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A Fast Recursive Algorithm For Spectrum Tracking in Power Grid Systems

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Abstract

A major challenge for future wide area measurement systems is how to efficiently monitor and control the power system, which requires accurate and real-time tracking of the phase, frequency, and Power Factor (PF) of the system. The presence of inter and sub harmonics, especially in a distribution system caused by widely used non-linear power loads, rectifiers, and inverters, will distort estimation of phase/frequency and make it difficult to track the voltage, current and frequency variations. In this paper we present a new approach to identify and track the harmonics, sub-harmonics and inter-harmonics, as well as observe their impact on the power system. We propose a two-stage processing approach that consists of a subspace-based estimation method to detect and identify all harmonic components, followed by a low-complexity fast tracking algorithm to monitor frequency variations of voltage and current signals in real-time with great accuracy. The simulation results show that the proposed approach can provide highly reliable estimation and fast tracking of the harmonic components, while avoiding the impact of time variance.

Index Term

Power Grid; Frequency Estimation and Tracking; Harmonic; Sub-harmonic; Inter-Harmonic; Subspace

I. Introduction

The increasing deployment of residential renewables and an extensive use of nonlinear devices in the distribution system can produce undesirable harmonic distortions. Such distortions can severely impact phase/frequency estimation in power systems, especially distributed grid systems. For grid monitoring for instance, the presence of harmonic distortions can degrade the performance of Phasor Measurement Units (PMUs) for tracking voltage, current, and frequency variations. Therefore, awareness and identification of all the harmonic spectral components, such as inter-harmonic and sub-harmonic, can greatly enhance the power system's reliability, hence reducing harm to the system [1, 2]. As a result, harmonic analysis for power grid has received considerable attention in recent years [3, 4]. While there is a great deal of effort involved in estimating the harmonics, study of inter-harmonics and sub-harmonics is still in its infancy. In particular, it is hard to pinpoint inter-

harmonic and sub-harmonic frequencies since they are discrete and not integer multiples of the fundamental frequency.

Well known frequency estimation techniques, such as zero crossing [5, 6], least squares error technique [7–9], Newton method [10], Kalman filter [11, 12], Prony method [13], artificial neural network [14], demodulation approaches [15–17], wavelet [18, 19], non-uniform sampling [20–27], and Fourier-based methods such as Discrete Fourier Transform (DFT) [16, 28–30] have been extensively studied in the past.

In addition, a Phase-Locked Loop (PLL)-based estimation algorithm has been investigated in [31]. While this algorithm is capable of extracting phase, frequency and amplitude of the fundamental component from a heavily distorted signal, it is unable to estimate and track the harmonic components. In [32], PLL and Adaptive Notch Filter (ANF) are considered mainly to handle the DC component in the power signal. An amplitude ANF based scheme is also proposed in [33] that can offer a high degree of immunity and insensitivity to power system disturbances. The authors in [34] propose an adaptive observer to estimate capacitor voltages based on the measurement of arm currents. In [35], several modal identification methods are invoked to investigate lightly damped electromechanical modes in powers systems. However, these methods generally lack the ability to identify and estimate inter-harmonics and sub-harmonics. Even the most widely used Fourier-based methods for harmonic estimation are unable to accurately estimate the inter-harmonic and sub-harmonic components.

This is mainly due to the unsuitability of synchronizing the sampling procedure to inter-harmonics and sub-harmonics, where a variable and long analysis window, under the assumption that the signal is stationary, would be required. These techniques also assume that harmonic magnitudes are constant during the estimation process. In reality harmonic frequencies and their magnitudes are not stationary due to sudden variations in power systems such as switching on-off a large load or other transient operations caused by faults. Under these conditions, it would be essential to develop a method that is capable of dynamically tracking not only the harmonic, but also inter-harmonic and sub-harmonic magnitudes, as well as the phase angles of each harmonic component. As a result, the task of developing a robust and reliable estimation method for real-time monitoring and analysis of harmonic variations is becoming a major challenge. In particular, we believe under transient conditions, any reliable approach should be based on first identifying the presence of all important harmonic components in order to accurately track their frequencies and magnitudes in a timely manner. Therefore, in this paper we present a novel two-tier detection and fast tracking scheme. For harmonic detection we developed a subspace-based method for loosely identifying polluted harmonic components. For each identified component, a low-complexity tracking algorithm is then deployed to accurately track each identified harmonic component. We should point out that a subspace-based method, in contrast to the Fourier-based methods, can achieve relatively high accuracy in frequency and amplitude estimation without requiring sampling synchronization with any inter/sub harmonic components.

