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Optimization of the Trade-Off Between Speckle Reduction and Axial Resolution in Frequency Compounding

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Abstract

We measured the reduction of speckle by frequency compounding using Gaussian pulses, which have the least time-bandwidth product. The experimental results obtained from a tissue mimicking phantom agree quantitatively with numerical simulations of randomly distributed point scatterers. For a fixed axial resolution, the amount of speckle reduction is found to approach a maximum as the number of bands increases while the total spectral range that they cover is kept constant. An analytical solution of the maximal speckle reduction is derived and shows that the maximum improves approximately as the inverse square root of the Gaussian pulse bandwidth. Since the axial resolution is proportional to the inverse of the pulse bandwidth, an optimized trade-off between speckle reduction and axial resolution is obtained. Considerations for the applications of the optimized trade-off are discussed.

Keywords

Ultrasound; image enhancement/restoration (noise and artifact reduction)

I. INTRODUCTION

ULTRASOUND imaging is the lowest cost of all deep tissue medical imaging modalities and is becoming an increasingly important diagnostic tool. Advantages include compact, real-time continuous imaging that can be used to guide injection and surgical procedures with no exposure to ionizing radiation to patients or clinicians. In addition, imaging modes such as color Doppler, shear wave and contrast agent labeling offer valuable additional information that complements x-ray, CT and MRI imaging. However, ultrasound imaging suffers from the presence of significant speckle noise, and useful resolution is severely degraded.

Speckle noise is the result of the coherent back-scattering of sound by the distribution of scatterers within each voxel. In each voxel, suppose we have scattering amplitudes $A_1(\vec{x}_1)$, $A_2(\vec{x}_2)$, $A_3(\vec{x}_3)$, These amplitudes can be either positive or negative, corresponding to positive and negative sound pressure. The amplitude of the detected signal is given by the coherent sum

$$|A| = |A_1(\vec{x}_1) + A_2(\vec{x}_2) + A_3(\vec{x}_3) + \dots| \quad (1)$$

If these amplitudes interfere constructively or destructively, $|A|$ can be either greater or less than the average amplitude, $\sqrt{|A_1(\vec{x}_1)|^2 + |A_2(\vec{x}_2)|^2 + |A_3(\vec{x}_3)|^2 + \dots}$, thus producing speckle.

In addition to ultrasound, speckle noise is present in synthetic aperture radar and optical coherence tomography.

A general strategy to reduce speckle is to compound (average) over a number of distinct speckle images of the same object [1]–[3]. The distinct speckle images can be obtained by varying the angle of incidence [4]–[7], the wavefront phase profile [8]–[10], or frequency [11]–[15]. There are also image processing methods for speckle reduction [16]–[21]. Since they largely rely on post-processing algorithms, any speckle reduction of the non-processed images can be used for further speckle reduction.

This work focuses on frequency compounding, where distinct speckle images using portions of the transducer bandwidth are averaged to obtain the compound image. When the sub-bands have negligible overlap, the corresponding speckle images are uncorrelated and the speckle in the compounded image is reduced by $\sqrt{N}$, where N is the number of subbands. In the presence of spectral overlap, however, the speckle patterns become correlated and speckle reduction is less than the uncorrelated limit. As N increases while the transducer bandwidth is fixed, the spectral overlap and the speckle correlation increase, suggesting a possible maximum in speckle reduction for a given axial resolution. Speckle correlation and speckle reduction by frequency compounding have been studied extensively [11]–[14]. It is yet unclear, however, what is the maximally achievable speckle reduction for given axial resolution. Furthermore, higher axial resolution is in general associated with broader bandwidth, which leads to larger spectral overlap, higher speckle correlation, and eventually less speckle reduction. A systematic assessment of this tradeoff between speckle reduction

and axial resolution is lacking. The trade-off is of particular interest in view of the significant improvement in transducer bandwidth over the years, which should enhance the outcome of frequency compounding.

In this work, we synthesize Gaussian pulses using an arbitrary waveform generator and used them to form speckle images of an ultrasound phantom. The Gaussian pulses achieve minimum product of bandwidth and pulse duration. We compare the speckle reduction obtained from the phantom with numerical simulations of randomly distributed point scatterers, as well as an analytical solution. Speckle reduction attained by Gaussian pulses is further compared to a random spectrum pulse that gives the same axial resolution. Considerations for the application of frequency compounding in practice are discussed.

