Propagation of pop ups in kirigami shells

Ahmad Rafsanjani^{a,b,1}, Lishuai Jin^{a,c,1}, Bolei Deng^a, and Katia Bertoldi^{a,d,e,2}

^aJohn A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138; ^bDepartment of Materials, ETH Zürich, 8093 Zürich, Switzerland; ^cDepartment of Mechanics, Tianjin University, Tianjin 300072, China; ^dKavli Institute, Harvard University, Cambridge, MA 02138; and ^eWyss Institute for Biologically Inspired Engineering, Harvard University, Cambridge, MA 02138

Edited by John A. Rogers, Northwestern University, Evanston, IL, and approved March 12, 2019 (received for review October 15, 2018)

Kirigami-inspired metamaterials are attracting increasing interest because of their ability to achieve extremely large strains and shape changes via out-of-plane buckling. While in flat kirigami sheets, the ligaments buckle simultaneously as Euler columns, leading to a continuous phase transition; here, we demonstrate that kirigami shells can also support discontinuous phase transitions. Specifically, we show via a combination of experiments, numerical simulations, and theoretical analysis that, in cylindrical kirigami shells, the snapping-induced curvature inversion of the initially bent ligaments results in a pop-up process that first localizes near an imperfection and then, as the deformation is increased, progressively spreads through the structure. Notably, we find that the width of the transition zone as well as the stress at which propagation of the instability is triggered can be controlled by carefully selecting the geometry of the cuts and the curvature of the shell. Our study significantly expands the ability of existing kirigami metamaterials and opens avenues for the design of the next generation of responsive surfaces as demonstrated by the design of a smart skin that significantly enhances the crawling efficiency of a simple linear actuator.

kirigami | buckling | propagative instability | metamaterials | phase transition

Kirigami—the Japanese art of cutting paper—has recently inspired the design of highly stretchable (1–8) and morphable (9–17) mechanical metamaterials that can be easily realized by embedding an array of cuts into a thin sheet. An attractive feature of these systems is that they are manufactured flat and then, exploit elastic instabilities to transform into complex 3D configurations (2–6, 13). Remarkably, the morphology of such buckling-induced 3D patterns can be tuned by varying the arrangement and geometry of the cuts (2–4) as well as the loading direction (13). However, in all kirigami systems proposed to date, the buckling-induced pop-up process occurs concurrently through the entire system, resulting in a simultaneous shape transformation.

The coexistence of two phases has been observed at both the microscopic and macroscopic scales in a variety of systems, including phase-transforming materials (18–22), dielectric elastomers (23, 24), and thin-walled elastic tubes (25, 26) (Movie S1). While these systems are very different in nature from each other, they all share a nonconvex free energy function that, for specific conditions, has two minima of equal height. When such a situation is reached, the homogeneous deformation becomes unstable, and a mixture of two states emerges. The new phase initially nucleates near a local imperfection and then, under prevailing conditions, propagates through the entire system (21, 24–26).

Here, we demonstrate via a combination of experiments and numerical/theoretical analyses that kirigami structures can also support the coexistence of two phases, the buckled one and the unbuckled one. Specifically, we show that, in thin cylindrical kirigami shells subjected to tensile loading, the buckling-induced pop-up process initially localizes near an imperfection and then, as the deformation is increased, progressively spreads through the cylinder at constant stress. We find that the curvature of the cylinder is the essential ingredient to observe this phenomenon, as it completely changes the deformation mechanism of the

hinges. In kirigami sheets, the initially flat hinges buckle out of plane, leading to a monotonic stress–strain relationship for the unit cell. By contrast, in kirigami shells, the initially bent ligaments snap to their second stable configuration, resulting in a nonmonotonic stress–strain curve typical of phase-transforming materials (18–22).

Experiments

We start by testing under uniaxial tension a kirigami flat sheet and a corresponding cylindrical shell. Both structures are fabricated by laser cutting triangular cuts arranged on a triangular lattice with lattice constants $l = 12$ mm and $\gamma = \pi/3$ separated by hinges with width $\delta = l/16$ (resulting in a rectangular unit cell with width $L = 2l \cos \pi/6$ and height $H = 2l \sin \pi/6$) into polyester plastic sheets (Artus Corporation) of thickness $t = 76.2 \mu\text{m}$ with Young's modulus $E = 4.33$ GPa and Poisson's ratio $\nu = 0.4$ (Fig. 1A). The flat kirigami sheet comprises an array of 2×20 cuts, while the cylindrical shell has 8×20 cuts and is created by bending an initially flat perforated sheet into a cylinder with radius $r \sim nL/(2\pi)$ (n denotes the number of unit cells along the circumference of the cylinder) (Fig. 1B) and gluing the two overlapping edges with a thin adhesive layer (SI Appendix, section 1 and Movie S2 have fabrication details).