By implementing an eigen decomposition, subspace-based estimation methods [36–40] can decompose the autocorrelation matrix of the noise-polluted signal into two subspaces; namely a signal subspace and a noise subspace. For a noisy power signal the fundamental and harmonic components will be grouped to the signal space with K eigenvalues bigger than σ^2 , where σ^2 is the noise variance. After retrieving information of the fundamental frequency and all its harmonic components, a low-complexity fast tracking algorithm, which is based on the method introduced in [41], is then considered for fast tracking phase/frequency in recursive mode. However, it is important to note that this method does not perform well due to its inability to accurately estimate each harmonic in the presence of other harmonics. Therefore, for the tracking stage we developed a modified version of the algorithm. The new algorithm is capable of accurately tracking the fundamental frequency and its harmonic as well as inter-harmonic components by using the information obtained during the subspace estimation stage.

The paper is organized as follows. In Section II we begin by presenting a low complexity amplitude and frequency tracking algorithm, which is applied to estimate the amplitude and frequency of each specific harmonic component. After providing a brief background of subspace-based estimation methods, we then discuss its application for power signal estimation in Section III. A subspace tracking algorithm is then presented and discussed in Section IV for estimation of fundamental frequency and all its spectral components. In Section V, we introduce a novel integrated spectral decomposition and tracking scheme with low complexity and a fast convergence rate. Section VI presents simulation results in terms of frequency and amplitude tracking under harmonic distortions, followed by the conclusion in Section VII.

II. Fast and Low-complexity Tracking algorithm

In power systems the sinusoid/cosine form of signal voltage and current may contain harmonics, decaying DC offset components and noise. In [41], a method to extract sinusoid signals and estimation from their parameters is presented. The method is based on a core unit, as depicted in Fig. 1. In this approach a multiplicity of core units in a parallel or cascaded manner [41, 42] can be used to decompose a multi-component input signal into its constituent sinusoidal components (i.e., Fig. 2.4 of [41]). The method is shown to be able to detect and track voltage sag more quickly than other methods, such as Root Mean Square (RMS), Fourier transform, and peak voltage detection methods [43]. Although this tracking scheme is very robust to decaying DC offset and noise, its performance is severely degraded in the presence of harmonics, as shown in Fig. 2. In other words, this method, with cascaded or parallel core structures, is unable to provide accurate estimation of harmonics, since the fundamental signal is treated as part of the error signal when estimating harmonics. Any such estimation inaccuracies of harmonics will consequently affect estimation of the fundamental signal.

To overcome such deficiencies, we have proposed a new multiple core unit structure to mitigate the impact of harmonics. Fig. 1 shows the block diagram of the proposed tracking scheme which is described below:

Let's assume that the power signal is expressed as

$$x(t) = \sum_{i=0}^{K-1} A_i \sin(2\pi f_i t + \varphi_i) + n(t), \quad i = 0, 1, \dots \quad (1)$$

where A_i , f_i , φ_i , and $n(t)$ correspond to the amplitude, frequency, the phase angle of the i th order of the harmonic component of the signal and the additive Gaussian white noise (AWGN), respectively. For $i = 0$, f_0 represents the fundamental frequency. In power systems faults and load changes result in the fluctuation of A_i , f_i and φ_i . Please note that the harmonics in this equation consist of classic harmonics (i.e., multiple integer of the fundamental frequency), sub-harmonics and inter-harmonics.

Gradient descent methods can be employed to minimize the least square error between the input signal $x(t)$ and the desired signal $A_i \sin(2\pi f_i t + \varphi_i)$ [44]. The manifold containing all sinusoidal signals in $x(t)$ can be defined as $\mathcal{M}$:

$$\mathcal{M} = \{A(t) \sin(2\pi f(t)t + \varphi(t))\} \quad (2)$$

where $A(t) \in [A_{min}, A_{max}]$, $f(t) \in [f_{min}, f_{max}]$, $\varphi(t) \in [\varphi_{min}, \varphi_{max}]$. The parameter vector belonging to parameter space $\Phi = [A, f, \varphi]$ can be expressed as:

$$\phi(t) = [A(t), f(t), \varphi(t)]^T \quad (3)$$

where T denotes matrix transposition. We define a desired sinusoidal component as follows:

$$y(t, \phi(t)) = A(t) \sin(2\pi f(t)t + \varphi(t)). \quad (4)$$

To extract any desired component, such as the i th order of harmonics from $x(t)$, would require identifying an optimum ϕ_i , $i = 0, 1, \dots, K-1$, according to the following equation

$$\phi_i = \arg \min_{\phi_i(t) \in \Phi} d[y(t, \phi_i(t)), (x(t) - \sum_{j=0, j \neq i}^{K-1} A_j \sin \psi_j)], \quad (5)$$

where $d[y(t, \phi_i(t)), (x(t) - \sum_{j=0, j \neq i}^{K-1} A_j \sin \psi_j)]$ is the distance function between $y(t, \phi_i(t))$ and $x(t) - \sum_{j=0, j \neq i}^{K-1} A_j \sin \psi_j$, while $A_j \sin \psi_j$ is the estimated component of the j th order of harmonics. In the case of the fundamental component ($i = 0$) we can show;

$$\phi_0 = \arg \min_{\phi_0(t) \in \Phi} d[y(t, \phi_0(t)), (x(t) - \sum_{j=1}^{K-1} A_j \sin \psi_j)]. \quad (6)$$

