Nonstationary model of oblique x-ray incidence in amorphous selenium detectors: II. Transfer functions

Raymond J. Acciavatti^{a)} and Andrew D. A. Maidment

Department of Radiology, Perelman School of Medicine at the University of Pennsylvania, Philadelphia, PA 19104-4206, USA

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Purpose: One limitation of experimental techniques for quantifying resolution and noise in detectors is that the measurement is made in a region-of-interest (ROI). With theoretical modeling, these properties can be measured at a point, allowing for quantification of spatial anisotropy. This paper calculates nonstationary transfer functions for amorphous selenium (*a*-Se) detectors in breast imaging. We use this model to demonstrate the performance advantage of a “next-generation” tomosynthesis (NGT) system, which is capable of x-ray source motion with more degrees of freedom than a clinical tomosynthesis system.

Methods: Using Swank’s formulation, the optical transfer function (OTF) and presampled noise power spectra (NPS) are determined based on the point spread function derived in Part I. The modulation transfer function (MTF) is found from the normalized modulus of the OTF. To take into account the presence of digitization, the presampled NPS is convolved with a two-dimensional comb function, for which the period along each direction is the reciprocal of the detector element size. The detective quantum efficiency (DQE) is then determined from combined knowledge of the OTF and NPS.

Results: First, the model is used to demonstrate the loss of image quality due to oblique x-ray incidence. The MTF is calculated along various polar angles, corresponding to different orientations of the input frequency. The MTF is independent of the incidence angle if the polar angle is perpendicular to the ray incidence direction. However, along other polar angles, oblique incidence results in MTF degradation at high frequencies. The MTF degradation is most substantial along the ray incidence direction. Unlike the MTF, the normalized NPS (NNPS) is independent of the incidence angle. To measure the relative signal-to-noise, the DQE is also calculated. Oblique incidence yields high-frequency DQE degradation, which is more pronounced than the MTF degradation. This arises because the DQE is proportionate with the square of the MTF. Ultimately, this model is used to evaluate how the image quality varies over the detector area. For various projection images, we calculate the variation in the incidence angle over this area. With the NGT system, the source can be positioned in such a way that this variation is minimized, and hence the DQE exhibits less anisotropy. To achieve this improvement in the image quality, the source needs to have a component of motion in the posteroanterior (PA) direction, which is perpendicular to the conventional direction of source motion in tomosynthesis.

Conclusions: In *a*-Se detectors, the DQE at high frequencies is degraded due to oblique incidence. The DQE degradation is more pronounced than the MTF degradation. This model is used to quantify the spatial variation in DQE over the detector area. The use of PA source motion is a strategy for minimizing this variation and thus improving the image quality. © 2018 American Association of Physicists in Medicine [https://doi.org/10.1002/mp.13312]

Key words: amorphous selenium, detective quantum efficiency (DQE), digital x-ray detectors, modulation transfer function (MTF), obliquity effect

1. INTRODUCTION

Oblique x-ray incidence is a source of resolution loss in amorphous selenium (*a*-Se) detectors. The loss of resolution can be quantified with the modulation transfer function (MTF). Que and Rowlands used theoretical modeling to derive a closed-form solution for the MTF in *a*-Se detectors.¹ This model was later validated by Hajdok and Cunningham with Monte Carlo simulations.² These simulations show that the MTF degradation is pronounced at high energy, since the

incident ray is less attenuated as it passes through the x-ray converter.

X-ray experiments in previous works have validated the MTF degradation due to oblique incidence. The MTF was calculated with a digital breast tomosynthesis (DBT) system using a tungsten edge.^{3,4} Previous works have also shown that increasing the thickness of selenium results in more substantial MTF degradation.^{3,5} This arises because the incident ray traverses a broader path length through the x-ray converter, yielding more blurring.

The purpose of this paper is to develop a nonstationary model of the MTF in a-Se detectors. This model is used to quantify the spatial variation in the image quality across the detector area. One limitation of experimental methods^{6–9} used to measure MTF is that spatial anisotropy cannot be quantified, since it is presumed that the MTF is stationary within a region-of-interest (ROI). We calculate the MTF using the model of the point spread function (PSF) developed in Part 1.¹⁰ This model includes the effects of the divergent-beam geometry and digitization.