In Fig. 1C and D, we show snapshots of the kirigami sheet and kirigami shell at different levels of applied deformation. We find that the responses of the two structures are remarkably different (Movie S3). In the kirigami sheet at a critical strain, all triangular features simultaneously pop up, forming a uniform 3D

Significance

Kirigami—the Japanese art of cutting paper—has become an emergent tool to realize highly stretchable devices and morphable structures. While kirigami structures are fabricated by simply perforating an array of cuts into a thin sheet, the applied deformation and associated instabilities can be exploited to transform them into complex 3D morphologies. However, to date, such reconfiguration always happen simultaneously through the system. By borrowing ideas from phase-transforming materials, we combine cuts and curvature to realize kirigami structures in which deformation-induced shape reconfiguration initially nucleates near an imperfection and then, under specific conditions, spreads through the system. We envision that such control of the shape transformation could be used to design the next generation of responsive surfaces and smart skins.

Author contributions: A.R., L.J., and K.B. designed research; A.R., L.J., and B.D. performed research; A.R., L.J., and B.D. analyzed data; B.D. developed the theoretical model; and A.R., L.J., B.D., and K.B. wrote the paper.

The authors declare no conflict of interest.

This article is a PNAS Direct Submission.

Published under the PNAS license.

¹A.R. and L.J. contributed equally to this work.

²To whom correspondence should be addressed. Email: bertoldi@seas.harvard.edu.

This article contains supporting information online at www.pnas.org/lookup/suppl/doi:10.1073/pnas.1817763116/-DCSupplemental.

Published online April 8, 2019.

Fig. 1. (A) Schematic of a kirigami structure that comprises an array of triangular cuts. The unit cell has width L and height H , and it is highlighted in gray. (B) Schematic of a kirigami cylindrical shell comprising $n = 8$ unit cells along the circumference. (C) Experimental images of a kirigami sheet with a triangular pattern characterized by $\delta/l = 0.0625$ (with $l = 12$ mm) at different levels of deformation. The pop-up process occurs concurrently through the entire system. (D) Experimental images of a kirigami sheet fabricated by rolling a sheet with a triangular pattern characterized by $\delta/l = 0.0625$ (with $l = 12$ mm and $n = 8$ unit cells along the circumference) at different levels of deformation. The pop-up process initiates at the top end of the sample and then, spreads toward the bottom one as $\bar{\epsilon}$ is increased. The thickness of all kirigami structures is $t = 76.2$ μm .

textured surface that becomes more accentuated for increasing deformation (Fig. 1C). By contrast, in the cylindrical kirigami shell, the pop-up process initiates at the top end of the sample and then, spreads toward the other end as the applied deformation is increased (Fig. 1D). Note that the pop-up process in our shells typically starts at one of the ends of the shell, since these act as imperfection. As a matter of fact, a local reduction in the size of the hinges has to be introduced to cause the propagation to start from a different location (*SI Appendix, Fig. S7* and *Movie S9*).

Next, to better characterize the response of our structures, during the tests, we monitor black circular markers located at the base of the triangular cuts (Fig. 1C and D) and use their position to determine both the applied strain, $\bar{\epsilon}$, and the local strain in longitudinal direction for the i th row of cuts, ϵ_i , as

$$\bar{\epsilon} = \frac{z_q - z_p}{Z_q - Z_p} - 1, \quad \epsilon_i = \frac{z_{i+1} - z_i}{Z_{i+1} - Z_i} - 1, \quad [1]$$

where z_i and Z_i denote the positions of the i th marker in the deformed and undeformed configurations, respectively. We choose $p = 3$ and $q = 18$ to minimize boundary effects. Moreover, we use a custom laser profilometer and track the deformation of a horizontal line passing through the hinges at different levels of applied deformation.