Based on (5), the corresponding cost function can be shown as

$$\begin{aligned}\mathcal{F}(t, \phi(t)) &= d^2(t, \phi(t)) \triangleq e^2(t) \quad (7) \\ &= \left[x(t) - \sum_{j=0}^{K-1} A_j \sin \psi_j \right]^2.\end{aligned}$$

The gradient decent method is then used to estimate parameter vector ϕ :

$$\frac{d\phi(t)}{dt} = -Y \frac{\partial[\mathcal{F}(t, \phi(t))]}{\partial \phi(t)}, \quad (8)$$

where the positive diagonal matrix Y is the algorithm regulating constant matrix. Using the mathematical proof in [41], a set of nonlinear differential equations for the i^{th} spectrum component can be derived as:

$$\dot{\hat{A}}_i = 2\mu_1^i e \sin \hat{\psi}_i \quad (9)$$

$$\dot{\hat{\omega}}_i = 2\mu_2^i e \hat{A}_i \cos \hat{\psi}_i \quad (10)$$

$$\dot{\hat{\psi}}_i = \hat{\omega}_i + 2\mu_2^i \mu_3^i e \hat{A}_i \cos \hat{\psi}_i \quad (11)$$

where $\hat{A}_i$ is the estimation of amplitude A_i , $\hat{\omega}_i$ is the estimation of frequency $\omega_i = 2\pi f_i$, $\hat{\psi}_i$ is the estimation of total phase $\psi_i = \omega_i t + \phi_i$, and $e(t) = x(t) - \sum_{i=0}^{K-1} \hat{A}_i \sin \hat{\psi}_i$ is the error signal between the input signal and its estimation. Step parameter μ_1^i is used to control the convergence speed and accuracy of the i^{th} component's amplitude, while step parameters μ_2^i and μ_3^i are pre-set to get a trade-off between convergence speed and accuracy of the i^{th} component's frequency. Based on the first order time derivative approximation, the discretized form of the Eqs. (9) – (11) can be written as:

$$A_i[n+1] = A_i[n] + 2T_s \mu_1^i e[n] \sin(\psi_i[n]), \quad (12)$$

$$\omega_i[n+1] = \omega_i[n] + 2T_s \mu_2^i e[n] A_i[n] \cos(\psi_i[n]), \quad (13)$$

$$\psi_i[n+1] = \psi_i[n] + T_s \omega_i[n] + 2T_s \mu_2^i \mu_3^i e[n] A_i[n] \cos(\psi_i[n]), \quad (14)$$

$$y_i[n] = A_i[n] \sin(\psi_i[n]), \quad (15)$$

while the error signal can be expressed as:

$$e[n] = x[n] - \sum_{i=0}^{K-1} y_i[n] = x[n] - \sum_{i=0}^{K-1} A_i[n] \sin(\psi_i[n]), \quad (16)$$

where n is the time step index and T_s is the sampling interval. The implementation of the proposed tracking algorithm is displayed in Fig. 1.

As shown in Fig. 7 of Section VI, the proposed algorithm achieves significant improvement over the tracking algorithm of [41]. Furthermore, different step parameters can be used to estimate the fundamental frequency and harmonics' frequency in order to attain a better performance. With prior knowledge of the fundamental frequency and harmonics' frequencies, the proposed algorithm is capable of simultaneously tracking multiple signal components, including inter harmonics and sub-harmonics, as demonstrated in Fig. 8 of Section VI. It is important to point out that having prior knowledge of the signal spectral components was the main factor in the versatility of the proposed tracking scheme. Without prior knowledge of the signal spectral components (e.g., without identifying them through subspace methods), it would be practically impossible to accurately track all the components by using the cascaded multiple core units of [41, 42]. More precisely, it is important to know the correct number of core units and their corresponding initial values.

To acquire such an advance knowledge would require identifying all spectral components of the power signal. To achieve this, we developed a subspace-based estimation method, which is described next.

III. Subspace based spectrum estimation

Subspace-based estimation methods have been widely applied to spectrum analysis and general parameter estimation. By implementing an eigen decomposition, subspace-based estimation methods can decompose the autocorrelation matrix of the noise-polluted signal into two subspaces, namely a signal subspace and a noise subspace. The signal subspace contains K orthonormal eigenvectors corresponding to the K largest eigenvalues of the autocorrelation matrix, while the noise subspace consists of $N-K$ orthonormal eigenvectors corresponding to the eigenvalue σ^2 of the autocorrelation matrix, where σ^2 is the noise variance. When applying subspace-based methods to the noisy power signal, the fundamental signal and harmonic components will be grouped to the signal space with an eigenvalue bigger than σ^2 . In contrast to Fourier-based methods, subspace-based methods

can achieve relatively high accuracy in frequency and amplitude estimation, without the effect of synchronizing with inter-harmonics and sub-harmonics.