II. MATHEMATICAL DESCRIPTION OF A GAUSSIAN PULSE

A Gaussian pulse can be described by its time-domain waveform $g(t)$ or by its frequency-domain spectrum $G(f)$, which are related by Fourier transform.

$$g(t) = g_0 e^{-\frac{t^2}{2\sigma_t^2}} \cos[2\pi f_c t], \quad (2)$$

$$G(f) = g_0 \sqrt{\frac{1}{2\pi\sigma_f^2}} e^{-\frac{f^2}{2\sigma_f^2}} * \left[\frac{\delta(f - f_c) + \delta(f + f_c)}{2} \right]. \quad (3)$$

The time domain waveform $g(t)$ is an amplitude-modulated sinusoidal wave, with center frequency f_c and peak amplitude g_0 . The amplitude modulation envelope is a Gaussian function that has a standard deviation σ_t . $G(f)$ contains two Gaussian bands centered at f_c and $-f_c$, respectively, and the symbol $*$ denotes the convolution operation. Because $g(t)$ is a real-valued function, the negative frequency spectrum of $G(f)$ is related to the positive frequency spectrum by complex conjugation $G(f) = \overline{G(-f)}$. For simplicity, we describe $G(f)$ by its Gaussian spectrum at positive frequency. The widths of the Gaussians in the frequency and time domains are related by $\sigma_f = \frac{1}{2\pi\sigma_t}$. Rearranging this relation yields the time-

frequency uncertainty relation for a Gaussian pulse $\sigma_t \sigma_f = \frac{1}{2\pi}$. Note that there are several equivalent expressions for the minimum time-frequency uncertainty relation [22], [23]. As a convention in this paper, we use σ_f to denote the bandwidth of the Gaussian bands. The full width at half maximum of a Gaussian is $2.35\sigma_f$.

We define the axial resolution by the distance z between two scatterers along the axial direction such that the echoes they produce are separated temporally by a full width at half maximum of the intensity. The axial resolution is then given by

$$\Delta z = \frac{2.35c\sigma_t}{2\sqrt{2}} = \frac{2.35c}{4\sqrt{2}\pi\sigma_f}, \quad (4)$$

which shows that the axial resolution is inversely related to the bandwidth of the Gaussian pulse. The factor of 2 in the denominator accounts for the round-trip delay. Dropping the factor of $\sqrt{2}$ in the denominator yields the corresponding resolution in terms of the amplitude.

III. SPECKLE IMAGING OF A TISSUE-MIMICKING PHANTOM

A. Experiment Setup

An ultrasound phantom (Computerized Imaging Reference Systems, Inc. Model 054GSE) is imaged in a pulse-echo configuration. A hyperechoic region (designed to generate stronger echo) in the phantom contains more than 100 scatterers per imaging voxel. Our numerical simulations described in Sect. IV indicates that this density is sufficient to produce random speckle accurately described by Rayleigh statistics. The depth z of the imaging voxel is determined from the delay of the echo T , *i.e.*, $z = cT/2$. A single-element transducer is used to transmit the sound pulse and detect the back-scattered sound. The transducer used in this work (NDT Systems Inc., model IBHF054) has a diameter of 1.3 cm and a focal distance of 5.0 cm. Electrical Gaussian pulses are synthesized by an arbitrary waveform generator, and then amplified by a gated RF amplifier. The gate control turns off the amplification after the transmission, thus reducing noise in the detected signal. The time duration of the prepared pulse signal is longer than $10\sigma_t$, so that the effect of a finite time support is negligible. Since the acoustic response of the transducer is broader than the width of Gaussian pulses, the electrical Gaussian pulses are transformed into acoustic Gaussian pulses with good fidelity. The echo amplitude along the depth (A-scan) is obtained from the detected waveform using a Hilbert transform technique [24]. Two-dimensional images are constructed by translating the phantom perpendicular to the A-scan direction. (Typically, a phased array is used to produce B-scans). We note that frequency compounding can be performed alternatively by filtering the echo of a broadband pulse using post data acquisition signal processing with digital spectral filters [25].