In Fig. 2A and B, we report the evolution of the local strain ϵ_i as a function of $\bar{\epsilon}$ for the kirigami sheet and shell, respectively. In full agreement with our previous observations, we find that, in the kirigami sheet, the local strain increases uniformly through the structure and is always very close to the applied deformation

(i.e., $\epsilon_i \sim \bar{\epsilon} \forall i$) (Fig. 2A). Differently, the contour map for the kirigami cylindrical shell shows a nonvertical boundary between popped/open (yellow in Fig. 2B) and unpopped/closed (blue in Fig. 2B) regions (Fig. 2B)—a clear signature of sequential opening. Furthermore, the constant slope of such boundary indicates that the pop-up process propagates at constant rate of applied deformation (*SI Appendix, section 2*).

To gain more insight into the physics behind the different behavior observed in the kirigami sheet and kirigami shell, we then investigate the deformation mechanism of their hinges. By inspecting their 3D-scanned profiles (Fig. 2C and D), we find that they deform in a very different way. In the kirigami sheet, the hinges are initially flat and act as straight beams (5, 13); for a critical level of applied deformation, they buckle and subsequently bend out of plane. By contrast, in the kirigami shell, the initially bent hinges behave as bistable arches (27) and snap to their second stable configurations, which are characterized by curvature inversion. This observation is fully consistent with the results of Figs. 1 and 2, since snapping is always accompanied by a highly nonlinear stress-strain response, which is typical of phase-transforming materials (18–22). As a matter of fact, while elastic structures comprising arrays of beams that buckle under the applied load have been shown to display homogeneous

Fig. 2. (A and B) Evolution of the local strain ϵ_i as a function of $\bar{\epsilon}$ for the (A) kirigami sheet shown in Fig. 1C and (B) the kirigami shell shown in Fig. 1D. (C and D) Projection in the xy plane of a line passing through hinges (such lines are indicated in orange in A and B) as reconstructed using a custom laser profilometer for (C) the kirigami sheet and (D) the kirigami shell. The deformation of the hinges is highlighted in orange. (E) Experimental stress-strain curves for the kirigami sheet and the kirigami shell considered in Fig. 1. Note that the amplitude of the peak observed for the kirigami shell is correlated to the size of imperfection that triggers the pop up (*SI Appendix, Fig. S20*).

Fig. 3. (A) FE snapshots of flat and rolled unit cells (characterized by $\delta/l = 0.0625$ with $l = 12$ mm) at $\bar{\epsilon} = 0$ and 0.1 . (B) Strain–stress curves of unit cells with $\delta/l = 0.0625$ as a function of n . (C) Strain–stress curves of unit cells with $n = 8$ as a function of δ/l . (D) Stress–strain response of unit cells with $\delta/l = 0.0625$ and $n = 8$. In the plot, we highlight the energy barrier S_1 , the steady-state propagation stress σ_p , and the critical strains ϵ_{p1} , ϵ_{p2} , and ϵ_{p3} . The thickness of all kirigami unit cells is $t = 76.2$ μm .

pattern transformations (13, 28), sequential events are typically observed in systems based on snapping units (29–31).

Finally, in Fig. 2E, we compare the stress–strain curves of the kirigami sheet and the kirigami shell. We find that the response of the kirigami sheet is typical of buckling-based structures (32) and that it is characterized by an initial linear regime (during which all hinges bend in plane) followed by a plateau stress (caused by the homogeneous buckling-induced pop-up process). The cylindrical kirigami shell also exhibits these two regimes, but the transition between them is more abrupt and characterized by a sharp load drop. At the peak, a small portion of the kirigami shell near the top end pops up, causing the unloading of the rest of the structure and a drop in stress. Subsequently, when the ligaments of the buckled region start to be stretched and become resistant to additional deformation, the pop-up process spreads sequentially through the entire structure, and the stress reaches a steady-state value $\sigma_p = 83.5$ kPa. Finally, after all units are fully popped up at $\bar{\epsilon} \sim 0.22$, the stress starts to rise again because of additional stretching of all hinges.