For the sake of spectral analysis, a general model of the power signal in complex form can be derived from Eq. (1) as

$$x(t) = A_0 e^{jw_0 t + \varphi_0} + \sum_{i=1}^{K-1} A_i e^{jw_i t + \varphi_i} + n(t), i = 0, 1, \dots \quad (17)$$

where, A_i , $w_i = 2\pi f_i$, φ_i and $n(t)$ correspond to the amplitude, angular frequency, the phase angle of the i^{th} order inter/intra harmonic component of the power signal and the additive Gaussian white noise (AWGN), respectively. For $i = 0$, f_0 represents the fundamental frequency. For a window of N samples, the corresponding power signal vector is denoted as:

$$\mathbf{x} = [x_1 \ x_2 \ \dots \ x_n \ \dots \ x_N]^T, \quad (18)$$

where $x_n = A_0 e^{jw_0(t - (N-n)T_s) + \varphi_0} + \sum_{i=1}^{K-1} A_i e^{jw_i(t - (N-n)T_s) + \varphi_i} + n(t - (N-n)T_s)$, $i = 0, 1, \dots$ and T_s is the sampling interval. The autocorrelation matrix $\mathbf{X}$ of the N -dimensional power signal vector $\mathbf{x}$ can be derived as

$$\mathbf{X} \triangleq E\{\mathbf{x}\mathbf{x}^H\} = \sum_{i=0}^{K-1} A_i^2 \mathbf{w}_i \mathbf{w}_i^H + \sigma^2 \mathbf{I}_N = \mathbf{W}\mathbf{A}\mathbf{W}^H + \sigma^2 \mathbf{I}_N, \quad (19)$$

where we have $\mathbf{w}_i = [e^{-jw_i(N-1)T_s} \dots \dots e^{-jw_i T_s} \ 1]^T$, $\mathbf{W} \triangleq [\mathbf{w}_0 \ \mathbf{w}_1 \ \dots \ \mathbf{w}_{K-1}]$, $\mathbf{A} \triangleq \text{diag}[A_0^2 \ A_1^2 \ \dots \ A_{K-1}^2]$ and $\mathbf{I}_N$ denotes the $N \times N$ -dimensional identity matrix. Please note that Eq. (19) is obtained based on the assumption that the N -dimensional vector $\{\mathbf{w}_i\}_{i=0}^{K-1}$ of the fundamental frequency and harmonic components are independent of each other.

The eigen-decomposition of the power signal's autocorrelation matrix $\mathbf{X}$ is given by

$$\mathbf{X} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H = [\mathbf{U}_s \ \mathbf{U}_n] \begin{bmatrix} \mathbf{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_n \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_n^H \end{bmatrix}, \quad (20)$$

where we have $\mathbf{U} = [\mathbf{U}_s \ \mathbf{U}_n]$ and $\mathbf{\Lambda} = \text{diag}[\mathbf{\Lambda}_s \ \mathbf{\Lambda}_n]$. In Eq. (20), $\mathbf{\Lambda}_s = \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_K]$ and $\mathbf{U}_s = [\mathbf{u}_1 \ \mathbf{u}_2 \ \dots \ \mathbf{u}_K]$ contain the largest K eigenvalues of the autocorrelation matrix $\mathbf{X}$ stored in descending order and the corresponding orthonormal eigenvectors. Matrix $\mathbf{U}_n = [\mathbf{u}_{K+1} \ \mathbf{u}_{K+2} \ \dots \ \mathbf{u}_N]$ consists of the $(N-K)$ orthonormal eigenvectors corresponding to the smallest eigenvalues σ^2 in $\mathbf{\Lambda}_n$. The range space of $\mathbf{U}_s$ is referred to as the signal space,

while its orthogonal complement, the noise space, is spanned by U_n . Eq. (20) physically states that the power signal's autocorrelation matrix X may be diagonalized by a unitary matrix U , resulting in a diagonal matrix Λ , where the entries are the eigenvalues of X . Furthermore, the unitary matrix U that is used to diagonalize X has constituting columns an orthonormal set of eigenvectors of X .