Unlike the MTF, the NPS is effectively independent of the incidence angle in a-Se detectors. This result was demonstrated by Hajdok and Cunningham with Monte Carlo simulations.² Hu et al. reached a similar conclusion in an experimental study.³ Based on the MTF and NPS results, it has been shown that the detective quantum efficiency (DQE) is degraded by oblique incidence. The DQE is proportionate with the square of the MTF, and hence, the DQE degradation is more pronounced than the MTF degradation.²

Ultimately, this model is used to demonstrate the performance advantage of a prototype “next-generation” tomosynthesis (NGT) system, which we are developing at the University of Pennsylvania.^{11–16} This system is capable of x-ray source motion with more degrees of freedom than a clinical DBT system. Our previous work showed that the NGT system offers an improvement in image quality using the Defrise phantom.^{13,16} This paper analyzes how the DQE varies over the detector area in different projection views supported by the NGT system. We demonstrate that the source can be positioned in such a way that there is less pronounced variation in DQE, and hence, the image quality is more isotropic.

2. MATERIALS AND METHODS

2.A. Modulation transfer function (MTF)

2.A.1 X-ray converter

To determine the MTF of the x-ray converter, it is first necessary to calculate the two-dimensional (2D) Fourier transform of P_1 ; that is, the PSF associated with the interaction point at the height z above the exit surface of the x-ray converter

$$\mathcal{F}_2 P_1 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_1 \cdot e^{-2\pi i(f_x x + f_y y)} dx dy \quad (1)$$

where $i = \sqrt{-1}$ and where f_x and f_y measure the frequency along the x and y directions, respectively. This integral can be transformed from the (x, y) coordinate system into the (v'_1, v'_2) coordinate system using the equations of Part 1.¹⁰

$$\mathcal{F}_2 P_1 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta[v'_1 - (l - z) \tan \theta] \delta(v'_2) \cdot e^{-2\pi i[f_x(\xi_1 + v'_1 \cos \Gamma - v'_2 \sin \Gamma) + f_y(\xi_2 + v'_1 \sin \Gamma + v'_2 \cos \Gamma)]} dv'_1 dv'_2 \quad (2)$$

$$\mathcal{F}_2 P_1 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta\left[v'_1 - \frac{r(l - z)}{d - l}\right] \delta(v'_2) \cdot e^{-2\pi i[f_x(\xi_1 + v'_1 \frac{\xi_1}{r} - v'_2 \frac{\xi_2}{r}) + f_y(\xi_2 + v'_1 \frac{\xi_2}{r} + v'_2 \frac{\xi_1}{r})]} dv'_1 dv'_2 \quad (3)$$

$$= e^{-2\pi i\left(\frac{\xi_1 f_x + \xi_2 f_y}{d - l}\right)(d - z)} \quad (4)$$

These equations presume that $r > 0$, as was the case in Part 1.¹⁰ Following Swank,¹⁷ the optical transfer function (OTF) of the x-ray converter is determined by integrating $\mathcal{F}_2 P_1$ over the thickness (l) of selenium, and weighting the integrand by the relative number of x-ray quanta at each depth z .

$$G_{Se} = \int_0^l N \cdot \mathcal{F}_2 P_1 \cdot dz \quad (5)$$

$$= C_1 e^{\frac{-2\pi i d(\xi_1 f_x + \xi_2 f_y)}{d - l}} \int_0^l e^{-\mu(l - z) \sqrt{1 + \left(\frac{r}{d - l}\right)^2} + \frac{2\pi i(\xi_1 f_x + \xi_2 f_y)z}{d - l}} dz \quad (6)$$

$$= \frac{e^{-2\pi i(\xi_1 f_x + \xi_2 f_y)}}{1 - e^{-\mu l \sqrt{1 + \left(\frac{r}{d - l}\right)^2}}} \left[\frac{1 - e^{-\mu l \sqrt{1 + \left(\frac{r}{d - l}\right)^2} + \frac{-2\pi i(\xi_1 f_x + \xi_2 f_y)l}{d - l}}}{1 + \frac{2\pi i(\xi_1 f_x + \xi_2 f_y)}{\mu r \sqrt{1 + \left(\frac{d - l}{r}\right)^2}}} \right] \quad (7)$$

The MTF of the x-ray converter can be determined from the normalized modulus of the OTF

$$\text{MTF}_{Se} = \frac{|G_{Se}|}{G_{Se}|_{\mathbf{f}=0}} \quad (8)$$

where $\mathbf{f}$ denotes the doublet with components f_x and f_y .