Modeling

Having understood how the imposed curvature affects the deformation mechanism as well as the response of our kirigami structures, we now use a combination of numerical and analytical tools to quantify this effect. To begin with, we conduct nonlinear finite element (FE) within Abaqus/Standard to investigate the effect of both the ligament width δ and the curvature $1/r$ (which is proportional to $1/n$) on the response of unit cells subjected to uniaxial tension (SI Appendix, section 4). We find that, for $\delta/l \in [0.025, 0.15]$ and $n \in [4, 24]$, the applied deformation always triggers a buckling instability that induces the pop up of the triangular features (Fig. 3A and Movie S7). However, the stress–strain response is found to be significantly affected by both δ and n (Fig. 3B and C). For large values of n (i.e., for small curvatures), all unit cells are characterized by monotonic stress–strain curves (Fig. 3B) irrespective of δ/l . Differently, below a critical n , the stress–strain response becomes nonmonotonic characterized by a peak, a subsequent drop in load, and final stiffening. Furthermore, we find that, by either decreasing n (at constant δ/l) (Fig. 3B) or increasing δ/l (at constant n) (Fig. 3C), the peak becomes more accentuated and is eventually followed by a sharp drop.

At this point, we want to emphasize that the nonmonotonic up–down–up behavior observed for most of our rolled unit cells is typical of elastic structures supporting propagative instabilities (25, 26). Remarkably, it has been shown that the Maxwell construction (33) can be applied to such stress–strain curves to determine several key parameters that characterize the behavior of our curved kirigami shell (25, 26). Specifically,

by equating the area of the two lobes formed by the $\sigma(\epsilon)$ curve (i.e., by imposing $S_1 = S_2$) (Fig. 3D), we can identify (i) the propagation stress σ_p , (ii) the energy barrier S_1 , and (iii) the critical strains ϵ_{p1} , ϵ_{p2} , and ϵ_{p3} (Fig. 3D). For $\bar{\epsilon} < \epsilon_{p1}$, the structure deforms homogeneously, and all triangular features are unpopped, whereas for $\epsilon_{p1} < \bar{\epsilon} < \epsilon_{p3}$, the pop-up process initiated at the top end of the sample spreads toward the other end.

While Maxwell construction enables us to easily determine several parameters, it does not provide any information on the width and the shape of the transition zone. This motivates the derivation of a more detailed model based on a 1D array of

Fig. 4. (A) Schematic of our model comprising a 1D array of nonlinear springs. The blue dashed lines indicate the strain gradient interactions that we account for. (B) Comparison between the strain distribution predicted by our model and measured in experiments for the cylindrical kirigami shell shown in Fig. 1B. (C) Phase diagram of the width W of the transition zone as predicted by our model. (D) Phase diagram of the energy barrier of the kirigami shells with triangular cut pattern. S_1 (shaded green area in Fig. 3D) was normalized by the total energy required for phase transition [i.e., $\sigma_p(\epsilon_{p3} - \epsilon_{p1})$] obtained from 441 unit cell FE simulations. The thickness of all kirigami structures is $t = 76.2$ μm .

Fig. 5. (A) Experimental images of a kirigami cylindrical shell with $n=20$ and linear cuts characterized by $\delta/l=0.2$ (with $l=12$ mm) at $\bar{\varepsilon} \approx 0.2$. (B) Experimental images of a kirigami cylindrical shell with $n=20$ and orthogonal cuts characterized by $\delta/l=0.08$ (with $l=6$ mm) at $\bar{\varepsilon} \approx 0.1$. The thickness of kirigami shells is $t=76.2$ μm .

N nonlinear springs (Fig. 4A) in which the response of the i th element is described as

$$F_i(u_i, u_{i+1}) = nLt \times \sigma(\varepsilon_i) = nLt \times \sigma\left(\frac{u_{i+1} - u_i}{H}\right), \quad [2]$$

where $\sigma(\varepsilon_i)$ is the nonlinear stress–strain response of the unit cell and $u_i = z_i - Z_i$ (Fig. 4A). We then write the strain energy of the system when subjected to a constant force $F_p = \sigma_p nLt$ as

$$\begin{aligned} \Pi = & U - F_p(u_{N+1} - u_1) \\ = & \sum_{i=1}^N nLt \int_0^{u_{i+1} - u_i} \sigma\left(\frac{u_{i+1} - u_i}{H}\right) d(u_{i+1} - u_i) \\ & - F_p(u_{N+1} - u_1) + \sum_{i=2}^N \frac{1}{2} nLt G (u_{i+1} - 2u_i + u_{i-1})^2, \end{aligned} \quad [3]$$

where the last term (and the coefficient G) is introduced to capture the effect of the strain gradient (34–36), which significantly affects the response of our kirigami structures given the strong coupling between their unit cells (SI Appendix, section 4). It follows that the equilibrium equations are given by

$$\sigma(\varepsilon_i) - \sigma(\varepsilon_{i-1}) - G(\varepsilon_{i+1} - 3\varepsilon_i + 3\varepsilon_{i-1} - \varepsilon_{i-2}) = 0, \quad [4]$$

for $i = 3, \dots, N-1$

where ε_i is defined in Eq. 1.