B. Measurement of Speckle Reduction

Speckle images of the hyperechoic region in the ultrasound phantom are acquired with Gaussian pulses of the same width $\sigma_f = 0.4$ MHz and at different center frequencies $f_c = 2.0, 2.4, 2.8, \dots, 6.8$ MHz (Fig. 1). The images are normalized to their average speckle amplitude within the hyperechoic region. The lateral resolution improves with higher frequency while the axial resolution that is determined by the pulse bandwidth remains constant. The speckle pattern is seen to vary with the center frequencies of the Gaussian pulses. This variation allows for the reduction of speckle when the speckle patterns are averaged in frequency compounding. The compounded images for the first $N (= 3, 6, 9, \text{ and } 12)$ speckle images are shown in Fig. 2.

We characterize speckle using the dimensionless quantity μ/σ , where μ and σ are the mean and standard deviation of the speckle amplitude. We further denote μ/σ as the signal to noise ratio (SNR), with higher SNR corresponding to less speckle. Speckle reduction is represented by the normalized SNR, defined as SNR/SNR_0 , where SNR_0 is the SNR before performing frequency compounding and equal to 1.91 for densely and randomly distributed scatterers [26], [27].

Fig. 3a shows the digital spectrum of Gaussian bands with $\sigma_f = 0.6$ MHz and $f_c = 2.0, 2.4, 2.8 \dots, 6.8$ MHz. They are used to produce the electrical waveforms that drive the transducer. Speckle reduction (normalized SNR) for the compounded image using the first $N (= 1, 2, \dots, 13)$ bands are plotted in Fig. 3b. When the bandwidth is reduced (Figs. 3c, e), the speckle reduction improves accordingly (Figs. 3d, f), a result of decreasing correlation among the neighboring bands. Since the axial resolution is proportional to the inverse of the pulse bandwidth (Eq. 4), this trend is a manifestation of the trade-off between axial resolution and speckle reduction.

IV. NUMERICAL SIMULATIONS OF SPECKLE REDUCTION

To verify our understanding of the measured speckle reduction, we obtain the expected speckle reduction from the statistics of 2000 A-scan simulations with different distributions of point scatterers. In each A-scan simulation, 1000 scatterers are placed randomly along a depth of 10 mm, corresponding to more than 20 scatterers per imaging voxel. The simulation shows an asymptotic behavior, *i.e.*, further improvements in speckle reduction is insignificant with further increase in the scatterer density. The frequency continuum is approximated as the sum of discretized frequencies that are distributed densely with a 0.02 MHz spacing. At each frequency f , the back-scattered sound amplitudes from all scatterers are added coherently to obtain the speckle amplitude at the center of the A-scan, which we use to compute the speckle statistics. For each scatterer, the back-scattered amplitude is calculated considering a round trip phase delay of $4\pi zf/c$, where z is the depth of the scatterer. The simulated speckle reduction shows good agreement with the experiment (Fig. 3b, d, f).

We optimize the trade-off between axial resolution and speckle reduction by maximizing speckle reduction while fixing the axial resolution. This is achieved by increasing the number of Gaussian bands with a fixed Gaussian width within the transducer bandwidth. The normalized SNR for different number of bands N are listed in Table 1. The band separations f_c is then given by $f_c = f/(N-1)$. Here $f = 4.8$ MHz is the difference between the center frequencies of the lowest frequency and the highest frequency bands. f also characterizes the total bandwidth of the transducer. According to the table, the change in speckle reduction becomes insignificant with increasing N when f_c is equal or smaller than σ_f . Consequently, we approximate the maximal speckle reduction for the examined cases using the last column of Table 1, which corresponds to center frequency spacings of $f_c = 0.1$ MHz. The optimized trade-off between speckle reduction and axial resolution is represented by the maximal normalized SNR and the axial resolution z , both calculated as functions of σ_f shown in Fig. 4.