In ROOT-MUSIC methods [39], an auxiliary vector is constructed as:

$$\mathbf{w} = [e^{-jw(N-1)T_s} \dots e^{-jwT_s} 1]^T = [Z^{-(N-1)} \dots Z^{-1} 1]^T, \quad (21)$$

where $z = e^{jwT_s}$. This auxiliary vector $\mathbf{w}$ is then projected onto noise space U_n to form the following equation:

$$\mathbf{w}^H U_n U_n^H \mathbf{w} = 0. \quad (22)$$

The root of Eq. (22) can be used to estimate fundamental frequency and harmonic components denoted by $f_0, f_1, \dots, f_{K-1}$. Specifically, the ROOT-MUSIC method defines the spectrum by using an inverse distance measure, which is given by

$$S_{music}(\mathbf{w}) = \frac{1}{\mathbf{w}^H U_n U_n^H \mathbf{w}} = \frac{1}{\sum_{i=K+1}^N |\mathbf{w}^H \mathbf{u}_i|^2}, \quad (23)$$

where, $\mathbf{u}_i (i = K+1, \dots, N)$ is the eigenvector of noise space U_n . As mentioned earlier, eigenvectors of the signal subspace corresponding to the fundamental signal and harmonic components are orthogonal to the noise subspace. Therefore, this spectrum displays sharp peaks at frequencies of the fundamental and harmonic components. In other words, the roots of $S_{music}(\mathbf{w})$ lying on the unit circle correspond to the K harmonic components and the remaining $N-K$ roots will fall inside the unit circle (and also at inverse complex conjugate positions outside the circle) [39].

After pinpointing frequencies of the fundamental and harmonic components, we can then calculate their power by using the following equation:

$$\mathbf{u}_i^H X \mathbf{u}_i = \mathbf{u}_i^H U_s \Lambda_s U_s^H \mathbf{u}_i + \sigma^2 \mathbf{u}_i^H U_n U_n^H \mathbf{u}_i = \lambda_i, \quad i = 1, 2, \dots, K. \quad (24)$$

Substituting $X = \sum_{i=1}^{K-1} A_i^2 \mathbf{w}_i \mathbf{w}_i^H + \sigma^2 I_N$, we obtain the power of each component: $A_i^2, i = 1, 2, \dots, K$.

Since calculating the roots of $S_{music}(\mathbf{w})$ is a much simpler operation and has much lower complexity than the eigenvalue decomposition [40], the computational complexity of the

ROOT-MUSIC method is mainly decided by the implementation of eigenvalue decomposition, which is in the order of $\mathcal{O}(N^3)$. This complexity is too high and hence, not practical to run in a recursive mode. The result in Fig. 9 shows that N should be at least 100 to get a reliable estimation.

IV. Subspace Tracking Algorithms

In order to reduce the complexity, subspace tracking algorithms [36, 37] have been proposed to recursively update the subspace on a sample-by-sample fashion, which aims at directly tracking the components of the eigenvalue decomposition, rather than carrying out the eigenvalue decomposition for each block (window) of the power signal samples. The Projection Approximation Subspace Tracking (PASTd) algorithm [36, 37] is one of such low-complexity tracking algorithms. It provides almost guaranteed global convergence to the true eigenvectors and eigenvalues of the signal. It has a low computational complexity in the order of $\mathcal{O}(NK)$, where N is the dimension of the received signal vector and K is the number of eigen components. Based on the estimated eigenvalues, the rank of the signal subspace can be estimated adaptively by using the Akaike Information Criterion (AIC) or Minimum Description Length (MDL) criterion [38], which directly gives the number of harmonic components involved.

The operation procedure of The PASTd algorithm is shown in Table I, where $\mathbf{x}(l)$ is the l^{th} power signal sample vector in the N -dimension, while $\lambda_i(l)$ and $\mathbf{u}_i(l)$ represent the i^{th} eigenvalue and i^{th} eigenvector at the l^{th} time instant, and finally, $\sigma^2(l)$ is the noise variance at the l^{th} time instant. Based on the so-called deflation technique [37], the PASTd algorithm sequentially estimates principal components such as eigenvalues and eigenvectors of the signal subspace. More specifically, the most dominant eigenvector is first updated (see Table I.) and then the projection of the current signal sample vector $\mathbf{x}(l)$ onto this eigenvector is removed from $\mathbf{x}(l)$ itself. Now the second most dominant eigenvector becomes the most dominant in the updated signal sample vector, which can be extracted in the same way as before. This procedure is applied repeatedly, until all desired eigen components have been estimated. After updating eigenvalues and eigenvectors of signal subspace, the noise variance $\sigma^2(l)$ will be updated followed by identification of the fundamental frequency and harmonic components $f_0(l), f_1(l), \dots, f_{K-1}(l)$, as well as the corresponding powers $A_i^2(l)$. The parameter $0 < \beta < 1$ is used to down-weight the previous data, for the sake of tracking the statistical variation of the observed data when working in a nonstationary environment.

