical image ensemble method without drift correction.

5.2. Experimental methods

To further test the proposed method, experimental measurements of SNR, NPS, and zero-frequency DQE were performed using a benchtop x-ray imaging system. The system uses a cadmium telluride (CdTe) based photon counting detector (XC-FLITE X-1, Xcounter, Sweden) and a rotating-anode x-ray tube (G-1592 with B-180H housing, Varian Medical Systems, Inc. Salt Lake City, UT). The photon counting detector (PCD) was manufactured by tiling twelve CdTe detector panels, each containing 128×128 pixels; each pixel has an area of $0.1 \times 0.1 \text{ mm}^2$. Other parameters of the PCD can be found in Ji *et al* (2018). As shown later in the results section, DQE_0 performance of these panels may be different, and thus it is necessary to perform measurement separately for each panel. The tube potential was set at 70 kVp. To produce the so-called RQA5 beam quality (IEC 61267:2005 2005), the x-ray beam was filtered by 21 mm of Al, which generated a half value layer of 7.1 mm Al. The beam was collimated to a $16 \times 16 \text{ cm}^2$ area on the PCD surface.

As shown in Ji *et al* (2018), DQE_0 of the PCD system is strongly dependent on its energy threshold and detection mode. To test the applicability of the proposed method, four different PCD modes were used, including two energy threshold levels (20 and 35 keV) and two detection modes (anti-charge sharing (ACS) and single pixel mode). For each configuration, 1000 image frames were acquired.

To measure SNR_{out} , pixel values within a 100×100 ROI in each detector panel were summed to get a sample of d_s . This process was repeated for every image in the image ensemble, then equations (5), (6), and (7) were used to calculate $\langle d_s \rangle$, $\bar{d}$, and $\sigma_{d_s}^2$,

respectively. To measure NPS, ensemble averaging was applied to the acquired 1000 images to estimate $\langle d_{m,n} \rangle$ then equation (14) was used to calculate 2D NPS.

To correct errors introduced by drift, the drift-contaminated autocovariance $K_{d'}(m', n')$ was measured for each detector and detection mode. The error term σ_f^2 was estimated using equation (23). The distance thresholds (l) were determined based on 2FWHM of the measured autocovariance function shown later in figure 4. Once σ_f^2 was estimated, corrections to the measured NPS and the large area noise variance $\sigma_{d_s}^2$ were applied using equations (24) and (25), respectively.

In addition to measuring SNR_{out} and $\text{NPS}(0)$, the calculation of the zero-frequency DQE also requires information about the input photon density q_0 . In order to estimate q_0 a 6 cm³ general purpose ion chamber (Model 10X6-6, Radcal Corp., Monrovia, CA) was placed in front of the detector to measure the ionizing radiation exposure level per image frame. A lead plate was placed between the ion chamber and the detector surface to reduce backscatter. The measured exposure level was then converted to air kerma, and a conversion factor recommended by IEC (IEC 62220-1 2003) was used to estimate q_0 from the air kerma. The measured exposure was 0.935 mR per image frame, and the estimated q_0 was $2.47 \times 10^5 \text{ mm}^{-2}$ per frame. With q_0 , equation (12) was used to calculate DQE_0 . Similarly, equation (3) was used to estimate $\text{DQE}(k)|_{k=0}$ from $\text{NPS}(0)$, q_0 , and $\bar{d}$.

6. Validation results

6.1. Simulation results

As shown in table 2, DQE_0 and $\text{DQE}(k)|_{k=0}$ generated by the proposed method were consistent with the ground truth value of 0.8. For all simulated imaging systems, the relative errors in the estimated zero-frequency DQE are no greater than 3%. As a comparison, DQE_0 and $\text{DQE}(k)|_{k=0}$ generated without drift correction were severely underestimated due to significant overestimation of $\text{NPS}(k)|_{k=0}$. Without drift correction, the relative error of the estimated zero-frequency DQE is no less than 97%.

Figure 2 shows how the outcome of the proposed measurement method depends on l. A range of l values centered at 2FWHM were used to estimate the drift-induced error term, and the resulting $\text{DQE}(k)|_{k=0}$ values were relatively stable when l was varied in the neighborhood of 2FWHM. Echoing the data in table 2, figure 2 shows that for the proposed method, the DQE estimation error increased with longer noise correlation length, as the tail of the autocovariance may contain more contribution from the true autocovariance.