Next, we take the continuum limit of Eq. 4, retain the nonlinear terms up to the third order, and integrate it with respect to Z to obtain

$$GH^2 \frac{d^2 \varepsilon}{dZ^2} = \sigma(\varepsilon) - \sigma_p, \quad [5]$$

where Z denotes the initial coordinate along the longitudinal direction and $\varepsilon(Z)$ is a continuous function of Z . We have assumed that, at $Z \rightarrow -\infty$, the unit cells are unpopped and subjected to a strain ε_{p1} . Eq. 5 is the continuum-governing equation for our kirigami structures, and given a stress–strain curve of the unit cell $\sigma(\varepsilon)$, it can be numerically solved to obtain the strain distribution $\varepsilon(Z)$ within the structure as a function of the applied strain. To test the relevance of our model, in Fig. 4B, we focus on the cylindrical kirigami shell of Fig. 1B and compare the evolution of the strain along its axes as predicted by our model and measured in experiments at $\bar{\varepsilon} = 0.06, 0.09$, and 0.12 . Note that the model predictions are obtained by numerically integrating Eq. 5 with $\sigma(\varepsilon)$ predicted by an FE simulation conducted on the unit cell, σ_p equal to the Maxwell stress and $G = 428$ kPa

(SI Appendix, section 4). Moreover, since the solution of Eq. 5 is translational invariant with respect to Z , the position of the propagation front Z_0 at a given applied strain $\bar{\varepsilon}$ can be determined from the compatibility condition

$$\bar{\varepsilon} = \frac{1}{Z_q - Z_p} \int_{Z_p}^{Z_q} \varepsilon(Z) dZ, \quad [6]$$

where $p=3$ and $q=18$ (Eq. 1). We find that our model accurately captures the shape, width, and amplitude of the transition zone as well as its position as a function of the applied strain, confirming the validity of our approach.

Effect of Geometry

While in Fig. 4B, we focus on a specific geometry, it is important to point out that our model can be used to efficiently characterize the propagation front as a function of the curvature of the shell, the hinge size, and the arrangement of the cuts. In Fig. 4C, we focus on kirigami structures with triangular cuts and report the evolution of the normalized width of the propagation front, W [which is defined as the width of region in which the strain changes by $0.9(\varepsilon_{p3} - \varepsilon_{p1})$] (Fig. 4B), as a function of δ/l and n . First, the results of our model indicate that, as the curvature of the kirigami shell increases (i.e., for larger n), W monotonically increases so that propagation of instability becomes less and less visible. In the limit of flat sheets, all unit cells are characterized by monotonic stress–strain curves, and only homogeneous pop up is possible (white region in Fig. 4C). Second, we find that, by increasing δ/l , at constant n , the propagation of instabilities becomes

Fig. 6. (A) Schematic of our patterned kirigami surface. (B and C) Experimental images of the heterogeneous (B) kirigami sheet and (C) kirigami shell at $\bar{\varepsilon} = 0.1$ and 0.2 . (D–F) Snapshots of our kirigami-skinned crawlers with triangular cuts characterized by (D) $\delta/l = 0.033$ (uniform pop up), (E) $\delta/l = 0.125$ (propagating pop up), and (F) $\delta/l = 0.125$ for the central units and $\delta/l = 0.033$ for first three units at two ends (programmed pop up).