2.A.2. Digital system

Analogous to Eq. (5), the net OTF of the digital system can be calculated by replacing P_1 with P_z .

$$G_{Net} = \int_0^l N \cdot \mathcal{F}_2 P_z \cdot dz \quad (9)$$

Since P_z is a convolution of two functions [Eq. (11), Acciavatti and Maidment¹⁰], the 2D Fourier transform can be evaluated using the convolution theorem

$$\mathcal{F}_2 P_z = \mathcal{F}_2 P_1 \cdot \mathcal{F}_2 P_{TFT} \quad (10)$$

where

$$\mathcal{F}_2 P_{TFT} = \text{sinc}(a_x f_x) \cdot \text{sinc}(a_y f_y) \quad (11)$$

and where

$$\text{sinc}(v) \equiv \frac{\sin(\pi v)}{\pi v} \quad (12)$$

Hence,

$$G_{\text{Net}} = \mathcal{F}_2 P_{\text{TFT}} \cdot \int_0^l N \cdot \mathcal{F}_2 P_1 \cdot dz \quad (13)$$

$$= \mathcal{F}_2 P_{\text{TFT}} \cdot G_{\text{Se}} \quad (14)$$

The MTF of the digital system can be determined with a normalization formula similar to the one described by Eq. (8) for the x-ray converter. In the special case of normal incidence ($r = 0$), G_{Se} is unity since there is no loss of image quality; hence, G_{Net} is given by 2D Fourier transform of P_{TFT} [Eq. (11)].

2.B. Normalized noise power spectra (NNPS)

2.B.1. X-ray converter

The NPS of the x-ray converter can be calculated from P_1 , the PSF associated with each interaction point in selenium. Since the Fourier transform of P_1 has a modulus of unity, the NPS is white (i.e., frequency-independent) at all incidence angles.

$$W_{\text{Se}} = \int_0^l N \cdot |\mathcal{F}_2 P_1|^2 \cdot dz \quad (15)$$

$$= \int_0^l N \cdot 1^2 \cdot dz \quad (16)$$

$$= 1 \quad (17)$$

The notation “ W ” comes from the alternate NPS term “Wiener spectra.”¹⁸ Eqs. (15)–(17) represent the normalized NPS (NNPS), as there is no weighting based on the x-ray fluence.^{19,20}

2.B.2. Digital system

In the digital system, the presampled NPS is calculated with the same approach, but P_1 is replaced with P_z to take into account the presence of the TFT array.

$$W_{\text{Pre}} = \int_0^l N \cdot |\mathcal{F}_2 P_z|^2 \cdot dz \quad (18)$$

$$= \int_0^l N \cdot |\mathcal{F}_2 P_1|^2 \cdot |\mathcal{F}_2 P_{\text{TFT}}|^2 \cdot dz \quad (19)$$

$$= |\mathcal{F}_2 P_{\text{TFT}}|^2 \cdot \int_0^l N \cdot 1^2 \cdot dz \quad (20)$$

$$= |\mathcal{F}_2 P_{\text{TFT}}|^2 \quad (21)$$

Previous work demonstrated that the digital NPS is found by convolving W_{Pre} with a 2D comb function. The period of the comb function is a_x^{-1} in the x direction and a_y^{-1} in the y direction, where a_x and a_y are the del dimensions in the two respective directions.

$$W_{\text{Net}}(f_x, f_y) = \sum_{k_y=-\infty}^{\infty} \sum_{k_x=-\infty}^{\infty} W_{\text{Pre}} *_2 \delta\left(f_x - \frac{k_x}{a_x}\right) \delta\left(f_y - \frac{k_y}{a_y}\right) \quad (22)$$

$$= \sum_{k_y=-\infty}^{\infty} \sum_{k_x=-\infty}^{\infty} \text{sinc}^2(a_x f_x) \quad (23)$$

$$\cdot \text{sinc}^2(a_y f_y) *_2 \delta\left(f_x - \frac{k_x}{a_x}\right) \delta\left(f_y - \frac{k_y}{a_y}\right)$$

$$= \sum_{k_x=-\infty}^{\infty} \text{sinc}^2\left[a_x \left(f_x - \frac{k_x}{a_x}\right)\right] \quad (24)$$

$$\cdot \sum_{k_y=-\infty}^{\infty} \text{sinc}^2\left[a_y \left(f_y - \frac{k_y}{a_y}\right)\right]$$

$$= 1 \quad (25)$$

The transition from Eq. (24) to Eq. (25) follows from an identity described in our previous work.²⁰ Eq. (25) indicates that the NNPS of the digital system is white, just as the NNPS of the x-ray converter is white.