To put our results into the context of earlier studies, we compare the maximal speckle reduction value we obtained with [13], where speckle reduction of 1.2 $\times$, 1.8 $\times$, and 2.0 $\times$ were obtained when the pulses were 1/2, 1/5, and 1/7 of the total bandwidth. In our case, the total bandwidth is 4.8 MHz, hence at 1/2, 1/5, and 1/7 bandwidths, the full widths at half maximum are 2.4 MHz, 1.0 MHz, and 0.7 MHz. The corresponding 1σ widths are 1.0 MHz, 0.4 MHz, and 0.28 MHz, and the maximal speckle reductions are 1.5 $\times$, 2.3 $\times$, and 2.7 $\times$, respectively. These results are in overall agreement with the previous study. The improvement in the obtained speckle reduction may be due to differences in the calibration of the pulse durations.

V. ANALYTICAL SOLUTION FOR MAXIMAL SPECKLE REDUCTION

Before deriving the analytical solution, we consider the behavior of the maximal speckle reduction in the limits of $f\sigma_f \ll 1$ and $\gg 1$. When $f\sigma_f \ll 1$, the frequency shifts of the Gaussian bands are much smaller than the bandwidth and they are completely overlapping. As a result, no significant speckle reduction is expected. When $f\sigma_f \gg 1$, the number of Gaussian bands that can fit within f for a given amount of correlation is proportional to $f\sigma_f$. Hence the optimized speckle reduction should scale as $(f\sigma_f)^{1/2}$.

For N Gaussian bands that produce speckle with equal average amplitudes, the expression for the normalized SNR is

$$\text{normalized SNR} = \sqrt{\frac{N^2}{\sum_{i,j} \rho_{ij}}}, \quad (5)$$

where ρ_{ij} is the cross-correlation coefficient of the speckle corresponding to the i^{th} and j^{th} bands. When $i = j$, ρ_{ii} is equal to 1. In the limit of $f\sigma_f \ll 1$, $\rho_{ij} \approx 1$ and Eq. 5 shows that normalized SNR ≈ 1 , confirming the expected behavior in this limit discussed at the beginning of this section. In the case where the N bands are well-separated, $\rho_{ij} = \delta_{ij}$, where δ_{ij} is the Kronecker delta function. The normalized SNR is then equal to $\sqrt{N}$.

ρ_{ij} can be expressed as a function of the frequency separation between the bands in the limit where the absolute frequencies of two bands are much greater than their separation ([12, eq. 26]). This is a reasonable approximation here as the Gaussian bands we consider have significant overlap only when their separations are smaller or comparable to their bandwidths, which are further smaller than their center frequencies. A general expression for the cross correlation coefficient in the k-space representation is given by Eq. 5 in [28].

Maximization of speckle reduction can be considered as a limiting process where N is increased while the spacings between adjacent bands are reduced toward zero. To evaluate the limit, we re-express the normalized SNR as

$$\text{normalized SNR} = \sqrt{\frac{\Delta f / \sigma_f}{\frac{\Delta f / \sigma_f}{N^2} \sum_{i,j} \rho_{ij}}} \quad (6)$$

For large N and equally spaced bands, $\frac{\Delta f / \sigma_f}{N^2} \sum_{i,j} \rho_{ij}$ turns into an integral $\int_{-\infty}^{\infty} \rho(f) h(f/M) df$ and

$$\text{maximal normalized SNR} = \sqrt{\frac{\Delta f / \sigma_f}{\int_{-\infty}^{\infty} \rho(f) h(f/M) df}}, \quad (7)$$

where f is the frequency normalized to σ_f , $\rho(f)$ is the cross-correlation coefficient between two bands separated by f , and h is the triangular function. Evaluation of Eq. 7 gives the maximal speckle reduction as a function of σ_f . When $f / \sigma_f \gg 1$, Eq. 7 can be approximated by

$$\text{maximal normalized SNR} \approx \sqrt{\frac{\Delta f / \sigma_f}{2.43}} \quad (\Delta f / \sigma_f \gg 1). \quad (8)$$

Note that Eq. 8 agrees with the qualitative analysis at the beginning of this section. In Fig. 4, the exact solution (Eq. 7) and the approximate solution (Eq. 8) are plotted along with the maximal speckle reduction obtained from the simulation. The exact solution is in excellent agreement with the numerical simulation and the accuracy of the approximation is better than 10%.