Fig. 7. Behavior surface for cylindrical kirigami shells with triangular cuts characterized by $l = 12$ mm, $\delta/l = 0.0625$, and $t = 76.2$ μm . The continuous sequence of states of equilibrium experienced by the shell on loading corresponds to a curve on the behavior surface. The behavior surface is smooth, but its projection on $n - \sigma$ contains two folds and a cusp. For shells with small curvature (i.e., for large n), the path does not reach the folds (blue line), and uniform pop ups occur at a critical strain ε_c associated with out-of-plane buckling of hinges. By increasing the curvature of the shell, the path eventually intersects the folds (red line). Inside the cusp, popped and unpopped regions coexist, and pop ups can propagate with a deformation path that passes from a stable unpopped strain ε_{p1} to another stable popped strain ε_{p3} through an unstable intermediate strain ε_{p2} . The projection of the ε_{p1} (or ε_{p2} or ε_{p3}) curves on the $\sigma - n$ plane represents the loci of propagation stress given by Maxwell construction.

more accentuated as the width W of the transition zone monotonically decreases (Fig. 4C). It is also interesting to note that the width of the transition zone is inversely proportional to the energy barrier S_1 (Fig. 4D). The largest values of S_1 are observed for unit cells with very large δ/l and very small n . For such units, the peak in stress is followed by a sharp drop, and the mechanical response is characterized by a discrete sequence of drops during propagation (*SI Appendix, Fig. S15D*), each corresponding to the opening of one row of cuts (*Movie S4*).

Notably, using our model in combination with FE analysis conducted on the unit cells, phase diagrams similar to those shown in Fig. 4 *C* and *D* can be constructed for any cut pattern (*SI Appendix*, Fig. S16). Such diagrams can then be used to identify regions in the parameter space where propagation of instability is triggered. As examples, in Fig. 5, we report snapshots of cylindrical kirigami shells with a staggered array of linear cuts (3–5) and an array of mutually orthogonal cuts (1, 13). Both images clearly show the coexistence of the popped and unpopped phases (*SI Appendix*, Figs. S5 and S6 and *Movies S5–S7*) and further indicate that the characteristics of the phase transition can be controlled by carefully selecting the geometry of the cuts as well as the curvature of the shell. The kirigami shell with the linear pattern is characterized by a sharp propagation front spanning across about one unit cell and a propagation stress $\sigma_p = 177$ kPa (*SI Appendix*, Fig. S18), whereas the orthogonal cuts lead to a wider front spreading across about four unit cells and $\sigma_p = 320$ kPa (*SI Appendix*, Fig. S19).

Finally, we find that the coexistence of the buckled and unbuckled phases observed in our kirigami cylindrical shells

provides opportunities to realize surfaces with complex behavior that can be programmed to achieve a desired functionality. To demonstrate this, we consider a kirigami surface with 20 rows of triangular cuts separated by hinges with two different sizes. Specifically, we choose $\delta/l = 0.033$ for three rows near the two ends and $\delta/l = 0.125$ for the central rows (Fig. 6A). If such a surface is planar, no clear signature of the two different δ/l is apparent on stretching. Since the critical strains associated with the two considered δ/l are very close to each other ($\bar{\epsilon}_c = 4.4275 \times 10^{-4}$ and 3.510×10^{-4} for $\delta/l = 0.033$ and 0.125 , respectively), all features pop up almost simultaneously and tilt fairly uniformly (Fig. 6B and *SI Appendix, Fig. S9B*). By contrast, if we use the heterogeneous kirigami sheet to form a cylinder with $n = 8$, on stretching, we find a clear sequence. First, the triangular features separated by ligaments with $\delta/l = 0.033$ pop up all together. Second, the pop ups propagate in the central region with $\delta/l = 0.125$ starting from the top (Fig. 6C, *SI Appendix, Fig. S9C*, and *Movie S8*). Remarkably, this sequencing achieved by simply patterning the sheet with regions characterized by different ligament widths can be exploited to design a smart skin that significantly enhances the crawling efficiency of a linear actuator (Fig. 6D–F). While all three kirigami-skinned crawlers advance on elongation and contraction of the actuator because of the anisotropic friction induced by the pop ups (7) (*SI Appendix, section 3*), the programmed pop up achieved in our patterned shell enhances the anchorage of the crawler to the substrate at two ends and significantly reduces the backslide (*SI Appendix, Fig. S11B*). As a result, the patterned crawler (Fig. 6F), which benefits from coexistence of popped and unpopped regions at desired locations, proceeds about twice as fast as the crawlers with a homogeneous array of triangular cuts with either $\delta/l = 0.033$ (Fig. 6D) or $\delta/l = 0.125$ (Fig. 6E).