2.C. Detective quantum efficiency (DQE)

Following Nishikawa and Yaffe,²¹ the DQE is calculated from the product of four terms

$$\text{DQE} = A_Q A_S R_C R_N, \quad (26)$$

where A_Q is the quantum detection efficiency (QDE).

$$A_Q = (1 - e^{-\mu l \sec \theta}) \cos \theta \quad (27)$$

$$= \frac{1 - e^{-\mu l \sqrt{1 + \left(\frac{r}{d-l}\right)^2}}}{\sqrt{1 + \left(\frac{r}{d-l}\right)^2}} \quad (28)$$

In Eq. (27), the quantity in parentheses is the percentage of x rays that are absorbed based on the attenuation coefficient (μ) of selenium. The term $\cos \theta$ models the degradation in x-ray fluence with increasing obliquity, which approaches zero at shearing incidence ($\theta = 90^\circ$).

Second, A_S is the Swank factor^{17,22}; it measures the relative signal-to-noise at zero frequency.

$$A_S = \frac{G^2|_{f=0}}{W|_{f=0}} \quad (29)$$

Since this formula applies to both the x-ray converter and the digital system, the subscripts “Se” and “Net” are being removed from the terms G and W .

Third, the term R_C is the Lubberts fraction; it quantifies the frequency dependence of the DQE.

$$R_C = \frac{1}{A_S} \cdot \frac{|G|^2}{W} \quad (30)$$

Finally, the term R_N is a relative measure of quantum noise power to total noise power. For the purpose of this paper, it is assumed that R_N is unity at all frequencies.

3. RESULTS

3.A. Calculation of MTF

3.A.1. Low x-ray energy

We model the NGT system^{11–16} with the same acquisition parameters as those described in Part 1¹⁰ for a digital mammography (DM) image (analogous to the central projection in DBT). To calculate the MTF along any polar angle (α), a coordinate transformation can be introduced

$$f_x = f_r \cos \alpha \quad (31)$$

$$f_y = f_r \sin \alpha, \quad (32)$$

where f_r denotes radial frequency. Figures 1(a) and 2(a) illustrate how the MTF at 15° incidence varies with radial frequency at an energy of 20 keV. The two plots differ in terms of the azimuthal angle (Γ) of the ray; that is, 90° in Fig. 1(a) and 45° in Fig. 2(a). The detector positions corresponding to these angles were illustrated in Part 1 (Acciavatti and Maidment,¹⁰ Figs. 3 and 6).

In a non-pixelated system, the MTF is perfect (1.0 at all frequencies) if the polar angle is perpendicular to the ray incidence direction; that is, 0° in Fig. 1(a) and 135° in Fig. 2(a). However, there is loss of resolution along the other polar angles, as one would expect.

It is also shown that the digital system has lower MTF than the x-ray converter. At a 0° or 90° polar angle, MTF_{Net} is zero at the frequency a^{-1} (11.8 mm⁻¹). By contrast, along 45° or 135° polar angles, the zero increases by a factor of $\sqrt{2}$, giving 16.6 mm⁻¹. These zeros are not dependent on the parameter Γ , which controls the direction of ray incidence.

3.A.2. High x-ray energy

At high energy, the x-ray beam is attenuated less quickly as it passes through the x-ray converter. As a result, the MTF

degradation due to the obliquity effect is more pronounced [Figs. 1(b) and 2(b)]. This finding is consistent with previous work.²

The plots at high energy show Gibbs ringing,²³ unlike the plots at low energy. This result is concordant with the work of Que and Rowlands,¹ who demonstrated that the frequency span of the MTF can be determined from the equation

$$f_g = \max \left\{ (l \tan \theta)^{-1}, \mu \csc \theta \right\} \quad (33)$$

At 20 keV, f_g is so large (78.2 mm⁻¹) that it exceeds the frequency range of the plot. However, at 40 keV, f_g is 18.7 mm⁻¹. This frequency is a perfect match to the local minimum if the polar angle (α) is aligned with the ray incidence direction; that is, 90° in Fig. 1(b) and 45° in Fig. 2(b). If the polar angle is at a 45° angle relative to the ray incidence direction, this frequency is scaled up by a factor of $\sqrt{2}$, giving 26.4 mm⁻¹.