The expression for the optimized trade-off between speckle reduction and the axial resolution can be obtained by rearranging Eq. 8,

$$\frac{\text{maximal normalized SNR}}{\sqrt{\Delta z / \lambda_0}} \approx \sqrt{3.1Q} \quad (\Delta f / \sigma_f \gg 1), \quad (9)$$

where λ_0 is the center wavelength of the transducer and $Q = \lambda / \lambda_0$ is the quality factor of the transducer. Here we have implicitly assumed that the useful frequency spectrum available is given by the Q of the transducer.

VI. RATIONALE FOR USING EQUALLY WEIGHTED AND EVENLY SPACED GAUSSIAN BANDS

In the experiment and the numerical analysis, we considered equally weighted and equally spaced Gaussian frequency bands. This simple scheme is adopted based on the fact that each band have the same contribution to the SNR (Eq. 5), neglecting the effects of the edge of the transducer bandwidth. Intuitively, variations in either the weight or spacing give rise to

uneven weighting of the frequencies over the bandwidth of the transducer and will tend to lower the normalized SNR. We performed simulations with randomly weighted Gaussian frequency bands, as well as random center frequency Gaussian bands, and found that the speckle reduction for these cases were indeed less than the equally weighted and evenly spaced bands.

We further compare the Gaussian pulses to the pulses with random spectra. The numerical waveforms of the random spectrum pulses are obtained by first creating a random spectrum in the frequency domain. This step does not guarantee a short pulse in the time domain. Hence in the second step, the spectrum is converted into the time domain through Fourier transform and then multiplied with a Gaussian envelop function to form a pulse. Note that due to the second step, the spectrum of the pulse is no longer completely random, but they are as random as possible under the constraint of the pulse width.

Fig. 5 shows the expected speckle reduction using up to 500 such random spectrum pulses with 13 equally weighted and equally spaced Gaussian pulses. The same axial resolution of $z = 0.5$ ($\sigma_z = 0.40$) is used for both cases. The simulation procedure for the random spectrum pulses is similar to that described in Sect. IV. In addition, 100 different permutations of the 500 random spectrum pulses are used to perform compounding. For each permutation, the normalized SNR is simulated for the compounded data associated with the first N random pulses. The normalized SNR's of the 100 permutations for each N are then averaged to obtain the expected speckle reduction. The averaging step over the permutations smooths out the fluctuations in the normalized SNR due to the randomness in the pulses and yields the expectation value of the normalized SNR. As shown in Fig. 5, the expected normalized SNR approaches an upper bound approximately equal to that obtained using the Gaussian pulses. While the speckle reduction with random frequency pulses is initially more significant than the Gaussian pulses, it takes significantly larger number of random frequency pulses to approach the maximal speckle reduction. Hence the equally weighted and equally spaced Gaussian frequency bands turn out to be efficient for optimizing speckle reduction in frequency compounding.

VII. CONSIDERATIONS FOR USING FREQUENCY COMPOUNDING IN PRACTICE

Frequency compounding can be potentially implemented by either transmitting the Gaussian pulses and forming several frames at different frequencies or by digital filtering of the radiofrequency signal of a single frame acquired by a broadband pulse. In the former case, the frame rate is reduced by the number of frames used for compounding. The latter implementation can be potentially real-time with fast electronics for digital spectral analysis.

We note further that transducer with broader bandwidth tend to allow for better speckle reduction as more Gaussians can be fitted within the bandwidth. Hence the speckle reduction is significantly improved for the same axial resolution with the development of wide bandwidth transducers that have fractional bandwidths approaching or even exceeding 100%. The transmission spectrum of the transducer can be flattened by driving the transducer with a signal that compensates for the natural response of the transducer, so that the signal power is approximately constant over the bandwidth of the transducer. In doing

so, the filtered frequency bands will have similar power density, which is desired for speckle reduction as discussed in the previous section.

The optimized trade-off between speckle reduction and axial resolution provides new options for the implementation of frequency compounding. If speckle reduction is more critical in an application, narrower bandwidth Gaussians can be used resulting in lower axial resolution; and if axial resolution is more desired, wider bandwidth can be used at the cost of speckle reduction. An adjustable parameter may thus be added to an ultrasound machine to let the users choose the optimal parameters for their applications. Alternatively, several modes may be implemented for resolution critical, signal to noise critical, or intermediate tasks.