However, as mentioned earlier, the PASTd algorithm sequentially estimates the harmonic components by employing the iterative deflation technique of [38]. This mechanism will accumulate round-off estimation errors and lead to poor estimates of the low-power components' eigenvalues and eigenvectors. Furthermore, the use of an inverse distance measurement to identify the frequencies of harmonic components involves inversion of the signal subspace's eigenvalues. Consequently, this exacerbates the effect of estimation errors for low-power components, resulting in a degraded performance.

V. Integrated spectrum estimation and tracking algorithm

In order to achieve a reliable performance while maintaining low complexity, our final approach is based on a combination of the subspace-based ROOT MUSIC method and a low-complexity tracking algorithm. Specifically, in the initial stage, the subspace-based ROOT MUSIC method is employed to pinpoint the harmonic components and their corresponding power. This information is then used by our proposed tracking algorithm to estimate the harmonic components in recursive mode. In the case of severe disturbance or voltage instability (e. g. new harmonics emerge and/or some harmonics cease), estimation of the fundamental frequency will display a significant fluctuation. Under this condition, the subspace-based ROOT MUSIC method will be invoked to re-locate the harmonic components. In this way, the computational complexity of this new algorithm is mainly decided by that of the proposed tracking algorithm, which is in the order of $\mathcal{O}(K)$.

As mentioned before, although the tracking algorithm of [41] can track voltage amplitude and detect voltage sag faster than traditional methods, it cannot track frequency quickly enough [43]. Overcoming such a disadvantage has been the main push of the proposed tracking algorithm. As shown in Fig. 3, where a power signal without harmonics is investigated, the estimation of the amplitude converges at around 0.04 seconds, while the estimation of the frequency converges at around 0.09 seconds. Although step parameters μ_2 and μ_3 can be adjusted to increase convergence speed, they can impact the estimation accuracy [41]. Fig. 4 shows the performances of the proposed algorithm employing different step parameter $s\bar{\mu}_2$, namely 40,000 and 80,000, where other step parameters remain the same. Obviously, increasing step parameter $\bar{\mu}_2$ from 40,000 to 80,000 can further degrade performance. Suitable step parameters μ_2 and μ_3 should be used to achieve a trade-off between the convergence speed and accuracy. More importantly, without any prior knowledge of harmonics, the algorithm cannot track harmonics, which could also affect the estimation accuracy of the fundamental frequency.

Therefore, the subspace-based ROOT MUSIC method can play a crucial role for firstly identifying the frequency and power of each harmonic component, as shown in Fig. 5. Our tracking algorithm can then utilize this information to estimate the fundamental frequency, as well as all other spectral components in recursive mode. Furthermore, to dynamically track the spectral components, the subspace-based ROOT MUSIC method is invoked to update the presence of any new set of harmonic components. Such invocation is triggered as soon as a change in the fundamental frequency estimate is detected (exceeding a pre-defined threshold).

VI. Performance evaluation

Our proposed spectrum tracking algorithm is examined in the presence of harmonics, inter-harmonics, sub-harmonics, and noise. A software-based grid network modelling using the Electro Magnetic Transients Program (EMTP)¹ software tool is employed to generate the

¹Certain commercial equipment, instruments, or materials are identified in this paper to foster understanding. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.

power signal. EMTF [45, 46] can provide almost all power system components, such as power plant, transformer, non-linear loads, and different types of faults. Therefore, we are able to generate power signals containing different sets of harmonics corrupted by added white Gaussian noise. The IEEE 34-bus distribution model is used to generate the power signal. The sampling rate of the power signals is 5000 Hz. We run a series of experiments that include evaluating our low complexity fast tracking algorithm with and without the deployment of various forms of sub-space spectrum analysis schemes. The sub-space schemes include recursive root music and Projection Approximation Subspace Tracking (PASTd) (without being used in combination with the tracking algorithm), as well as complexity one-time sub-spaced root music, which is used in combination with the tracking algorithm.

In the first set of experiments we evaluate the tracking scheme by assuming that only rough frequency information of the fundamental signal and harmonics (not their amplitudes) are available. For a 60 Hz power signal with 10 % of the 3rd harmonic, Fig. 6 shows significant performance improvement with the proposed tracking algorithm compared with the method used in [41]. Such superiority is the result of estimating the fundamental frequency and the 3rd harmonic frequency separately, where each can then be recursively subtracted from the input signal (see Fig. 1). In these experiments we use different step parameters; for example, $\mu_1 = 100$, $\mu_2 = 10000$ and $\mu_3 = 0.02$ for the fundamental frequency, while $\bar{\mu}_1 = 100$, $\bar{\mu}_2 = 40000$ and $\bar{\mu}_3 = 0.02$ for estimating the third harmonic. In a similar experiment (see Fig. 7) the power signal not only contains 3rd harmonic (30 %), but also 5th harmonic (5 %), 35 Hz sub-harmonic (20 %), and 130 Hz inter-harmonic (10 %). Fig. 7 verifies the ability of the proposed algorithm to track multiple signals, including inter-harmonics and sub-harmonics. However, the convergence rate is low, since only rough frequency information (without any information about their amplitudes) has been made available.