3.A.3. Dependency on incidence angle

Figure 3 illustrates how the MTF is degraded as a function of the incidence angle, assuming that $f_r = 5.0$ mm⁻¹ and $\Gamma = 90^\circ$. The incidence angle is varied by increasing the distance from the chest wall along the +y direction. This plot presumes that the x-ray energy is 20 keV, as is the case in all subsequent plots unless explicitly stated. The MTFs are shown for both a digital and a non-pixelated system. The MTFs of these two systems differ by a constant factor, which is given by the MTF of the detector sampling function [Eq. (11)].

If frequency is perpendicular to the ray incidence direction ($\alpha = 0^\circ$), the MTF is independent of the incidence angle. However, there is MTF degradation along the other polar angles. The most pronounced resolution loss is seen along the 90° polar angle (parallel to the ray incidence direction).

To generalize Fig. 3 to more than one frequency, Fig. 4 shows the MTF at 20 keV as a function of both incidence angle and frequency. This figure presumes that $\Gamma = \alpha = 90^\circ$, so the polar angle is aligned with the ray incidence direction. Frequencies up to the alias frequency $0.5a^{-1}$ (5.88 mm⁻¹)

FIG. 1. The MTF at 15° incidence is plotted as a function of frequency. The ray incidence direction is at a 90° azimuthal angle (Γ), so the most MTF degradation is seen along this direction. The MTF degradation is more pronounced at high energy. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 2. The MTF is calculated under assumptions similar to Fig. 1, but the azimuthal angle (Γ) is 45° . Along the perpendicular direction ($\alpha = 135^\circ$), there is no MTF degradation due to oblique incidence. [Color figure can be viewed at [wileyonlinelibrary.com](#)]

FIG. 3. For a 20 keV x-ray beam, the MTF at 5.0 mm^{-1} is plotted as a function of the incidence angle, assuming $\Gamma = 90^\circ$. The MTF is unaffected by oblique incidence if frequency is measured along the polar angle perpendicular to the ray incidence direction ($\alpha = 0^\circ$). However, the MTF is degraded by oblique incidence along the other polar angles shown. [Color figure can be viewed at [wileyonlinelibrary.com](#)]

are considered. At low obliquity ($\theta \sim 0^\circ$), the MTF of a non-pixelated system [Fig. 4(a)] is effectively constant, yet the MTF of a digital system [Fig. 4(b)] is degraded at high frequencies. As the incidence angle increases, both the non-pixelated and digital systems show MTF degradation at high frequencies.

3.B. Calculation of DQE

3.B.1. Effect of x-ray energy and polar angle

Under assumptions similar to Figs. 1 and 2, the DQE is plotted as a function of frequency in Figs. 5 and 6. Following the approach used in previous work, the DQE is plotted up to the alias frequency.^{20,24} Along the 0° and 90° polar angles,

the alias frequency is $0.5a^{-1}$ (5.88 mm^{-1}). By contrast, along the 45° and 135° polar angles, the alias frequency increases by a factor of $\sqrt{2}$, giving 8.32 mm^{-1} .

Using Eq. (29) and the equations for OTF and NPS, it can be shown that the Swank factor (A_S) is unity for both the non-pixelated and digital systems. Therefore, the DQE at zero-frequency is equivalent to A_Q (the QDE); namely, 0.951 at 20 keV and 0.449 at 40 keV. There is lower DQE at high energy, since fewer x rays are absorbed by the detector. These results hold for both the non-pixelated and digital systems.

The DQE degradation is most pronounced along the polar angle aligned with the ray incidence direction; that is, 90° in Fig. 5 and 45° in Fig. 6. These are consistent with the polar angles corresponding to the most pronounced MTF degradation.

In a non-pixelated system, the DQE is not degraded along the polar angle perpendicular to the ray incidence direction. This arises because the MTF and NNPS of the x-ray converter are independent of frequency along this direction. By contrast, in the digital system, the DQE is frequency-dependent along all polar angles.

It should be noted that digitization introduces electronic noise not modeled in this paper. Based on our previous work,²⁰ including this noise source in the model would re-scale the curves in Figs. 5 and 6 by a constant factor, reducing the DQE.