6.2. Experimental results

Figure 3 demonstrates the presence of drift in the experimental images. Within a given 128×128 detector panel, pixels at different spatial locations demonstrated similar drift behavior. In comparison, pixels from different detector panels shows different drift curves, suggesting the need to apply the drift correction separately for each panel. Note that based on the vertical axis of the plots shown in figure 3, the drifting of detector output over time is quite small. One possible cause of the system drift is temporal fluctuation of the x-ray tube output, although certain physical properties of the PCD system may also fluctuate over time due to mechanisms such as temperature variation and charge trapping (Knoll 2010).

Despite the small magnitude of drift, its impact on the measured autocovariance function is evident (figure 4). For each detection mode tested, there was a plateaued tail in the measured autocovariance function. Because autocovariance is supposed to decrease towards zero, this plateaued tail sitting well above zero does not reflect the intrinsic noise properties of the imaging system and should be attributed to the σ_f^2 term defined in equation (20). The estimated values of σ_f^2 for the four detection modes are listed in table 3, and the corrected autocovariance functions are plotted in figure 4.

Figure 5 compares radial profiles of 2D NPS generated with and without drift correction. Both methods led to similar NPS results at frequencies above zero. However, as predicted by the theoretical analysis, NPS measured without drift correction demonstrated a discontinuity and sudden rise at $k = 0$. Using the proposed drift correction, the discontinuity at $k = 0$ was successfully removed.

As shown in table 3, DQE_0 measured using equation (1) without drift correction is very close to zero for all detection modes tested, which is apparently nonphysical. Similarly, $DQE(k)|_{k=0}$ calculated using the drift-contaminated NPS is close to zero. Using the proposed measurement method, $DQE(k)|_{k=0}$ is consistent with DQE_0 for each PCD mode tested.

Since the proposed method does not involve shifting the ROI location to collect the ROI ensemble, it enables panel-specific or local DQE assessment. As an example, table 4 shows error-corrected zero-frequency DQE results of three neighboring PCD panels, each of which has an area of $1.3 \times 1.3 \text{ cm}^2$. Depending on the energy threshold and ACS mode, these panels often have slightly different zero-frequency DQE values. These difference would be difficult to observe without drift correction, since drift-induced overestimation of NPS at $k = 0$ may make all panels' $DQE(k)|_{k=0}$ close to zero.

7. Discussion

As described in Shaw (1963), the concept of the frequency-dependent DQE does not exclude the point at $k = 0$, and by definition, $DQE(k)|_{k=0}$ is identical to DQE_0 . This work demonstrates that in practice, experimentally measured $DQE(k)|_{k=0}$ and DQE_0 may deviate from the true value due to the presence of system drift. Through a theoretical analysis, this work shows that temporal variation of system output leads to a positive global offset σ_f^2 in the measured autocovariance function, and the offset can be significantly amplified after being transferred to the zero-frequency NPS. Since $DQE(k)$ is inversely proportional to $NPS(k)$, the positive error in the measured $NPS(k)|_{k=0}$ would eventually lead to an underestimate of $DQE(k)|_{k=0}$. Because of the error at $k = 0$, the value of $DQE(k)|_{k=0}$ is often discarded, not shown, or borrowed from neighboring low frequency levels through data extrapolation. Extra error may be introduced if the true $DQE(k)$ curve deviates from the extrapolation model.

One alternative approach to mitigating the impact of system drift is to reduce the total number of repeated image acquisitions to shorten the total imaging time. From each acquired image, multiple large-area and uncorrelated ROIs are extracted to increase the total

number of data samples for large-area SNR and NPS measurements. For an imaging system whose detector covers a large spatial area and has homogeneous DQE performance across the entire area, augmenting the image ensemble with an ROI ensemble is an effective and efficient strategy to measure SNR, NPS, and DQE, as long as the covariance between the ROIs is negligible. Meanwhile, for systems with small-area detectors, or systems with independent small detector modules, there is often a need to bypass the ROI ensemble method and fully employ the image ensemble method. Consequentially, the total number of image acquisitions can be quite large, resulting in a large drift-induced error.

To correct this error, this work developed a method to estimate σ_f^2 based on the autocovariance data themselves. Next, according to a quantitative relationship between σ_f^2 and $\text{NPS}(k)|_{k=0}$ derived in this work, drift-induced error in $\text{NPS}(k)|_{k=0}$ can be corrected. Finally, the corrected $\text{NPS}(k)|_{k=0}$ is used to calculate $\text{DQE}(k)|_{k=0}$ based on equation (3). Different from other drift correction methods, the goal of the proposed method is to correct the impact of drift to noise rather than to signal, and it does not require any modeling of the drift function.