In order to attain complete information about the harmonics, the subspace-based root music method described earlier is applied to the same power signal as in Fig. 7. Fig. 8 shows the new results which indicate that the subspace-based root music method can provide all the harmonic frequency components and their amplitudes. Nonetheless, as can be observed from Fig. 8(b), for a window size of $N = 100$, the subspace-based root music method lacks estimation accuracies for components with low amplitudes. For the fundamental signal, for instance, Fig. 9 shows that a larger window size (e.g., $N > 100$) can enhance estimation accuracy [40]. However, taking into consideration the computation complexity of the subspace-based root music method, (in the order of $O(N^3)$), the algorithm is not practical to run in a recursive mode. As a result, the PASTd algorithm is then employed to track spectrum components with a lower complexity in the order of $O(NK)$. However, because of the employed iterative deflation technique and inverse distance measurement, the PASTd algorithm is not able to provide an accurate estimation [38]. Fig. 10 illustrates the results of the tracking algorithm with or without the one-time sub-spaced Root Music, as well as the PASTd algorithm. Without a loss of generality, only the frequency tracking performance of the fundamental signal and the sub-harmonic are displayed in this figure. Based on one-time subspace-based root music spectrum mapping, the proposed integrated spectrum estimation and tracking approach achieves the best performance. This demonstrates that such a

combination is capable of accurately tracking all the harmonic components (phase, frequency and amplitude) with low complexity and a fast convergence rate.

In Fig. 11 the integrated spectrum detection and tracking algorithm is examined using a power signal in a situation when a disturbance occurs at time, $t = 5$ s. In this scenario, a 60 Hz power signal containing a 3rd harmonic (30 %), a 5th harmonic (5 %), and a 35 Hz sub-harmonic (30 %) is impacted by a disturbance causing the harmonic components to change from 30 % to 20 % of the 3rd harmonic, 5 % to 8 % of the 5th harmonic, and from 35 Hz (30 %) to 40 Hz (20 %) sub-harmonic. It can be seen from Fig. 11 that a disturbance occurring at $t = 5$ s causes a fluctuation in the estimation of the fundamental signal, which results in activating the one-time subspace-based root music in order to provide fresh estimates of the spectral comments. Fig. 11 verifies that integration of the sub-space spectrum estimation and tracking algorithm is capable of reliably tracking the fundamental signal, as well as harmonics, inter-harmonics and sub-harmonics, even in the case of a sudden disturbance.

In Fig. 12 we investigate the proposed tracking algorithm in the presence of a very high frequency harmonic component, such as 12,000 Hz. As shown in Fig. 12, the algorithm is still capable of tracking very high frequency components despite using a sub-Nyquist sampling rate of 5,000 Hz. In Fig. 13, a 60 Hz power signal corrupted by a 30 % of 3rd harmonic, a 0.5 % of 5th harmonic and a 20 % of 50 Hz sub-harmonic is considered. Without loss of generality, only the frequency tracking performance of the fundamental signal and the sub-harmonic are displayed in this figure. Two different sets of initial values are used to examine the performance of the proposed integrated scheme. In the first set, an initial value of 50 Hz is set for the fundamental signal, while an initial value of 40 Hz is considered for the sub-harmonic. In the second set, an initial value of 60 Hz is used for the fundamental signal, while an initial value of 50 Hz is set for the sub-harmonic. Fig. 13 demonstrates that the proposed algorithm can still track the spectrum components, although more offshoot initial values can impact the convergence rate.

In our next experiment we investigate the impact of different types of noise on the performance of the proposed algorithm in the presence of the same harmonic components as in our previous experiment (see Fig 13). Five different types of noises: pink, white (AWGN), brown, purple and blue noises are considered. In each case, the Signal to Noise Ratio (SNR) is set at 27 dB. The results are presented in Fig. 14. For the sake of simplicity, only the frequency tracking performance of the fundamental signal (60 Hz) are displayed in this figure. As can be observed, the proposed algorithm is highly robust to different types of noise. In addition, thermal noise is also considered but not included in this figure due to its marginal effect on the performance.

We also compare the performance of our two tier algorithm with the standard DFT scheme using the same distorted power signal. As shown in Fig. 15, the proposed tracking algorithm demonstrates a significant advantage over the standard DFT algorithm in the presence of harmonics and noise distortions.

In Fig. 16, the proposed algorithm is used to track a 60 Hz power signal in the presence of a damped 3rd harmonic with decaying amplitude: $0.3e^{-t}$. It can be seen from this figure that the proposed algorithm is capable of not only accurately tracking the diluted fundamental frequency, but also the 3rd damped harmonic.