3.B.2. Obliquity effect

Figure 7 shows the dependency of DQE on incidence angle (under similar assumptions as Fig. 3). Oblique incidence results in DQE degradation along all polar angles. This includes the polar angle perpendicular to the ray incidence direction ($\alpha = 0^\circ$), even though there is no MTF degradation along this direction. To understand why there is DQE degradation along this direction, it is important to recall the QDE formula [Eq. (27)]. The formula consists of two terms. The term $(1 - e^{-\mu l \sec \theta})$ is an increasing function of the incidence

FIG. 4. The MTF at 20 keV is plotted as a function of both the incidence angle and frequency, assuming that $\Gamma = \alpha = 90^\circ$. At low frequency, the MTF is effectively independent of the incidence angle. However, at high frequency, the MTF is more strongly dependent on the incidence angle. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 5. The DQE is calculated under assumptions similar to Fig. 1; that is, $\Gamma = 90^\circ$. Unlike the DQE of the x-ray converter, the DQE of the digital system is frequency-dependent along all polar angles, including the polar angle perpendicular to the ray incidence direction ($\alpha = 0^\circ$). Increasing the energy results in less x-ray absorption and hence lower DQE. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 6. The DQE is calculated under assumptions similar to Fig. 2. Hence, the direction corresponding to the most DQE degradation is aligned with the 45° polar angle (α). [Color figure can be viewed at wileyonlinelibrary.com]

angle; yet, the term $\cos \theta$ is a decreasing function. The latter term has a more dominant effect on QDE.

Figure 7 also illustrates that the DQE is degraded by oblique incidence more strongly than the MTF. The DQE degradation is most pronounced along the ray incidence direction ($\alpha = 90^\circ$); for example, at 15° incidence, the percent change in MTF relative to normal incidence is 5.6%, yet the percent change in DQE is 13.7%. These percent differences are the same for both the x-ray converter and the digital system.

Figure 8 is an extension of Fig. 7 to a broader range of frequencies, assuming that frequency is measured along the ray incidence direction. At low frequency ($f \sim 0 \text{ mm}^{-1}$), the DQE is given by the QDE and thus is degraded by oblique incidence. This differs from the MTF (Fig. 4), which is unaffected by oblique incidence at low frequency. At high

frequencies, the MTF and DQE are both degraded by oblique incidence, although the DQE degradation is greater.

3.C. An application in tomosynthesis system design

3.C.1. Acquisition geometry

We now explore how the obliquity effect gives rise to spatial variation in image quality. This calculation is used to demonstrate an advantage of the NGT design; namely, that the image quality shows less spatial anisotropy.

In the NGT system, the source is capable of arbitrary motions in a plane parallel to the breast support; for example, T-shaped motion (Fig. 9). This motion includes a component in the conventional scan direction (x). There is also a component of motion in the perpendicular direction (y); that is, the posteroanterior (PA) direction. To demonstrate the advantage of this design, the image quality can be quantified in different projection views.

3.C.2. Spatial anisotropy in the incidence angle

For image quality to be high, the incidence angle should be as close to 0° as possible. Figure 10 illustrates how the incidence angle varies over the active area of the detector ($304.64 \text{ mm} \times 239.36 \text{ mm}$) in the central and oblique projection images of a conventional system design. The central projection [Fig. 10(a)] uses the same acquisition geometry considered previously for DM imaging. By contrast, in the oblique projection [Fig. 10(b)], the source is shifted by -100.0 mm in the x direction. The incidence angle increases at positions distal to the (x, y) coordinate of the source. It should be noted that, in the NGT system, the z -coordinate of the source is the same in all projections; this differs from a clinical system, in which there is circular motion in the xz plane.

Figure 11 shows the benefit of introducing source motion in the PA direction. For the purpose of this figure, it is

FIG. 7. The DQE is degraded by oblique incidence. Similar to the MTF, the degradation is most pronounced along the ray incidence direction ($\alpha = 90^\circ$). There is also DQE degradation along the perpendicular direction ($\alpha = 0^\circ$); this is due to degradation in QDE. The assumptions made in this figure are similar to Fig. 3. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 8. Figure 7 is generalized to a range of frequencies under assumptions similar to Fig. 4. Increasing the incidence angle results in DQE degradation at all frequencies (including zero-frequency due to loss of QDE). [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 9. In the NGT system, there are two degrees of freedom in the motion of the source (x and y). This differs from a clinical DBT system in which there is no source motion in the y direction; that is, posteroanteriorly. An example of a motion supported by the NGT system is T-shaped. This figure is not to scale.