Discussion and Conclusions

To summarize, we have shown that, in cylindrical kirigami shells, the buckled and unbuckled phases can coexist, with the pop-up process initially starting near an end and then, propagating along the cylinder at constant stress. In contrast to flat kirigami sheets, which can only support continuous phase transitions, by introducing curvature, the buckling-induced transformation exhibits discontinuity in the first derivative of the free energy, resulting in the coexistence of two phases (37). This remarkable difference in behavior arises, because the curvature transforms the ligaments from straight columns that buckle to bistable arches that snap. It should be also noted that such response is completely different from that of porous cylindrical shells, which under compression, exhibit uniform buckling-induced shape transformation (38–40), whereas it shares similarities with structures consisting of an array of beams resting on flexible supports, which have recently been shown to exhibit a very rich response (41, 42). The behavior of our system can be further understood by looking at its behavior surface (Fig. 7 shows a triangular pattern with $\delta/l = 0.125$). We find that, by increasing the curvature of the shell (i.e., by decreasing n), a cusp catastrophe emerges (43–45). The increase in curvature causes a progressively larger divergence between the top and bottom faces of the fold, making the discontinuous phase transition more pronounced. As such, the behavior surface in Fig. 7 further confirms that the curvature is the essential ingredient to trigger propagation of pop ups. However, we also find that the stresses introduced to bend the sheets into cylinders play an important role, as they increase the energy barrier and make the propagation more pronounced (*SI Appendix, Fig. S14*). Finally, we have shown that the characteristics of discontinuous phase transition can be tuned by carefully selecting the geometry of the kirigami structure. With such control on the phase transition in kirigami structures, we envision that these mechanical metamaterials could be used to design the

next generation of responsive surfaces as shown by the design of a smart skin that enhances the crawling efficiency of a linear actuator.

Materials and Methods

Details of fabrication of kirigami shells are described in *SI Appendix, section 1*. The protocol for experiments and additional experimental data for kirigami shells with triangular, linear, and orthogonal cut patterns are provided in *SI Appendix, section 2*. Principles of kirigami-skinned crawlers are presented in *SI Appendix, section 3*. Details of FE simulations and theoretical models are presented in *SI Appendix, section 4*.