Today's commercial systems use various signal processing methods and spatial compounding. Frequency compounding may be implemented as an additional layer before these signal processing methods to achieve further speckle reduction. An interesting direction for future study is to combine frequency compounding with the other techniques that are currently used in ultrasound imaging.

While speckle is generally regarded as an undesirable artifact in ultrasound imaging, the characteristics of speckle are sometimes found to be useful in clinical applications such as the diagnostics of liver cirrhosis [29], where the statistics of speckle deviates from the Rayleigh statistics [30], [31]. An extension of this work is to examine whether the reduction of speckle using frequency compounding will actually improve or weaken the ability to use speckle to diagnose cirrhosis and other conditions.

VIII. CONCLUSION

In summary, we optimized the trade-off between speckle reduction and axial resolution in frequency compounding using evenly spaced and equally weighted Gaussian pulses. Speckle reduction is measured from the speckle images of a tissue-mimicking phantom for a number of axial resolutions. The measurements agree with numerical simulations of randomly distributed point scatterers. As the number of bands increases and the band separations decrease, the simulated speckle reduction is found to approach a maximum. As a rule of thumb, the improvement in speckle reduction becomes insignificant when the adjacent bands are separated by less than σ_f . An analytical solution for the maximal speckle reduction is derived and a simple approximation is found for $f\sigma_f \gg 1$. Equally weighted and evenly spaced Gaussians give better speckle reduction than randomly weighted or randomly spaced Gaussians. Further, it requires less Gaussian pulses than pulses with random spectra to approach the optimized trade-off. Considerations in the practical implementation of frequency compounding are discussed.

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Fig. 1.

False-color speckle images measured with Gaussian pulse excitation. The pulses are centered at 2.0, 2.4, 2.8, ..., 6.8 MHz. The spectral widths of the pulses are $\sigma_f = 0.4$ MHz.

Fig. 2.
Compounded speckle images averaging 3, 6, 9, and 12 sub-band images in Fig. 1.

Fig. 3.

Gaussian bands centered at $f_c = 2.0, 2.4, 2.8, \dots, 6.8$ MHz, with $\sigma_f = 0.6$ MHz (a), 0.4 MHz (c), and 0.2 MHz (e). Normalized SNR as a function of the number of frequency bands used in frequency compounding, with $\sigma_f = 0.6$ MHz (b), 0.4 MHz (d), and 0.2 MHz (f).

Fig. 4.

Left axis: Simulated maximal speckle reduction (normalized SNR) versus σ_f for a fixed total bandwidth $f = 4.8$ MHz. The exact analytical solution is also shown. In the regime where

$f / \sigma_f \gg 1$, the maximal speckle reduction can be approximated as $\sqrt{(\Delta f / \sigma_f) / 2.43}$. Right axis: the axial resolution Δz versus σ_f . For a speckle reduction of ~ 4 , the optimal axial resolution is 1.4 mm. The ovals with arrows indicate the y-axes of the corresponding simulations and functional curves.

Fig. 5. Comparison of the speckle reduction (normalized SNR) using equally weighted and evenly spaced Gaussian pulses and random frequency pulses.

TABLE 1

NORMALIZED SNR (SIMULATED) FOR DIFFERENT BANDWIDTHS σ_f , AXIAL RESOLUTIONS z , AND BAND SEPARATIONS f_C

Bandwidth (MHz)	Axial resolution (mm)	$N = 13$ ($f_c = 0.4$ MHz)	$N = 25$ ($f_c = 0.2$ MHz)	$N = 49$ ($f_c = 0.1$ MHz)
$\sigma_f = 0.10$	$z = 2.0$	3.6	4.4	4.5
$\sigma_f = 0.14$	$z = 1.4$	3.5	3.8	3.8
$\sigma_f = 0.20$	$z = 1.0$	3.1	3.2	3.2
$\sigma_f = 0.28$	$z = 0.7$	2.7	2.7	2.7
$\sigma_f = 0.40$	$z = 0.5$	2.3	2.3	2.3
$\sigma_f = 0.60$	$z = 0.3$	1.9	1.9	1.9
$\sigma_f = 1.00$	$z = 0.2$	1.5	1.5	1.5