Despite the capability of the presented method to correct drift-induced error, this work contains the following limitations. The plateaued tail in the measured autocovariance curve is attributed entirely to system drift. However, for imaging systems generating non-negligible noise covariance at long spatial distances (i.e. at large m' or n' values), the tail of the autocovariance may be attributed to both drift and true autocovariance, making it more difficult to isolate the contribution of σ_f^2 . Meanwhile, for systems whose long range autocovariance is introduced by software-based apodization, this limitation can be addressed by working on the pre-filtered data. Another limitation is related to the assumption of the local shift invariance of the drift function f_t . As shown by the experimental results, this assumption is valid within a given PCD panel in our imaging system. However, its soundness needs to be further tested for other systems. Finally, this work focuses on planar x-ray imaging systems and does not evaluate the applicability of the proposed method to the measurement of CT DQE, as its formula (Wagner *et al* 1979, Hanson 1979) differs from equation (3). In addition, CT images may contain long range autocovariance due to data filtering and interpolation during tomographic reconstruction, which increases the complexity in isolating the drift error from the true autocovariance.

8. Conclusions

An experimental method was developed to correct drift-induced error in large area SNR, $\text{NPS}(k)|_{k=0}$, and $\text{DQE}(k)|_{k=0}$ of 2D x-ray imaging systems. With the proposed correction, the measured value of $\text{DQE}(k)$ at $k = 0$ is consistent with DQE_0 defined in equation (1).

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Appendix. Impact of system drift to the autocovariance function

With the signal model in equation (18), an ensemble average of the acquired T image frames is given by

$$\langle d'_{m,n} \rangle = \frac{1}{T} \sum_{t=1}^T d'_{m,n,t} = \langle d_{m,n} \rangle + \frac{1}{T} \sum_{t=1}^T f_t. \quad (\text{A.1})$$

Deviation of the i th time frame from the ensemble average is

$$\begin{aligned} \Delta d'_{m,n,t} &= d'_{m,n,t} - \langle d'_{m,n} \rangle \\ &= (d_{m,n} - \langle d_{m,n} \rangle) + \left(f_t - \frac{1}{T} \sum_{t=1}^T f_t \right) \\ &= \Delta d_{m,n} + \Delta f_t. \end{aligned} \quad (\text{A.2})$$

The autocovariance of the drift-contaminated image data d' is given by

$$K_{d'}(m, n; m + m', n + n') \quad (\text{A.3})$$

$$\begin{aligned} &= \langle \Delta d'_{m,n} \Delta d'_{m+m', n+n'} \rangle \\ &= \frac{1}{T-1} \sum_{t=1}^T \Delta d'_{m,n,t} \Delta d'_{m+m', n+n', t} \\ &= \frac{1}{T-1} \sum_{t=1}^T (\Delta d_{m,n} + \Delta f_t) (\Delta d_{m+m', n+n'} + \Delta f_t) \\ &= \frac{1}{T-1} \sum_{t=1}^T [\Delta d_{m,n} \Delta d_{m+m', n+n'} + \Delta d_{m,n} \Delta f_t + \Delta d_{m+m', n+n'} \Delta f_t + (\Delta f_t)^2] \\ &= \frac{1}{T-1} \sum_{t=1}^T (\Delta d_{m,n} \Delta d_{m+m', n+n'}) + 0 + 0 + \frac{1}{T-1} \sum_{t=1}^T (\Delta f_t)^2 \\ &= K_d(m, n; m + m', n + n') + \sigma_f^2. \end{aligned}$$

$$= K_d(m, n; m + m', n + n') + \sigma_f^2. \quad (\text{A.4})$$

In (A.3), we utilized the fact that the drift function and pixel noise are uncorrelated. In equation (A.4), $K_d(m, n; m + m', n + n')$ denotes the true autocovariance.

By averaging $K_d'(m, n; m+m', n+n')$ over all detector pixels, its dependence on the absolute pixel location (m, n) can be removed:

$$\begin{aligned} K_d'(m', n') &= \frac{1}{MN} \sum_{m=1}^M \sum_{n=1}^N K_d'(m, n; m+m', n+n') \\ &= \frac{1}{MN} \sum_{m=1}^M \sum_{n=1}^N K_d(m, n; m+m', n+n') + \frac{1}{MN} \sum_{m=1}^M \sum_{n=1}^N \sigma_f^2 \\ &= K_d(m', n') + \sigma_f^2. \end{aligned} \quad (\text{A.5})$$

Compared with the true autocovariance function $K_d(m', n')$, the measured autocovariance function $K_d'(m', n')$ contains an error term σ_f^2 that is introduced by system drift.