1. Cho Y, et al. (2014) Engineering the shape and structure of materials by fractal cut. *Proc Natl Acad Sci USA* 111:17390–17395.
2. Zhang Y, et al. (2015) A mechanically driven form of Kirigami as a route to 3D mesostructures in micro/nanomembranes. *Proc Natl Acad Sci USA* 112:11757–11764.
3. Shyu TC, et al. (2015) A kirigami approach to engineering elasticity in nanocomposites through patterned defects. *Nat Mat* 14:785–789.
4. Blees MK, et al. (2015) Graphene kirigami. *Nature* 524:204–207.
5. Isobe M, Okumura K (2016) Initial rigid response and softening transition of highly stretchable kirigami sheet materials. *Sci Rep* 6:24758.
6. Dias MA, et al. (2017) Kirigami actuators. *Soft Matter* 13:9087–9092.
7. Rafsanjani A, Zhang Y, Liu B, Rubinstein SM, Bertoldi K (2018) Kirigami skins make a simple soft actuator crawl. *Sci Robotics* 3:eaar7555.
8. Hwang D-G, Trent K, Bartlett MD (2018) Kirigami-inspired structures for smart adhesion. *ACS Appl Mater Inter* 10:6747–6754.
9. Sussman DM, et al. (2015) Algorithmic lattice kirigami: A route to pluripotent materials. *Proc Natl Acad Sci USA* 112:7449–7453.
10. Neville RM, Scarpa F, Pirrera A (2016) Shape morphing Kirigami mechanical metamaterials. *Sci Rep* 6:31067.
11. Konaković M, et al. (2016) Beyond developable: Computational design and fabrication with auxetic materials. *ACM Trans Graph* 35:89.
12. Castle T, Sussman DM, Tanis M, Kamien RD (2016) Additive lattice kirigami. *Sci Adv* 2:e1601258.
13. Rafsanjani A, Bertoldi K (2017) Buckling-induced kirigami. *Phys Rev Lett* 118:084301.
14. Zhao Y, et al. (2017) Carbon nanotubes kirigami mechanical metamaterials. *Phys Chem Phys* 19:11032.
15. Yang N, Silverberg JL (2018) Decoupling local mechanics from large-scale structure in modular metamaterials. *Proc Natl Acad Sci USA* 112:11757–11764.
16. Liu Z, et al. (2018) Nano-kirigami with giant optical chirality. *Sci Adv* 4:eaat4436.
17. Yang Y, Dias MA, Holmes DP (2018) Multistable kirigami for tunable architected materials. *Phys Rev Mater* 2:110601.
18. Ericksen JL (1975) Equilibrium of bars. *J Elast* 5:191–201.
19. James RD (1979) Co-existent phases in the one-dimensional static theory of elastic bars. *Arch Ration Mech Anal* 72:99–140.
20. Abeyaratne R, Knowles JK (1991) Kinetic relations and the propagation of phase boundaries in solids. *Arch Ration Mech Anal* 114:119–154.
21. Shaw JA, Kyriakides S (1997) On the nucleation and propagation of phase transformation fronts in a NiTi alloy. *Acta Mater* 45:683–700.
22. Feng P, Sun QP (2006) Experimental investigation on macroscopic domain formation and evolution in polycrystalline NiTi microtubing under mechanical force. *J Mech Phys Sol* 54:1568–1603.
23. Zhao X, Hong W, Suo Z (2007) Electromechanical hysteresis and coexistent states in dielectric elastomers. *Phys Rev B* 76:134113.
24. Zhou J, Hong W, Zhao X, Zhang Z, Suo Z (2008) Propagation of instability in dielectric elastomers. *Int J Sol Struct* 45:3739–3750.
25. Chater E, Hutchinson JW (1984) On the propagation of bulges and buckles. *J Appl Mech* 51:269–277.
26. Kyriakides S (1993) Propagating instabilities in structures. *Adv Appl Mech* 30:67–189.
27. Plaut RH (2015) Snap-through of arches and buckled beams under unilateral displacement control. *Int J Sol Struct* 63:109–113.
28. Kang SH, et al. (2014) Complex ordered patterns in mechanical instability induced geometrically frustrated triangular cellular structures. *Phys Rev Lett* 112:098701.
29. Shan S, et al. (2015) Multistable architected materials for trapping elastic strain energy. *Adv Mater* 27:4296–4301.
30. Rafsanjani A, Akbarzadeh A, Pasini D (2015) Snapping mechanical metamaterials under tension. *Adv Mater* 27:5931–5935.
31. Restrepo D, Mankame ND, Zavattieri PD (2016) Phase transforming cellular materials. *Extreme Mech Lett* 27:5931–5935.
32. Bertoldi K, Reis PM, Willshaw S, Mullin T (2010) Negative Poisson's ratio behavior induced by an elastic instability. *Adv Mater* 22:361–366.
33. Maxwell JC (1875) On the dynamical evidence of the molecular constitution of bodies. *Nature* 11:357–359.
34. Mindlin RD (1964) Micro-structure in linear elasticity. *Arch Ration Mech Anal* 16:51–78.
35. Polyzos D, Fotiadis DI (2012) Derivation of Mindlin's first and second strain gradient elastic theory via simple lattice and continuum models. *Int J Sol Struct* 49:470–480.
36. Audoly B, Hutchinson JW (2016) Analysis of necking based on a one-dimensional model. *J Mech Phys Sol* 97:68–91.
37. Jaeger G (1998) The Ehrenfest classification of phase transitions: Introduction and evolution. *Arch Hist Exact Sci* 53:51–81.
38. Matsumoto EA, Kamien RD (2012) Patterns on a roll: A method of continuous feed nanoprinting. *Soft Matter* 8:11038–11041.
39. Lazarus A, Reis PM (2015) Soft Actuation of structured cylinders through auxetic behavior. *Adv Eng Mater* 17:815–820.
40. Javid F, et al. (2016) Mechanics of instability-induced pattern transformations in elastomeric porous cylinders. *J Mech Phys Sol* 96:1–17.
41. Xu S, et al. (2015) Assembly of micro/nanomaterials into complex, three-dimensional architectures by compressive buckling. *Science* 347:154–159.
42. Abdullah AM, Nan K, Rogers JA, Hsia KJ (2016) Mismatch strain programmed shape transformation of curved bilayer-flexible support assembly. *Extreme Mech Lett* 7:34–41.
43. Zeeman EC (1976) Catastrophe theory. *Sci Am* 234:65–83.
44. Thompson JMT (1975) Experiments in catastrophe. *Nature* 254:392–395.
45. Lu T, Cheng S, Li T, Wang T, Suo Z (2016) Electromechanical catastrophe. *Int J Appl Mech* 8:1640005.