FIG. 10. The incidence angle varies over the detector area. (a) In the central projection of a conventional DBT system, the angle increases up to 23.6°. (b) In the oblique projection, the angle increases up to 28.2°. [Color figure can be viewed at wileyonlinelibrary.com]

assumed that the x -coordinate of the source is the same as the central projection of the conventional design [Fig. 10(a)] but the y -coordinate is displaced halfway between the chest wall and the opposite end of the detector (i.e., 119.7 mm anterior to the chest wall). Compared against the projections in the conventional design, the incidence angle varies over a smaller range of values, and hence one would expect more isotropic image quality across the detector area.

3.C.3. Spatial anisotropy in DQE

Figure 12 illustrates how the net DQE at 5.0 mm^{-1} varies over the detector area for these three projections, assuming 20 keV x rays. For better visualization of the surface, the plots are prepared with a different orientation

than Figs. 10 and 11. The benefit of PA source motion (bottom row) is that there is less spatial variation in DQE. This can be illustrated with the results along a 90° polar angle (α). The DQE range (difference between max and min) is 0.13 for the central projection and 0.14 for the oblique projection in the conventional design. Yet, the DQE range is only 0.047 for the projection obtained with PA source motion.

Figure 12 also illustrates how the DQE is sensitive to the polar angle (α) of the input frequency. In the conventional design, the spatial anisotropy is more pronounced along a 45° polar angle (DQE ranges of 0.16 and 0.21 for the central and oblique projections, respectively) than along a 90° polar angle. The use of PA source motion minimizes the anisotropy (DQE range of 0.087).

FIG. 11. The NGT system supports PA source motion. There is a smaller range of incidence angles (up to 16.6°) compared against projections obtained with the conventional design (Fig. 10). [Color figure can be viewed at wileyonlinelibrary.com]

4. DISCUSSION

This paper develops a nonstationary model of the obliquity effect in *a*-Se detectors. We show that the MTF is degraded by oblique incidence. This result is consistent with previous works on both direct- and indirect-converting detectors.^{1–5,9,25,26} The MTF degradation is most pronounced along the ray incidence direction. There is no MTF degradation along the perpendicular direction. Although these results were illustrated with the AXS-2430 detector, similar results are expected in other detector applications.

We follow the approach used by Que and Rowlands¹ to model the obliquity effect; namely, we consider this effect separate from other x-ray interactions in selenium. With this approach, we find that the NPS is white. Previous work has demonstrated that, at low energy, the NPS is effectively white.⁴ However, in high-energy applications, it has been shown that the NPS exhibits more pronounced dependency on frequency.^{2–4} This arises from x-ray interactions not included in our model; for example, Compton scattering.²

This paper shows that the DQE is degraded by oblique incidence, similar to the MTF. The DQE degradation is most pronounced along the ray incidence direction. The DQE is more strongly dependent on the incidence angle than the MTF, since the DQE is proportionate with the square of the MTF. There is additional DQE degradation due to the dependency of QDE on $\cos \theta$ [Eq. (27)].

In breast imaging, calcifications are high-frequency structures that can act as an early sign of cancer. Based on Monte Carlo simulations of DQE, Hajdok and Cunningham concluded that oblique incidence gives rise to poorer calcification visibility.² However, their work is limited in that they did not calculate how the DQE varies over the detector area. Furthermore, they did not model the presence of detector pixelation (i.e., digitization). Our results suggest that calcification visibility does indeed fluctuate over the detector area, and that the image quality at a given detector position differs between projection views

(Fig. 12). Exploring the anisotropies in calcification imaging should be the subject of future work.

In the work by Hajdok and Cunningham, the DQE was calculated assuming that frequency is oriented along the ray incidence direction.² By contrast, in this paper, the input frequency is modeled along an arbitrary polar angle (α). In studying the spatial variation in DQE, we show that the net DQE variation over the detector area is dependent on the polar angle.

This model is a tool for evaluating the image quality in a prototype NGT system. This system is capable of source motion in the PA direction, unlike a clinical system. In various projection images supported by this system, we analyze how the incidence angle varies over the detector area. The use of PA source motion minimizes the range of angles over this area, and hence anisotropies in the DQE are minimized. This results in an improvement in image quality.