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Figure 1.
Plot of the drift function used in the numerical simulation study.

Figure 2. Measured DQE(0) of the simulated x-ray systems by using different l for correction. (a) System with white noise. (b) System with apodization (width = 4 pixels). (c) System with apodization (width = 20 pixels).

Figure 3.

Experimental demonstration of system drift. (a) Comparison of drift function f_d measured from four quadrants of a 100×1000 ROI in a detector panel. (b) Comparison of f_t measured in two ROIs from two different detector panels in the PCD system.

Figure 4.

Radial profiles of 2D autocovariance functions of the experimental system. The horizontal wavy lines indicate broken vertical axes. (a) ACS off; 20 keV. (b) ACS off; 35 keV. (c) ACS on; 20 keV. (d) ACS on; 35 keV.

Figure 5.

Radial profiles of the 2D NPS of the experimental system. NPS measured without drift correction demonstrated abrupt elevation at the zero frequency. The proposed drift correction generated smooth NPS within the entire bandwidth, including the point of $k = 0$. The horizontal wavy lines indicate broken vertical axes. (a) ACS off; 20 keV. (b) ACS off; 35 keV. (c) ACS on; 20 keV. (d) ACS on; 35 keV.

Table 1.

Parameters used in the simulation. Definitions of these parameters can be found in section 5.1.

Parameter	Value
DQE_0^{truth}	0.8
ϵ	0.8
q_0	$1 \times 10^6 \text{ (mm}^{-2}\text{)}$
T	1000
$M \times N$	256×256
x and y	0.1 mm
a	0.01
β	8
c_0	-2.40
c_1	-3.20×10^{-3}
c_2	4.80×10^{-6}
c_3	-6.40×10^{-9}

Table 2.

Summary of simulation results. Units of NPS are (mm^{-2}). DQE is dimensionless.

	Zero-frequency NPS			Zero-frequency DQE		
	0	4	20	0	4	20
Apodization filter width (σ , in (pixels))	0	4	20	0	4	20
Ground truth		8×10^1			0.8	
DQE(k)-based measurement w/o drift correction	2.8×10^3	2.8×10^3	2.9×10^3	0.023	0.023	0.022
DQE(k)-based measurement with drift correction	8.0×10^1	8.1×10^1	8.2×10^1	0.80	0.79	0.78
DQE ₀ -based measurement w/o drift correction	<i>Not Applicable</i>			0.023	0.023	0.022
DQE ₀ -based measurement with drift correction				0.80	0.79	0.78

Table 3.

Summary of experimental results for a given $1.3 \times 1.3 \text{ cm}^2$ PCD module. Units of NPS are (mm^{-2}). Other results are dimensionless.

Detection mode	ACS off	ACS on	ACS off	ACS on
Energy threshold (keV)	35	35	20	20
Drift error (σ_f^2)	1.0	0.93	24	0.61
SNR _{out} (w/o drift correction)	870	1510	435	2400
SNR _{out} (w. drift correction)	3020	3930	4120	4350
NPS(k) _{$k=0$} (w/o drift correction)	110	110	2500	88
NPS(k) _{$k=0$} (with drift correction)	9.3	16	27	27
$\bar{d}$	920	1600	2160	2300
DQE ₀ (w/o drift correction)	3.1×10^{-2}	9.2×10^{-2}	7.7×10^{-3}	0.23
DQE ₀ (w. drift correction)	0.37	0.62	0.69	0.77
DQE(k) _{$k=0$} (w/o drift correction)	3.1×10^{-2}	9.2×10^{-2}	7.7×10^{-3}	0.23
DQE(k) _{$k=0$} (with drift correction)	0.37	0.62	0.69	0.77

Table 4.

Comparison of zero-frequency DQE (with drift correction) for three neighboring PCD panels.

Detection mode	ACS off	ACS on	ACS off	ACS on
Energy threshold (keV)	35	35	20	20
Panel A	0.37	0.62	0.69	0.77
Panel B	0.42	0.66	0.71	0.79
Panel C	0.41	0.65	0.69	0.77