Further experiments were carried out to compare the proposed tracking algorithm with the method of cascading units proposed in [41, 42]. In these experiments we use a 60 Hz power signal in the presence of a regular 3rd harmonic with an amplitude of 0.3. The results, which are shown in Fig. 17, show the versatility of our proposed method. Again, such superiority is the result of estimating the fundamental frequency and the 3rd harmonic component separately, where each can then be recursively subtracted from the input signal (see Fig. 1).

VII. Conclusion

With increasing harmonic distortion in the power system, real-time monitoring and analysis of harmonic variations has become a challenging issue. In this paper we investigate harmonic, inter-harmonic and sub-harmonic estimation and tracking for real-time power grid monitoring. We have demonstrated that an integrated spectrum estimation and tracking algorithm can provide a fast and reliable performance while maintaining low complexity. A one-time ROOT MUSIC spectrum estimation is first employed to detect the presence of any harmonic, inter-harmonic and sub-harmonic components in the power signal. This information is then utilized to accurately track all detected spectral components. The simulation results demonstrate the robustness of the proposed combined spectrum estimation and tracking algorithm under dynamic conditions.

VIII. References

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Figure 1.
Block diagram of the proposed tracking algorithm.

Figure 2.
The frequency estimation performance of the tracking algorithm [36] with or without harmonics.

Figure 3. The convergence rate of frequency and amplitude estimation of the proposed tracking algorithm of [41].

Figure 4.

The frequency estimation performance of the proposed algorithm with different step parameters of $\bar{\mu}_2$.

Figure 5.
Flowchart of the proposed integrated spectrum estimation and tracking algorithm.

Figure 6. Frequency tracking performance of the proposed algorithm and the tracking algorithm of [41].

Figure 7.
Frequency and amplitude tracking performance of the proposed algorithm

Figure 8.

The frequency and amplitude tracking performance of the subspace-based recursive root music method where the window size is $n=100$.

Figure 9.

The subspace-based recursive root music method with different sample-by-sample sliding window size: N.

Figure 10.

Frequency tracking performance of the PASTd algorithm, the proposed tracking algorithm and the integrated spectrum estimation and tracking algorithm.

Figure 11.

Performance of the integrated sub-space spectrum estimation and tracking algorithm in the scenario of power disturbance.

Figure 12.

The performance of the integrated spectrum estimation and tracking algorithm in the presence of a harmonic at 12000 Hz.

Figure 13.

The frequency tracking performance of the proposed algorithm with difference initial value.

Figure 14.

The fundamental frequency tracking performance of the proposed algorithm in the presence of different noises.

Figure 15.

Frequency tracking performance of the proposed tracking algorithm and the standard DFT algorithm, where SNR is 27 dB.

Figure 16.

The amplitude tracking performance of the proposed algorithm in the presence of regular and damped 3rd harmonic.

Figure 17.

The fundamental frequency tracking performance of the tracking algorithm [41], the cascaded unites [41, 42] and the proposed tracking algorithm.

TABLE I

Procedure of the PASTd algorithm

<p>Updating the eigenvalues and eigenvectors of signal space λ_i, u_i</p> <p>Updating noise variance σ^2 and matrix $C = U_n U_n^H$</p> <p>Give $\lambda_i(0)$, $u_i(0)$, and $\sigma^2(0)$ a suitable value;</p> <p>For $l = 1, 2, \dots$ Do</p> <p style="padding-left: 20px;">$x_1(l) = x(l)$;</p> <p style="padding-left: 20px;">For $i = 1, 2, \dots, K$ Do</p> <p style="padding-left: 40px;">$y_i(l) = u_i^H(l-1)x_i(l)$; projection operation</p> <p style="padding-left: 40px;">$\lambda_i(l) = \beta\lambda_i(l-1) + y_i(l) ^2$; updating eigenvalue</p> <p style="padding-left: 40px;">$u_i(l) = u_i(l-1) + \frac{[x_i(l) - u_i(l-1)y_i(l)]y_i^*(l)}{\lambda_i(l)}$;</p> <p style="padding-left: 20px;">$x_{i+1}(l) = x_i(l) - u_i(l)y_i(l)$;</p> <p>END</p> <p style="padding-left: 20px;">$\sigma^2(l) = \beta\sigma^2(l-1) + \frac{\ x_{K+1}(l)\ ^2}{N-K}$; updating noise variance</p> <p style="padding-left: 20px;">$C(l) = \frac{X - U_S \Lambda_S U_S^H}{\sigma^2(l)} = \frac{x(l)x^H(l) - U_S \Lambda_S U_S^H}{\sigma^2(l)}$;</p> <p>Identify the fundamental frequency and harmonic components $f_0(l), f_1(l), \dots, f_{K-1}(l)$ by calculating the roots of Eq. (23);</p> <p>Derive the corresponding powers $A_f^2(l)$ through Eq. (24).</p>
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