While this paper proposes a model of image quality in an individual projection image, future work should perform these calculations in the reconstruction. In addition, the three-dimensional transfer functions should simulate the reconstruction filter.^{27,28}

This paper shows that the Swank factor (A_S) is unity for all incidence angles, and thus the DQE at zero-frequency is determined by the QDE [Eq. (26)]. This result differs from our previous work modeling indirect-converting detectors.^{25,26} We demonstrated that A_S does show angular dependence if an optical dye is added to a scintillator. The purpose of the optical dye is to absorb some of the visible light produced by the scintillator, and thus minimize the lateral spread of light due to optical scatter.

As was discussed in Section 3.B.1., electronic noise has not been included in the modeling assumptions of this paper. This noise source re-scales the DQE in Figs. 5 and 6 by a constant factor,²⁰ resulting in a loss of image quality. While this paper presumes that the x-ray system is quantum-limited, future work should explore additional sources of noise²⁹ affecting the term R_N in the DQE formula [Eq. (26)].

5. CONCLUSION

In Part 1, the consequences of oblique incidence in *a*-Se detectors were analyzed in the spatial domain.¹⁰ By contrast, this paper focuses on the Fourier domain. We show that there is MTF and DQE degradation resulting from oblique incidence.

This model is ultimately used to quantify spatial variation in image quality over the detector area. We calculate the DQE in projection images supported by the NGT system. The benefit of PA source motion is that it minimizes the spatial variation in DQE, and hence the image quality is more isotropic.

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FIG. 12. The net DQE varies over the detector area. With PA source motion (bottom row), there is less pronounced variation and hence more isotropic image quality. The shape of these surface plots is dependent on the polar angle (α) of the input frequency. These calculations presume an input frequency of 5.0 mm^{-1} and an energy of 20 keV. [Color figure can be viewed at wileyonlinelibrary.com]

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CONFLICTS OF INTEREST

Andrew D. A. Maidment is a scientific advisor to RTT, and his spouse is an employee and shareholder of RTT.

APPENDIX: NOMENCLATURE

Symbol	Meaning
$\ast_2$	2D convolution operator
$\mathcal{F}_2$	2D Fourier operator
α	Polar angle (defined in Part 1 ¹⁰)
Γ	Azimuthal angle of ray (defined in Part 1 ¹⁰)
δ	Delta function
θ	Incidence angle (defined in Part 1 ¹⁰)
μ	Linear attenuation coefficient of selenium
ξ_1, ξ_2	Coordinates of entrance point of ray in x-ray converter (defined in Part 1 ¹⁰)
A_Q	Abbreviation for QDE
A_S	Swank factor
a_x, a_y	Del dimensions; if $a_x = a_y$, the dimension is abbreviated a
C_1	Normalization term for P_1 (defined in Part 1 ¹⁰)
d	Distance between source and exit surface of x-ray converter (defined in Part 1 ¹⁰)
DBT	Digital breast tomosynthesis
Del	Detector element
DM	Digital mammography
DQE	Detective quantum efficiency
DQE _{Net}	DQE of digital system
DQE _{Se}	DQE of x-ray converter
f_g	Frequency span of MTF [Eq. (33)], as shown by Que and Rowlands ¹
f_r	Radial frequency
f_x, f_y	Frequency along x and y directions
FS	Focal spot
G_{Net}	OTF of digital system
G_{Se}	OTF of x-ray converter
l	Selenium thickness
MTF	Modulation transfer function
MTF _{Net}	MTF of digital system
MTF _{Se}	MTF of x-ray converter
N	Relative number of x-ray quanta (defined in Part 1 ¹⁰)
NNPS	Normalized NPS
NPS	Noise power spectra
OTF	Optical transfer function
P_1	PSF associated with arbitrary ionization point in x-ray converter
P_{TFT}	PSF of TFT array
P_z	Depth-dependent PSF (combined effect of P_1 and P_{TFT})
PA	Posteroanterior
PSF	Point spread function
QDE	Quantum detection efficiency
r	$\sqrt{\xi_1^2 + \xi_2^2}$ as defined in Part 1 ¹⁰
R_C	Lubberts fraction
R_N	Ratio of quantum noise power to total noise power
TFT	Thin-film transistor
v'_1	Position along ray incidence direction (defined in Part 1 ¹⁰)
v'_2	Position perpendicular to ray incidence direction (defined in Part 1 ¹⁰)
W_{Net}	NNPS of digital system
W_{Pre}	Presampled NNPS
W_{Se}	NNPS of x-ray converter

^{a)} Author to whom correspondence should be addressed. Electronic mail: raymond.acciavatti@uphs.upenn.edu; Telephone: +1-215-746-8759; Fax: +1-215-746-8764.

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